DC Amplifier

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A DC amplifier has an open loop gain of 1000 and two poles, a dominant one at 1kHz and a high frequency one whose location can be controlled. It is required to connect this amplifier in a negative feedback loop that provides a DC closed loop gain of 10 and a maximally flat response.

1. Find the required value of H.
Solution: Table 1 summarises the given information. The open loop gain can be expressed as

$$G(s) = \frac{G_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} \tag{1.1}$$

$$\implies G(0) = G_0 \tag{1.2}$$

The closed loop gain

$$T(s) = \frac{G(s)}{1+G(s)H} \tag{1.3}$$

$$\implies T(0) = \frac{G_0}{1+G_0H} \tag{1.4}$$

Substituting from Table 1,

$$\frac{1000}{1+1000H} = 10 \tag{1.5}$$

$$\implies H = 0.099 \tag{1.6}$$

Parameter	Value
dc open loop gain	1000
dominant pole	-1000Hz
insignificant pole	-p_2
dc closed loop gain	10

TABLE 1: 1

$$G_0 = 1000 \tag{1.7}$$

$$\text{Therefore.,} G(s) = \frac{1000}{(1+\frac{s}{p_1})(1+\frac{s}{p_2})} \tag{1.8}$$

2. Find p_2.
Solution: From (1.3) and (1.1),

$$T(s) = \frac{p_1p_2G_0}{s^2+(p_1+p_2)s+(HG_0+1)p_1p_2} \tag{2.1}$$

$$= \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{2.2}$$

$$\omega_n = \sqrt{(HG_0+1)p_1p_2}$$

$$\implies \zeta = \frac{p_1+p_2}{2\sqrt{(HG_0+1)p_1p_2}} \tag{2.3}$$

using the standard formulation for a second order system. Also, for maximally flat response, the quality factor

$$Q = \frac{1}{2\zeta} = \frac{1}{\sqrt{2}} \tag{2.4}$$

$$\implies \zeta = \frac{1}{\sqrt{2}} \tag{2.5}$$

$$\implies \frac{p_1+p_2}{2\sqrt{(HG_0+1)p_1p_2}} = \frac{1}{\sqrt{2}} \tag{2.6}$$

$$\implies \sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}} = \sqrt{2(HG_0+1)} \tag{2.7}$$

The above equation is of the form

$$x + \frac{1}{x} = a \tag{2.8}$$

$$\implies x = \frac{a \pm \sqrt{a^2-4}}{2} \tag{2.9}$$

where

$$x = \sqrt{\frac{p_2}{p_1}} \tag{2.10}$$

$$a = \sqrt{2(HG_0+1)}, \tag{2.11}$$

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Thus, from (2.10), (2.11) and (2.9),

$$p_2 = p_1 \left[\frac{\sqrt{2}\,(HG_0+1) \pm \sqrt{2\,(HG_0+1)-4}}{2} \right]^2 \tag{2.12}$$

From the following code,

```
codes/ee18btech11005/ee18btech11005_1.py
```

$$p_2 = 1244038.9567529503 \text{ and } 31.734068607786863 \tag{2.13}$$

3. Draw the equivalent circuit system diagram.
Solution: The equivalent circuit system is shown in the figure.3

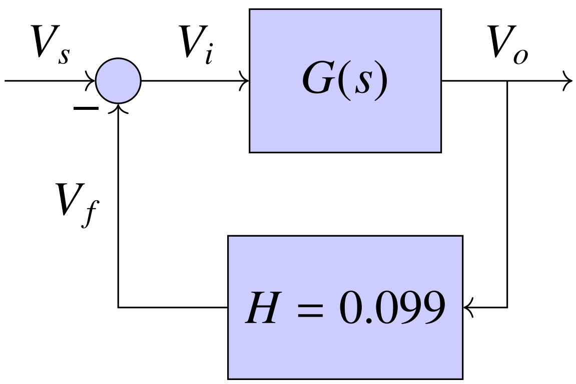

Fig. 3: 1

4. Obtain $G(s)$ and $T(s)$
Solution: Substituting the value of p_2 in (1.1) and (2.1),

$$G(s) = \frac{1000}{(1+\frac{s}{2\pi 10^3})(1+\frac{s}{1.244\times 10^6})} \tag{4.1}$$

$$T(s) = \frac{10}{0.128\times 10^{-11}s^2 + 1.599\times 10^{-6}s + 1} \tag{4.2}$$

5. Verify from the Bode plot of above closed loop transfer function that it has maximally flat response.
Solution: The following code generates the bode plot of the transfer function in Fig. 5.

```
codes/ee18btech11005/ee18btech11005_2.py
```

6. Find the step response of $T(s)$
Solution: The following code generates the desired response of in Fig. 6.

```
codes/ee18btech11005/ee18btech11005_3.py
```

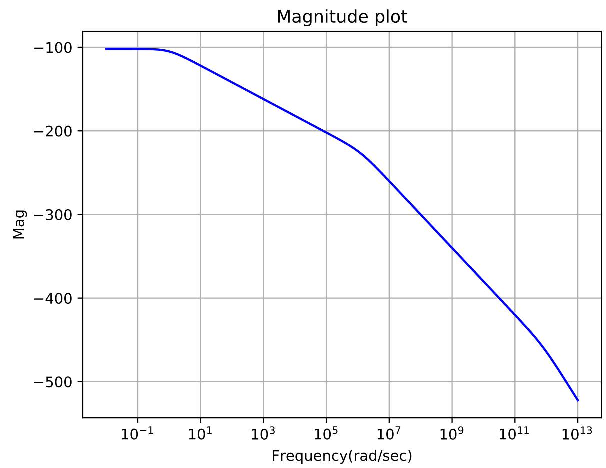

Fig. 5

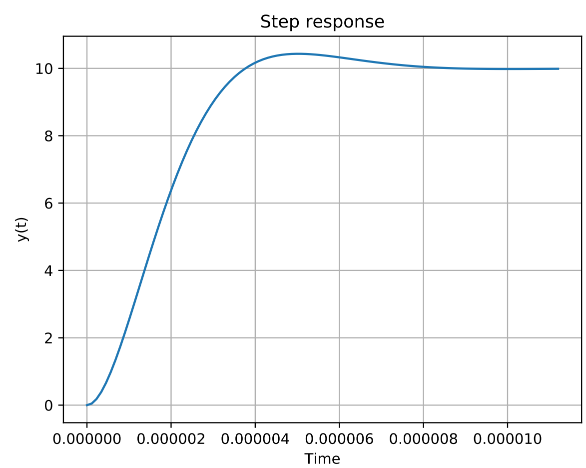

Fig. 6

7. Design a circuit that represents the above transfer function.
Solution: The circuit can be designed using operational amplifiers having negative feedback.Consider the circuit shown in figure.7:1.Assume the gain of all the amplifiers are large.And assume no zero state response.Take the parameters in s-domain.
For the first amplifier,

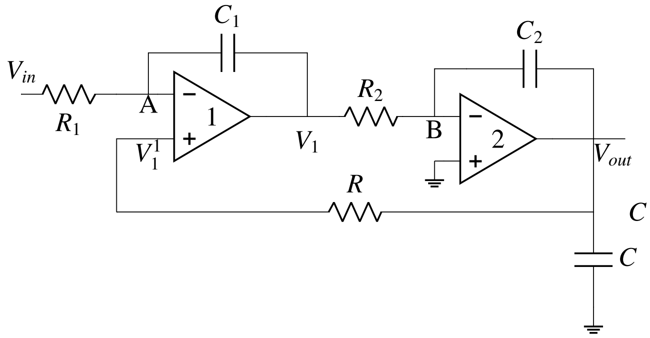

Fig. 7: 1

$$\frac{V_{in} - V_1^1}{R_1} = \frac{V_1^1 - V_1}{\frac{1}{sC_1}} \tag{7.1}$$

$$\frac{V_{in}}{R_1} = \frac{V_1^1}{R_1} + sC_1V_1^1 - sC_1V_1 \tag{7.2}$$

$$\frac{V_{in}}{R_1} = V_1^1\left[sC_1 + \frac{1}{R_1}\right] - sC_1V_1 \tag{7.3}$$

$$V_{in} = V_1^1(sC_1R_1 + 1) - sC_1R_1V_1 \tag{7.4}$$

For the second amplifier.,

$$\frac{V_1 - V_b}{R_2} = (V_b - V_{out})sC_2 \tag{7.5}$$

$$\text{Since.,} V_b = 0 \tag{7.6}$$

$$\implies V_1 = -sC_2R_2Vout \tag{7.7}$$

Voltage division at node C.,

$$\frac{V_1^1}{V_{out}} = \frac{1 + \frac{1}{sC}}{\frac{1}{sC}} \tag{7.8}$$

$$\implies V_1^1 = (sCR + 1)V_{out} \tag{7.9}$$

From eq:(7.4) ,eq:(7.7) ,eq:(7.9)

$$V_{in} = ((sCR + 1)(sC_1R_1 + 1) + s^2C_1R_1C_2R_2)V_{out} \tag{7.10}$$

$$\tag{7.11}$$

$$\frac{V_{in}}{V_{out}} = s^2(CRC_1R_1{+}C_1R_1C_2R_2){+}s(CR{+}C_1R_1){+}1$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2(CRC_1R_1 + C_1R_1C_2R_2) + s(CR + C_1R_1) + 1} \tag{7.12}$$

Comparing the equation.(4.2) and equation.(7.12)

$$C_1R_1(CR + C_2R_2) = 0.128 \times 10^{-11} \tag{7.13}$$

$$CR + C_1R_1 = 1.599 \times 10^{-6} \tag{7.14}$$

$$\text{Let.,} CR = 10^{-6} \tag{7.15}$$

$$\implies C_1R_1 = 0.599 \times 10^{-6} \tag{7.16}$$

$$0.599 \times 10^{-6}(10^{-6} + C_2R_2) = 0.128 \times 10^{-11} \tag{7.17}$$

$$C_2R_2 = 0.681 \times 10^{-6} \tag{7.18}$$

The parameters can be chosen as shown in the TABLE:7

The final circuit is shown in the figure.7

Parameter	Value
R_1	1000 Ω
R_2	1000 Ω
R	1000 Ω
C_1	0.1 nF
C_2	0.681 nF
C	0.599 nF

TABLE 7

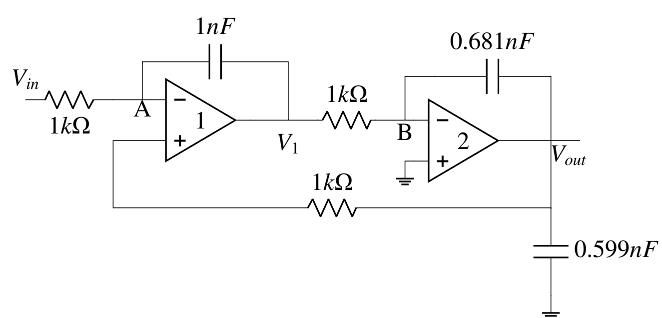

Fig. 7

8. Draw the block diagram of the closed loop system.
   Solution: The block diagram is shown in figure.8
9. Find H_2.
   Solution: H_2 can be calculated as follows..,

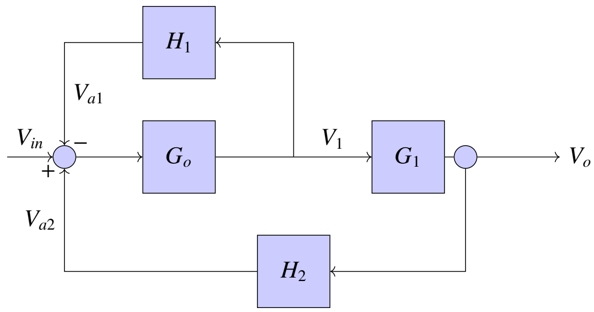

Fig. 8

$$H_2 = \frac{V_{a2}}{V_o} \tag{9.1}$$

$$\tag{9.2}$$

But from the figure.7.,

$$\frac{V_{a2}}{V_o} = \frac{V_1^1}{V_o} \tag{9.3}$$

$$\frac{V_{a2}}{V_o} = \frac{R + \frac{1}{sC}}{\frac{1}{sC}} \tag{9.4}$$

$$\frac{V_{a2}}{V_o} = 1 + sCR \tag{9.5}$$

$$H_2 = 1 + sCR \tag{9.6}$$

10. Find equivalent open loop gain of the system without positive feedback.
Solution: The figure.10 shows the equivalent block diagram.
The resultant open loop gain after removing

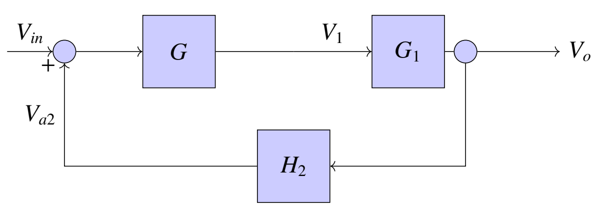

Fig. 10

positive feedback is GG_1.Where G can be calculated as follows.,

$$G = \frac{G_o}{1 + G_o H_1} \tag{10.1}$$

$$G = \frac{1}{\frac{1}{G_o} + H_1} \tag{10.2}$$

Since,Go is very large.,

$$G \approx \frac{1}{H_1} \tag{10.3}$$

For finding H_2.,ground V_{in}.,

$$H_1 = \frac{V_{a1}}{V_1} \tag{10.4}$$

$$H_1 = \frac{R_1}{R_1 + \frac{1}{sC_1}} \tag{10.5}$$

$$H_1 = \frac{sC_1R_1}{sC_1R_1 + 1} \tag{10.6}$$

Finding G_1.,

$$G_1 = \frac{V_o}{V_1} \tag{10.7}$$

$$G_1 = -\frac{1}{sC_2R_2} \tag{10.8}$$

Therefore.,The open loop gain without positive feedback is given by.,

$$OLG = GG_1 \tag{10.9}$$

$$OLG = \frac{G_1}{H_1} \tag{10.10}$$

$$OLG = -\frac{\frac{1}{sC_2R_2}}{\frac{sC_1R_1}{sC_1R_1+1}} \tag{10.11}$$

$$OLG = -\frac{sC_1R_1 + 1}{s^2C_1R_1C_2R_2} \tag{10.12}$$

11. Find the approximate closed loop transfer function.
Solution: The overall system is in positive feedback.The closed loop transfer function is given by.,

$$T(s) = \frac{OLG}{1 - OLG \times H_2} \tag{11.1}$$

$$T(s) = \frac{\frac{sC_1R_1+1}{s^2C_1R_1C_2R_2}}{1 + \left[\frac{sC_1R_1+1}{s^2C_1R_1C_2R_2}\right](1 + sCR)} \tag{11.2}$$

$$T(s) = \frac{sC_1R_1 + 1}{s^2C_1R_1C_2R_2 + (sC_1R_1 + 1)(sCR + 1)} \tag{11.3}$$

Since C_1R_1 is very small compared to 1. We can assume the zero lies far away from origin.

$$T(s) \approx \frac{1}{s^2C_1R_1C_2R_2 + (sC_1R_1 + 1)(sCR + 1)} \tag{11.4}$$

The above equation is similar to equation.7.12 Hence verified.

12. Find the block diagram and circuit diagram for H_2.
Solution: The block diagram is shown in figure.12 The circuit diagram is shown in the

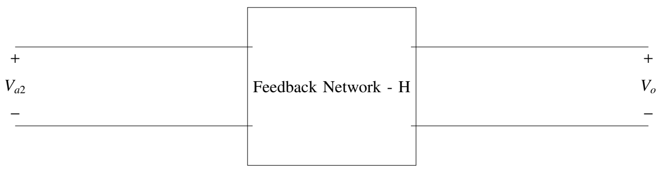

Fig. 12: Block diagram

fig.12

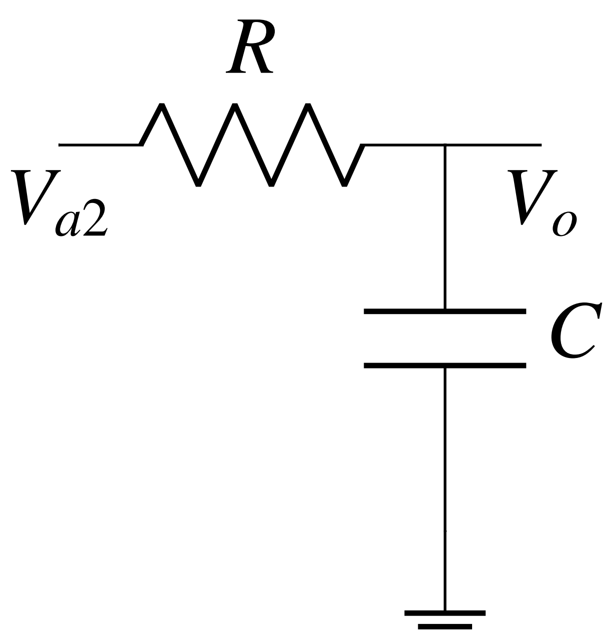

Fig. 12: Circuit diagram

13. Draw the block diagram for open loop gain GG_1 and find R_{11} and R_{22}.
Solution: The figure.13 Finding R11., short V_o

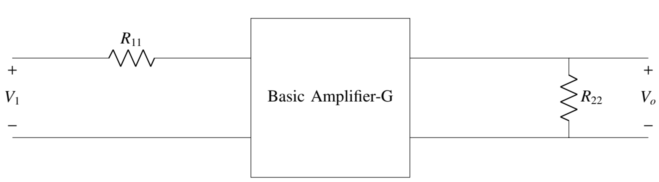

Fig. 13: Circuit diagram

to ground.,

$$R_{11} = R \tag{13.1}$$

Finding R22., short V_{a2} to ground.,

$$R_{22} = R||\frac{1}{sC} \tag{13.2}$$

14. Verify the closed loop DC gain using NGSPICE simulator.
Solution: The following README file gives the procedure to be followed.

```
codes/ee18btech11005/spice/README
```

From equation.4.2. The DC closed loop gain is 10.
The following netlist file, gives the DC gain of the closed loop function.

```
codes/ee18btech11005/spice/gvv_ngspice.net
```

We can observe from simulation that the value of DC closed loop gain is 9.997.
Error analysis:-
ERROR in DC GAIN = 10-9.993 = 0.007 Thus, the predicted value in ngspice is almost accurate. Therefore, the value is verified using ngspice.

15. Verify the step response of the output from ngspice simulation.
Solution: The following netlist file does the transient anaylsis and store the Vout values with respect to time in a dat file.

```
codes/ee18btech11005/spice/gvv_ngspice.net
```

Following python code is to plot the step response.

```
codes/ee18btech11005/spice/
    ee18btech11005_spice.py
```

The step response obtained is shown in the figure.15.The graph has steady state value equal to 10.

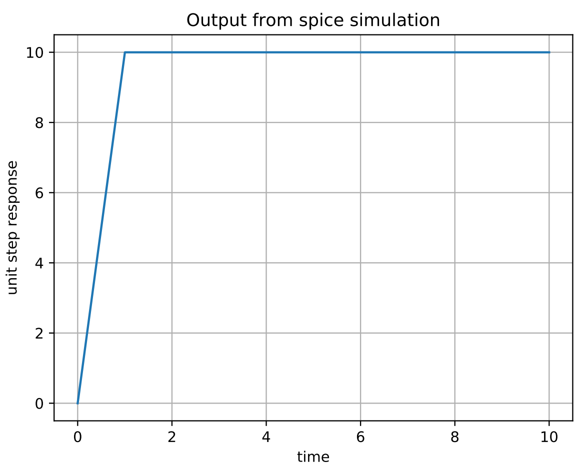
Output from spice simulation
unit step response
time
0
2
4
6
8
10

Fig. 15