GATE 2026

Algorithm’s Short Notes with PYQ’s and Test

With some Hints and Advices

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1. Short Notes

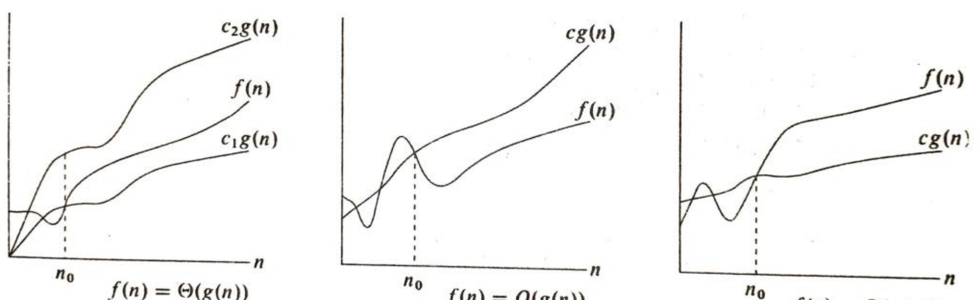
* Average Case deals with Probability
* In General, B.C $\leq$ A.C $\leq$ W.C
    - ↳ B.C < [A.C = W.C] → L.S , B.S
    - ↳ [B.C = A.C] < W.C → Q.S
    - ↳ [B.C = A.C = W.C] → M.S, H.S, Selection Sort

* $\Theta(g(n)) = \{f(n) \mid \exists$ positive constants c_1, c_2 and n_0 such that $0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \ \forall n \geq n_0\}$

* $O(g(n)) = \{f(n) \mid \exists$ positive constants c and n_0 such that $0 \leq f(n) \leq c \cdot g(n) \ \forall n \geq n_0\}$

* $\Omega(g(n)) = \{f(n) \mid \exists$ positive constants c and n_0 such that $0 \leq c \cdot g(n) \leq f(n) \ \forall n \geq n_0\}$

3.1 Asymptotic notation 45



$f(n) = \Theta(g(n))$ (a) $\quad f(n) = O(g(n))$ (b) $\quad f(n) = \Omega(g(n))$ (c)

Theorem :- For any two functions $f(n)$ & $g(n)$, we have $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

$o(g(n)) = \{f(n) :$ for any positive constant $c > 0$, there exists a constant $n_0 > 0$ such that $0 \le f(n) < cg(n)$ for all $n \ge n_0\}$.

For example, $2n = o(n^2)$, but $2n^2 \ne o(n^2)$.

The definitions of O-notation and o-notation are similar. The main difference is that in $f(n) = O(g(n))$, the bound $0 \le f(n) \le cg(n)$ holds for *some* constant $c > 0$, but in $f(n) = o(g(n))$, the bound $0 \le f(n) < cg(n)$ holds for *all* constants $c > 0$. Intuitively, in o-notation, the function $f(n)$ becomes insignificant relative to $g(n)$ as n approaches infinity; that is,

$$\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = 0$$

$\omega(g(n)) = \{f(n) :$ for any positive constant $c > 0$, there exists a constant $n_0 > 0$ such that $0 \le cg(n) < f(n)$ for all $n \ge n_0\}$.

For example, $n^2/2 = \omega(n)$, but $n^2/2 \ne \omega(n^2)$. The relation $f(n) = \omega(g(n))$ implies that

$$\lim_{n \to \infty} \left(\frac{f(n)}{g(n)} \right) = \infty$$

Comparing Functions

Transitivity:

$$\begin{aligned}
f(n) &= \Theta(g(n)) \text{ and } g(n) = \Theta(h(n)) \text{ imply } f(n) = \Theta(h(n)) , \\
f(n) &= O(g(n)) \text{ and } g(n) = O(h(n)) \text{ imply } f(n) = O(h(n)) , \\
f(n) &= \Omega(g(n)) \text{ and } g(n) = \Omega(h(n)) \text{ imply } f(n) = \Omega(h(n)) , \\
f(n) &= o(g(n)) \text{ and } g(n) = o(h(n)) \text{ imply } f(n) = o(h(n)) , \\
f(n) &= \omega(g(n)) \text{ and } g(n) = \omega(h(n)) \text{ imply } f(n) = \omega(h(n)) .
\end{aligned}$$

Reflexivity:

$$\begin{aligned}
f(n) &= \Theta(f(n)) , \\
f(n) &= O(f(n)) , \\
f(n) &= \Omega(f(n)) .
\end{aligned}$$

Symmetry:

$$f(n) = \Theta(g(n)) \text{ if and only if } g(n) = \Theta(f(n)) .$$

Transpose symmetry:

$$\begin{aligned}
f(n) &= O(g(n)) \text{ if and only if } g(n) = \Omega(f(n)) , \\
f(n) &= o(g(n)) \text{ if and only if } g(n) = \omega(f(n)) .
\end{aligned}$$

	Reflexive	Symmetric	Transitive	Transpose Symmetry
O	✓	✗	✓	✓
Ω	✓	✗	✓	✓
$\boldsymbol{\theta}$	✓	✓	✓	✗
o	✗	✗	✓	✓
$\boldsymbol{\omega}$	✗	✗	✓	✓

In group

In group

NOTE:- Transpose Symmetry is Important

Because these properties hold for asymptotic notations, we can draw an analogy between the asymptotic comparison of two functions f and g and the comparison of two real numbers a and b:

$$
\begin{array}{lll}
f(n) = O(g(n)) & \text{is like} & a \le b\,, \\
f(n) = \Omega(g(n)) & \text{is like} & a \ge b\,, \\
f(n) = \Theta(g(n)) & \text{is like} & a = b\,, \\
f(n) = o(g(n)) & \text{is like} & a < b\,, \\
f(n) = \omega(g(n)) & \text{is like} & a > b\,.
\end{array}
$$

We say that $f(n)$ is ***asymptotically smaller*** than $g(n)$ if $f(n) = o(g(n))$, and $f(n)$ is ***asymptotically larger*** than $g(n)$ if $f(n) = \omega(g(n))$.

One property of real numbers, however, does not carry over to asymptotic notation:

Trichotomy: For any two real numbers a and b, exactly one of the following must hold: $a < b$, $a = b$, or $a > b$.

Although any two real numbers can be compared, not all functions are asymptotically comparable. That is, for two functions $f(n)$ and $g(n)$, it may be the case that neither $f(n) = O(g(n))$ nor $f(n) = \Omega(g(n))$ holds. For example, we cannot compare the functions n and $n^{1+\sin n}$ using asymptotic notation, since the value of the exponent in $n^{1+\sin n}$ oscillates between 0 and 2, taking on all values in between.

- **Look at this question to understand Trichotomy Property:-** https://gateoverflow.in/8501/gate-cse-2015-set-3-question-42
- This Property has also been asked on IISC Interview

3.1-7] (page 53) Prove that $\boldsymbol{o(g(n)) \cap \omega(g(n))}$ is the empty set?

* See page 18 and 19 for mathematical properties and geometric sum formula

➢ See gate 2023 question **'∈'** is used instead of **'= '.**

Sorting

1. **Bubble Sort: -**
   - Logic: - In every i^{th} pass/iteration, place the i^{th} max element at its correct position
   - It is INPLACE
   - Stale (Yes/No) ___________
   - Every pass **(n-1) comparision**

	No. of Comparison	No. of Swaps
Best Case (Increasing Order)	$(n-1)(n-1)$ $= (n-1)^2$	$0 \rightarrow \Omega(n)$
Worst Case (Decreasing Order)	$(n-1)^2$	$\frac{n(n-1)}{2} \rightarrow O(n^2)$

$\Theta(n^2)$

Optimised Bubble Sort: - If there are no swaps in any of the pass, then the array has got sorted already

2. **Selection Sort: -**
   - **Steps**
     - In the i^{th} pass, find the index/position of the i^{th} smallest element.
     - Swap it with the element that is present in the i^{th} index.
     - Keep repeating until the array is sorted.
   - Inplace + Unstable
   - Always takes (n-1) passes

- Every pass exactly 1 swap, so in total (n-1) swaps to get sorted
- Among other Algorithm it takes least amount of swaps

3. **Insertion Sort: -**
    - **Idea** -> Take on element from sorted part and insert it to its correct position in sorted part.
    - **Insertion** -> It is achieved by swapping or updating the adjacent element
    - **Stable and Inplace**
    - **Best Case is when input is already sorted in required order. Every pass 1 Comparison and 0 Swaps**
    - There are always (n-1) passes always
    - Total (n-1) Comparison and 0 Swaps
    - Always takes (n-1) passes
    - T.C = O(n-1) = O(n)
    - **Worst Case is when input sorted in opposite order. T.C = $O(n^2)$**

4. **Radix Sort: -**
    - **No. of Passes = No. of digit in Max element of given input**
    - **T.C = O(d * (n+b)) = O(n*d),**
        - b = base of the given input
        - d = # digits in max element
        - n = # elements in given array
    - **Stable but not Inplace**

Divide and Conquer

Symmetric D & C :- $T(n) = a * T\left(\frac{n}{b}\right) + F(n)$, $a \geq 1$, $b > 1$

#Sub-Problem

Size of each Sub-Problem

$F(n) \rightarrow$ time Function

Asymmetric D & C :-

Form ① $T(n) = T(\alpha n) + T((1-\alpha)n) + g(n)$

Form ② $T(n) = T(\alpha n) + T(\beta n) + T(\delta n) + \ldots + g(n)$

Min-Max Divide and Conquer Algorithm

★ **Time Complexity analysis**

- **Best Case**:- Input in Decreasing Order (Only if condition get's executed
  - **No. Of Comparisons:- (n-1)**
- **Worst Case**:- Input is in Increasing order
  - **No. Of Comparisons:- 2*(n-1)**
- **Average Case:-** On Average, if condition get's executed half times

$$(n-1) + \frac{n}{2} = \frac{3n}{2} - 1$$

	If	Else if	Total Comparisons
Best Case	$(n-1)$	0	$(n-1)$
Worst Case	$(n-1)$	$(n-1)$	$2(n-1)$
Average Case	$(n-1)$	$n/2$	$3n/2 - 1$

$\rightarrow \Omega(n)$

$\rightarrow O(n)$

$\Big\} \; \theta(n)$

★ **Performance of Divide and Conquer based solution**

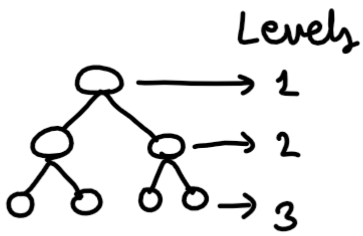
T(n) = 0 , n = 1

T(n) = 1 , n = 2

T(n) = $2T(n/2) + 2$, n > 2 $= O(n)$

Value of Recurrence $= \frac{3n}{2} - 2$

Binary Search



↳ Max^{m} # nodes at any level 'i' in Full Binary Tree $= 2^{(i-1)}$

↳ Range of Height of a Binary Tree

$$\boxed{\log_2 n \leq h \leq n}$$

where n = # nodes

* $T.C = O(\log_2 n)$

Merge Sort

★ ***Problem Statement:-*** Given two sorted list/array $L_1 \to n_1$ and $L_2 \to n_2$ such that $n_1 \le n_2$, Merge them into single sorted list/array.

* # Comparisions :- Minimum = $\min(n_1, n_2) = n_1$

  Maximum = $n_1 + n_2 - 1$

* T.C Conquer = $O(n)$

* T.C M.S = $2T(n/2) + n$
  $= \theta(n \log n)$

* S.C = Aux $[O(n)]$ + Stack $[O(\log n)]$
  $= O(n) \leftarrow$ Not Inplace

* It is Stable

Quick Sort

★ The running time of Quick Sort depends on whether the partitioning is balanced or unbalanced, which in turn depends on which elements are used for partitioning

Insertion Sort ≤ Quick Sort ≤ Merge Sort

★ W.C Partitioning -> T(n-1) + $\theta(n) = O(n^2)$

B.C Partitioning -> $2T\left(\frac{n}{2}\right) + \theta(n) = \theta(n * \log n)$

★ Running time of QS in W.C when

1. all the elements are same: $\underline{O(n^2)}$
2. Array contain distinct elements and is sorted in decreasing order: $\underline{O(n^2)}$
3. Only Middle elements as Pivot: $\underline{O(n^2)}$
4. Median is always selected as Pivot: $\underline{\theta(n * \log n)}$

Matrix Multiplication

★ Recurrence for Divide and Conquer based

T(n) = c ; n ≤ 2

T(n) = $8T\left(\frac{n}{2}\right) + b * n^2 ; n > 2, b > 2$

= $O(n^3)$

★ SC = $O(\log n)$

★ Recurrence for Stressen's Matrix Multiplication

T(n) = c ; n ≤ 2

T(n) = $7T\left(\frac{n}{2}\right) + b * n^2 ; n > 2, b > 2$

= $O(n^{2.81})$

★ SC = $O(n^2 + \log n) = O(n^2)$

Master's Theorem

★ Only applicable for:-

$$T(n) = a * T\left(\frac{n}{b}\right) + f(n)$$

Such the a ≥ 1, b > 1, $f(n)$ is a Positive function

★ Case 1:-
If f(n) is $O\left(n^{(\log_b a - \varepsilon)}\right)$ then

$\boxed{\text{T(n)} = \theta\left(n^{\log_b a}\right)}$

★ Case 2:-
If f(n) is $\theta\left(n^{\log_b a} * (\log n)^k\right)$ for some k, Such that
(a) k ≥ 0, then

$\boxed{\text{T(n)} = \theta\left(n^{\log_b a} * (\log n)^{(k+1)}\right)}$

(b) k = -1, then

$\boxed{\text{T(n)} = \theta\left(n^{\log_b a} * \log\log n\right)}$

★ Case 3:-
If f(n) is $\Omega\left(n^{(\log_b a + \varepsilon)}\right)$ for some $\varepsilon > 0$ AND
$a * f(n) \le \delta * f(n)$ for some $\delta < 1$ then

$\boxed{\text{T(n)} = \theta(f(n))}$

2. Revision Through PYQ's

48

Let $W(n)$ and $A(n)$ denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is **ALWAYS TRUE**?

A. $A(n) = \Omega(W(n))$
B. $A(n) = \Theta(W(n))$
C. $A(n) = O(W(n))$
D. $A(n) = o(W(n))$

2012

https://gateoverflow.in/50/gate-cse-2012-question-18

$B.C \leq AC \leq W.C$
$\quad\quad\quad A(n) \quad W(n)$

(a) $A(n) \geq W(n)$ ✗ (c) $A(n) \leq W(n)$ ✓
(b) $A(n) = W(n)$ ✗ (d) $A(n) < W(n)$ ✗

47

Consider the following function:

```
int unknown(int n){

int i, j, k=0;
for (i=n/2; i<=n; i++)
    for (j=2; j<=n; j=j*2)
        k = k + n/2;
 return (k);

}
```

2013

The return value of the function is

A. $\Theta(n^2)$
B. $\Theta(n^2 \log n)$
C. $\Theta(n^3)$
D. $\Theta(n^3 \log n)$

https://gateoverflow.in/1542/gate-cse-2013-question-31

$i = \frac{n}{2}$ times
$\hookrightarrow j = \log_2 n$ times
$K = \frac{n}{2}\left(\frac{n}{2} \log_2 n\right) = \frac{n^2}{4} \log_2 n = \Theta(n^2 \log n)$

$\therefore$ C is the correct answer

36

What is the number of swaps required to sort n elements using selection sort, in the worst case?

A. $\Theta(n)$

B. $\Theta(n \log n)$

C. $\Theta(n^2)$

D. $\Theta(n^2 \log n)$

2009

https://gateoverflow.in/1303/gate-cse-2009-question-11

- Find for several other sorting algorithm.

for Comparision based Sorting two factors play's the role.

1] # Swaps ⟶ for S.S → $(n-1)$ swaps

2] # Comparisions.

∴ a is the correct answer

54

Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001n^2$ time units and package B requires $10n \log_{10} n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

A. 12
B. 10
C. 6
D. 5

2010

https://gateoverflow.in/2185/gate-cse-2010-question-12

$10^k \rightarrow$ Records

$t(A) = 0.0001n^2$ $\quad t(B) = 10n\log_{10} n$

for, n =

$10 \rightarrow A = 0.01$, $B = 100$	$100 \rightarrow A = 1$, $B = 200$
$\underline{10^3} \rightarrow A = 100$, $B = 30000$	$10^4 \rightarrow A = 10000$, $B = 4*10^5$
$10^5 \rightarrow A = 10^6$, $B = 5*10^6$	$10^6 \rightarrow A = 10^8$, $B = 6*10^7$

$\therefore$ $\underline{C}$ is the Correct answer

<u>Sir</u>

$$\left.\begin{aligned} 10\cancel{n}\log_{10} n &< 10^{-4} n^{\cancel{2}} \\ \Rightarrow 10\log_{10} n &< 10^{-4} n \end{aligned}\right] \text{Take Care}$$

⋮

$$\Rightarrow K * 10 < 10^{(k-4)}$$

* Look options :- $\underbrace{12, 10, 6, 5}$

→ Check only these

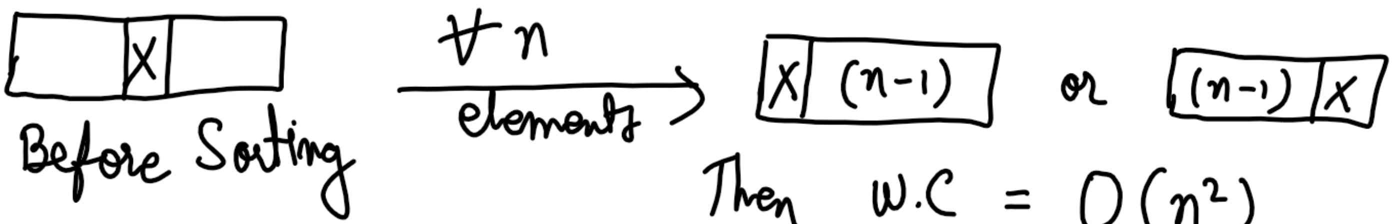
75

You have an array of n elements. Suppose you implement quicksort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst case performance is

A. $O(n^2)$
B. $O(n \log n)$
C. $\Theta(n \log n)$
D. $O(n^3)$

2014(3)

https://gateoverflow.in/2048/gate-cse-2014-set-3-question-14



Then W.C = $O(n^2)$

$\therefore$ a is the correct answer

25

Consider the following array.

23	32	45	69	72	73	89	97

Which algorithm out of the following options uses the least number of comparisons (among the array elements) to sort the above array in ascending order?

A. Selection sort
B. Mergesort
C. Insertion sort
D. Quicksort using the last element as pivot

2021(1)

https://gateoverflow.in/357443/gate-cse-2021-set-1-question-9

- Table of No. of Comparisons of all sorting algorithm

Already sorted

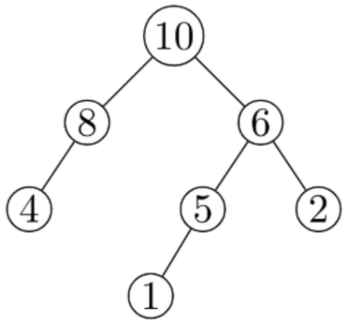
(a) $O(n^2)$ so # Comparisions always (d) $O(n^2)$ [W.C]

(b) $\Theta(n \log n)$ always (c) $O(n-1) \approx O(n)$

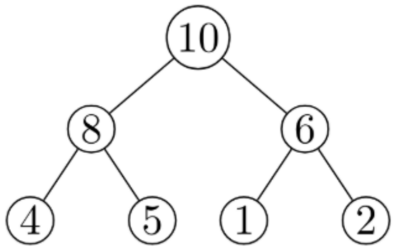
$\therefore$ c is the correct answer.

24 A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?

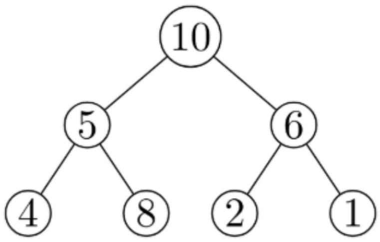
2011



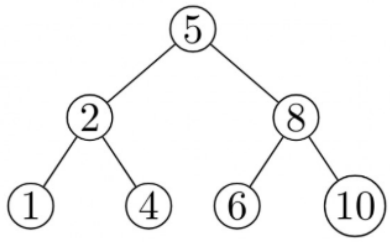
A.



B.



C.



D.

https://gateoverflow.in/2125/gate-cse-2011-question-23

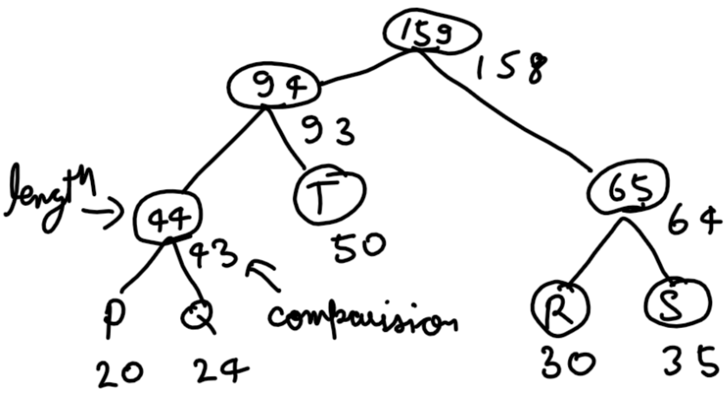
- (a) is not a CBT
- Hence (b) is the correct answer

107 Suppose P, Q, R, S, T are sorted sequences having lengths $20, 24, 30, 35, 50$ respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ____.

2014(2)

https://gateoverflow.in/1997/gate-cse-2014-set-2-question-38

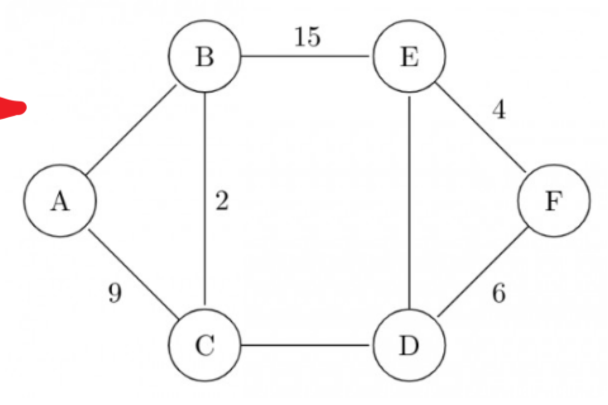
- Pick them in **<u>SHORTED ORDER ONLY</u>**



Total # Comp $= 158 + 93 + 64 + 43$

$\boxed{= 358}$

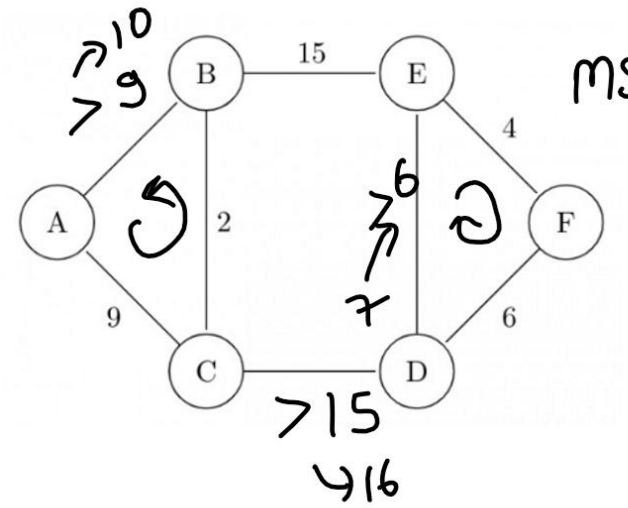
77 The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (**MST**) is of weight 36 and contains the edges: $\{(A, C), (B, C), (B, E), (E, F), (D, F)\}$. The edge weights of only those edges which are in the **MST** are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is_______________.



2015(1)

https://gateoverflow.in/8313/gate-cse-2015-set-1-question-43

- **Read Information Carefully**
  - **Distinct integer edge weights:- earlier my answer was 66(9 + 15 + 6)**



MST Cost = 36

Min^m Possible Sum = 36 + 10 + 16 + 7

= 69

30 Consider a max heap, represented by the array: $40, 30, 20, 10, 15, 16, 17, 8, 4$.

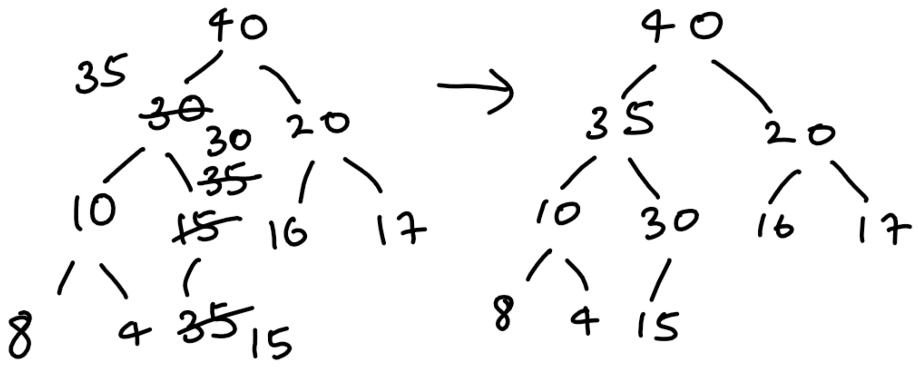
2015(1)

Array index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

A. $40, 30, 20, 10, 15, 16, 17, 8, 4, 35$
B. $40, 35, 20, 10, 30, 16, 17, 8, 4, 15$
C. $40, 30, 20, 10, 35, 16, 17, 8, 4, 15$
D. $40, 35, 20, 10, 15, 16, 17, 8, 4, 30$

https://gateoverflow.in/8273/gate-cse-2015-set-1-question-32



↓ Storage in array

40, 35, 20, 10, 30, 16, 17, 8, 4, 15

∴ b is the correct answer

73

Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?

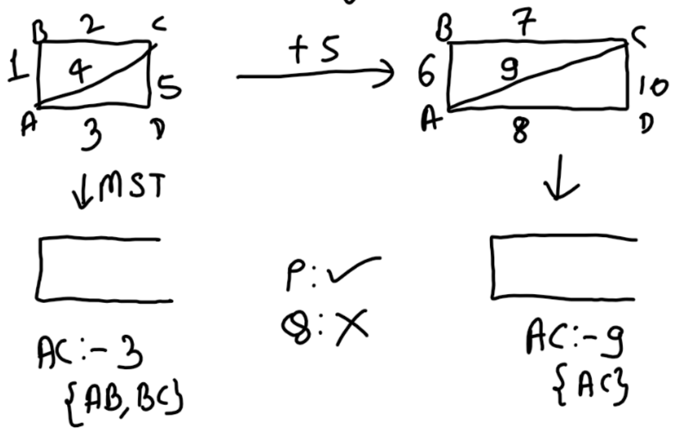
- P: Minimum spanning tree of G does not change.
- Q: Shortest path between any pair of vertices does not change.

A. P only
B. Q only
C. Neither P nor Q
D. Both P and Q

2016(1)

https://gateoverflow.in/39673/gate-cse-2016-set-1-question-14

* Distinct Positive Edge Weights
↳ Increased by same value | TRUE



- **Tricks:**
  - Come up with ONE example to contradict this fact

51

Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ______.

2015 (3)

https://gateoverflow.in/8499/gate-cse-2015-set-3-question-40

100 V & 300 E | MST Cost = 500

To connect 100 V ⟶ 99 edges needed

↓ Each edge cost ↑es by 5

$\therefore$ New MST Cost $= 500 + 99 * 5$

$= 500 + 495$

$= 995$

23

Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H?

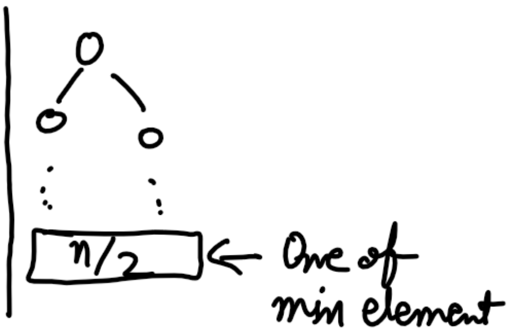
A. $\Theta(1)$
B. $\Theta(\log n)$
C. $\Theta(n)$
D. $\Theta(n \log n)$

2021 (2)

https://gateoverflow.in/357538/gate-cse-2021-set-2-question-2

* Min - Heap

* W.C T.C of Optimal Algo to find element H (min^{m} element) = ?



$\therefore$ W.C T.C $= \Theta(n)$

$\therefore$ <u>C</u> is the correct answer

22

Consider the string abbccddeee. Each letter in the string must be assigned a binary code satisfying the following properties:

1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

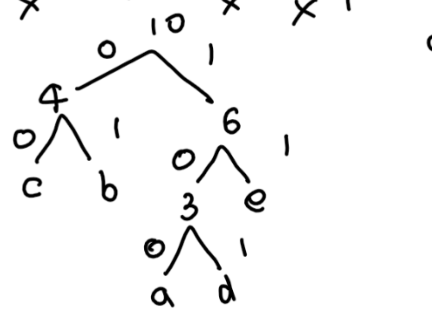
Among the set of all binary code assignments which satisfy the above two properties, what is the minimum length of the encoded string?

2021 (2)

A. 21
B. 23
C. 25
D. 30

https://gateoverflow.in/357514/gate-cse-2021-set-2-question-26

a	b	c	d	e			
1	2	2	2	3	3	4	6
x			x	x			



a → 100
b → 01
c → 00
d → 101
e → 11

abbccddeee → 3+4+4+6+6
= 23

∴ b is the correct answer

- Some more question to practice
- Take care of the frequencies.

39

Suppose the letters a, b, c, d, e, f have probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}$, respectively.

What is the average length of the Huffman code for the letters a, b, c, d, e, f?

2007

A. 3
B. 2.1875
C. 2.25
D. 1.9375

https://gateoverflow.in/43513/gate-cse-2007-question-77

49

A message is made up entirely of characters from the set $X = \{P, Q, R, S, T\}$. The table of probabilities for each of the characters is shown below:

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

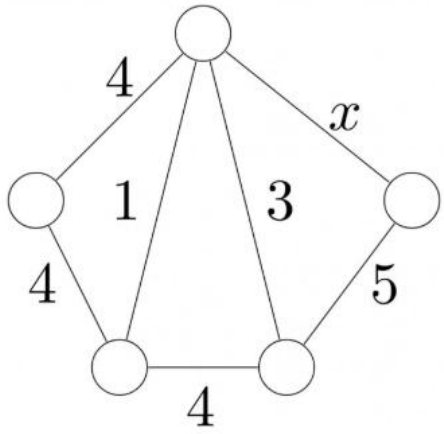
2017(2)

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is ______.

https://gateoverflow.in/118395/gate-cse-2017-set-2-question-50

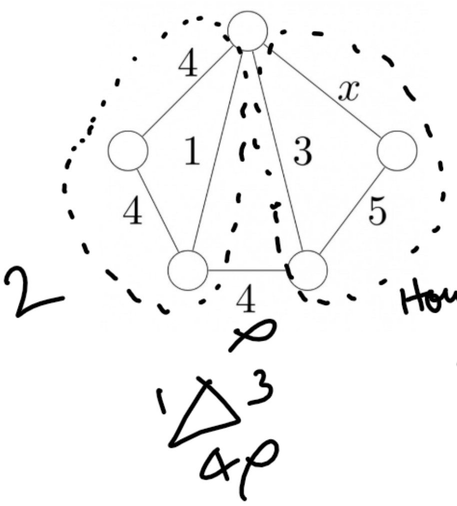
46

Consider the following undirected graph G:



2018

Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is ____.



$\therefore$ Total # MWST's Possible = 4

How to Create two Sub-MST

for $\rightarrow x = 3$ (only 1)

$\rightarrow x = 5$

48

Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item number	Weight (in Kgs)	Value (in rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}.

2018

The value of $V_{opt} - V_{greedy}$ is ____

https://gateoverflow.in/204123/gate-cse-2018-question-48

Item number	Weight (in Kgs)	Value (in rupees)	V/W	
1	10	60	6	2
2	7	28	4	4
3	4	20	5	3
4	2	24	12	1

$m = 11$

OPT	Greedy
Highest Value first ↓	Highest V/W first ↓
$V_{OPT} = 60$	4 $2 < 11$
	1 $10 > 9$ ✗
	3 $4 < 9$ ✓

$\cancel{11}\ \cancel{9}\ 5$

$V_G = 24 + 20 = 44$

2 $7 > 5$ ✗

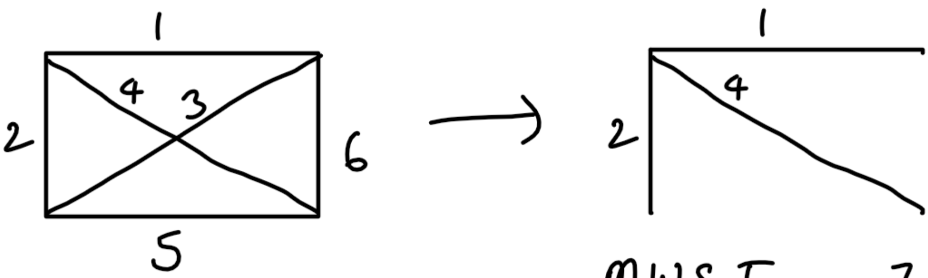
$\therefore \boxed{V_{OPT} - V_{greedy} = 60 - 44 = 16}$

95

Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being $1, 2, 3, 4, 5,$ and 6. The maximum possible weight that a minimum weight spanning tree of G can have is __________

2016 (1)

https://gateoverflow.in/39725/gate-cse-2016-set-1-question-39



MWST = 7

- Minimum Cost Spanning Tree is always UNIQUE iff all the edge weight are DISTINCE
- While constructing Minimum Cost Spanning Tree if a cycle get's formed then we remove the heaviest cost edge of that cycle.

35

Consider the following array of elements.

$\langle 89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100 \rangle$

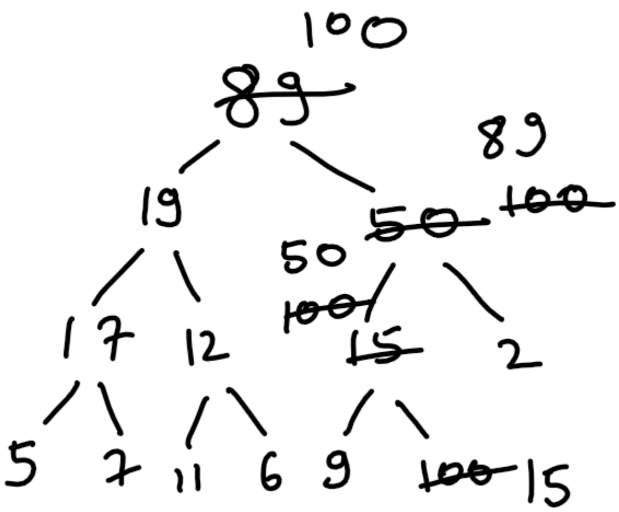
The minimum number of interchanges needed to convert it into a max-heap is

A. 4
B. 5
C. 2
D. 3

2015(3)

https://gateoverflow.in/8418/gate-cse-2015-set-3-question-19

https://gateoverflow.in/2740/gate-cse-1996-question-2-11



$15 \longleftrightarrow 100$ (1)

$100 \longleftrightarrow 50$ (2)

$89 \longleftrightarrow 100$ (3)

∴ d is the correct answer

70

Assume that a mergesort algorithm in the worst case takes 30 seconds for an input of size 64. Which of the following most closely approximates the maximum input size of a problem that can be solved in 6 minutes?

A. 256
B. 512
C. 1024
D. 2018

2015(3)

https://gateoverflow.in/8480/gate-cse-2015-set-3-question-27

$n = 64$ | Merge Sort $= c \cdot n \log n$

As

$30 \text{ sec} = c \cdot 64 \log_2 2^6$

$\Rightarrow c = \frac{\cancel{30}\ 5}{16 \times 4 \times \cancel{6}} = \frac{5}{64}$

for 6 min (360 sec), $n = ?$

$\cancel{360}^{72} = \frac{\cancel{5}}{64} n \log n$

$\Rightarrow n \log n = 9 \times 8 \times 64$

$\Rightarrow n \log n = 512 * 9$

$\Rightarrow n \log n = 512 * \log_2 2^9$

$\therefore$ B is the correct answer.

38 Four Matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is $pqr + prs + pst$.

If $p = 10, q = 100, r = 20, s = 5$ and $t = 80$, then the minimum number of scalar multiplications needed is

A. 248000
B. 44000
C. 19000
D. 25000

2011

https://gateoverflow.in/2140/gate-cse-2011-question-38

ABCD

$[(AB)C]D$	$[(AB)(CD)]$	$[A(BC)]D$
$(AB)_{p \times r} = pqr = 20000$	$AB = 20000$	$BC = qrs = 10,000$
$+$ $[(AB)C]_{p \times s} = prs = 1000$	$CD_{r \times t} = rst = 8000$	$ABC = pqs = 5,000$
$(ABCD)_{p \times t} = pst = 4000$	$(ABCD) = prt = 16000$	$ABCD = pst = 4,000$
Total $= 25,000$	Total $= 44,000$	Total $= 19,000$

$\therefore$ C is the correct answer

- Some of the best solutions are:-
  - https://gateoverflow.in/2140/gate-cse-2011-question-38?show=6997#a6997
  - https://gateoverflow.in/2140/gate-cse-2011-question-38?show=344541#a344541

63

Which one of the following statements is TRUE for all positive functions $f(n)$?

A. $f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial

B. $f(n^2) = o(f(n)^2)$

C. $f(n^2) = O(f(n)^2)$, when $f(n)$ is an exponential function

D. $f(n^2) = \Omega(f(n)^2)$

2022

https://gateoverflow.in/371935/gate-cse-2022-question-1

Soln:- https://gateoverflow.in/371935/gate-cse-2022-question-1?show=371951#a371951

for A

$f(n) = n^2$

$f(n^2) = n^4$

$\underbrace{f(n)^2 = n^4}$

$f(n^2) = \theta(f(n)^2)$

True

for C

$f(n) = 2^n$

$f(n^2) = 2^{n^2}$

$\underbrace{f(n)^2 = 2^{2n}}$

$f(n^2) \neq O(f(n)^2)$

False

for B & D

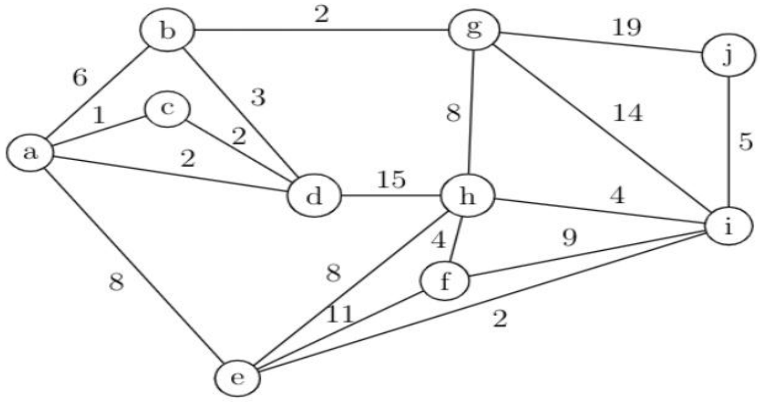
The two will fail because these completely depends on $f(n)$

Revision Through PYQ Link :- https://www.youtube.com/live/2kD0FxX_vD8

3. Some More PYQ's

32

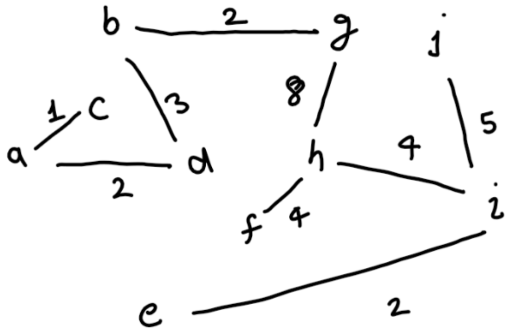
What is the weight of a minimum spanning tree of the following graph?



2003

A. 29
B. 31
C. 38
D. 41

https://gateoverflow.in/955/gate-cse-2003-question-68

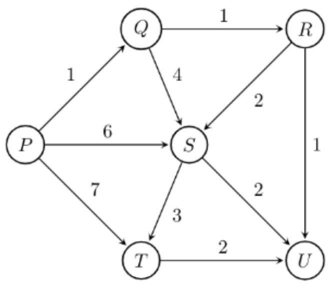


Total Cost = 1+2+3+
2+4+4+
5+2+8
= 31

∴ B is the correct answer

42

Suppose we run Dijkstra's single source shortest path algorithm on the following edge-weighted directed graph with vertex P as the source.



2004

In what order do the nodes get included into the set of vertices for which the shortest path distances are finalized?

A. P, Q, R, S, T, U
B. P, Q, R, U, S, T
C. P, Q, R, U, T, S
D. P, Q, T, R, U, S

https://gateoverflow.in/1041/gate-cse-2004-question-44

	Q	R	S	T	U
P	1	∞	6	7	∞
P, Q	1	2_Q	4_R	7	3_R
P, Q, R	1	2	4_R	7	3_R
P, Q, R, U	1	2	4_R	7	3
P, Q, R, U, S	1	2	4	7	3
P, Q, R, U, S, T	1	2	4	7	3

$P \to Q \to R \to U \to S \to T$

$\therefore$ B is the correct answer

75

Let $f(n) = n$ and $g(n) = n^{(1+\sin\, n)}$, where n is a positive integer. Which of the following statements is/are correct?

I. $f(n) = O(g(n))$
II. $f(n) = \Omega(g(n))$

A. Only I
B. Only II
C. Both I and II
D. Neither I nor II

2015 (3)