Assignment 1

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Download all python codes from

https://github.com/varunireddy/EE3025_IDP/codes

and latex-tikz codes from

https://github.com/varunireddy/EE3025_IDP

1 Problem

(5.3) The system h(n) is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

2 Solution

We know the system is defined by.,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.1}$$

$$y(n) = 0 \text{ for } y < 0 \tag{2.0.2}$$

For a system to be stable the output should be bounded for every bounded input.(BIBO stability). Since the input sequence x(n) is bounded we have.,

$$|x(n)| < B_x < \infty \tag{2.0.3}$$

From convolution property,

$$|y(n)| = \left| \sum_{-\infty}^{\infty} h(k)x(n-k) \right| \tag{2.0.4}$$

$$|y(n)| \leq \sum_{-\infty}^{\infty} |h(k)| \, |x(n-k)| \tag{2.0.5}$$

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |h(k)| \tag{2.0.6}$$

$$\text{if.., } \sum_{-\infty}^{\infty} |h(n)| < \infty \tag{2.0.7}$$

$$\implies |y(n)| \leq B_y < \infty \tag{2.0.8}$$

We can therefore say that y(n) is bounded for all bounded input when h(n) is absolutely summable.

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{2.0.9}$$

Applying Z-Transform,

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.0.10}$$

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{2.0.11}$$

$$H(z) = \frac{2(z^2 + 1)}{z(2z + 1)} \tag{2.0.12}$$

We know,

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{2.0.13}$$

for the system to be stable. The above equation can be rewritten as.,

$$\sum_{n=-\infty}^{\infty} |h(n)| \left| z^{-n} \right|_{|z|=1} < \infty \tag{2.0.14}$$

$$\sum_{n=-\infty}^{\infty} \left| h(n) z^{-n} \right|_{|z|=1} < \infty \tag{2.0.15}$$

We know from triangle inequality.,

$$\sum_{n=-\infty}^{\infty} \left| h(n) z^{-n} \right|_{|z|=1} < \left| \sum_{n=-\infty}^{\infty} h(n) z^{-n} \right|_{|z|=1} \tag{2.0.16}$$

$$\implies |H(n)|_{|z|=1} < \infty \tag{2.0.17}$$

Therefore, the ROC (region of convergence) should include the unit circle.
Since, h(n) is right sided the ROC is outside the outermost pole. From the equation.2.0.12

$$\textbf{Poles:-}\ z = -\frac{1}{2}, 0 \tag{2.0.18}$$

From the above poles, ROC is $|z| > \frac{1}{2}$ H(z) is plotted in z-plane in python. From the figure.0, we can observe that ROC includes the unit circle $|z| = 1$,

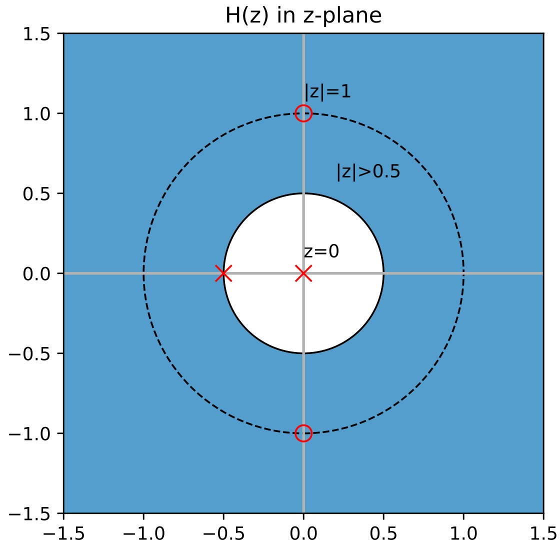

Fig. 0: H(z) in z-plane

which implies the given IIR filter is stable, because h(n) is absolutely summable.
Verification:- Given bounded input x(n).,

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{2.0.19}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.20}$$

Maximum value of x(n) is 4 and minimum value

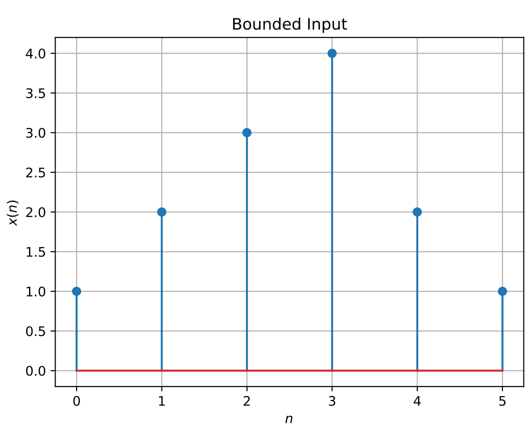

Fig. 0: Given bounded input

of x(n) is 0. x(n) is bounded.
From the python code,the maximum value of y is 4.375. Minimum value of y is -0.449. y(n) vanishes to zero as n tends to infinity. Therefore, output is bounded.

The system returns bounded output for the given

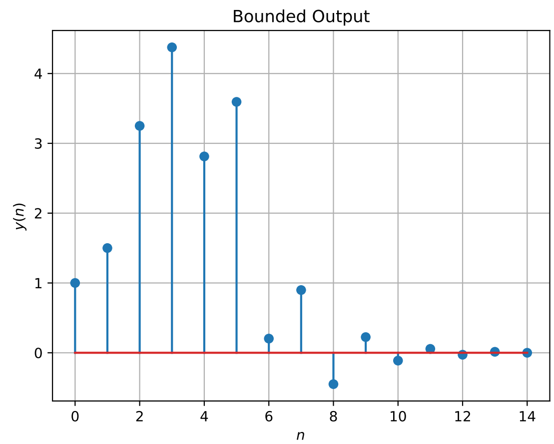

Fig. 0: Bounded output

bounded input. Implies, the system is stable. Hence, verified.