SAAKAAR

FOR IIT JAM 2025

Lecture-01

Linear Algebra

Introduction to Vector Spaces

By-Sanjeev sir





ODICS to be covered

- **Motivation to Vector Spaces**
- What is a Field?
- Definition of Vector space





Motivation to Vector Spaces



$$\widehat{a} = a_1 \widehat{1} + a_2 \widehat{j}$$

$$\widehat{b}_1 \widehat{i} + b_2 \widehat{j}$$

Nector addition

$$0,\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

①
$$\vec{a}' + (\vec{b} + \vec{c}') = (\vec{a}' + \vec{b}) + \vec{c}'$$

There exist
$$\vec{o} = 0\hat{i} + 0\hat{j}$$

Such that $\vec{a}' + \vec{o}' = \vec{a}$

$$\vec{a}' + \vec{b}' = (q_1 + k_1) \hat{i} + (q_2 + k_3) \hat{j}$$

Scalar multiplication:
 \vec{q} $\vec{q} \in \mathbb{R}$, $\vec{q} = (xa_1) \hat{i} + (xa_2) \hat{j}$

(IV) for all verter
$$\vec{a} = a_1 \cdot 1 + a_2 \cdot \vec{\gamma}$$

there exist $(-\vec{a}) = (-a_1)(1 + 1 - a_2)$
 $\vec{a} + (-\vec{a}') = 0$ ($+ 0$) = $\vec{0}$

$$(VI) (\alpha + \beta) \vec{a}$$

$$= \alpha \vec{a} + \beta \vec{a}$$

$$(VII) (\alpha + \beta) \vec{a}$$

Linear Algebra
done right"

2. linear Algebra
By Friedberg

3 linear Algebra Vikas Bist vevek Sahan





What is a Field?



```
A non-empty Set F
    jogether with two binary operations
    + ( Called addition). ( Called Multiplication)
  is Souid to be a field of following conditions
   are satisfied
    ath = b + a for all 0, b ∈ F (Commutation of addition and Multiplication)
0) \quad a+b=b+a
```

Associativity g + and.

When $a,b,c \in F$ a+(b+c)=(a+b)+c

a (b() = (ab)c

Existence of addition

and multiplicative 2 dentisty

There exist DEF:

9+0=9 Vaef

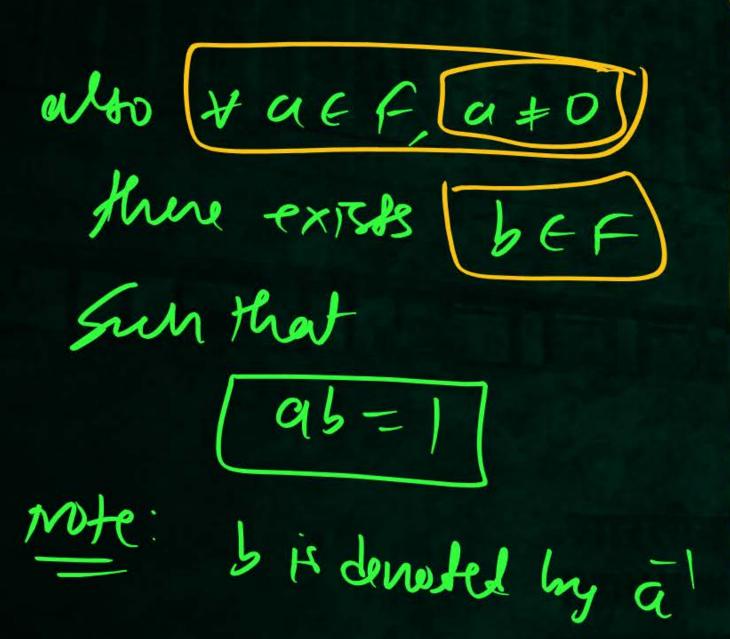
There exists IEF3

Existence of addition and multiplicative soverse.

for all at F there exists

be F such that

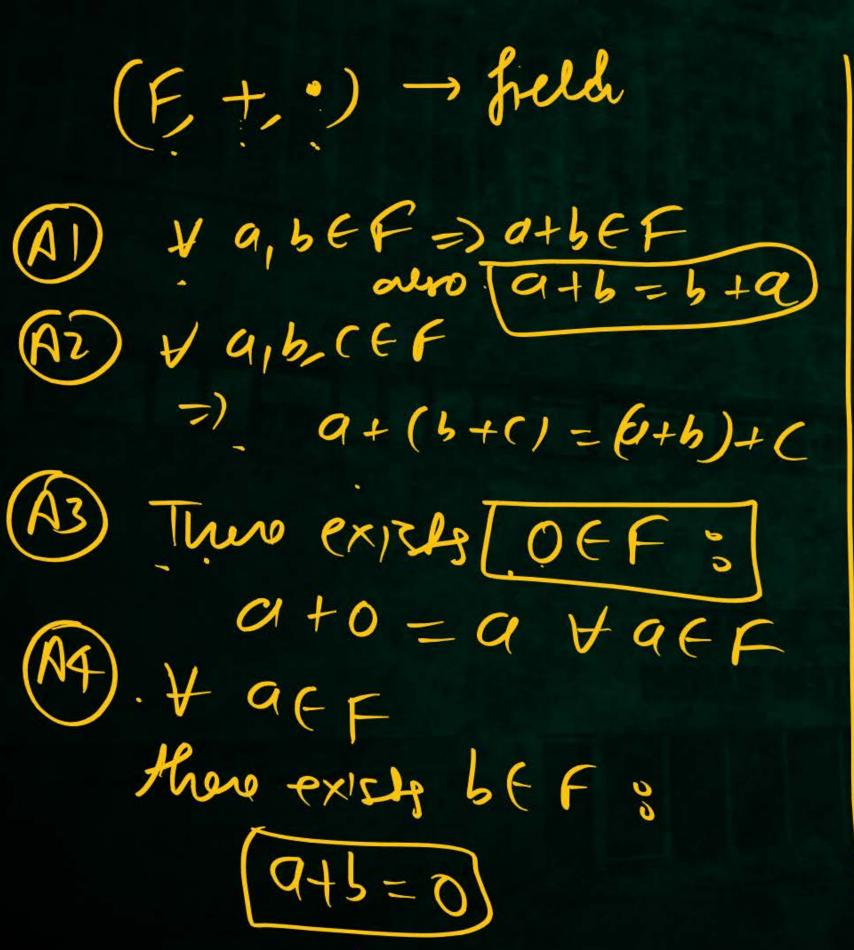
Note. 675 demoted by -a.





1 Distributivety

a(b+c)-ab+ac (a+b)c-ac+bc freedl a,bcef.



MI) tabefzabef alro ab - ba (M2) +a,b,cef -) a(bc) = (ab) c (M3) There exists IEF: 1-a = a + a E F (mf) tock, at 0, shere exists bef: (ab-1)



Distributivety

$$a(b+c) = ab+ac$$
 $(a+b)(-ac+bc)$
 $\forall a,b,c \in F$

(E. Q(5)-40+bs | 9,660) 答 (R,+,·)

J. is a field
$$x = a_1 + b_1 s_2, y = a_1 + b_2 s_2$$

So a field $x = a_1 + b_1 s_2, y = a_1 + b_2 s_2$

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 $x = a_1$

Mez (¢,+,·)

ti o field

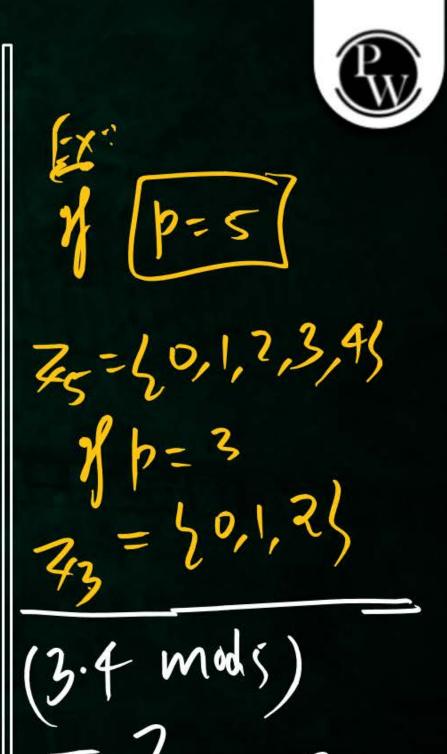




(N, +, ·) is not field Sinle 0¢ IN (7.1 has no additione Thursday) Ex (Z,+,) is not field for 2 = 2 = 0 But 26= = 1 = 1 = 1 = 1

Example of a finite field -: let p be prime. Depre Zp = {0,1,2,3,4,-,b-1} and Binary operations (1) a+b = least non-negative $(a+b) = (a+b) \mod b$ / When at b is divided by $(3.4 \mod 5)$ ab = least ven-vegative remainder when (3+4) and 2

ab = (ab) mod b is divided to b.



$$(3.4 \text{ mod s})$$

= 2
 $(3+4)$ mod s
 $(3+4)$ mod s



EX

$$(96) - (96) \text{ med } 3$$

| •. | 0 | | (2) |
|----|---|---|-----|
| 0. | 0 | D | 0 |
| ļ. | 0 | | 2 |
| 2: | 0 | 2 | |
| | | | |

2.2. mudz) = (7.2 mudz) = 4 mudz



fields:

$$EX in (R_{1}, +, ')$$



$$D(a_1+b_1i) + (a_2+b_2i)$$

$$= (a_1+a_2) + i(b_1+b_2)$$



2 Mins Summary



- 1 Motivation to Vector Spaces
- 2 What is a Field?



THANKYOU



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