

GATE

Linear Algebra

PYS's and Solution

VARUN KUMAR
7-31-2025

Table of Contents

CSE	3
2025	3
2024	4
2023	6
2022	7
2021, S-1	9
2021, S-2	10
2020	10
2019	10
2018	11
ECE	12
2025	12
2024	12
2023	13
2022	14
2021	15
2020	15
2019	16
2018	16
EEE	17
2025	17
2024	18
2023	19
2022	19
2021	20
2020	21
2019	21
2018	22
Civil	23
2025	23
2024	25
2023	26
2022	28
2021	29
2020	31

2019	32
2018	32
Mechanical	34
2025	34
Instrumentation Engineering	35
2025	35
Data Science and Artificial Intelligence	36
2025	36

CSE

2025

S1

Q. Consider the given system of linear equations for variables x and y , where k is a real-valued constant. Which of the following option(s) is/are CORRECT?

$$x + ky = 1$$

$$kx + y = -1$$

- There is exactly one value of k for which the above system of equations has no solution.
- There exist an infinite number of values of k for which the system of equations has no solution.
- There exists exactly one value of k for which the system of equations has exactly one solution.
- There exists exactly one value of k for which the system of equations has an infinite number of solutions.

ANS: - a, d

Q. Let A be a 2×2 matrix as given.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

What are the eigenvalues of the matrix A^{13} ?

- 1, -1
- $2\sqrt{2}, -2\sqrt{2}$
- $4\sqrt{2}, -4\sqrt{2}$
- $64\sqrt{2}, -64\sqrt{2}$

ANS: - d

S2

Q. If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ then which ONE of the following is A^8 ?

- | | |
|---|---|
| a. $\begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}$ | c. $\begin{pmatrix} 625 & 0 \\ 0 & 625 \end{pmatrix}$ |
| b. $\begin{pmatrix} 125 & 0 \\ 0 & 125 \end{pmatrix}$ | d. $\begin{pmatrix} 3125 & 0 \\ 0 & 3125 \end{pmatrix}$ |

ANS: - c

Q. Let L , M , and N be non-singular matrices of order 3 satisfying the equations

$$L^2 = L^{-1}, M = L^8, \quad \text{and } N = L^2$$

Which ONE of the following is the value of the determinant of $(M - N)$?

- a. 0
- b. 1
- c. 2
- d. 3

ANS: - a

Q. Consider a system of linear equations $PX = Q$ where $P \in \mathbb{R}^{3 \times 3}$ and $Q \in \mathbb{R}^{3 \times 1}$. Suppose P has an LU decomposition, $P = LU$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Which of the following statement(s) is/are TRUE?

- a. The system $PX = Q$ can be solved by first solving $LY = Q$ and then $UX = Y$.
- b. If P is invertible, then both L and U are invertible
- c. If P is singular, then at least one of the diagonal elements of U is zero.
- d. If P is symmetric, then both L and U are symmetric.

ANS: - a, b, c

2024

Q.12 The product of all eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

- (A) -1
 - (B) 0
 - (C) 1
 - (D) 2
-

Q.49 Let A be any $n \times m$ matrix, where $m > n$. Which of the following statements is/are TRUE about the system of linear equations $Ax = \mathbf{0}$?

- (A) There exist at least $m - n$ linearly independent solutions to this system
 - (B) There exist $m - n$ linearly independent vectors such that every solution is a linear combination of these vectors
 - (C) There exists a non-zero solution in which at least $m - n$ variables are 0
 - (D) There exists a solution in which at least n variables are non-zero
-

S2

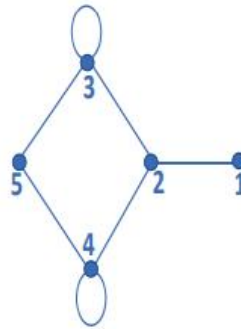
Q.47 Let A be an $n \times n$ matrix over the set of all real numbers \mathbb{R} . Let B be a matrix obtained from A by swapping two rows. Which of the following statements is/are TRUE?

- (A) The determinant of B is the negative of the determinant of A
 - (B) If A is invertible, then B is also invertible
 - (C) If A is symmetric, then B is also symmetric
 - (D) If the trace of A is zero, then the trace of B is also zero
-

2023

Q.18	<p>Let</p> $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ <p>and</p> $B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}.$ <p>Let $\det(A)$ and $\det(B)$ denote the determinants of the matrices A and B, respectively.</p> <p>Which one of the options given below is TRUE?</p>
(A)	$\det(A) = \det(B)$
(B)	$\det(B) = -\det(A)$
(C)	$\det(A) = 0$
(D)	$\det(AB) = \det(A) + \det(B)$

Q.30 Let A be the adjacency matrix of the graph with vertices $\{1, 2, 3, 4, 5\}$.



Let $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5 be the five eigenvalues of A . Note that these eigenvalues need not be distinct.

The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$ _____.

2022

Q.20 Consider the following two statements with respect to the matrices $A_{m \times n}$, $B_{n \times m}$, $C_{n \times n}$ and $D_{n \times n}$.

Statement 1: $tr(AB) = tr(BA)$

Statement 2: $tr(CD) = tr(DC)$

where $tr()$ represents the trace of a matrix. Which one of the following holds?

(A) Statement 1 is correct and Statement 2 is wrong.

(B) Statement 1 is wrong and Statement 2 is correct.

(C) Both Statement 1 and Statement 2 are correct.

(D) Both Statement 1 and Statement 2 are wrong.

Q.37	Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of
(A)	A^3
(B)	A^3 divided by 2
(C)	A^3 divided by 3
(D)	A^3 divided by 6

Q.45	<p>Consider solving the following system of simultaneous equations using LU decomposition.</p> $\begin{aligned}x_1 + x_2 - 2x_3 &= 4 \\x_1 + 3x_2 - x_3 &= 7 \\2x_1 + x_2 - 5x_3 &= 7\end{aligned}$ <p>where L and U are denoted as</p> $L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$ <p>Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?</p>
(A)	$L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$
(B)	$L_{32} = 2, U_{33} = 2, x_1 = -1$
(C)	$L_{32} = -\frac{1}{2}, U_{33} = 2, x_1 = 0$
(D)	$L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$

Q.53 Which of the following is/are the eigenvector(s) for the matrix given below?

$$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$

(A) $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

(C) $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$

(D) $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

2021, S-1

Q.52 Consider the following matrix.

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The largest eigenvalue of the above matrix is _____.

ANS: - 3

2021, S-2

Q.24 Suppose that P is a 4×5 matrix such that every solution of the equation $P\mathbf{x} = \mathbf{0}$ is a scalar multiple of $[2 \ 5 \ 4 \ 3 \ 1]^T$. The rank of P is _____.

ANS: - 4

2020

Q.No. 27 Let A and B be two $n \times n$ matrices over real numbers. Let $\text{rank}(M)$ and $\det(M)$ denote the rank and determinant of a matrix M , respectively. Consider the following statements.

- I. $\text{rank}(AB) = \text{rank}(A) \text{rank}(B)$
- II. $\det(AB) = \det(A) \det(B)$
- III. $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$
- IV. $\det(A + B) \leq \det(A) + \det(B)$

Which of the above statements are TRUE?

- (A) I and II only
- (B) I and IV only
- (C) II and III only
- (D) III and IV only

ANS: - C

2019

Q.9 Let X be a square matrix. Consider the following two statements on X .

- I. X is invertible.
- II. Determinant of X is non-zero.

Which one of the following is TRUE?

- (A) I implies II; II does not imply I.
- (B) II implies I; I does not imply II.
- (C) I does not imply II; II does not imply I.
- (D) I and II are equivalent statements.

ANS: - D

Q.44 Consider the following matrix:

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of R is _____.ANS: - 12

2018

Q.17 Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that v^T denotes the transpose of v . The largest eigenvalue of A is _____.

ANS: - 3

Q.26 Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements.

- (I) P does not have an inverse
- (II) P has a repeated eigenvalue
- (III) P cannot be diagonalized

Which one of the following options is correct?

- (A) Only I and III are necessarily true
- (B) Only II is necessarily true
- (C) Only I and II are necessarily true
- (D) Only II and III are necessarily true

ANS: - D

ECE

2025

Q.11 Consider the matrix A below:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

For which of the following combinations of α , β , and γ , is the rank of A at least three?

- (i) $\alpha = 0$ and $\beta = \gamma \neq 0$.
- (ii) $\alpha = \beta = \gamma = 0$.
- (iii) $\beta = \gamma = 0$ and $\alpha \neq 0$.
- (iv) $\alpha = \beta = \gamma \neq 0$.

(A) Only (i), (iii), and (iv)

(B) Only (iv)

(C) Only (ii)

(D) Only (i) and (iii)

ANS: - A

2024

Q.30 Let \mathbb{R} and \mathbb{R}^3 denote the set of real numbers and the three dimensional vector space over it, respectively. The value of α for which the set of vectors

$$\{[2 \ -3 \ \alpha], [3 \ -1 \ 3], [1 \ -5 \ 7]\}$$

does not form a basis of \mathbb{R}^3 is _____.

ANS: - 5 (Check the answer)

Q.55 Consider the matrix $\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$, where k is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(A) $\begin{bmatrix} 1 \\ -\sqrt{2/k} \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ \sqrt{2/k} \end{bmatrix}$

(C) $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

ANS: - A, B (check the answer)

2023

Q.15 Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{\mathbf{v}_k \mid 1 \leq k \leq n\}$, respectively. For any invertible matrix P , the sets of eigenvalues and eigenvectors of the matrix A , where $B = P^{-1}AP$, respectively, are

(A) $\{\lambda_k \det(A) \mid 1 \leq k \leq n\}$ and $\{P\mathbf{v}_k \mid 1 \leq k \leq n\}$

(B) $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{\mathbf{v}_k \mid 1 \leq k \leq n\}$

(C) $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{P\mathbf{v}_k \mid 1 \leq k \leq n\}$

(D) $\{\lambda_k \mid 1 \leq k \leq n\}$ and $\{P^{-1}\mathbf{v}_k \mid 1 \leq k \leq n\}$

ANS: -

Q.38	Let \mathbf{x} be an $n \times 1$ real column vector with length $l = \sqrt{\mathbf{x}^T \mathbf{x}}$. The trace of the matrix $P = \mathbf{x}\mathbf{x}^T$ is
(A)	l^2
(B)	$\frac{l^2}{4}$
(C)	l
(D)	$\frac{l^2}{2}$

ANS: -

2022

Q.12	Consider a system of linear equations $Ax = b$, where $A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$ This system of equations admits _____.
(A)	a unique solution for x
(B)	infinitely many solutions for x
(C)	no solutions for x
(D)	exactly two solutions for x

ANS: -

2021

- Q.16** | If the vectors $(1.0, -1.0, 2.0)$, $(7.0, 3.0, x)$ and $(2.0, 3.0, 1.0)$ in \mathbb{R}^3 are linearly dependent, the value of x is _____

ANS: -

- Q.36** | A real 2×2 non-singular matrix A with repeated eigenvalue is given as
- $$A = \begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$
- where x is a real positive number. The value of x (rounded off to one decimal place) is _____

ANS: -

2020

- Q.No. 1** If $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_6$ are six vectors in \mathbb{R}^4 , which one of the following statements is FALSE?
- (A) It is not necessary that these vectors span \mathbb{R}^4 .
- (B) These vectors are not linearly independent.
- (C) Any four of these vectors form a basis for \mathbb{R}^4 .
- (D) If $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_5, \mathbf{v}_6\}$ spans \mathbb{R}^4 , then it forms a basis for \mathbb{R}^4 .

ANS: - C

- Q.No. 26** Consider the following system of linear equations.

$$x_1 + 2x_2 = b_1 \quad ; \quad 2x_1 + 4x_2 = b_2 \quad ; \quad 3x_1 + 7x_2 = b_3 \quad ; \quad 3x_1 + 9x_2 = b_4$$

Which one of the following conditions ensures that a solution exists for the above system?

- (A) $b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$
- (B) $b_3 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$
- (C) $b_2 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$
- (D) $b_3 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

ANS: - A

2019

Q.17 The number of distinct eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to _____.

ANS: - 3

2018Q.11 Let \mathbf{M} be a real 4×4 matrix. Consider the following statements:S1: \mathbf{M} has 4 linearly independent eigenvectors.S2: \mathbf{M} has 4 distinct eigenvalues.S3: \mathbf{M} is non-singular (invertible).

Which one among the following is TRUE?

(A) S1 implies S2

(B) S1 implies S3

(C) S2 implies S1

(D) S3 implies S2

ANS: - C

Q.22 Consider matrix $\mathbf{A} = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions is _____.

ANS: - 2

EEE

2025

Q.12	Let \mathbf{v}_1 and \mathbf{v}_2 be the two eigenvectors corresponding to distinct eigenvalues of a 3×3 real symmetric matrix. Which one of the following statements is true?
(A)	$\mathbf{v}_1^T \mathbf{v}_2 \neq 0$
(B)	$\mathbf{v}_1^T \mathbf{v}_2 = 0$
(C)	$\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{0}$
(D)	$\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{0}$

ANS: - B

Q.13	Let $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \end{bmatrix}$. Then, the system of linear equations $\mathbf{Ax} = \mathbf{b}$ has
(A)	a unique solution.
(B)	infinitely many solutions.
(C)	a finite number of solutions.
(D)	no solution.

ANS: - B

Q.14	Let $P = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and let I be the identity matrix. Then P^2 is equal to
(A)	$2P - I$
(B)	P
(C)	I
(D)	$P + I$

ANS: - A

2024

Q.11	Which one of the following matrices has an inverse?
(A)	$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$
(B)	$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$
(C)	$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$
(D)	$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$

ANS: -

Q.32 | The sum of the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is _____ (rounded off to the nearest integer).

ANS: -

2023

Q.11 | For a given vector $\mathbf{w} = [1 \ 2 \ 3]^T$, the vector normal to the plane defined by $\mathbf{w}^T \mathbf{x} = 1$ is

(A) $[-2 \ -2 \ 2]^T$

(B) $[3 \ 0 \ -1]^T$

(C) $[3 \ 2 \ 1]^T$

(D) $[1 \ 2 \ 3]^T$

ANS: -

2022

Q.20 | Consider a 3×3 matrix A whose (i, j) -th element, $a_{i,j} = (i - j)^3$. Then the matrix A will be

(A) symmetric.

(B) skew-symmetric.

(C) unitary.

(D) null.

ANS: -

Q.42 Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix.

Then $(c + d)$ is equal to

- (A) 5
- (B) 17
- (C) -6
- (D) 11

ANS: -

2021

Q.1	Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are
------------	---

- | | |
|-----|----------------|
| (A) | 1 and 1 |
| (B) | 1 and -1 |
| (C) | j and $-j$ |
| (D) | pq and $-pq$ |

ANS: - B

Q.38	Let A be a 10×10 matrix such that A^5 is a null matrix, and let I be the 10×10 identity matrix. The determinant of $A + I$ is _____.
-------------	--

ANS: - 1

2020

Q.No. 42 The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is _____.

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

- (A) 5
- (B) 6
- (C) 8
- (D) 9

ANS: - MTA

2019

Q.2 M is a 2×2 matrix with eigenvalues 4 and 9. The eigenvalues of M^2 are

- (A) 4 and 9 (B) 2 and 3 (C) -2 and -3 (D) 16 and 81

ANS: - D

Q.24 The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, is _____.

ANS: - 3

Q.26 Consider a 2×2 matrix $M = [v_1 \ v_2]$, where, v_1 and v_2 are the column vectors. Suppose

$M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$, where u_1^T and u_2^T are the row vectors. Consider the following statements:

Statement 1: $u_1^T v_1 = 1$ and $u_2^T v_2 = 1$

Statement 2: $u_1^T v_2 = 0$ and $u_2^T v_1 = 0$

Which of the following options is correct?

- (A) Statement 1 is true and statement 2 is false
- (B) Statement 2 is true and statement 1 is false
- (C) Both the statements are true
- (D) Both the statements are false

ANS: - C

2018

- Q.17 Consider a non-singular 2×2 square matrix A . If $\text{trace}(A) = 4$ and $\text{trace}(A^2) = 5$, the determinant of the matrix A is _____ (up to 1 decimal place).

ANS: - 5.5

- Q.44 Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix. The determinant of B is _____ (up to 1 decimal place).

ANS: - 0.9 - 1.1

Civil

2025

CE 1

Q.11	Suppose λ is an eigenvalue of matrix A and x is the corresponding eigenvector. Let x also be an eigenvector of the matrix $B = A - 2I$, where I is the identity matrix. Then, the eigenvalue of B corresponding to the eigenvector x is equal to
(A)	λ
(B)	$\lambda + 2$
(C)	2λ
(D)	$\lambda - 2$

ANS: - D

Q.12	Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ -2 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. For $Ax = b$ to be solvable, which one of the following options is the <i>correct</i> condition on b_1, b_2 , and b_3 :
(A)	$b_1 + b_2 + b_3 = 1$
(B)	$3b_1 + b_2 + 2b_3 = 0$
(C)	$b_1 + 3b_2 + b_3 = 2$
(D)	$b_1 + b_2 + b_3 = 2$

ANS: - B

CE – 2

Q.11	For the matrix $[A]$ given below, the transpose is _____.
	$[A] = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$
(A)	$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$
(B)	$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}$
(C)	$\begin{bmatrix} 4 & 2 & 3 \\ 5 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$
(D)	$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$

ANS: - A

Q.45	Pick the CORRECT eigen value(s) of the matrix $[A]$ from the following choices.
	$[A] = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$
(A)	10
(B)	4
(C)	-2
(D)	-10

ANS: - A, C

2024

S1

Q.36 What are the eigenvalues of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$?

- (A) 1, 2, 5
- (B) 1, 3, 4
- (C) -5, 1, 2
- (D) -5, -1, 2

ANS: -

S2

Q.12 The statements P and Q are related to matrices **A** and **B**, which are conformable for both addition and multiplication.

P: $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$

Q: $(\mathbf{AB})^T = \mathbf{A}^T \mathbf{B}^T$

Which one of the following options is CORRECT?

- (A) P is TRUE and Q is FALSE
- (B) Both P and Q are TRUE
- (C) P is FALSE and Q is TRUE
- (D) Both P and Q are FALSE

ANS: -

Q.48 Consider two matrices $\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}$.

The determinant of the matrix **AB** is _____ (in integer).

ANS: -

2023

S1

Q.24	If \mathbf{M} is an arbitrary real $n \times n$ matrix, then which of the following matrices will have non-negative eigenvalues?
(A)	\mathbf{M}^2
(B)	$\mathbf{M}\mathbf{M}^T$
(C)	$\mathbf{M}^T\mathbf{M}$
(D)	$(\mathbf{M}^T)^2$

ANS: - B, C

Q.47	For the matrix $[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ which of the following statements is/are TRUE?
(A)	The eigenvalues of $[A]^T$ are same as the eigenvalues of $[A]$
(B)	The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$
(C)	The eigenvectors of $[A]^T$ are same as the eigenvectors of $[A]$
(D)	The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of $[A]$

ANS: - A, B, D

S2

Q.25	For the matrix $[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ which of the following statements is/are TRUE?
(A)	$[A]\{x\} = \{b\}$ has a unique solution
(B)	$[A]\{x\} = \{b\}$ does not have a unique solution
(C)	$[A]$ has three linearly independent eigenvectors
(D)	$[A]$ is a positive definite matrix

ANS: - B,C

Q.37	Two vectors $[2 \ 1 \ 0 \ 3]^T$ and $[1 \ 0 \ 1 \ 2]^T$ belong to the null space of a 4×4 matrix of rank 2. Which one of the following vectors also belongs to the null space?
(A)	$[1 \ 1 \ -1 \ 1]^T$
(B)	$[2 \ 0 \ 1 \ 2]^T$
(C)	$[0 \ -2 \ 1 \ -1]^T$
(D)	$[3 \ 1 \ 1 \ 2]^T$

ANS: - A

Q.38	<p>Cholesky decomposition is carried out on the following square matrix $[A]$.</p> $[A] = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$ <p>Let l_{ij} and a_{ij} be the $(i,j)^{\text{th}}$ elements of matrices $[L]$ and $[A]$, respectively. If the element l_{22} of the decomposed lower triangular matrix $[L]$ is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22}?</p>
(A)	5
(B)	7
(C)	9
(D)	11

ANS: - B

2022

S1

Q.25	<p>The matrix M is defined as</p> $M = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ <p>and has eigenvalues 5 and -2. The matrix Q is formed as</p> $Q = M^3 - 4M^2 - 2M$ <p>Which of the following is/are the eigenvalue(s) of matrix Q?</p>
(A)	15
(B)	25
(C)	-20
(D)	-30

ANS: - A, C

S2

Q.25	P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?
(A)	If P and Q are invertible, then $[\mathbf{PQ}]^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$.
(B)	If P and Q are invertible, then $[\mathbf{QP}]^{-1} = \mathbf{P}^{-1}\mathbf{Q}^{-1}$.
(C)	If P and Q are invertible, then $[\mathbf{PQ}]^{-1} = \mathbf{P}^{-1}\mathbf{Q}^{-1}$.
(D)	If P and Q are not invertible, then $[\mathbf{PQ}]^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$.

ANS: - A, B

Q.45	Let y be a non-zero vector of size 2022×1 . Which of the following statement(s) is/are TRUE ?
(A)	\mathbf{yy}^T is a symmetric matrix.
(B)	$\mathbf{y}^T \mathbf{y}$ is an eigenvalue of \mathbf{yy}^T .
(C)	\mathbf{yy}^T has a rank of 2022.
(D)	\mathbf{yy}^T is invertible.

ANS: - A, B

2021

S1

Q.1	The rank of matrix $\begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$ is
(A)	1
(B)	2
(C)	3
(D)	4

ANS: - B

Q.2	If $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $Q^T P^T$ is
(A)	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
(B)	$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
(C)	$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$
(D)	$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

ANS: - D

S2

Q.2	The rank of the matrix $\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ is
(A)	1
(B)	2
(C)	3
(D)	4

ANS: - C

Q.4	If A is a square matrix then orthogonality property mandates
(A)	$AA^T = I$
(B)	$AA^T = 0$
(C)	$AA^T = A^{-1}$
(D)	$AA^T = A^2$

ANS: - A

Q.27	The smallest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 2 & -2 \\ -1 & 6 \end{bmatrix}$, respectively, are
(A)	1.55 and $\begin{Bmatrix} 2.00 \\ 0.45 \end{Bmatrix}$
(B)	2.00 and $\begin{Bmatrix} 1.00 \\ 1.00 \end{Bmatrix}$
(C)	1.55 and $\begin{Bmatrix} -2.55 \\ -0.45 \end{Bmatrix}$
(D)	1.55 and $\begin{Bmatrix} 2.00 \\ -0.45 \end{Bmatrix}$

ANS: - A

2020

S1

40 Consider the system of equations

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & -3 \\ 4 & 4 & -6 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

The value of x_3 (round off to the nearest integer), is _____.

ANS: - 3

S2

Q. 27 A 4×4 matrix $[P]$ is given below

$$[P] = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

The eigenvalues of $[P]$ are

- (A) 0, 3, 6, 6
- (B) 1, 2, 3, 4
- (C) 3, 4, 5, 7
- (D) 1, 2, 5, 7

ANS: -

2019

S2

Q.1 Euclidean norm (length) of the vector $[4 \ -2 \ -6]^T$ is

(A) $\sqrt{12}$

(B) $\sqrt{24}$

(C) $\sqrt{48}$

(D) $\sqrt{56}$

ANS: - D

Q.35

The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

(B) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -\frac{4}{5} & -\frac{9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & -\frac{1}{5} & -\frac{6}{5} \end{bmatrix}$

ANS: - C

2018

S1

Q.1 Which one of the following matrices is singular?

(A) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

ANS: - C

Q.2 For the given orthogonal matrix Q,

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

The inverse is

(A) $\begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

(B) $\begin{bmatrix} -3/7 & -2/7 & -6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

(C) $\begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

(D) $\begin{bmatrix} -3/7 & 6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

ANS: - C

S2

Q.26 The matrix $\begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix}$ has

- (A) real eigenvalues and eigenvectors
- (B) real eigenvalues but complex eigenvectors
- (C) complex eigenvalues but real eigenvectors
- (D) complex eigenvalues and eigenvectors

ANS: - D

Q.28 The rank of the following matrix is

$$\begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$$

- (A) 1 (B) 2 (C) 3 (D) 4

ANS: - B

Mechanical

2025

Q.11	Let A and B be real symmetric matrices of same size. Which one of the following options is correct?
(A)	$\mathbf{A}^T = \mathbf{A}^{-1}$
(B)	$\mathbf{AB} = \mathbf{BA}$
(C)	$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$
(D)	$\mathbf{A} = \mathbf{A}^{-1}$

ANS: - C

Instrumentation Engineering

2025

Q.11 A $2n \times 2n$ matrix $A = [a_{ij}]$ has its elements as

$$a_{ij} = \begin{cases} \beta & \text{if } (i+j) \text{ is odd,} \\ -\beta & \text{if } (i+j) \text{ is even,} \end{cases}$$

where n is any integer greater than 2 and β is any non-zero real number. The rank of A is

- (A) 1
- (B) 2
- (C) n
- (D) $2n$

ANS: - A

Q.32

If one of the eigenvectors of the matrix $A = \begin{bmatrix} -1 & -1 \\ x & -4 \end{bmatrix}$ is along the direction of $\begin{bmatrix} \alpha \\ 2\alpha \end{bmatrix}$, where α is any non-zero real number, then the value of x is _____ (in integer).

ANS: - 2

Data Science and Artificial Intelligence

2025

- Q. 13 The sum of the elements in each row of $A \in \mathbb{R}^{n \times n}$ is 1. If $B = A^3 - 2A^2 + A$, which one of the following statements is correct (for $x \in \mathbb{R}^n$)?
- (A) The equation $Bx = 0$ has no solution
 - (B) The equation $Bx = 0$ has exactly two solutions
 - (C) The equation $Bx = 0$ has infinitely many solutions
 - (D) The equation $Bx = 0$ has a unique solution

ANS: - C

- Q. 25 Which of the following statements is/are correct?
- (A) \mathbb{R}^n has a unique set of orthonormal basis vectors
 - (B) \mathbb{R}^n does not have a unique set of orthonormal basis vectors
 - (C) Linearly independent vectors in \mathbb{R}^n are orthonormal
 - (D) Orthonormal vectors in \mathbb{R}^n are linearly independent

ANS: - B, D

- Q. 28 Let $A = I_n + xx^\top$, where I_n is the $n \times n$ identity matrix and $x \in \mathbb{R}^n$, $x^\top x = 1$. Which of the following options is/are correct?
- (A) Rank of A is n
 - (B) A is invertible
 - (C) 0 is an eigenvalue of A
 - (D) A^{-1} has a negative eigenvalue

ANS: - A, B

- Q. 37 Let $A \in \mathbb{R}^{n \times n}$ be such that $A^3 = A$. Which one of the following statements is ALWAYS correct?
- (A) A is invertible
 - (B) Determinant of A is 0
 - (C) The sum of the diagonal elements of A is 1
 - (D) A and A^2 have the same rank

ANS: - D


- Q. 38 Let $\{x_1, x_2, \dots, x_n\}$ be a set of linearly independent vectors in \mathbb{R}^n . Let the (i, j) -th element of matrix $A \in \mathbb{R}^{n \times n}$ be given by $A_{ij} = x_i^\top x_j$, $1 \leq i, j \leq n$. Which one of the following statements is correct?
- (A) A is invertible
 - (B) 0 is a singular value of A
 - (C) Determinant of A is 0
 - (D) $z^\top A z = 0$ for some non-zero $z \in \mathbb{R}^n$

ANS: - A

- Q. 50 Let x_1, x_2, x_3, x_4, x_5 be a system of orthonormal vectors in \mathbb{R}^{10} . Consider the matrix $A = x_1 x_1^\top + \dots + x_5 x_5^\top$. Which of the following statements is/are correct?
- (A) Singular values of A are also its eigenvalues
 - (B) Singular values of A are either 0 or 1
 - (C) Determinant of A is 1
 - (D) A is invertible

ANS: - A, B

Q. 52 An $n \times n$ matrix A with real entries satisfies the property: $\|Ax\|^2 = \|x\|^2$, for all $x \in \mathbb{R}^n$, where $\|\cdot\|$ denotes the Euclidean norm. Which of the following statements is/are ALWAYS correct?

- (A) A must be orthogonal
 - (B) $A = I$, where I denotes the identity matrix, is the only solution
 - (C) The eigenvalues of A are either $+1$ or -1
 - (D) A has full rank
- 

ANS: - A, D
