

# Theory of Computation $\rightarrow$ mathematical part of CSE

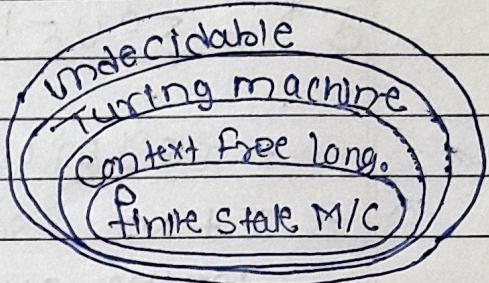
Step by step process to execute a given task

TOC is mainly about what kind of things can you really compute mechanically, how fast and how much space does it take to do so

FSM  $\rightarrow$  Finite State Machine

CF L  $\rightarrow$  Context Free Language  
 Set of strings

Automata is abstract model  
of Computational Devices.



finite State Machine (prerequisites)

- Symbol  $\rightarrow a, b, c, 0, 1, 2, 3$  (Smallest Building Block)
- Alphabet  $(\Sigma) \rightarrow$  A collection of symbols. Ex:  $\{a, b\}, \{0, 1, 2\}$
- String  $\rightarrow$  Sequence of symbols. Ex:  $a, b, 0, 1, aa, bb$ .
- Language - set of strings Ex:  $\Sigma = \{0, 1\}$

$L_1$  = set of all strings of length 2

$$L_1 = \{00, 01, 10, 11\}$$

$L_2$  = set of all strings of length 3

$$= \{000, 001, 010, 011, 100, 101, 110, 111\}$$

$L_3$  = set of all strings that begin with 0

$$= \{0, 00, 01, 010, 011, 000, 001, 010, 011, 0000, 0001, 0010, 0011, \dots\}$$

Powers of Sigma ( $\Sigma$ ):  $\Sigma = \{0, 1\}$

$\Sigma^0$  = set of all strings of length 0;  $\Sigma^0 = \{\} \quad \{ \in \}$

$\Sigma^1$  = set of all strings of length 1;  $\Sigma^1 = \{0, 1\}$

$\Sigma^2$  = set of all strings of length 2;  $\Sigma^2 = \{00, 01, 10, 11\}$

$\Sigma^3$  =  $\{000, 001, 010, 011, 100, 101, 110, 111\}$

Fuzzy Logic → for Real life average accuracy  
propositional logic → for Ideal 99.99% accuracy

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$\Sigma^n$  = Set of all strings of length n.

Cardinality - No. of Elements in a set.

$$\hookrightarrow \Sigma^n = 2^n$$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots \text{ Kleen Star}$$

$\Sigma^*$  CLOSURE

$$= \{ \epsilon \} \cup \{ 0, 1 \} \cup \{ 00, 01, 10, 11 \} \cup \dots$$

= It is a set of all possible strings of all lengths over {0, 1} → (Infinite set)

## Finite State Machines

also known as. **Finite Automata**

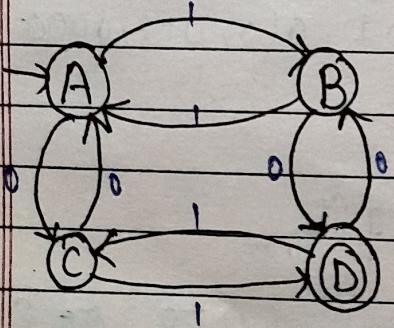
With O/P

Moore  
m/c

Without O/P

DFA  
NFA  
ε-NFA

- DFA → Deterministic Finite Automata
  - It is the simplest model of computation
  - It has very limited memory



① States :- These circles represented by A, B, C, D are known as states

② Edge :- These 0, 1 lines (edges) are transitions,

When I am in state (A) get input of (0), So my state (A) goes from (A) to (B)

- Memory of FSM is very limited
- It cannot store or count strings

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③ Initial state:- whenever you see an arrow coming from nowhere pointing to a state, i.e., the initial state or the starting state.

④ Final state:- whenever you see a double circle i.e., the terminating means final state.

Every DFA defined using these following tuples

a)  $Q$  = Set of all states.

b)  $\Sigma$  = inputs.

c)  $q_0$  = Start state / Initial state

d)  $F$  = set of final states.

e)  $\delta$  = Transition func<sup>n</sup> from  $Q \times \Sigma \rightarrow Q$

Tuples values for above DFA.

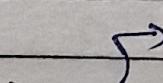
$$Q = \{A, B, C, D\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = \text{█████} A$$

$$F = \{D\}$$

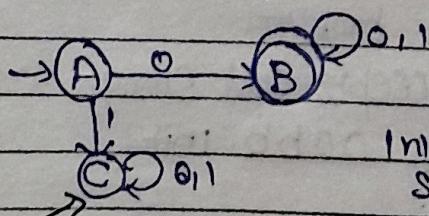
$$\delta =$$



	0	1
A	C	B
B	P	A
C	A	D
D	B	C

### Deterministic finite Automata Ex:-

L<sub>1</sub> = Set of all strings that start with '0'  
 $= \{0, 00, 01, 000, 010, \dots\}$



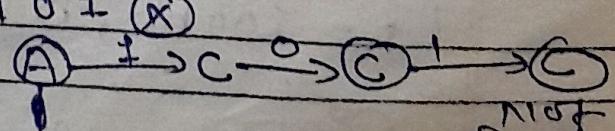
Ex:- 001

Initial state  $\textcircled{A} \xrightarrow{0} \textcircled{B} \xrightarrow{0} \textcircled{B} \xrightarrow{1} \textcircled{B}$  final state

Dead State  
or  
Trap State

Ex:- 101

Initial state

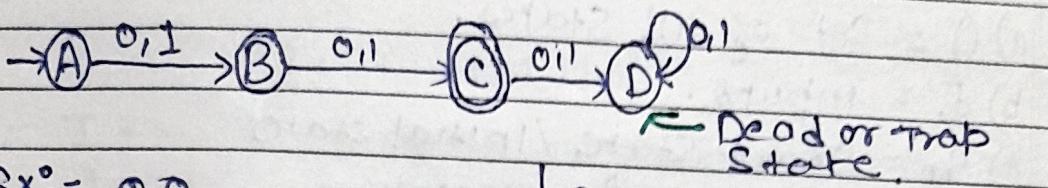


Not  
final  
state.

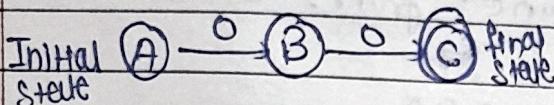
Q Construct a DFA that accepts sets of all strings over  $\{0, 1\}$  of length 2.

$$\Sigma = \{0, 1\}$$

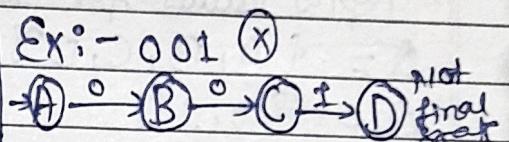
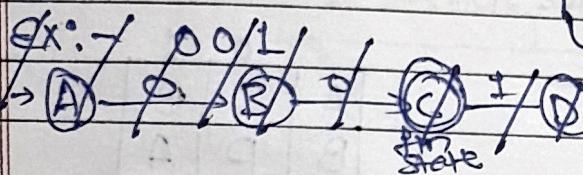
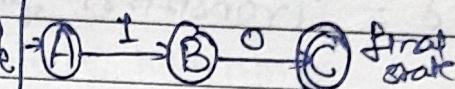
$$L = \{00, 01, 10, 11\}$$



Ex:- 00 ✓ accepted.



Ex:- 10



Ex:- 1 (X)



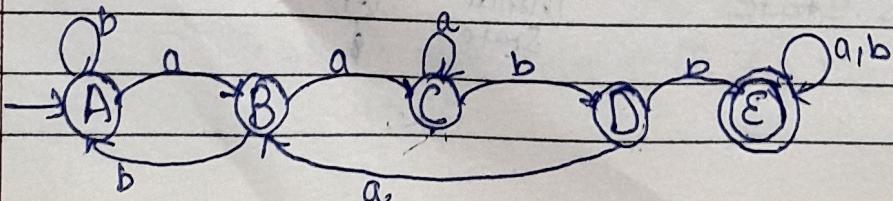
B. Construct a DFA that accepts any strings over  $\{a, b\}$  that does not contain the string aabb.

$$\Sigma = \{a, b\}$$

(\*) Try to design a simpler problem

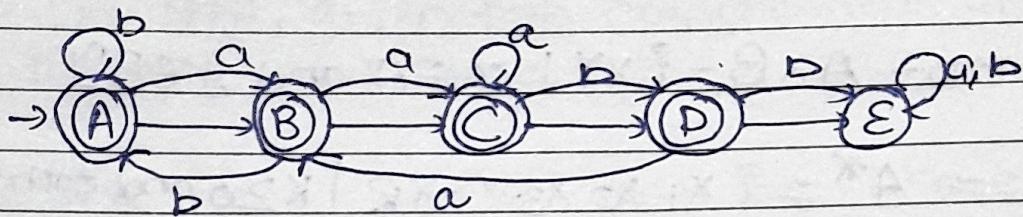
Let us construct DFA that accepts all strings over  $\{a, b\}$  that contains the string aabb init

$$L = \{aabb, baabb, ba baabb, \dots\}$$

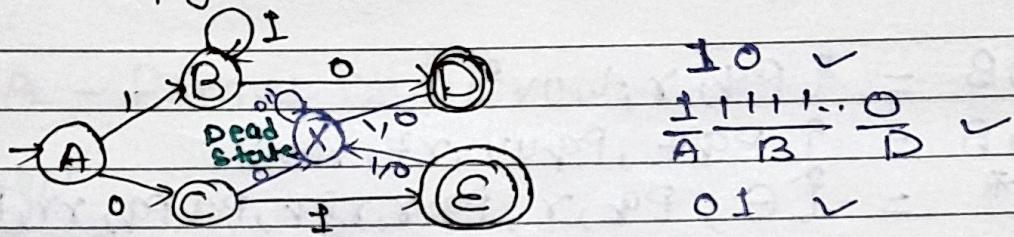


Now flip the States

- ①  $\Rightarrow$  make the final state into non-final state
- ②  $\Rightarrow$  Make the non-final states into final states.



How to figure out what a DFA recognizes?



$10 \checkmark$   
 $\frac{11111\ldots}{AB} \frac{0}{D} \checkmark$   
 $01 \checkmark$

$L = \{ \text{Accepts the string } 0^+ \text{ or a string of at least '1' followed by a '0'} \}$

Ex:- 001, 010, 011, 1101, 1100 (All falls to Dead state in end)  
X X X X X  
 $\times \rightarrow \text{Dead state}$

## Regular Languages

A language is said to be a Regular language if and only if some finite state machine recognizes it.

The languages

- $\gg$  which are not recognized by any FSM
- $\gg$  which require memory

Ex:- ababb, ababb;  $a^N b^N$ .

FSM can't keep them in memory. FSM can't keep count of no. of 'a's

## Operations on Regular Languages

Union -  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$  (such that)

Concatenation -  $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

STAR -  $A^* = \{x_1 x_2 x_3 \dots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$

Ex:-  $A = \{pq, r\}$ ,  $B = \{t, uv\}$ .

$$A \cup B = \{pq, r, t, uv\}$$

$$A \circ B = \{pqt, pquv, rt, ruv\}$$

$$A^* = \{\epsilon, pq, r, pqr, rpr, pqrpq, rr, Pq, Pq, Pq, rr, \dots\}$$

Theorem 1: The class of Regular languages is closed under UNION.

Theorem 2: The class of Regular languages is closed under concatenation.

## NFA - Non-deterministic Finite Automata

### Deterministic Finite Automata

$\downarrow$   
Determinism

→ In DFA, given the current state

we know what the next state will be.

→ It has

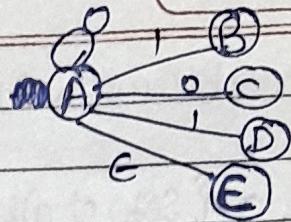
→ It has only one unique next state

→ It has no choice or randomness

→ It is simple and easy to design

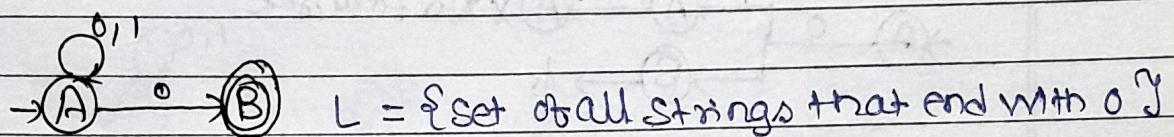
## Nondeterministic Finite Automata

Non-determinism



- In NFA, given the current state there could be multiple next states.
- The next state may be chosen at random or
- All the next states may be chosen in parallel

## NFA - Formal Definition



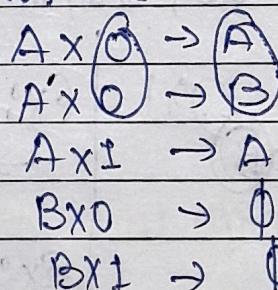
$(Q, \Sigma, q_0, F, \delta)$

- $Q$  = Set of all states  $- \{A, B\}$
- $\Sigma$  = Inputs  $- \{0, 1\}$
- $q_0$  = Start state / Initial state  $- A$
- $F$  = Set of final states  $- \{B\}$
- $\delta = Q \times \Sigma \rightarrow 2^Q$

Another ex:-

3 states A, B, C  
 $A^1 = \emptyset, A, B, C$   
 $AB, AC, BC, ABC$

From the above given ex:-



$\therefore A$  can go to  $A, B, AB, \emptyset$   
 Some goes for B

So we can say that

when we have

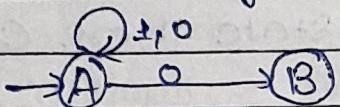
2 states  $\Rightarrow 4$  possibilities.

3 states  $\Rightarrow 8$  possibilities

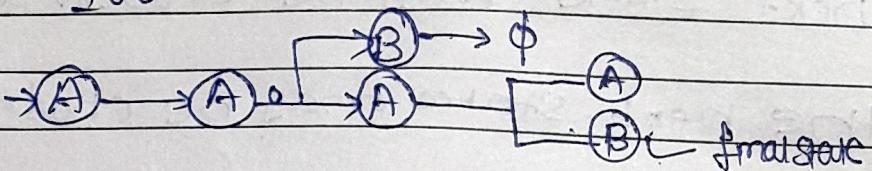
$\therefore 2^n$  states  $\Rightarrow n$ . no. of possibilities.

Q. Create an NFA whose all strings end with 0

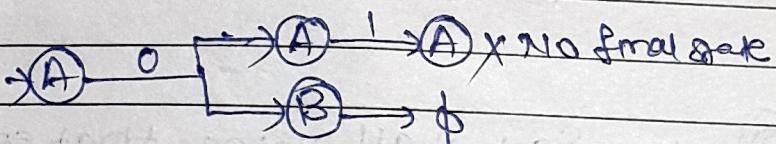
$L = \{ \text{Set of all strings that end with } 0 \}$



Ex:- 100



Ex:- 01

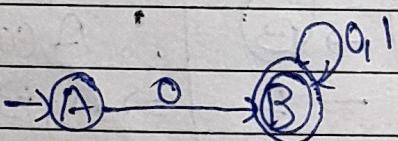


How can we formally write down when a string will be accepted

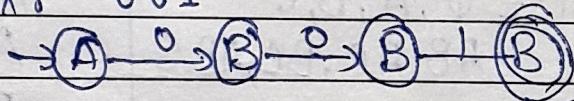
If there is any way to run the machine that ends in any set of states out of which at least one state is a final state, then the NFA accepts.

Q.  $L = \{ \text{Set of all strings that starts with } 0 \}$

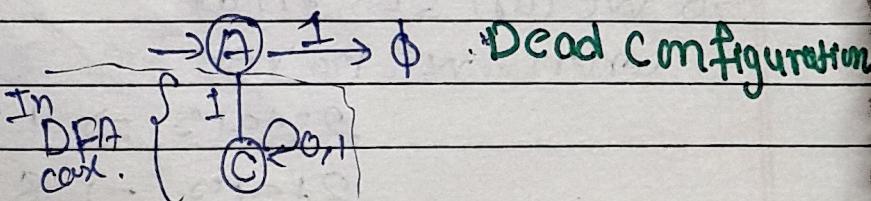
$\{ 0, 00, 01, 000 \dots \}$



Ex:- 001



Ex:- 101

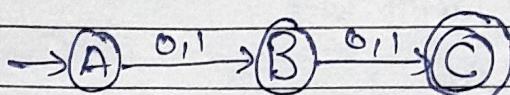


Q. Construct a NFA that accepts sets of all strings over  $\{0,1\}$  of length 2.

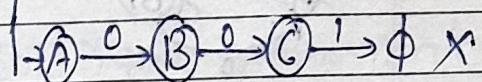
$$\Sigma = \{0,1\}$$

$$L = \{00, 01, 10, 11\}$$

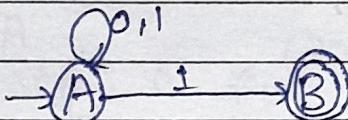
Ex:- 00



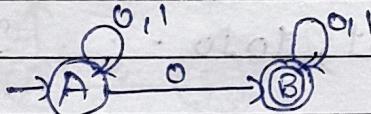
Ex:- 001



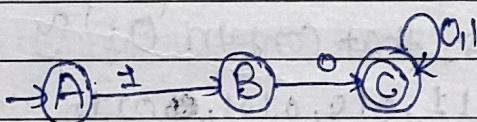
Q.  $L_1 = \{ \text{Set of all strings that ends with '1'} \}$



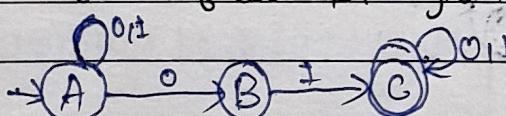
Q.  $L_2 = \{ \text{Set of all strings that contain '0'} \}$



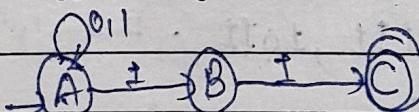
Q.  $L_3 = \{ \text{Set of all strings that starts with '10'} \}$



Q.  $L_4 = \{ \text{Set of all strings that contain '01'} \}$



Q.  $L_5 = \{ \text{Set of all strings that ends with '11'} \}$

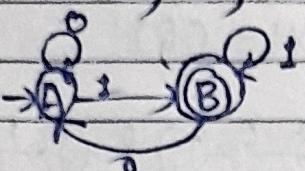


HW

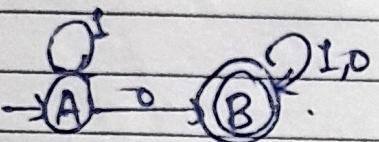
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Construct Equivalent DFAs for the L<sub>i</sub>'s  
 Then tell how many minimum no. of states you  
 would use for the construction of each DFA.

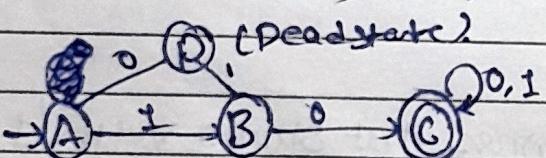
- Q. L<sub>1</sub> = {Set of all strings that end with '1' }  
 $= \{ \varnothing, 01, 11, 001, 0101, \dots \}$



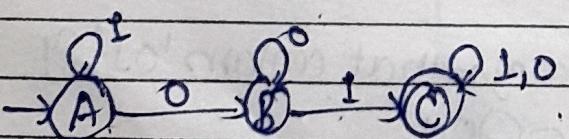
- Q. L<sub>2</sub> = {Set of all strings that contains '0' }  
 $= \{ \varnothing, 0, 01, 10, 110, 1001, \dots \}$



- Q. L<sub>3</sub> = {Set of all strings that starts with '10' }  
 $= \{ 10, 100, 101, 1001, 1010, \dots \}$

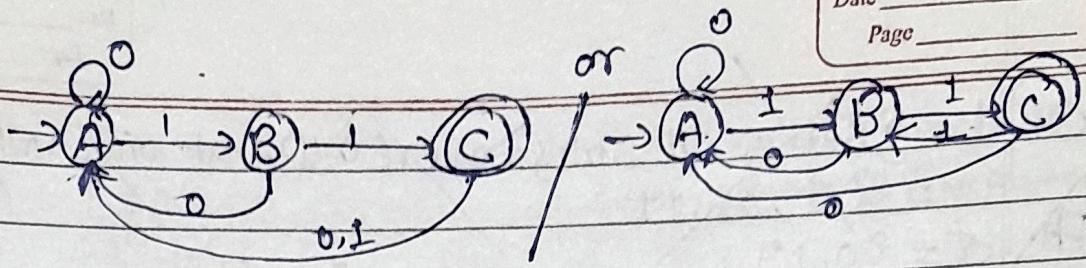


- Q. L<sub>4</sub> = {Set of all strings that contain '01' }  
 $= \{ 101, 101, 001, 011, 010, 0101, 0010, \dots \}$



- Q. L<sub>5</sub> = {Set of all strings that ends with '11' }  
 $= \{ 11, 011, 111, 0111, 1011, \dots \}$



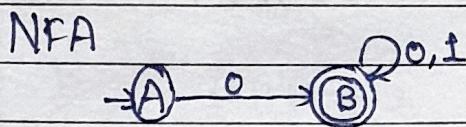


## Conversion of NFA to DFA

Every DFA is an NFA, but not vice versa

But there is an equivalent DFA for every NFA

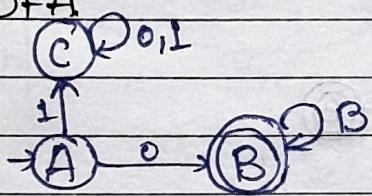
$$\begin{aligned}
 L &= \{\text{set of all strings over } \{0,1\} \text{ that starts with '0'}\} \\
 &= \{0, 01, 001, 010, \dots\} \\
 \Sigma &= \{0,1\}
 \end{aligned}$$



Transitions ~~Table~~ Table

	0	1
A	B	$\emptyset$
B	B	B

DFA



	0	1	C - Dead state
A	B	$\emptyset$	C
B	B	B	-
C	C	C	-

DFA

- a) It has 5 components they are  $Q, \Sigma, \delta, q_0, F$

b) Transition func<sup>n</sup> ( $\delta$ ) =  $Q \times \Sigma \Rightarrow Q^Q$

- c) All states must be defined at all I/p

d) only 1 transition possible on single I/p. Ex: -

- e) Null transition ( $\epsilon$ ) not allowed

NFA.

- It also has 5 components they are  $Q, \Sigma, \delta, q_0, F$

Transition func<sup>n</sup> ( $\delta$ ) =  $Q \times \Sigma \Rightarrow 2^Q$

- f) Not compulsory to define all I/p

more than 1 transition possible on single I/p. Ex: -

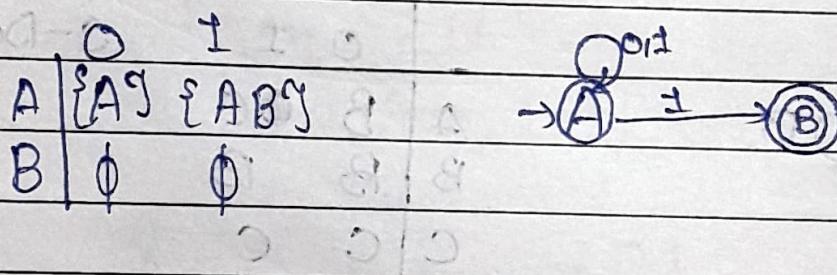
- g) Null transition ( $\epsilon$ ) allowed

Q.  $L = \{ \text{set of all strings over } \{0,1\} \text{ that ends with } 1^*\}$

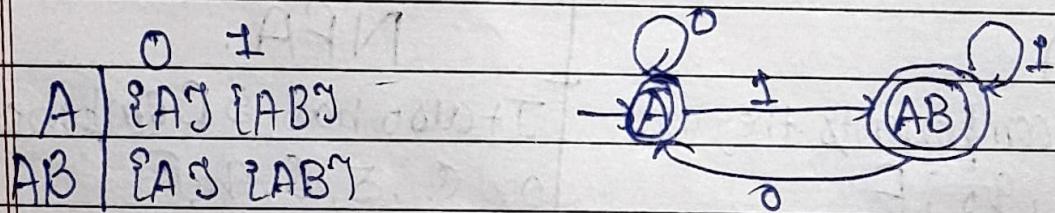
$$L = \{ \_, 01, 11, \dots \}$$

$$\Sigma = \{0, 1\}$$

NFA.



DFA.



Note:-

AB - is now single state

∴ we can't see B state

in making transition table

∴ No Need of B state

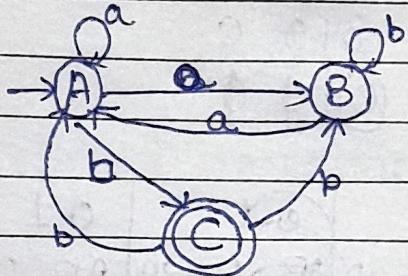
Above method is called as

Subset construction method

- Q. Find the equivalent DFA for the NFA given by  
 $M = [ \{A, B, C\}, \{a, b\}, S, A, \{c\} ]$  where  $S$  &  $\delta$  are given by:

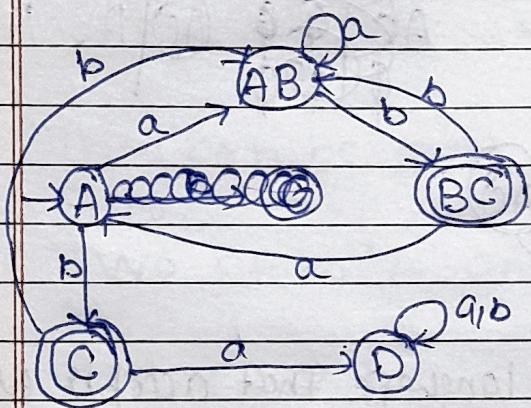
$\delta$  (states)     $\epsilon$  (T/F)    Transition Table    Initial State    Final State

NFA Transition diag.



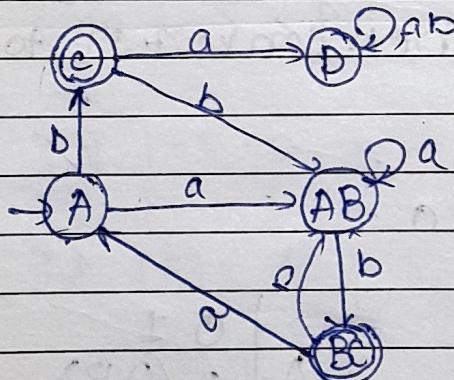
	a	b
$\rightarrow A$	A, B	C
B	A	B
(C)	-	A, B

DFA



a	b.
$\rightarrow A$	$\{AB\}$
AB	$\{AB\}$ $\{BC\}$
(BC)	A $\{AB\}$
(C)	$\{AB\}$
D	D

or.



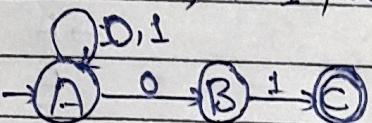
- Q. Find out what does this NFA and its equivalent DFA accept

1 :- Should not start with 'ba' and must have odd no. of b's at the end

Q L = {set of all strings over {0,1} that ends with '01'}<sup>Y</sup>. Construct its equivalent DFA.

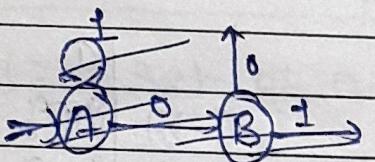
NFA.

$$L = \{01, 101, 001\}$$

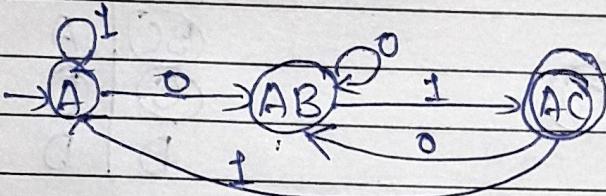


	0	1
A	AB	A
B	∅	C
C	∅	∅

DFA.



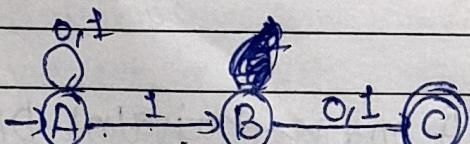
	0	1	01
A	AB	A	AB A
B	AB	AC	AB AC
C	AC	AB	AB A



Q Design an NFA for a language that accepts all strings over {0,1} in which the second last symbol is always '1'. Then convert it to its equivalent DFA.

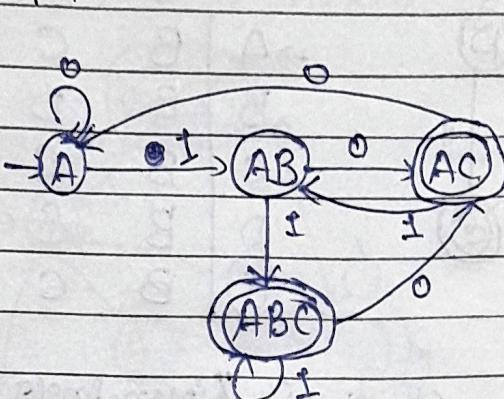
Ans NFA

$$L = \{10, 010, 011, 110\}$$



	0	1
A	A(AB)	
B	C	C
C	∅	∅

DFA.



	O 1
→ A	A AB
AB	AC ABC
AC	A AB
ABC	AC ABC

## Minimization of DFA

Minimization of DFA is required to obtain the minimal version of any DFA which consists of the minimum no. of states possible

This can be done by combination of equivalent states.

TWO STATES 'A' and 'B' are said to be equivalent if.

$$S(A, x) \rightarrow F \quad S(A, x) \not\rightarrow F$$

OR

when 'x' is  
any I/p string

$$S(B, x) \rightarrow F \quad S(B, x) \not\rightarrow F$$

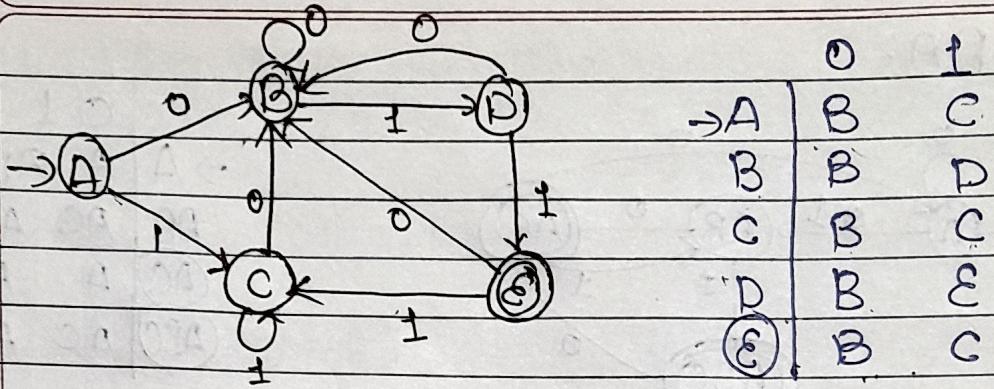
If  $|x| = 0$ , then A and B are said to be equivalent.

$$J \neq 1 \times 1 = 2 \quad , \quad 1 \quad 2 \quad 2 \quad 1 \quad 2 \quad 1$$

1 2 3 4 5 6 7 8 9 10 11 12

$$\text{If } |x|=n \quad (1, 1, 1, 1, \dots, n, 1, 1)$$

If not same IP state  
Check set, if not in set  
then not Eigenvalue



0	1
→ A	B C
B	B D
C	B C
D	B E
E	B C

Q Equivalence:  $\{ABCD\} \not\equiv \{E\}$  // Non-final states in set

1 Equivalence:  $\{ABC\} \not\equiv \{D\} \not\equiv \{E\}$

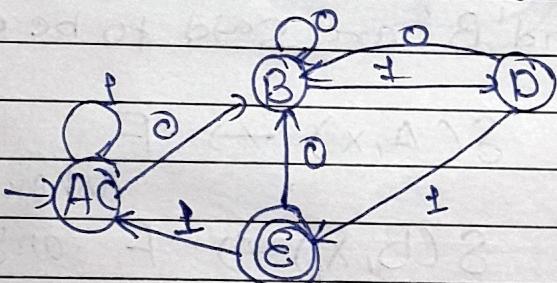
2  $\{A, B, C\} \not\equiv \{B\} \not\equiv \{D\} \not\equiv \{E\}$

3  $\{A, C\} \not\equiv \{B\} \not\equiv \{D\} \not\equiv \{E\}$

Same

When 2 states are giving consecutively the same result so end the process of equivalence.

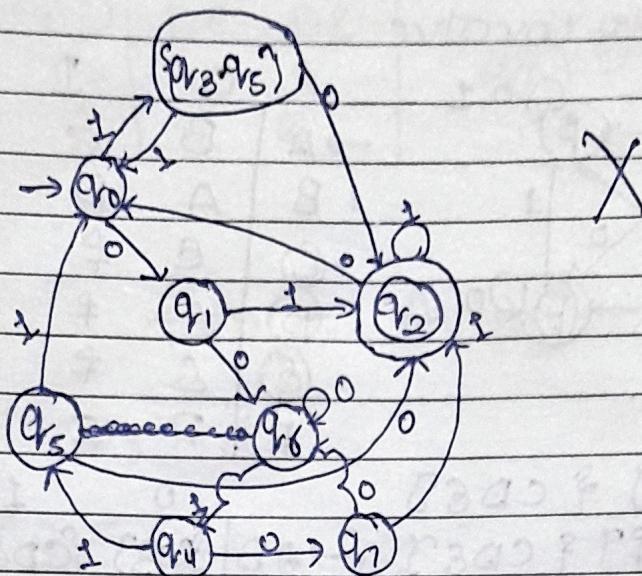
$\{A, C\}$  now as a single state.



Q. Construct a minimum DFA equivalent to the DFA described by.

	0	1	
→ $q_0$	$q_1$	$q_5$	0 Equiv: $\{q_0, q_1, q_3, q_4, q_5, q_6, q_7\} \not\equiv \{q_2\}$
$q_1$	$q_6$	$q_2$	1 Equiv: $\{q_1, q_6\} \not\equiv \{q_2\} \not\equiv \{q_3\} \not\equiv \{q_4\}$
$q_2$	$q_0$	$q_2$	2 Equiv: $\{q_6\} \not\equiv \{q_3\} \not\equiv \{q_0, q_3, q_4\} \not\equiv \{q_2, q_3, q_4, q_5\} \not\equiv \{q_2, q_3, q_4\}$
$q_3$	$q_2$	$q_0$	3 Equiv: $\{q_0\} \not\equiv \{q_1\} \not\equiv \{q_2\} \not\equiv \{q_3\} \not\equiv \{q_4\} \not\equiv \{q_5\} \not\equiv \{q_6\} \not\equiv \{q_7\}$
$q_4$	$q_1$	$q_5$	
$q_5$	$q_2$	$q_0$	Here 2 Equiv and 3 Equiv are giving
$q_6$	$q_6$	$q_4$	Same result So end the process
$q_7$	$q_6$	$q_2$	

$\{q_0 q_3 q_5\}$  is now a single state.



0 Equi :-  $\{q_0 q_1 q_3 q_4 q_5 q_6 q_7\} \{q_0\}$

1 Equi :-  $\{q_0 q_4, q_6\} \{q_1, q_7\} \{q_3 q_5\} \{q_2\}$

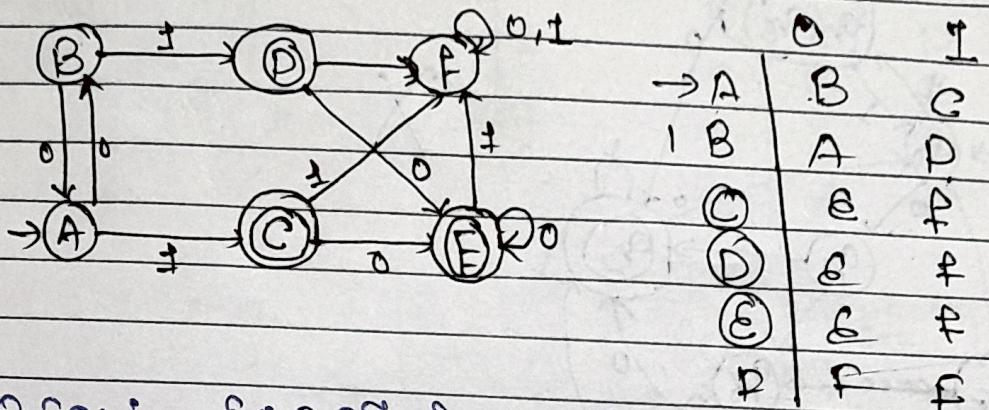
2 Equi :-  $\{q_0, q_4\} \{q_6\} \{q_1, q_7\} \{q_3, q_5\} \{q_2\}$

3 Equi :-  $\{q_0, q_4\} \{q_6\} \{q_1, q_7\} \{q_3 q_5\} \{q_2\}$

Stop as 2 consecutive equivalent same

	0	1
$\{q_0, q_4\}$	$q_0 q_1$	$\{q_3 q_5\}$
$\{q_6\}$	$q_6$	$q_0 q_4$
$\{q_1, q_7\}$	$q_6$	$q_2$
$\{q_3 q_5\}$	$q_2$	$q_6$
$\{q_2\}$	$q_0 q_4$	$q_2$

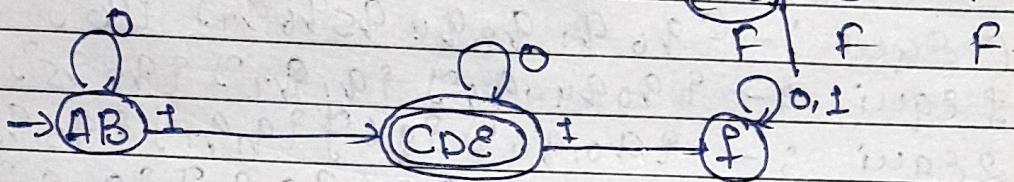
Q. minimize DFA when there are more than one final states involve.



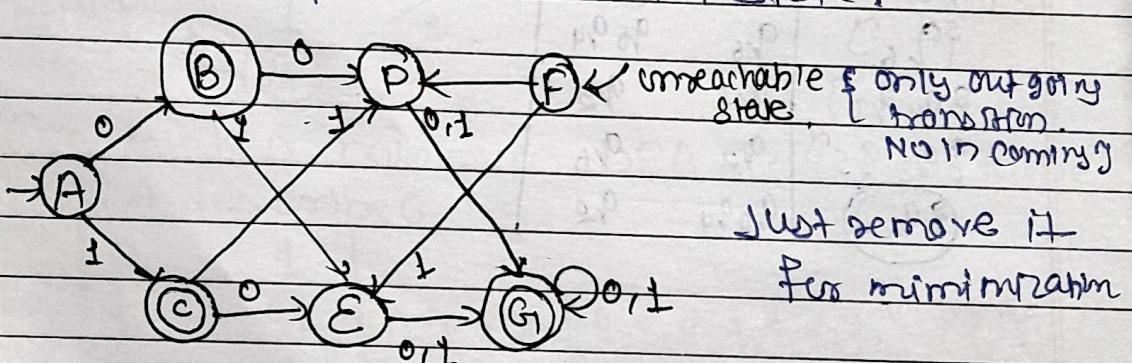
$$0\text{Equi} = \{ABF\} \triangleq CDE\bar{F}$$

$$1\text{Equi} = \{AB\}\{F\} \triangleq CDE\bar{F} \rightarrow AB \{A\}\{B\} \{CDE\}$$

$$2\text{Equi} = \{AB\}\{F\} \triangleq CDE\bar{F} \quad \begin{matrix} FCDE \\ CDE \\ F \end{matrix} \quad P$$

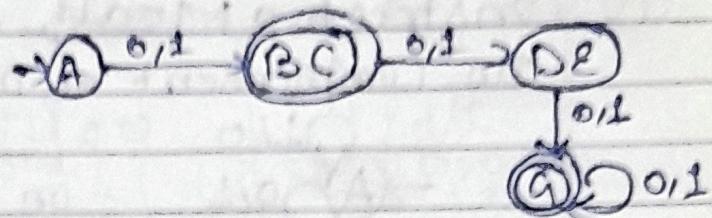


**Unreachable State :-** A state is said to be unreachable if there is no way it can be reached from the initial state.



	0	1	
$\rightarrow A$	B C	D	0 Equi :- {ADE}\{BCG\}
B	D	E	1 Equi :- {ADE}\{BCG\}\{G\}
C	E	D	2 Equi :- {A}\{DE\}\{BCG\}\{G\}
D	a	a	3 Equi :- {A}\{DE\}\{BCG\}\{G\}
E	a	a	Stop for same consecutive equivalence.
G	a	a	

	$\Sigma$	$\Delta$
$\rightarrow A$	BC	BC
(B)	DE	DE
DE	q	q
a	q	q



## Finite Automata with output

Mealy M/C

$$(Q, \Sigma, \Delta, \delta, \lambda, q_0)$$

where

$Q$  = finite set of states

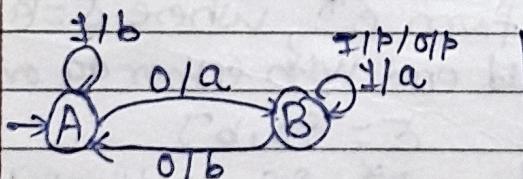
$\Sigma$  = finite non-empty set of I/Ps alphabets.

$\Delta$  = the set of output alphabets.

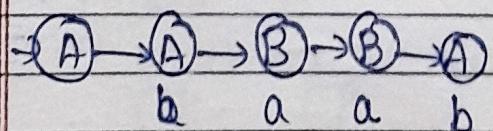
$\delta$  = Transition function  $Q \times \Sigma \rightarrow Q$

$\lambda$  = Output function  $\Sigma \times Q \rightarrow \Delta$   $\lambda = O/P$  funcn:  $Q \rightarrow \Delta$

$q_0$  = Initial state / start state  $\rightarrow$  same.



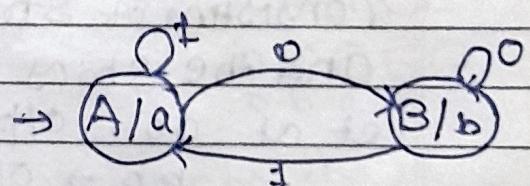
Eg. 1010



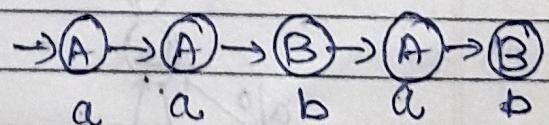
So

Length of I/P String = O/P String  
 $n = n$

I/P O/P.



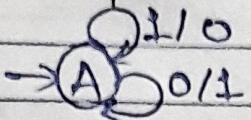
Ex:- 1010



Length of I/P String = length of O/P + 1  
 $n \Rightarrow n+1$

I/P O/P

- Q. Construct a Mealy machine that produces  
the Complement of binary strings.

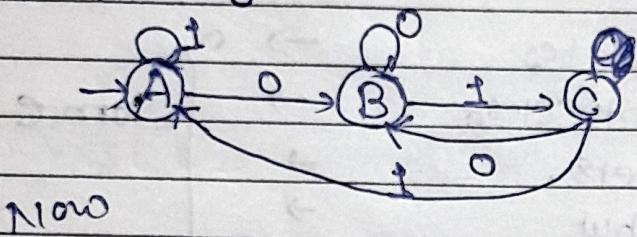


on 1/0  $\Rightarrow$  gets 0/p 1  
on 0/1  $\Rightarrow$  gets 0/p 0

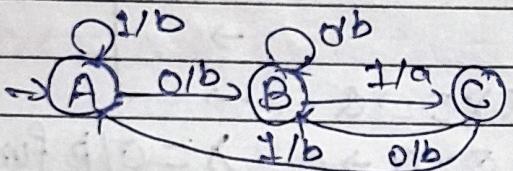
- Q. Construct a mealy machine that prints 'a' whenever the sequence '01' is encountered in any T/p binary string.

$$\Sigma = \{0, 1\} \quad \Delta = \{a, b\}$$

DFA Try.



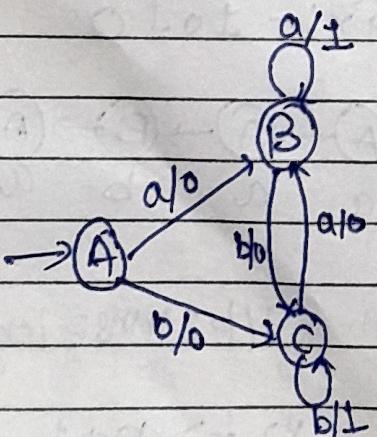
Now



- Q. Design a Mealy M/c accepting the language consisting of strings from  $\Sigma^*$ , where  $\Sigma = \{a, b\}$  and the string should end with either aa or bb

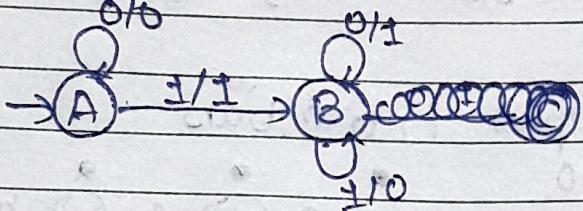
Let at  $a \rightarrow 0/p \text{ is } 1 \quad \Sigma = \{a, b\}$

$b \rightarrow 0/p \text{ is } 1 \quad \Sigma^* = \{\epsilon, aa, bb, ab, ba\}$



Construct a Mealy M/C that gives 2's complement of any binary I/P (Assume that the last carry bit is neglected).

$$\text{Ex:- } \begin{array}{r} 10100 \\ 01011 \\ \hline 01100 \end{array} \quad | \quad \begin{array}{r} 11100 \\ 00011 \\ \hline 00100 \end{array} \quad | \quad \begin{array}{r} 1111 \\ 0000 \\ \hline 0001 \end{array}$$

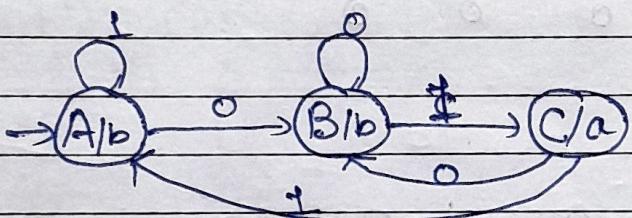


So we get

that upon 2's complement up till we get 1 we have same complement.

$$\begin{array}{r} 10100 \\ \textcircled{B} \textcircled{B} \textcircled{B} \textcircled{A} \textcircled{A} \\ 01100 \end{array}$$

Construct a Moore M/C that prints b' whenever the sequence '01' is encountered in any I/P binary strings.



$$\Sigma = \{0, 1\}$$

$$\Delta = \{a, b\}$$

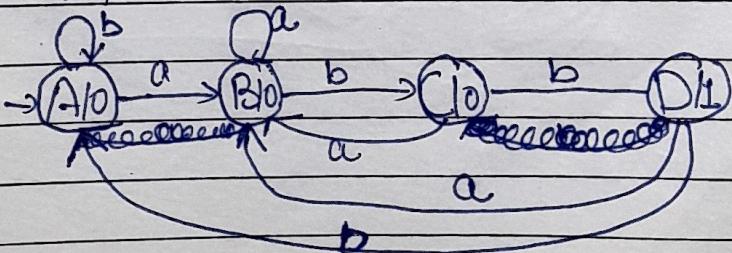
$$\text{Ex:- } 0101$$

$$\textcircled{A} \textcircled{B} \textcircled{C} \textcircled{B} \textcircled{C}$$

Q) Construct a Moore M/C that counts the occurrences of the sequence 'abb' in any input strings over  $\{a, b\}$ .

$$\Sigma = \{a, b\} \Delta = \{0, 1\}$$

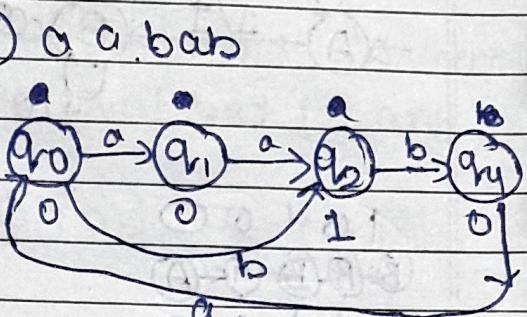
Let whenever we encounter abb we will get 0/p1.



Q. For the construction following Moore M/C  
the I/P alphabet is  $\Sigma = \{a, b\}$  and the O/P  
alphabet is  $\Delta = \{0, 1\}$ . Run the following  
input sequences and find the respective outputs.

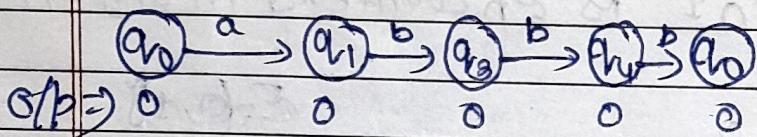
i) aa bab ii) ab bb iii) ab abb

States	a	b	outputs
$q_{r0}$	$q_{r1}$	$q_{r2}$	0
$q_{r1}$	$q_{r2}$	$q_{r3}$	0
$q_{r2}$	$q_{r3}$	$q_{r4}$	$\pm$
$q_{r3}$	$q_{r4}$	$q_{r4}$	0
$q_{r4}$	$q_0$	$q_0$	0

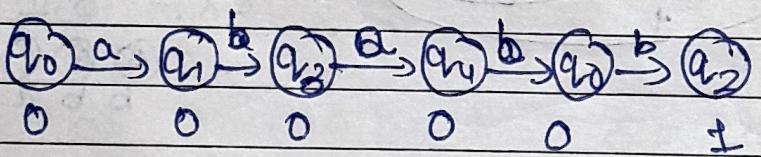


$$O/P = 00 \pm 001$$

ii) ab bb



iii) ababb

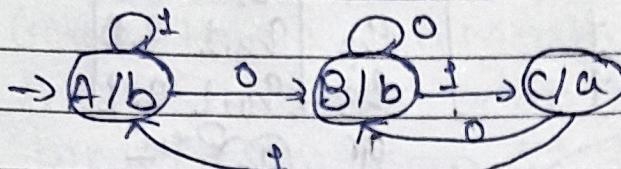


# Conversion of Moore M/C to Mealy M/C

Q Construct a Moore M/C that prints a whenever the sequence '01' is encountered in any 1/p binary string and then convert it to its equivalent Mealy M/C.

Moore

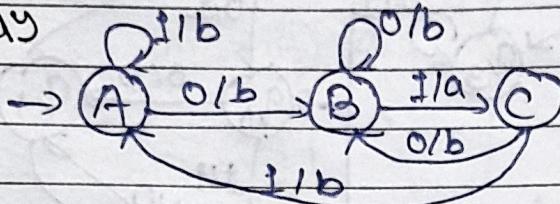
$$\Sigma = \{0, 1\} \quad \Delta = \{0, 1\}$$



Moore M/C  $\rightarrow$  Mealy M/C

state	0	1	output
$\rightarrow A$	B	A	b
B	B	C	b
C	B	A	a

Mealy

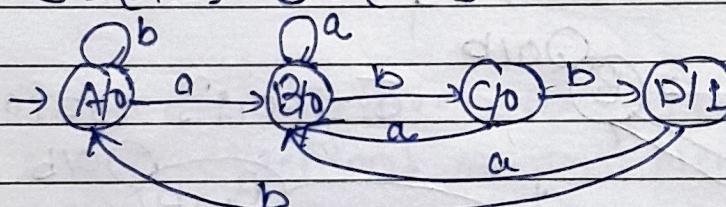


state	0/b	1/a	0/b
$\rightarrow A$	0/b	1/a	0/b
B			
C			

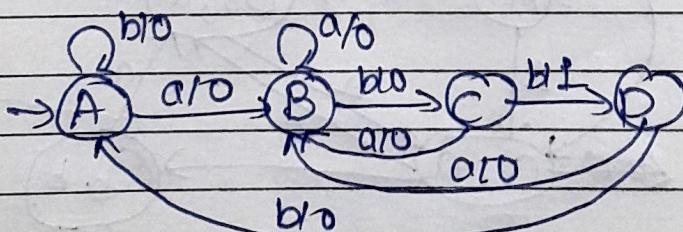
Q

The given Moore M/C counts the occurrences of the sequence 'abb'. In any 1/p binary string over  $\{a, b\}$ . Convert it to its equivalent Mealy M/C

$$\Sigma = \{a, b\} \quad \Delta = \{0, 1\}$$



Mealy .



Moore .

state	a	b	
$\rightarrow A$	B	A	0
B	B	C	0
C	B	D	0
D	B	A	1

Mealy .

state	a	b	
$\rightarrow A$	B, 0	A, 0	
B	B, 0	C, 0	
C	B, 0	D, 1	
D	B, 0	A, 1	

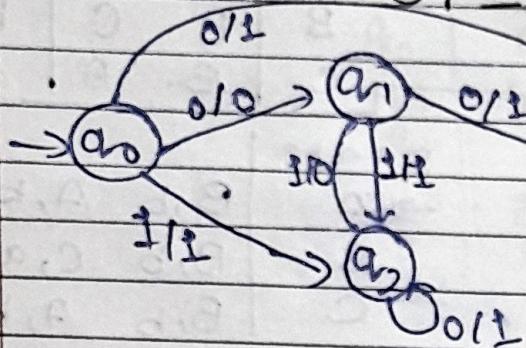
Mealy  $\rightarrow$  Q/Ps are associated with transitions  
 Moore  $\rightarrow$  Q/Ps are associated with states.

Page

Q. Convert the given Moore M/C to its equivalent Mealy M/C.

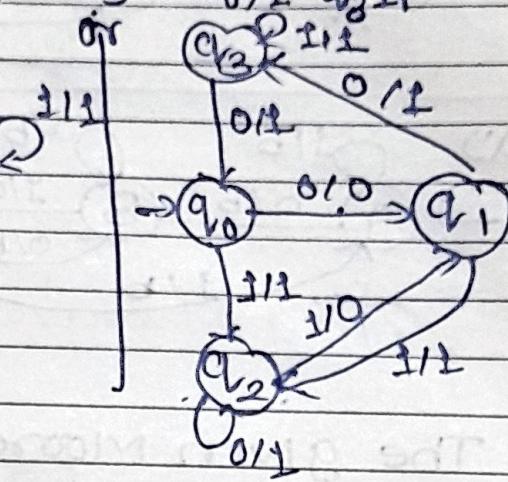
Moore.

STATE	0	1	Output
$q_0$	$q_1$	$q_2$	1
$q_1$	$q_3$	$q_2$	0
$q_2$	$q_0$	$q_1$	1
$q_3$	$q_0$	$q_3$	1



Mealy.

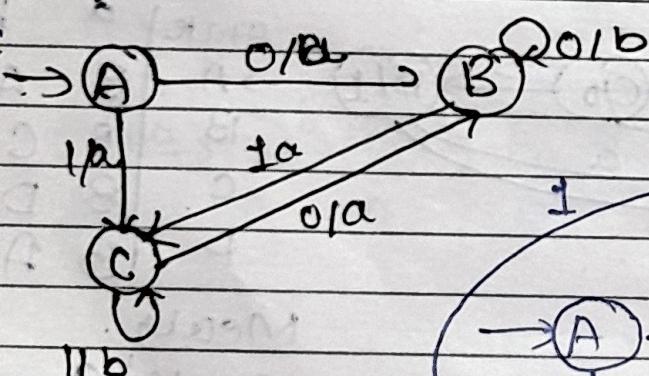
STATE	0	1
$q_0$	$q_1, 0$	$q_2, 1$
$q_1$	$q_3, 1$	$q_0, 1$
$q_2$	$q_2, 1$	$q_1, 0$
$q_3$	$q_0, 1$	$q_3, 1$



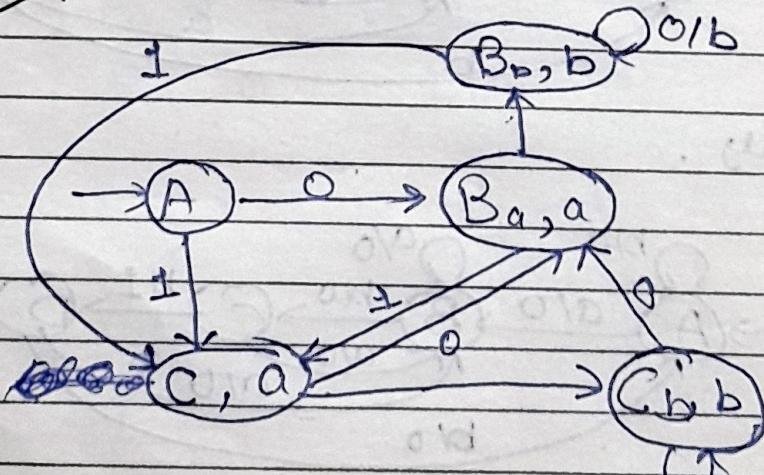
Mealy to Moore

Q. Convert the following Mealy M/C to its equivalent Moore M/C

Mealy:



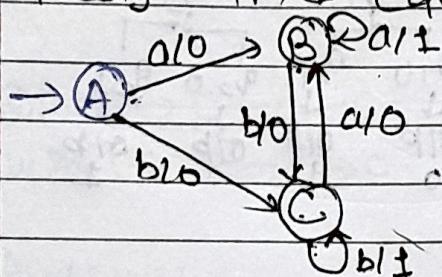
$$\Sigma = \{0, 1\} \quad D = \{a, b\}$$



Moel

Moore  $\rightarrow$  Mealy  $\Rightarrow$  No. of States were Same  
 Mealy  $\rightarrow$  Moore  $\Rightarrow$  No. of States increased  
 $\downarrow$   
 $x$  and  $y$        $(x \times y)$  no. of states at max.  
 States      outputs.

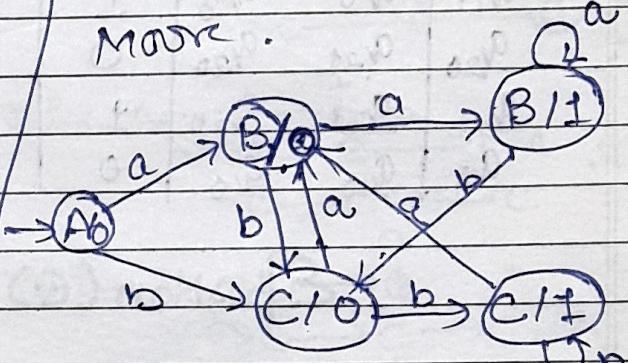
- Q Given below is a Mealy M/C that prints '1' whenever the sequence 'aa' or 'bb' is encountered in any 11-bit binary string from  $\Sigma^*$  where  $\Sigma = \{a, b\}$ . Design the equivalent Moore M/C for it.



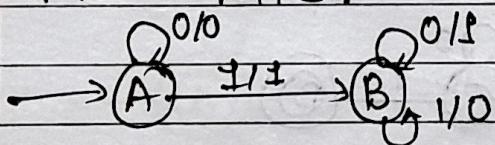
$$\Sigma = \{0, 1\} \quad \Delta = \{a, b\}$$

Test Run.

'aa'    ab    abaa  
001    000    00001

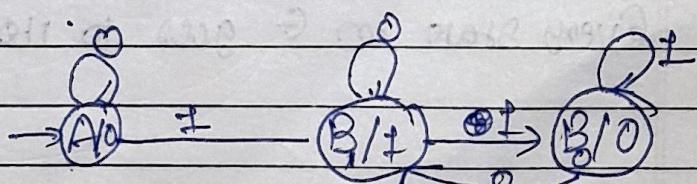


- Q. Convert the given mealy m/c that gives the 2's complement of any binary 11-bit to its equivalent Moore m/c.



$$\Sigma = \{0, 1\} \quad \Delta = \{0, 1\}$$

Test run.  
 $10100$   
 $01011$   
 $+ 1$   
 $-----$   
 $01100$



Mealy  
 $10100$   
 $B B B A A$

Moore  
 $10100$   
 $B_0 B_1 B_1 A_0 A_0$

Q. Convert the given Mealy M/C to its equivalent moore m/c.

State	a	b
$\rightarrow q_0$	$q_3, 0$	$q_1, 1$
$q_1$	$q_0, 1$	$q_3, 0$
$q_2$	$q_2, 1$	$q_2, 0$
$q_3$	$q_1, 0$	$q_0, 1$

moore m/c.

State	a	b	output	$q_1$	$q_2$
$\rightarrow q_0$	$q_3$	$q_{11}$	1	$q_{10}$	$q_{21}$
$q_{10}$	$q_0$	$q_3$	0	$0/P$	$0/P$
$q_{11}$	$q_0$	$q_3$	1	$0/P$	$0/P$
$q_{20}$	$q_{21}$	$q_{20}$	0	$0/P$	$0/P$
$q_{21}$	$q_{21}$	$q_{20}$	1	$0/P$	$0/P$
$q_3$	$q_{10}$	$q_0$	0		

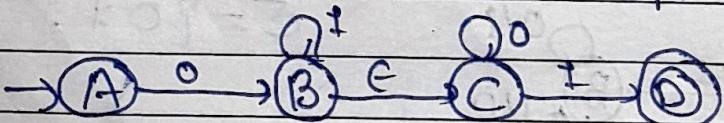
## Epsilon (ε) NFA

ε-NFA

↳ empty symbols.

5 tuples  $\{Q, \Sigma, q_0, S, F\}$

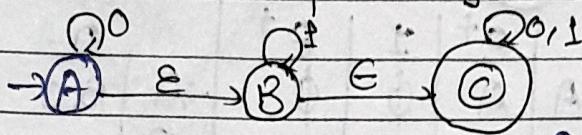
$$\delta \quad \delta : Q \times \Sigma \cup \epsilon \rightarrow 2^Q$$



Every state on G goes to itself.

## Conversion of G-NFA to NFA

Q Convert the following G-NFA to its equivalent NFA.



$\epsilon$ -closure ( $\epsilon^+$ ) - All the states

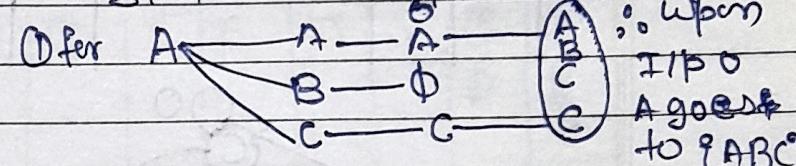
that can be reached from a particular state only by seeing the ' $\epsilon$ ' symbol.

use the following framework.

	0	1
(A)	{ABC}	{BC}
(B)	C	{BC}
(C)	C	C

State  $\epsilon^+$  Input  $\epsilon^*$ .

i) For  $C \rightarrow C - C - C$   
ii) For  $C \rightarrow C - \overset{1}{C} \rightarrow C$

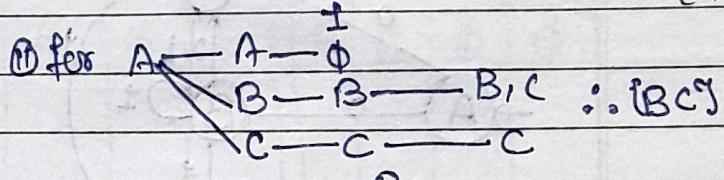


Final State:-

Any state that can reach the final state only by seeing  $\epsilon$

$\therefore ABC$  are final

state.



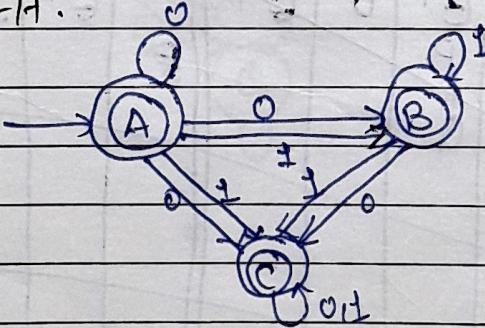
iii) For  $B \rightarrow B \rightarrow \emptyset$

$\rightarrow C \rightarrow C \rightarrow C \therefore C$

iv) For  $B \rightarrow B \rightarrow \overset{1}{B} \rightarrow B$

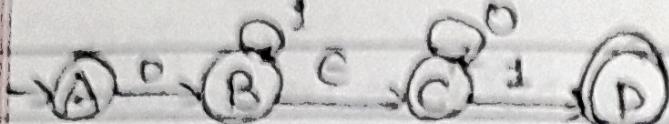
$\rightarrow C \rightarrow C \rightarrow C \therefore \{BC\}$

NFA :-

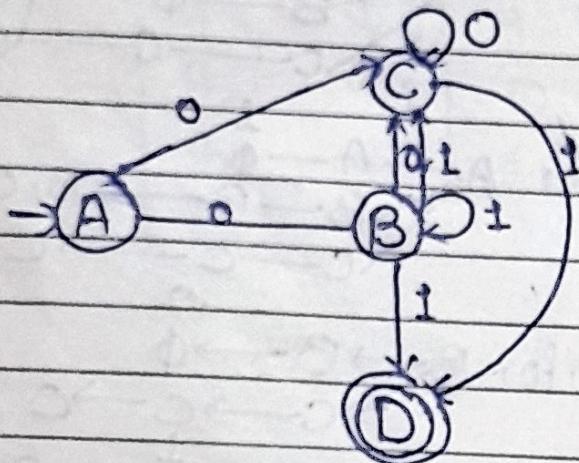


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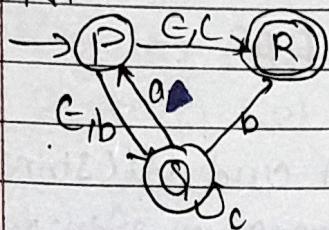
Q) Convert the following CNFA to its equivalent NFA.



	$\epsilon^+$	O	C	D		$\epsilon^+$	y	$\epsilon^+$	NFA
A	A	B	$B, \epsilon$	.	A	A	$\emptyset$	$\emptyset$	$\rightarrow A$   O $\epsilon$ 1
B	B	B	$\emptyset$	$\emptyset$	B	B	B	$\emptyset$	B   B, C $\emptyset$
C	C	C	C	C	C	D	$\emptyset$	C   C B, C, D	
D	C	C	C	C	C	D	D	D   C $\emptyset$	
	D	D	$\emptyset$	$\emptyset$	D	D	$\emptyset$	$\emptyset$	

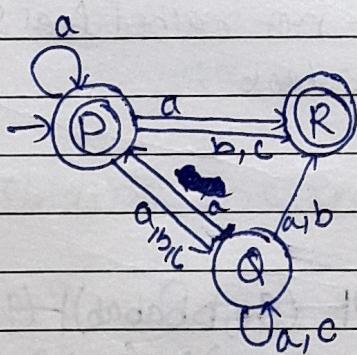


Convert the following  $\epsilon$ -NFA to its equivalent NFA.



	a	b	c
P	$\{PQR\}$	$\{QR\}$	$\{Q,R\}$
Q	$\{PQS\}$	$\{RS\}$	$\{Q\}$
R	$\{Q\}$	$\emptyset$	$\emptyset$
S	$\emptyset$	$\emptyset$	$\emptyset$

	$\epsilon^+$	a	$\epsilon^+$			$\epsilon^+$	b	$\epsilon^+$		$\epsilon^+$	c	$\epsilon^+$
P	P	$\emptyset$	P	P	$\emptyset$	P	$\emptyset$	P	$\emptyset$	R	R	
R	$\emptyset$	P	$\emptyset$	R	$\emptyset$	$\emptyset$	P	$\emptyset$	R	$\emptyset$	$\emptyset$	
Q	P	$\emptyset$	Q	Q	R	R	R	Q	Q	Q	Q	
S	$\emptyset$	R	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	



If we are reaching the final state from initial state that string is called ~~accepted string~~  
Accepted string.

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Some First topics

## Acceptability of a String by F.A

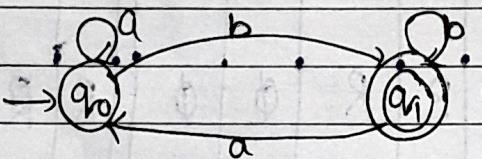
If we have a given F.A and W(string) we have to check whether the given string will be accepted by the mc or not by parsing the whole string.

$$M = \{Q, \Sigma, S, q_0, F\}$$

$\Sigma = \{q_0, w\} \quad | \quad \text{or more transition}$

$| \quad F$   
at last reaches final state

Ex:-



$$\textcircled{1} \quad w = aabbab$$

$q_0 q_0 q_1 q_1 q_0 F$

$\therefore$  String(w) will be accepted by the given mc final state

$$\textcircled{2} \quad w = aabbba$$

$\therefore w \text{ is not acceptable}$

$q_0 q_1 q_1 q_0$  as it not reached final state.

in the last

How to represent it.

$$w_3 = aa bba bab$$

$(q_0, aabbabab) \rightarrow (q_0, abbabab) \rightarrow (q_0, bbabab) \rightarrow (q_1, babab)$   
 $\rightarrow (q_1, abab) \rightarrow (q_0, bab) \rightarrow (q_1, ab) \rightarrow (q_0, b) \rightarrow (q_1) \in F$

$\rightarrow q_0 \xrightarrow{a} (q_0) \xrightarrow{a} (q_0) \xrightarrow{b} (q_1) \xrightarrow{b} (q_1) \xrightarrow{a} (q_0) \xrightarrow{b} (q_1) \xrightarrow{a} (q_0) \xrightarrow{b} (q_1)$  Accepted string

$\therefore w \text{ is accepted}$