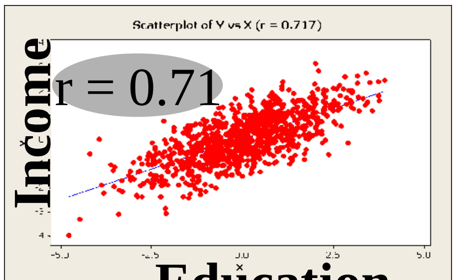
Moran's *I* and Correlation Coefficient *r*

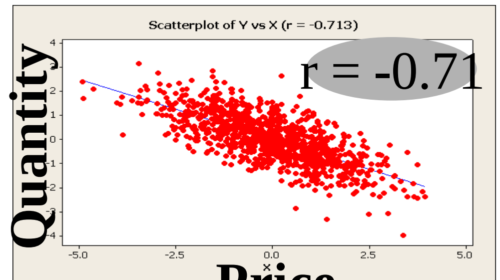
Differences and Similarities

Correlation Coefficient *r*

- Relationship between <u>two</u> variables



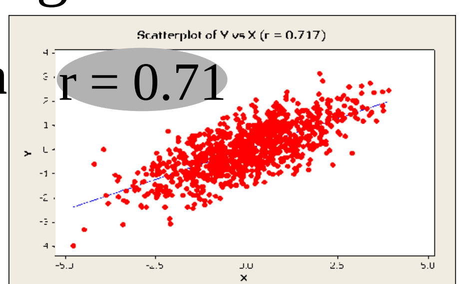
or

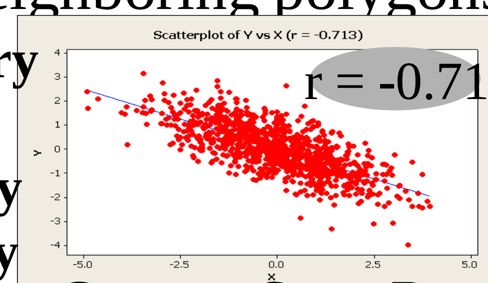


Moran's I

– Involves <u>one</u> variable only

– Correlation between variable, X, and the "spatial lag" of X formed by averaging all the values of X for the neighboring polygons





Formula for Moran's I

$$I = \frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\right)\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- Where:

N is the number of observations (points or polygons)
$\bar{x}$ is the mean of the variable
X_i is the variable value at a particular location
X_j is the variable value at another location
W_{ij} is a weight indexing location of *i* relative to *j*

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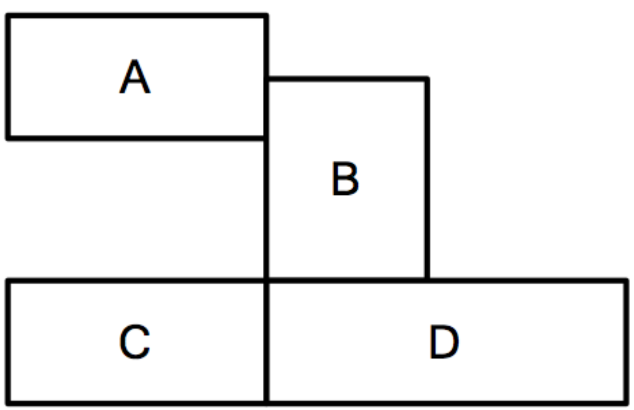
Candidate Neighbor Relationship: W-Matrix for Moran's I

- Diagonal Elements are always set to zero

The w-matrix can be specified in many ways:

- The weight for any two different locations is a constant.
- All observations within a specified distance have a fixed weight.
- K nearest neighbors have a fixed weight, and all others are zero.
- Weight is proportional to inverse distance, inverse distance squared, or inverse distance up to a specified distance.



(a) Map

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}
\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}
$$

(b) Boolean W

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}
\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.3 & 0 & 0.3 & 0.3 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 \end{bmatrix} \end{array}
$$

(c) Row-normalized W

$$\frac{\sum_{i=1}^{n} 1(y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Correlation Coefficient

Note the similarity of the numerator (top) to the measures of spatial association discussed earlier if we view Yi as being the Xi for the neighboring polygon

(see next slide)

$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Spatial auto-correlation

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Correlation Coefficient

$$\frac{\sum_{i=1}^{n} 1 (y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}}$$

Spatial weights

Yi is the Xi for the neighboring polygon

$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\right)\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i-\bar{x})(x_j-\bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}}$$

Moran's I

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