DC Amplifier

Buereddy Varuni*

A DC amplifier has an open loop gain of 1000 and two poles, a dominant one at 1kHz and a high frequency one whose location can be controlled. It is required to connect this amplifier in a negative feedback loop that provides a DC closed loop gain of 10 and a maximally flat response.

1. Find the required value of H.
Solution: Table 1 summarises the given information. The open loop gain can be expressed as

$$G(s) = \frac{G_0}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)} \tag{1.1}$$

$$\implies G(0) = G_0 \tag{1.2}$$

The closed loop gain

$$T(s) = \frac{G(s)}{1 + G(s)H} \tag{1.3}$$

$$\implies T(0) = \frac{G_0}{1 + G_0 H} \tag{1.4}$$

Substituting from Table 1,

$$\frac{1000}{1 + 1000H} = 10 \tag{1.5}$$

$$\implies H = 0.099 \tag{1.6}$$

Parameter	Value
dc open loop gain	1000
dominant pole	-1000Hz
insignificant pole	-p_2
dc closed loop gain	10

TABLE 1: 1

$$G_0 = 1000 \tag{1.7}$$

$$\text{Therefore.,} G(s) = \frac{1000}{(1 + \frac{s}{p_1})(1 + \frac{s}{p_2})} \tag{1.8}$$

2. Find p_2.
Solution: From (1.3) and (1.1),

$$T(s) = \frac{p_1 p_2 G_0}{s^2 + (p_1 + p_2)s + (HG_0 + 1)p_1 p_2} \tag{2.1}$$

$$= \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2.2}$$

$$\omega_n = \sqrt{(HG_0 + 1)p_1 p_2}$$

$$\implies \zeta = \frac{p_1 + p_2}{2\sqrt{(HG_0 + 1)p_1 p_2}} \tag{2.3}$$

using the standard formulation for a second order system. Also, for maximally flat response, the quality factor

$$Q = \frac{1}{2\zeta} = \frac{1}{\sqrt{2}} \tag{2.4}$$

$$\implies \zeta = \frac{1}{\sqrt{2}} \tag{2.5}$$

$$\implies \frac{p_1 + p_2}{2\sqrt{(HG_0 + 1)p_1 p_2}} = \frac{1}{\sqrt{2}} \tag{2.6}$$

$$\implies \sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}} = \sqrt{2(HG_0 + 1)} \tag{2.7}$$

The above equation is of the form

$$x + \frac{1}{x} = a \tag{2.8}$$

$$\implies x = \frac{a \pm \sqrt{a^2 - 4}}{2} \tag{2.9}$$

where

$$x = \sqrt{\frac{p_2}{p_1}} \tag{2.10}$$

$$a = \sqrt{2(HG_0 + 1)}, \tag{2.11}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India.

Thus, from (2.10), (2.11) and (2.9),

$$p_2 = p_1 \left[\frac{\sqrt{2\left(HG_0 + 1\right)} \pm \sqrt{2\left(HG_0 + 1\right) - 4}}{2} \right]^2 \tag{2.12}$$

From the following code,

```
codes/ee18btech11005/ee18btech11005_1.py
```

$$\begin{aligned} p_2 &= 1244038.9567529503 \\ &\quad \text{and } 31.734068607786863 \end{aligned} \tag{2.13}$$

3. Draw the equivalent circuit system diagram.
Solution: The equivalent circuit system is shown in the figure.3

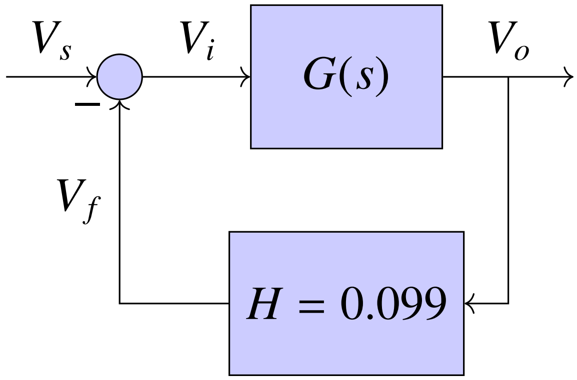

Fig. 3: 1

4. Obtain $G(s)$ and $T(s)$
Solution: Substituting the value of p_2 in (1.1) and (2.1),

$$G(s) = \frac{1000}{(1 + \frac{s}{2\pi 10^3})(1 + \frac{s}{1.244 \times 10^6})} \tag{4.1}$$

$$T(s) = \frac{10}{0.128 \times 10^{-11} s^2 + 1.599 \times 10^{-6} s + 1} \tag{4.2}$$

5. Verify from the Bode plot of above closed loop transfer function that it has maximally flat response.
Solution: The following code generates the bode plot of the transfer function in Fig. 5.

```
codes/ee18btech11005/ee18btech11005_2.py
```

6. Find the step response of $T(s)$
Solution: The following code generates the desired response of in Fig. 6.

```
codes/ee18btech11005/ee18btech11005_3.py
```

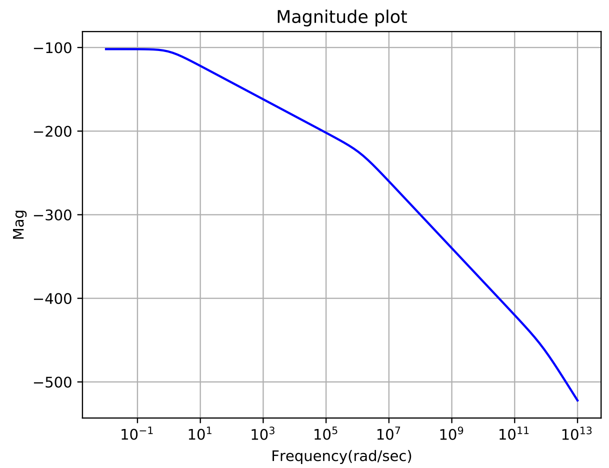

Fig. 5

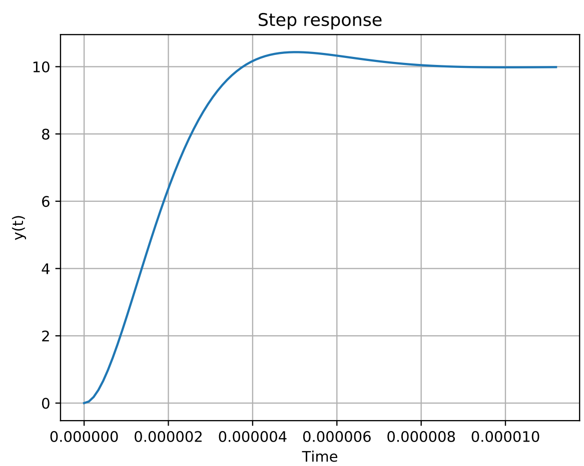

Fig. 6

7. Design a circuit that represents the above transfer function.
Solution: The circuit can be designed using an operational amplifiers having negative feedback.Consider the circuit shown in figure.7:1.Assume the gain of all the amplifiers are large.And assume no zero state response.Take the parameters in s-domain.
For the first amplifier.., Applying KCL at

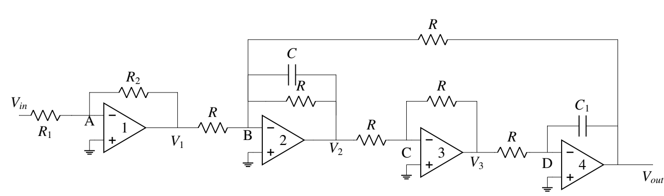

Fig. 7: 1

node A., Since, the opamp has large gain, potential at node A is assumed to be zero due to virtual short at node A.

$$\frac{0-V_{in}(s)}{R_1}+\frac{0-V_1(s)}{R_2}=0 \tag{7.1}$$

$$\frac{V_{in}(s)}{R_1}=\frac{V_1(s)}{R_2} \tag{7.2}$$

$$\implies V_{in}=-\frac{V_1(s)R_1}{R_2} \tag{7.3}$$

For the second amplifier.., Applying KCL at node B.., Similarly potential at node B is zero.

$$\frac{-V_1(s)}{R}+\frac{-V_2(s)}{R}-sCV_2(s)+\frac{-V_{out}(s)}{R}=0 \tag{7.4}$$

$$\frac{-V_1(s)}{R}+\frac{-V_2(s)}{R}-sCV_2(s)=\frac{V_{out}(s)}{R} \tag{7.5}$$

$$\frac{-V_1(s)}{R}=V_2(s)\left[sC+\frac{1}{R}\right]+\frac{Vout(s)}{R} \tag{7.6}$$

For the third amplifier.., Potential at node C is zero(Due to high gain of amplifier).Applying KCL at node C.

$$\frac{-V_2(s)}{R}+\frac{-V_3(s)}{R}=0 \tag{7.7}$$

$$\implies V_2(s)=-V_3(s) \tag{7.8}$$

For the Fourth amplifier., Potential at node D is zero.Applying KCL at node D.

$$\frac{-V_3(s)}{R}+sC_1(-V_{out}(s))=0 \tag{7.9}$$

$$V_3(s)=-sC_1RV_{out}(s) \tag{7.10}$$

From equation.7.10 and equation. 7.8..,

$$V_2(s)=sC_1RV_{out}(s) \tag{7.11}$$

Substituting the equation.7.6 and equation.7.11,

$$\frac{-V_1(s)}{R}=(s^2C_1CR+sC_1)V_{out}(s)+\frac{V_{out}(s)}{R} \tag{7.12}$$

$$V_1(s)=-(s^2C_1CR^2+sC_1R+1)V_{out}(s) \tag{7.13}$$

from equation.7.3 and equation.7.13.

$$V_1(s)=\frac{R_1}{R_2}(s^2C_1CR^2+sC_1R+1)V_{out}(s) \tag{7.14}$$

$$\frac{V_{out}(s)}{V_{in}(s)}=\frac{R_2}{R_1(s^2C_1CR^2+sC_1R+1)} \tag{7.15}$$

Comparing equation.4.2 and equation.7.15

$$\frac{R_2}{R_1}=10 \tag{7.16}$$

$$C_1CR^2=0.128x10^{-11} \tag{7.17}$$

$$C_1R=1.599x10^{-6}F \tag{7.18}$$

$$\text{Let.,}R=1000\Omega \tag{7.19}$$

$$\implies C_1=1.599x10^{-9} \tag{7.20}$$

$$\text{and.,}C_1CR^2=0.128x10^{-11} \tag{7.21}$$

$$\implies C=0.8005x10^{-9}F \tag{7.22}$$

$$\text{Let..,}R_1=100\Omega \tag{7.23}$$

$$\implies R_2=1000\Omega \tag{7.24}$$

From Table.7:1. The Final circuit is shown in

Parameter	Value
R_1	100 Ω
R_2	1000 Ω
R	1000 Ω
C	0.8005 nF
C_1	1.599 nF

TABLE 7: 1

figure.7

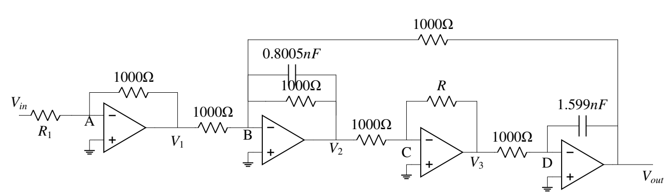

Fig. 7: 2

8. Draw the equivalent block diagram of the above circuit.
Solution: For a circuit shown in figure.8:1 The equivalent G is found to be

$$\frac{V_1}{V_2}=\frac{-Z_2}{Z_1} \tag{8.1}$$

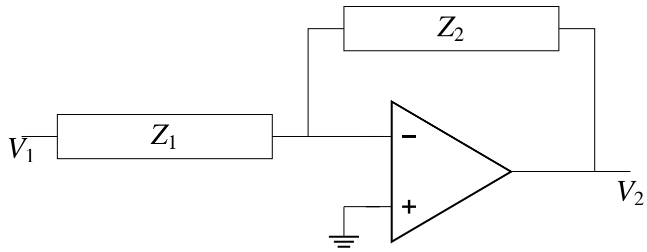

Fig. 8: 1

The control system block is shown in the Fig:8:2. Consider the first opamp from the

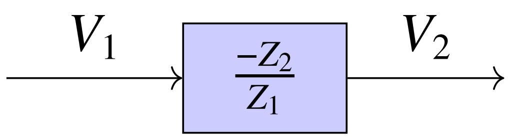

Fig. 8: 2

Fig.7:1 For the first opamp having negative feedback.

$$G_1 = \frac{-R_2}{R_1} \tag{8.2}$$

For the second, third and fourth opamp each having individual open loop gain G2,G3,G4.

$$G_2 = \frac{-R}{R(sRC + 1)} \tag{8.3}$$

$$G_3 = \frac{-R}{R} = -1 \tag{8.4}$$

$$G_4 = \frac{-1}{sC_1R} \tag{8.5}$$

$$G(s) = G_2 \times G_3 \times G_4 \tag{8.6}$$

Now , H can be computed as follows from

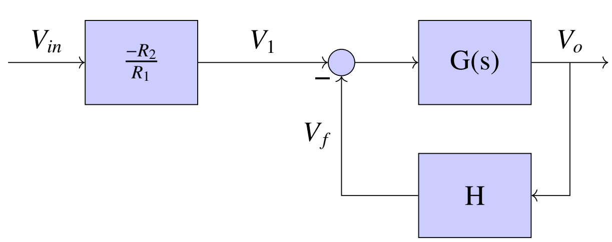

Fig. 8: 3

Fig:8:4.,

$$H = \frac{V_1}{V_o} = \frac{R}{R} \tag{8.7}$$

$$H = 1 \tag{8.8}$$

This feedback is given across G2,G3,G4.

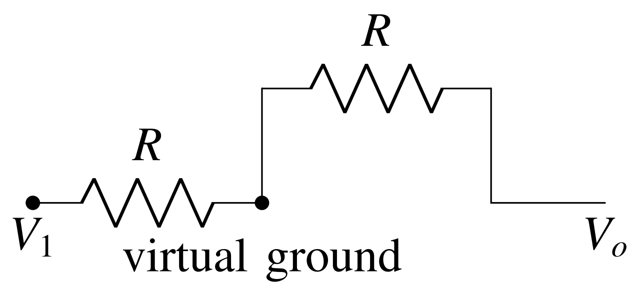

Fig. 8: 4

The equivalent block diagram is shown in the Fig.8:5.

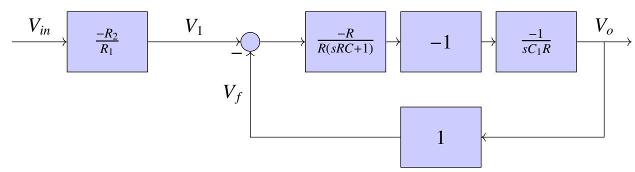

Fig. 8: 5

9. Draw the block diagram for G(s).
Solution: The block diagram is shown in the Fig.9.

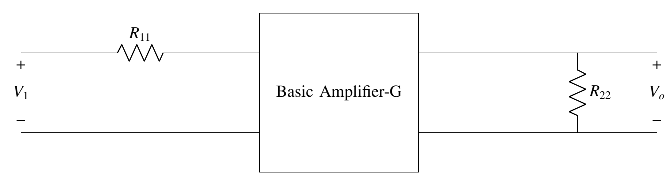

Fig. 9

10. Find R_{11} and R_{22} in the Fig.9.
Solution: For R_{11}, short Vo and find the equivalent resistance in Fig.8:5.

$$R_{11} = R + R = 2R \tag{10.1}$$
$$\tag{10.2}$$

For R_{22}, short V1 and find the equivalent resistance in Fig.8:5.

$$R_{22} = R + R = 2R \tag{10.3}$$
$$\tag{10.4}$$

The TABLE:10 shows obtained vales of the block diagram.

11. Verify the closed loop DC gain using NGSPICE simulator.

Parameter	Value
G_1	$\frac{-R_2}{R_1}$
G_2	$\frac{-1}{1+sCR}$
G_3	-1
G_4	$\frac{-1}{sC_1R}$
H	1

TABLE 10: 1

Solution: The following README file gives the procedure to be followed.

codes/ee18btech11005/spice/README

From equation.4.2. The DC closed loop gain is 10.
The following netlist file, gives the DC gain of the closed loop function.

codes/ee18btech11005/spice/gvv_ngspice.net

We can observe from simulation that the value of DC closed loop gain is 9.997.
Error analysis:-
ERROR in DC GAIN = 10-9.993 = 0.007
Thus, the predicted value in ngspice is almost accurate. Therefore, the value is verified using ngspice.

12. Verify the step response of the output from ngspice simulation.
Solution: The following netlist file does the transient anaylsis and store the Vout values with respect to time in a dat file.

codes/ee18btech11005/spice/gvv_ngspice2.net

Following python code is to plot the step response.

codes/ee18btech11005/spice/ee18btech11005_spice.py

The step response obtained is shown in the figure.12.The graph has steady state value equal to 10.

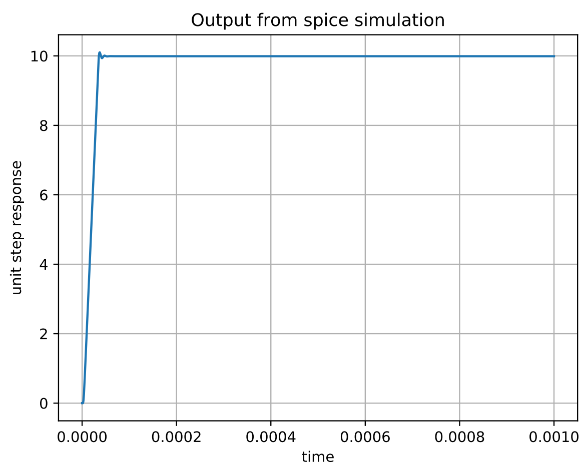

Fig. 12