SAKAR

FOR IIT JAM 2025

Lecture-03

Linear Algebra

Examples of Vector Spaces, and Subspaces

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RECCIO of previous lecture

- Definition of Vector space
- 2 Examples of vector spaces





TODICS to be covered

- Examples of vector spaces
- Properties of vector spaces
- Subspace
- **Properties of Subspace**

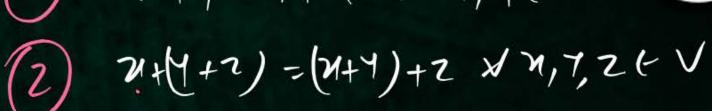


V(F) is a vector space

- $\rightarrow V \neq \phi$ - F K o field
- -> There exist
 - 1 veiter addition

2 SCalar multiplication Y YEF, XEV, YXEVI unique)

(1) 7+7 = 7+2 × 7,7+1 (1)



(3) There exist 0.6 V : 21+0= XXXXV

XXFV BYEV: X+Y - 0

(S) d(n+1)-dn+14 Agef AnileA

を+B)カーイオ+BNYJBEFVNEV

(dB) 7 = d(13x) Yd, BEF X4FV

I.Y = X YXEV.

Properties of vector spales: ly V(F) be a V.5.

(1)
$$Q(x-y) = dx - dy + diff$$

(2) $(q-13)x = dx - ix$
 $y = dx - ix$





Subspaces



let V(F) be any vector Space , and when men-empty subsed of V then W is Said to be a subspace of V OFW (OF O Vertor) (II) + x, y (-W -> 21-y (-W) (III) Hat F, Hretw =) artw

V={(a,a)|a,,2+1R} K. Ve vor addition (a, a)+(b, b)=(a+b), SCalar mulhiplication d (a, a) = (da, da) V(F) is a rector space 0=(0,0)

 $W_1 = \frac{1}{2}(a_1, a_1) | a_1 - a_1$ W1 = {(K,K) | KE|R) / (D) 0 - (0,0) EW, // (2+62) (1) 21 = (K,K), Y=(K,K) 21-4-(K-K), K-K) (W) / (III) LER, REW 27 = 4(1C,K1=(4K,4K)(W Wijk a Ghabau y.

Result -I let V(F) be a v-s and \$p\$ WCV. Then wis a subspace of (=) W(F) is a vector space W. 8-t vector addition and S(alar mulh) literation that whole U(f) a voctor space.

See all proofs in Friedborg



Subspace Test-I

All V(F) be a vector space, and $\phi + W \subseteq V$, then W is a subspace of $V \in V$

(1) Y M, Y EW =) R+ Y EW (1) Y MEF Y X EW =) MX EW



Subspace Fest-I

lid V(F) be a vector spall , and \$ \pm \cup V, then Wis a subspace of V E) Hd, BEF HU, YEW 一) インナドタール

Subfield ->:

let (F,+,) be any field a non- Empty subset K of Fis Soud to be a Subfield of F 9 () + 9,5 (K =) Q-6 (K (1) ¥ 9, 6 ∈ K, 6 ≠ 0 =) a 6 ∈ K

((() - { a + 1 > 5 | a + 5 & a | Co ソークーー(2) はりんしの (1)21-7-(4-4)+(b,-1) & EQ((2) (1) Y+0 =) 9x+0 or b2 +0

2xy'= 2xy = (ax+bx(2) x 3xbx(2)

(ax+bx(2) x 3xbx(2)

 $\frac{(a_1a_2-b_1b_1\cdot \iota)+(\iota(a_2b_1-a_1b_1)}{(a_1(\iota))(a_2)}$



(R) Q((2) Subfields of ¢ Q is subfiell of R Q à suffell of Q(s) O(i) is suffer of t Q & Edfield & Q(1) a(1/2) Hi a subfield of R.

Q(i)-- {a+bi | a, b ∈ Q } D(a+ib) +((+id) = (a+()+(b+d) ((+id) ((+id) - (ac-bd)+i(ad+bc)

FACT -:

Every field is a subfield of 1+suf.



Example -

of is a field , and K is a subfield of F then F(K) is a vector space R(Q) is vector space $\mathbb{R}(q(r_2))$, 9(E)(Q) ,,,

(Pw)

```
C(IR) is a vertor space
¢(Q) / / //
¢(Q(1/2)) ////
¢(0(i)) / / /-/
R(R) - -,
¢(¢) / //
 Q (Q)
Q(I_2)(Q(I_2))
```



Space of n Tuples



Let
$$F$$
 be any field, and K be a subfield of F .

Define $V = F^n = \{(a_1, a_2, a_3, -, a_n) \mid a_1, a_2, -, a_n \in F\}$.

Then $V(K)$ is a vector $S \mid 2a(k)$.

We then addition $(a_1, a_2, a_3, -, a_n) + (b_1, b_2, -, b_n) = (a_1 + b_1, a_2 + b_2, -, a_1 + b_n)$.

Zero-vector $S = \{(a_1, a_2, a_3, -, a_n) + (b_1, b_2, -, b_n) = (a_1 + b_1, a_2 + b_2, -, a_1 + b_n)$.

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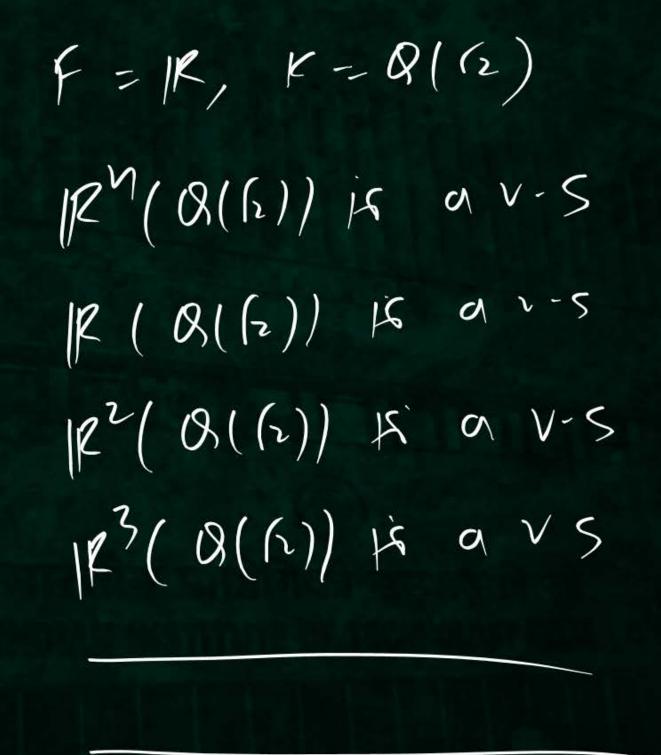
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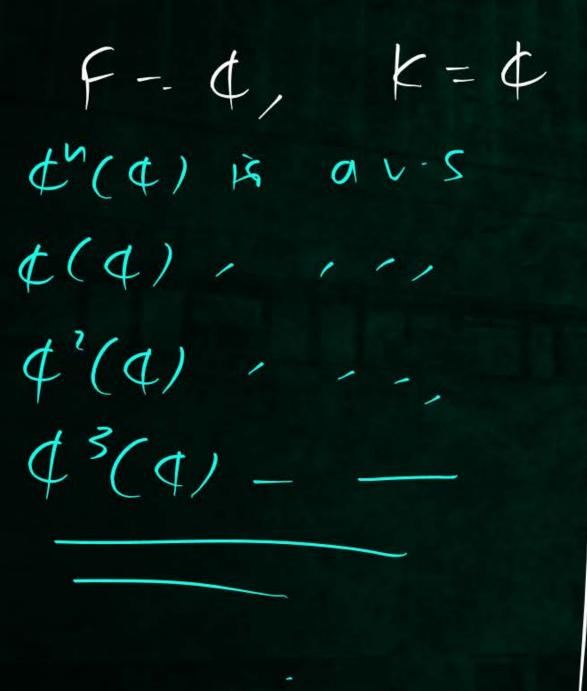
of Fix a field

K is subfield of F then F(K) is a vertor space F=IR, K=IR IRM(IR) is a vertor space |R?(|R), -R3(1R) -

1R"-14 (a, a, - an) | a, a, - ant | R) R2=4(a,2)|a, ack) 123-4(a, 4, 23) | a, a, a, a, e, R







F-4, K-- |R Homy - work 4=4, K-Q

Heme-work





Space of Matrices



Space of Matrices

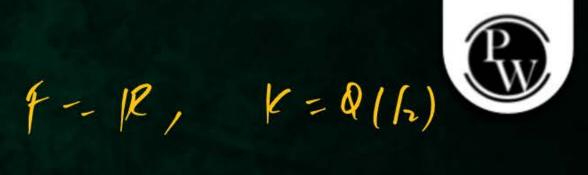
A F be any field, and K be a subfield of F.

for
$$m, n \in \mathbb{N}$$
, define $V = \{A = (ais)_{m \times n} | ais \in F \}$

Then V is a vector space over
$$K$$
 w x : f is least addition $(a_{1})_{m \times n} + (b_{1})_{m \times n} = (a_{1})_{m \times n}$ f is f in f

$$F = |R|, |K = |R|$$
 $|R|^{2\times3}(|R|)$ is a $V.S$
 $|R^{2\times3}(|R|)$ is a $V.S$
 $|R^{2\times3} = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{21} & a_{31} \end{pmatrix} \middle| \begin{array}{c} a_{31} \in |R| \\ 1 = 1.2 \\ 7 = 1.1.5 \end{array} \right\}$
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$$F-|R, K=Q$$



f = 4, K = t $t^{\text{mxn}}(t)$ is a v - s $t^{\text{nxn}}(t)$ is a v - s

F-4, K-1R T m x n (1R) is a v-s

Inxu(R) is a vs

than (Q(i)) is av-s



2 Mins Summary



- 1 Examples of vector spaces
- 2 Properties of vector spaces
- 3 Subspace
- 4 Properties of Subspace



THANKYOU



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