GATE

# Linear Algebra

PYS’s and Solution

VARUN KUMAR
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# CSE

## 2025

### S1

**Q.** Consider the given system of linear equations for variables $x$ and $y$, where $k$ is a real-valued constant. Which of the following option(s) is/are CORRECT?

$$x + ky = 1$$

$$kx + y = -1$$

a. There is exactly one value of $k$ for which the above system of equations has no solution.
b. There exist an infinite number of values of $k$ for which the system of equations has no solution.
c. There exists exactly one value of $k$ for which the system of equations has exactly one solution.
d. There exists exactly one value of $k$ for which the system of equations has an infinite number of solutions.

ANS: - a, d

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**Q.** Let $A$ be a $2 \times 2$ matrix as given.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

What are the eigenvalues of the matrix $A^{13}$?

a. $1, -1$
b. $2\sqrt{2}, -2\sqrt{2}$
c. $4\sqrt{2}, -4\sqrt{2}$
d. $64\sqrt{2}, -64\sqrt{2}$

**ANS: - d**

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### S2

**Q**. If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ then which ONE of the following is $A^8$?

a. $\begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}$

b. $\begin{pmatrix} 125 & 0 \\ 0 & 125 \end{pmatrix}$

c. $\begin{pmatrix} 625 & 0 \\ 0 & 625 \end{pmatrix}$

d. $\begin{pmatrix} 3125 & 0 \\ 0 & 3125 \end{pmatrix}$

ANS: - c

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**Q.** Let $L$, $M$, and $N$ be non-singular matrices of order 3 satisfying the equations

$$L^2 = L^{-1}, M = L^8, \qquad and\ N = L^2$$

Which ONE of the following is the value of the determinant of $(M - N)$?

a. 0
b. 1
c. 2
d. 3

ANS: - a

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**Q.** Consider a system of linear equations $PX = Q$ where $P \in \mathbb{R}^{3\,X\,3}$ and $Q \in \mathbb{R}^{3\,X\,1}$. Suppose $P$ has an LU decomposition, $P = LU$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Which of the following statement(s) is/are TRUE?

a. The system $PX = Q$ can be solved by first solving $LY = Q$ and then $UX = Y$.
b. If $P$ is invertible, then both $L$ and $U$ are invertible
c. If $P$ is singular, then at least one of the diagonal elements of $U$ is zero.
d. If $P$ is symmetric, then both $L$ and $U$ are symmetric.

ANS: - a, b, c

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## 2024

Q.12 The product of all eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

(A) $-1$

(B) 0

(C) 1

(D) 2



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Q.49 Let $A$ be any $n \times m$ matrix, where $m > n$. Which of the following statements is/are TRUE about the system of linear equations $Ax = \mathbf{0}$ ?

(A) There exist at least $m - n$ linearly independent solutions to this system

(B) There exist $m - n$ linearly independent vectors such that every solution is a linear combination of these vectors

(C) There exists a non-zero solution in which at least $m - n$ variables are 0

(D) There exists a solution in which at least $n$ variables are non-zero

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## S2

Q.47 Let $A$ be an $n \times n$ matrix over the set of all real numbers $\mathbb{R}$. Let $B$ be a matrix obtained from $A$ by swapping two rows. Which of the following statements is/are TRUE?

(A) The determinant of $B$ is the negative of the determinant of $A$

(B) If $A$ is invertible, then $B$ is also invertible

(C) If $A$ is symmetric, then $B$ is also symmetric

(D) If the trace of $A$ is zero, then the trace of $B$ is also zero

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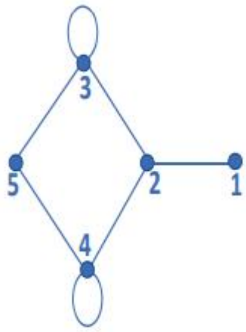
## 2023

| | |
|---|---|
| Q.18 | Let $$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$ and $$B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}.$$ Let $\texttt{det}(A)$ and $\texttt{det}(B)$ denote the determinants of the matrices $A$ and $B$, respectively. Which one of the options given below is TRUE? |
| (A) | $\texttt{det}(A) = \texttt{det}(B)$ |
| (B) | $\texttt{det}(B) = -\texttt{det}(A)$ |
| (C) | $\texttt{det}(A) = 0$ |
| (D) | $\texttt{det}(AB) = \texttt{det}(A) + \texttt{det}(B)$ |

| | |
|---|---|
| Q.30 | Let $A$ be the adjacency matrix of the graph with vertices $\{1, 2, 3, 4, 5\}$.<br><br>Let $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$, and $\lambda_5$ be the five eigenvalues of $A$. Note that these eigenvalues need not be distinct.<br>The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$ ________. |



## 2022

| | |
|---|---|
| Q.20 | Consider the following two statements with respect to the matrices $A_{m\times n}$, $B_{n\times m}$, $C_{n\times n}$ and $D_{n\times n}$.<br><br>Statement 1: $tr(AB) = tr(BA)$<br>Statement 2: $tr(CD) = tr(DC)$<br><br>where $tr()$ represents the trace of a matrix. Which one of the following holds? |
| (A) | Statement 1 is correct and Statement 2 is wrong. |
| (B) | Statement 1 is wrong and Statement 2 is correct. |
| (C) | Both Statement 1 and Statement 2 are correct. |
| (D) | Both Statement 1 and Statement 2 are wrong. |

| | |
|---|---|
| Q.37 | Consider a simple undirected unweighted graph with at least three vertices. If $A$ is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of |
| (A) | $A^3$ |
| (B) | $A^3$ divided by 2 |
| (C) | $A^3$ divided by 3 |
| (D) | $A^3$ divided by 6 |

| | |
|---|---|
| Q.45 | Consider solving the following system of simultaneous equations using LU decomposition. $$\begin{aligned} x_1 + x_2 - 2x_3 &= 4 \\ x_1 + 3x_2 - x_3 &= 7 \\ 2x_1 + x_2 - 5x_3 &= 7 \end{aligned}$$ where $L$ and $U$ are denoted as $$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$ Which one of the following is the correct combination of values for $L_{32}$, $U_{33}$, and $x_1$? |
| (A) | $L_{32} = 2,\ U_{33} = -\frac{1}{2},\ x_1 = -1$ |
| (B) | $L_{32} = 2,\ U_{33} = 2,\ x_1 = -1$ |
| (C) | $L_{32} = -\frac{1}{2},\ U_{33} = 2,\ x_1 = 0$ |
| (D) | $L_{32} = -\frac{1}{2},\ U_{33} = -\frac{1}{2},\ x_1 = 0$ |

| | |
|---|---|
| Q.53 | Which of the following is/are the eigenvector(s) for the matrix given below? $$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$ |
| (A) | $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ |
| (B) | $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ |
| (C) | $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$ |
| (D) | $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$ |

## 2021, S-1

| | |
|---|---|
| Q.52 | Consider the following matrix. $$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$ The largest eigenvalue of the above matrix is __________. |

ANS: - 3

## 2021, S-2

Q.24 Suppose that $P$ is a $4 \times 5$ matrix such that every solution of the equation $P\mathbf{x} = \mathbf{0}$ is a scalar multiple of $[2\ \ 5\ \ 4\ \ 3\ \ 1]^T$. The rank of $P$ is ________.

ANS: - 4

## 2020

Q.No. 27 Let $A$ and $B$ be two $n \times n$ matrices over real numbers. Let $\text{rank}(M)$ and $\det(M)$ denote the rank and determinant of a matrix $M$, respectively. Consider the following statements.

I. $\text{rank}(AB) = \text{rank}(A)\,\text{rank}(B)$

II. $\det(AB) = \det(A)\det(B)$

III. $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$

IV. $\det(A + B) \leq \det(A) + \det(B)$

Which of the above statements are TRUE?

(A) I and II only

(B) I and IV only

(C) II and III only

(D) III and IV only

ANS: - C

## 2019

Q.9 Let $X$ be a square matrix. Consider the following two statements on $X$.

I. $X$ is invertible.

II. Determinant of $X$ is non-zero.

Which one of the following is TRUE?

(A) I implies II; II does not imply I.

(B) II implies I; I does not imply II.

(C) I does not imply II; II does not imply I.

(D) I and II are equivalent statements.

ANS: - D

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Q.44 Consider the following matrix:

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of $R$ is ________.

ANS: - 12

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## 2018

Q.17 Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that $v^T$ denotes the transpose of $v$. The largest eigenvalue of $A$ is _____.

ANS: - 3

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Q.26 Consider a matrix $\boldsymbol{P}$ whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements.

(I) $\boldsymbol{P}$ does not have an inverse
(II) $\boldsymbol{P}$ has a repeated eigenvalue
(III) $\boldsymbol{P}$ cannot be diagonalized

Which one of the following options is correct?

(A) Only I and III are necessarily true
(B) Only II is necessarily true
(C) Only I and II are necessarily true
(D) Only II and III are necessarily true

ANS: - D

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# ECE

## 2025

Q.11 Consider the matrix $A$ below:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

For which of the following combinations of $\alpha, \beta$, and $\gamma$, is the rank of $A$ at least three?

(i) $\alpha = 0$ and $\beta = \gamma \neq 0$.
(ii) $\alpha = \beta = \gamma = 0$.
(iii) $\beta = \gamma = 0$ and $\alpha \neq 0$.
(iv) $\alpha = \beta = \gamma \neq 0$.

(A) Only (i), (iii), and (iv)

(B) Only (iv)

(C) Only (ii)

(D) Only (i) and (iii)

ANS: - A

## 2024

Q.30 Let $\mathbb{R}$ and $\mathbb{R}^3$ denote the set of real numbers and the three dimensional vector space over it, respectively. The value of $\alpha$ for which the set of vectors

$$\{[2 \quad -3 \quad \alpha], \quad [3 \quad -1 \quad 3], \quad [1 \quad -5 \quad 7]\}$$

does not form a basis of $\mathbb{R}^3$ is ________.

ANS: - 5 (Check the answer)

Q.55 Consider the matrix $\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$, where $k$ is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(A) $\begin{bmatrix} 1 \\ -\sqrt{2/k} \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ \sqrt{2/k} \end{bmatrix}$

(C) $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

ANS: - A, B (check the answer)

## 2023

| | |
|---|---|
| Q.15 | Let the sets of eigenvalues and eigenvectors of a matrix $B$ be $\{\lambda_k \mid 1 \le k \le n\}$ and $\{\boldsymbol{v}_k \mid 1 \le k \le n\}$, respectively. For any invertible matrix $P$, the sets of eigenvalues and eigenvectors of the matrix $A$, where $B = P^{-1}AP$, respectively, are |
| | |
| (A) | $\{\lambda_k \det(A) \mid 1 \le k \le n\}$ and $\{P\boldsymbol{v}_k \mid 1 \le k \le n\}$ |
| (B) | $\{\lambda_k \mid 1 \le k \le n\}$ and $\{\boldsymbol{v}_k \mid 1 \le k \le n\}$ |
| (C) | $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P\boldsymbol{v}_k \mid 1 \le k \le n\}$ |
| (D) | $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P^{-1}\boldsymbol{v}_k \mid 1 \le k \le n\}$ |

ANS: -

| | |
|---|---|
| Q.38 | Let $\boldsymbol{x}$ be an $n \times 1$ real column vector with length $l = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$. The trace of the matrix $P = \boldsymbol{x}\boldsymbol{x}^T$ is |
| | |
| (A) | $l^2$ |
| (B) | $\frac{l^2}{4}$ |
| (C) | $l$ |
| (D) | $\frac{l^2}{2}$ |

ANS: -

## 2022

| | |
|---|---|
| Q.12 | Consider a system of linear equations $Ax = b$, where<br>$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$<br>This system of equations admits __________. |
| | |
| (A) | a unique solution for $x$ |
| (B) | infinitely many solutions for $x$ |
| (C) | no solutions for $x$ |
| (D) | exactly two solutions for $x$ |

ANS: -

## 2021

**Q.16** **If the vectors $(1.0,\ -1.0,\ 2.0)$, $(7.0,\ 3.0,\ x)$ and $(2.0,\ 3.0,\ 1.0)$ in $\mathbb{R}^3$ are linearly dependent, the value of $x$ is ________**

ANS: -

**Q.36** **A real $2 \times 2$ non-singular matrix A with repeated eigenvalue is given as**

$$\mathbf{A} = \begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

**where $x$ is a real positive number. The value of $x$ (*rounded off to one decimal place*) is ________**

ANS: -

## 2020

**Q.No. 1** If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_6$ are six vectors in $\mathbb{R}^4$, which one of the following statements is FALSE?

(A) It is not necessary that these vectors span $\mathbb{R}^4$.

(B) These vectors are not linearly independent.

(C) Any four of these vectors form a basis for $\mathbb{R}^4$.

(D) If $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_5, \mathbf{v}_6\}$ spans $\mathbb{R}^4$, then it forms a basis for $\mathbb{R}^4$.

ANS: - C

**Q.No. 26** Consider the following system of linear equations.

$$x_1 + 2x_2 = b_1 \ ; \quad 2x_1 + 4x_2 = b_2 \ ; \quad 3x_1 + 7x_2 = b_3 \ ; \quad 3x_1 + 9x_2 = b_4$$

Which one of the following conditions ensures that a solution exists for the above system?

(A) $b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

(B) $b_3 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

(C) $b_2 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

(D) $b_3 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

ANS: - A

## 2019

Q.17 The number of distinct eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to ______.

ANS: - 3

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## 2018

Q.11 Let $\mathbf{M}$ be a real $4 \times 4$ matrix. Consider the following statements:

S1: $\mathbf{M}$ has 4 linearly independent eigenvectors.

S2: $\mathbf{M}$ has 4 distinct eigenvalues.

S3: $\mathbf{M}$ is non-singular (invertible).

Which one among the following is TRUE?

(A) S1 implies S2
(B) S1 implies S3
(C) S2 implies S1
(D) S3 implies S2

ANS: - C

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Q.22 Consider matrix $\mathbf{A} = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of $k$ for which the equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions is _______.

ANS: - 2

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# EEE

## 2025

| | |
|---|---|
| Q.12 | Let $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ be the two eigenvectors corresponding to distinct eigenvalues of a $3 \times 3$ real symmetric matrix. Which one of the following statements is true? |
| (A) | $\boldsymbol{v}_1^T\boldsymbol{v}_2 \neq 0$ |
| (B) | $\boldsymbol{v}_1^T\boldsymbol{v}_2 = 0$ |
| (C) | $\boldsymbol{v}_1 + \boldsymbol{v}_2 = \boldsymbol{0}$ |
| (D) | $\boldsymbol{v}_1 - \boldsymbol{v}_2 = \boldsymbol{0}$ |



ANS: - B

| | |
|---|---|
| Q.13 | Let $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, and $\boldsymbol{b} = \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \end{bmatrix}$. Then, the system of linear equations $\boldsymbol{Ax} = \boldsymbol{b}$ has |
| (A) | a unique solution. |
| (B) | infinitely many solutions. |
| (C) | a finite number of solutions. |
| (D) | no solution. |

ANS: - B

| | |
|---|---|
| Q.14 | Let $P = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and let $I$ be the identity matrix. Then $P^2$ is equal to |
| (A) | $2P - I$ |
| (B) | $P$ |
| (C) | $I$ |
| (D) | $P + I$ |

ANS: - A

## 2024

| | |
|---|---|
| Q.11 | Which one of the following matrices has an inverse? |
| | |
| (A) | $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$ |
| (B) | $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$ |
| (C) | $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ |
| (D) | $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$ |

ANS: -

| | |
|---|---|
| Q.32 | The sum of the eigenvalues of the matrix A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is ______ (rounded off to the nearest integer). |

ANS: -

## 2023

| Q.11 | For a given vector $\mathbf{w} = [1\ 2\ 3]^T$, the vector normal to the plane defined by $\mathbf{w}^T\mathbf{x} = 1$ is |
|---|---|
| (A) | $[-2 \quad -2 \quad 2]^T$ |
| (B) | $[3 \quad 0 \quad -1]^T$ |
| (C) | $[3 \quad 2 \quad 1]^T$ |
| (D) | $[1 \quad 2 \quad 3]^T$ |

ANS: -

## 2022

Q.20 Consider a 3 x 3 matrix $A$ whose $(i, j)$-th element, $a_{i,j} = (i - j)^3$. Then the matrix $A$ will be

(A) symmetric.

(B) skew-symmetric.

(C) unitary.

(D) null.

ANS: -

Q.42 Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix $A$ satisfies the equation $6A^{-1} = A^2 + cA + dI$, where $c$ and $d$ are scalars and $I$ is the identity matrix.

Then $(c + d)$ is equal to

(A) 5

(B) 17

(C) $-6$

(D) 11

ANS: -

## 2021

| | |
|---|---|
| **Q.1** | **Let $p$ and $q$ be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are** |
| (A) | 1 and 1 |
| (B) | 1 and -1 |
| (C) | $j$ and $-j$ |
| (D) | $pq$ and $-pq$ |

ANS: - B

| | |
|---|---|
| **Q.38** | **Let $\mathbf{A}$ be a $10 \times 10$ matrix such that $\mathbf{A}^5$ is a null matrix, and let $\mathbf{I}$ be the $10 \times 10$ identity matrix. The determinant of $\mathbf{A} + \mathbf{I}$ is ______________.** |

ANS: - 1

## 2020

Q.No. 42 The number of purely real elements in a lower triangular representation of the given $3 \times 3$ matrix, obtained through the given decomposition is __________.

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(A) 5
(B) 6
(C) 8
(D) 9

ANS: - MTA

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## 2019

Q.2 $M$ is a $2 \times 2$ matrix with eigenvalues 4 and 9. The eigenvalues of $M^2$ are

(A) 4 and 9 (B) 2 and 3 (C) -2 and -3 (D) 16 and 81

ANS: - D

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Q.24 The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, is _________.

ANS: - 3

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Q.26 Consider a $2 \times 2$ matrix $\boldsymbol{M} = [\boldsymbol{v_1} \quad \boldsymbol{v_2}]$, where, $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are the column vectors. Suppose $\boldsymbol{M^{-1}} = \begin{bmatrix} \boldsymbol{u_1^T} \\ \boldsymbol{u_2^T} \end{bmatrix}$, where $\boldsymbol{u_1^T}$ and $\boldsymbol{u_2^T}$ are the row vectors. Consider the following statements:

Statement 1: $\boldsymbol{u_1^T v_1 = 1}$ and $\boldsymbol{u_2^T v_2 = 1}$
Statement 2: $\boldsymbol{u_1^T v_2 = 0}$ and $\boldsymbol{u_2^T v_1 = 0}$

Which of the following options is correct?

(A) Statement 1 is true and statement 2 is false
(B) Statement 2 is true and statement 1 is false
(C) Both the statements are true
(D) Both the statements are false

ANS: - C

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# 2018

Q.17 Consider a non-singular $2\times2$ square matrix $\mathbf{A}$. If $trace(\mathbf{A}) = 4$ and $trace(\mathbf{A}^2) = 5$, the determinant of the matrix $\mathbf{A}$ is _________(up to 1 decimal place).

ANS: - 5.5

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Q.44 Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where $I$ is the $3\times3$ identity matrix. The determinant of $B$ is ________ (up to 1 decimal place).

ANS: - 0.9 - 1.1

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# Civil

## 2025

CE 1

| | |
|---|---|
| Q.11 | Suppose $\lambda$ is an eigenvalue of matrix $A$ and $x$ is the corresponding eigenvector. Let $x$ also be an eigenvector of the matrix $B = A - 2I$, where $I$ is the identity matrix. Then, the eigenvalue of $B$ corresponding to the eigenvector $x$ is equal to |
| (A) | $\lambda$ |
| (B) | $\lambda + 2$ |
| (C) | $2\lambda$ |
| (D) | $\lambda - 2$ |

ANS: - D

| | |
|---|---|
| Q.12 | Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ -2 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. For $Ax = b$ to be solvable, which one of the following options is the *correct* condition on $b_1, b_2$, and $b_3$: |
| (A) | $b_1 + b_2 + b_3 = 1$ |
| (B) | $3b_1 + b_2 + 2b_3 = 0$ |
| (C) | $b_1 + 3b_2 + b_3 = 2$ |
| (D) | $b_1 + b_2 + b_3 = 2$ |

ANS: - B

CE – 2

| Q.11 | For the matrix $[A]$ given below, the transpose is ______. |
|---|---|
| | $[A] = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$ |
| (A) | $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$ |
| (B) | $\begin{bmatrix} 4 & 3 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ |
| (C) | $\begin{bmatrix} 4 & 2 & 3 \\ 5 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$ |
| (D) | $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$ |

ANS: - A

| Q.45 | Pick the **CORRECT** eigen value(s) of the matrix $[A]$ from the following choices. |
|---|---|
| | $[A] = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$ |
| (A) | 10 |
| (B) | 4 |
| (C) | -2 |
| (D) | -10 |

ANS: - A, C

# 2024

## S1

Q.36 What are the eigenvalues of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$?

(A) $1, 2, 5$

(B) $1, 3, 4$

(C) $-5, 1, 2$

(D) $-5, -1, 2$

ANS: -

---

## S2

Q.12 The statements P and Q are related to matrices **A** and **B**, which are conformable for both addition and multiplication.

P: $(\mathbf{A} + \mathbf{B})^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}} + \mathbf{B}^{\mathrm{T}}$

Q: $(\mathbf{AB})^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}$

Which one of the following options is CORRECT?

(A) P is TRUE and Q is FALSE

(B) Both P and Q are TRUE

(C) P is FALSE and Q is TRUE

(D) Both P and Q are FALSE

ANS: -

---

Q.48 Consider two matrices $\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}$.

The determinant of the matrix **AB** is __________ (*in integer*).

ANS: -

---

# 2023

**S1**

| Q.24 | If $\boldsymbol{M}$ is an arbitrary real $n \times n$ matrix, then which of the following matrices will have non-negative eigenvalues? |
|---|---|
| (A) | $\boldsymbol{M}^2$ |
| (B) | $\boldsymbol{M}\boldsymbol{M}^T$ |
| (C) | $\boldsymbol{M}^T\boldsymbol{M}$ |
| (D) | $(\boldsymbol{M}^T)^2$ |

ANS: - B, C

| Q.47 | For the matrix $$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$ which of the following statements is/are TRUE? |
|---|---|
| (A) | The eigenvalues of $[A]^T$ are same as the eigenvalues of $[A]$ |
| (B) | The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$ |
| (C) | The eigenvectors of $[A]^T$ are same as the eigenvectors of $[A]$ |
| (D) | The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of $[A]$ |

ANS: - A, B, D

**S2**

| | |
|---|---|
| Q.25 | For the matrix<br>$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$<br>which of the following statements is/are TRUE? |
| (A) | $[A]\{x\} = \{b\}$ has a unique solution |
| (B) | $[A]\{x\} = \{b\}$ does not have a unique solution |
| (C) | $[A]$ has three linearly independent eigenvectors |
| (D) | $[A]$ is a positive definite matrix |

ANS: - B,C

| | |
|---|---|
| Q.37 | Two vectors $[2 \quad 1 \quad 0 \quad 3]^T$ and $[1 \quad 0 \quad 1 \quad 2]^T$ belong to the null space of a $4 \times 4$ matrix of rank 2. Which one of the following vectors also belongs to the null space? |
| (A) | $[1 \quad 1 \quad -1 \quad 1]^T$ |
| (B) | $[2 \quad 0 \quad 1 \quad 2]^T$ |
| (C) | $[0 \quad -2 \quad 1 \quad -1]^T$ |
| (D) | $[3 \quad 1 \quad 1 \quad 2]^T$ |

ANS: - A

| | |
|---|---|
| Q.38 | Cholesky decomposition is carried out on the following square matrix $[A]$. $$[A] = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$ Let $l_{ij}$ and $a_{ij}$ be the (i,j)$^{th}$ elements of matrices $[L]$ and $[A]$, respectively. If the element $l_{22}$ of the decomposed lower triangular matrix $[L]$ is 1.968, what is the value (rounded off to the nearest integer) of the element $a_{22}$? |
| (A) | 5 |
| (B) | 7 |
| (C) | 9 |
| (D) | 11 |

ANS: - B

## 2022

S1

| | |
|---|---|
| Q.25 | The matrix $\boldsymbol{M}$ is defined as $$\boldsymbol{M} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$ and has eigenvalues **5** and $-$ **2**. The matrix $\boldsymbol{Q}$ is formed as $$\boldsymbol{Q} = \boldsymbol{M}^3 - 4\boldsymbol{M}^2 - 2\boldsymbol{M}$$ Which of the following is/are the eigenvalue(s) of matrix $\boldsymbol{Q}$ ? |
| (A) | 15 |
| (B) | 25 |
| (C) | –20 |
| (D) | –30 |

ANS: - A, C

S2

| Q.25 | **P** and **Q** are two square matrices of the same order. Which of the following statement(s) is/are correct? |
|---|---|
| (A) | If **P** and **Q** are invertible, then $[\mathbf{PQ}]^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$. |
| (B) | If **P** and **Q** are invertible, then $[\mathbf{QP}]^{-1} = \mathbf{P}^{-1}\mathbf{Q}^{-1}$. |
| (C) | If **P** and **Q** are invertible, then $[\mathbf{PQ}]^{-1} = \mathbf{P}^{-1}\mathbf{Q}^{-1}$. |
| (D) | If **P** and **Q** are not invertible, then $[\mathbf{PQ}]^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$. |

ANS: - A, B

| Q.45 | Let $\boldsymbol{y}$ be a non-zero vector of size $2022 \times 1$. Which of the following statement(s) is/are **TRUE**? |
|---|---|
| (A) | $\boldsymbol{yy}^T$ is a symmetric matrix. |
| (B) | $\boldsymbol{y}^T\boldsymbol{y}$ is an eigenvalue of $\boldsymbol{yy}^T$. |
| (C) | $\boldsymbol{yy}^T$ has a rank of 2022. |
| (D) | $\boldsymbol{yy}^T$ is invertible. |

ANS: - A, B

## 2021

S1

| **Q.1** | **The rank of matrix** $\begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$ **is** |
|---|---|
| (A) | 1 |
| (B) | 2 |
| (C) | 3 |
| (D) | 4 |

ANS: - B

| Q.2 | If $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $Q^T P^T$ is |
|---|---|
| (A) | $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ |
| (B) | $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ |
| (C) | $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ |
| (D) | $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ |

ANS: - D

S2

| Q.2 | The rank of the matrix $\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ is |
|---|---|
| (A) | 1 |
| (B) | 2 |
| (C) | 3 |
| (D) | 4 |

ANS: - C

| Q.4 | If $A$ is a square matrix then orthogonality property mandates |
|---|---|
| (A) | $AA^T = I$ |
| (B) | $AA^T = 0$ |
| (C) | $AA^T = A^{-1}$ |
| (D) | $AA^T = A^2$ |

ANS: - A

| Q.27 | The smallest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix} 2 & -2 \\ -1 & 6 \end{bmatrix}$, respectively, are |
|---|---|
| (A) | 1.55 and $\begin{Bmatrix} 2.00 \\ 0.45 \end{Bmatrix}$ |
| (B) | 2.00 and $\begin{Bmatrix} 1.00 \\ 1.00 \end{Bmatrix}$ |
| (C) | 1.55 and $\begin{Bmatrix} -2.55 \\ -0.45 \end{Bmatrix}$ |
| (D) | 1.55 and $\begin{Bmatrix} 2.00 \\ -0.45 \end{Bmatrix}$ |

ANS: - A

## 2020

S1

40 Consider the system of equations

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & -3 \\ 4 & 4 & -6 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

The value of $x_3$ (*round off to the nearest integer*), is ___________.

ANS: - 3

S2

Q. 27 A 4×4 matrix $[P]$ is given below

$$[P] = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

The eigenvalues of $[P]$ are

(A) 0, 3, 6, 6

(B) 1, 2, 3, 4

(C) 3, 4, 5, 7

(D) 1, 2, 5, 7

ANS: -

## 2019

S2

Q.1 Euclidean norm (length) of the vector $[4 \quad -2 \quad -6]^T$ is

(A) $\sqrt{12}$ (B) $\sqrt{24}$ (C) $\sqrt{48}$ (D) $\sqrt{56}$

ANS: - D

---

Q.35 The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$ (B) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -\frac{4}{5} & -\frac{9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & -\frac{1}{5} & -\frac{6}{5} \end{bmatrix}$

ANS: - C

---

## 2018

S1

Q.1 Which one of the following matrices is singular?

(A) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

ANS: - C

---

Q.2 For the given orthogonal matrix Q,

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

The inverse is

(A) $\begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

(B) $\begin{bmatrix} -3/7 & -2/7 & -6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

(C) $\begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

(D) $\begin{bmatrix} -3/7 & 6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

ANS: - C

S2

Q.26 The matrix $\begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix}$ has

(A) real eigenvalues and eigenvectors
(B) real eigenvalues but complex eigenvectors
(C) complex eigenvalues but real eigenvectors
(D) complex eigenvalues and eigenvectors

ANS: - D

Q.28 The rank of the following matrix is

$$\begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$$

(A) 1 (B) 2 (C) 3 (D) 4

ANS: - B

# Mechanical

## 2025

| | |
|---|---|
| Q.11 | Let **A** and **B** be real symmetric matrices of same size. Which one of the following options is correct? |
| | |
| (A) | $\mathbf{A}^T = \mathbf{A}^{-1}$ |
| (B) | $\mathbf{AB} = \mathbf{BA}$ |
| (C) | $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$ |
| (D) | $\mathbf{A} = \mathbf{A}^{-1}$ |

ANS: - C

## 2024

Q.14 Consider the system of linear equations

$$x + 2y + z = 5$$

$$2x + ay + 4z = 12$$

$$2x + 4y + 6z = b$$

The values of $a$ and $b$ such that there exists a non-trivial null space and the system admits infinite solutions are

(A) $a = 8, b = 14$

(B) $a = 4, b = 12$

(C) $a = 8, b = 12$

(D) $a = 4, b = 14$

Q.36 The matrix $\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}$ (where $a > 0$) has a negative eigenvalue if $a$ is greater than

(A) $\frac{3}{8}$

(C) $\frac{1}{4}$

(B) $\frac{1}{8}$

(D) $\frac{1}{5}$

ANS: -

## 2022

**SET – 1**

Q.15 If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is ____________________ .

(A) **8**

(B) **5**

(C) **−0.4**

(D) $\frac{\mathbf{1+\sqrt{1561}}}{\mathbf{12}}$

ANS: -

Q.46 The system of linear equations in real $(x, y)$ given by

$$(x \quad y)\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0 \quad 0)$$

involves a real parameter $\alpha$ and has infinitely many non-trivial solutions for special value(s) of $\alpha$. Which one or more among the following options is/are non-trivial solution(s) of $(x, y)$ for such special value(s) of $\alpha$?

(A) $x = 2, \quad y = -2$

(B) $x = -1, \quad y = 4$

(C) $x = 1, \quad y = 1$

(D) $x = 4, \quad y = -2$

**SET – 2**

Q.46 A is a 3×5 real matrix of rank 2. For the set of homogeneous equations **Ax = 0**, where **0** is a zero vector and **x** is a vector of unknown variables, which of the following is/are true?

(A) The given set of equations will have a unique solution.

(B) The given set of equations will be satisfied by a zero vector of appropriate size.

(C) The given set of equations will have infinitely many solutions.

(D) The given set of equations will have many but a finite number of solutions.

Q.48 If the sum and product of eigenvalues of a 2 × 2 real matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and −1 respectively, then $|p|$ is _______ (in integer).

ANS: -

---

## 2021

SET – 2

Q.1 Consider an $n \times n$ matrix A and a non-zero n × 1 vector p. Their product Ap = a²p, where a ∈ R and a ∈ {-1, 0, 1}. Based on the given information, the eigen value of A2 is:

**(A) α**

**(B) $a^2$**

**(C) $\sqrt{a}$**

**(D) $a^4$** (D) is the correct answer

---

## 2020

**SET – 1**

Q. 1 Multiplication of real valued squared matrices of same dimension is

(A) Associative (B) Commutative

(C) always positive definite (D) not always possible to compute

ANS: - A

---

**SET – 2**

Q. 2 A matrix P is decomposed into its symmetric part S and skew-symmetric part V.

If

$$S = \begin{bmatrix} -4 & 4 & 2 \\ 4 & 3 & 7/2 \\ 2 & 7/2 & 2 \end{bmatrix}, and\ V = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 7/2 \\ -3 & -7/2 & 0 \end{bmatrix}$$

Then Matrix P is:

(A) $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} 4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$ (D) $\begin{pmatrix} -2 & 9/2 & -1 \\ -1 & 81/4 & 11 \\ -2 & 45/2 & 73/4 \end{pmatrix}$

ANS: - B

Q. 19 Let I be a 100-dimensional identity matrix and E be set of distinct (no value appears more than once in E) real Eigen Values. The number of elements in E ____________ .

ANS: - 1

## 2019

**SET – 1**

Q.1 Consider the matrix

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The number of distinct eigenvalues of P is

(A) 0 (B) 1 (C) 2 (D) 3

ANS: - B

Q.26 The set of equations

$$\begin{aligned} x + y + z &= 1 \\ ax - ay + 3z &= 5 \\ 5x - 3y + az &= 6 \end{aligned}$$

has infinite solutions, if $a =$

(A) $-3$ (B) 3 (C) 4 (D) $-4$

ANS: - C

**SET – 2**

Q.1 In matrix equation $[A]\{X\}=\{R\}$,

$$[A]=\begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix},\ \{X\}=\begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\}=\begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}.$$

One of the eigenvalues of matrix $[A]$ is

(A) 4 (B) 8 (C) 15 (D) 16

ANS: - C

## 2018

**SET – 1**

Q.2

The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is

(A) 1 (B) 2 (C) 3 (D) 4

ANS: - B

---

**SET – 2**

Q.19

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is __________ (correct to two decimal places).

ANS: - 0.25

---

# Instrumentation Engineering

## 2025

| | |
|---|---|
| Q.11 | A $2n \times 2n$ matrix $A = [a_{ij}]$ has its elements as $$a_{ij} = \begin{cases} \beta & \text{if } (i+j) \text{ is odd,} \\ -\beta & \text{if } (i+j) \text{ is even,} \end{cases}$$ where $n$ is any integer greater than 2 and $\beta$ is any non-zero real number. The rank of $A$ is |
| | |
| (A) | 1 |
| (B) | 2 |
| (C) | $n$ |
| (D) | $2n$ |

ANS: - A

| | |
|---|---|
| Q.32 | If one of the eigenvectors of the matrix $A = \begin{bmatrix} -1 & -1 \\ x & -4 \end{bmatrix}$ is along the direction of $\begin{bmatrix} \alpha \\ 2\alpha \end{bmatrix}$, where $\alpha$ is any non-zero real number, then the value of $x$ is ______ **(in integer).** |

ANS: - 2

## 2024

| | |
|---|---|
| Q.19 | A matrix $M$ is constructed by stacking three column vectors $v_1, v_2, v_3$ as $$M = [v_1 \quad v_2 \quad v_3].$$ Choose the set of vectors from the following options such that $rank(M) = 3$. |
| | |
| (A) | $v_1 = \begin{bmatrix}1\\0\\1\end{bmatrix},\ v_2 = \begin{bmatrix}0\\-1\\0\end{bmatrix},\ v_3 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$ |
| (B) | $v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix},\ v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix},\ v_3 = \begin{bmatrix}0\\0\\0\end{bmatrix}$ |
| (C) | $v_1 = \begin{bmatrix}1\\0\\1\end{bmatrix},\ v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix},\ v_3 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$ |
| (D) | $v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix},\ v_2 = \begin{bmatrix}-1\\1\\-1\end{bmatrix},\ v_3 = \begin{bmatrix}0\\-1\\0\end{bmatrix}$ |

ANS: -

| | |
|---|---|
| Q.26 | A $3 \times 3$ matrix $P$ with all real elements has eigenvalues $\frac{1}{4}$, 1, and $-2$. The value of $\lvert P^{-1}\rvert$ is ________ (**rounded off to nearest integer**). |

ANS: -

## 2023

| | |
|---|---|
| Q.11 | Choose solution set $S$ corresponding to the systems of two equations<br>$x - 2y + z = 0$<br>$x - z = 0$<br>**Note**: $\Re$ denotes the set of real numbers |
| | |
| (A) | $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle\vert\, \alpha \in \Re \right\}$ |
| (B) | $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle\vert\, \alpha, \beta \in \Re \right\}$ |
| (C) | $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle\vert\, \alpha, \beta \in \Re \right\}$ |
| (D) | $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle\vert\, \alpha \in \Re \right\}$ |

ANS: - A

## 2022

| | |
|---|---|
| Q.24 | Given M = $\begin{bmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{bmatrix}$, which of the following statement(s) is/are correct? |
| (A) | The rank of M is 2 |
| (B) | The rank of M is 3 |
| (C) | The rows of M are linearly independent |
| (D) | The determinant of M is 0 |

ANS: - A, D

| | |
|---|---|
| Q.49 | The matrix $A = \begin{bmatrix} 4 & 3 \\ 9 & -2 \end{bmatrix}$ has eigenvalues $-5$ and 7. The eigenvector(s) is/are ________ |
| (A) | $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ |
| (B) | $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ |
| (C) | $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ |
| (D) | $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$ |

ANS: - A, C

## 2021

| | |
|---|---|
| **Q.1** | **Consider the row vectors $v = (1, 0)$ and $w = (2, 0)$. The rank of the matrix $M = 2v^T v + 3w^T w$, where the superscript $T$ denotes the transpose, is** |
| (A) | 1 |
| (B) | 2 |
| (C) | 3 |
| (D) | 4 |

ANS: - A

| | |
|---|---|
| **Q.25** | **The determinant of the matrix M shown below is ____.** $$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$ |

ANS: - 4

**Q.38** **Given A = $\begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix}$. The value of the determinant $|A^4 - 5A^3 + 6A^2 + 2I|$ = ___.**

ANS: - 4

## 2020

3] A set of linear equations is given in the form $Ax = b$, where $A$ is a $2 \times 4$ matrix with real number entries and $b \neq 0$. Will it be possible to solve for $x$ and obtain a **unique solution** by multiplying both left and right sides of the equation by $A^T$ (the super script $T$ denotes the transpose) and inverting the matrix $A^T A$ ? Answer is ___

(A) Yes, it is always possible to get a unique solution for any $2 \times 4$ matrix A.

(B) No, it is not possible to get a unique solution for any $2 \times 4$ matrix A.

(C) Yes, can obtain a unique solution provided the matrix $A^T A$ is well conditioned

(D) Yes, can obtain a unique solution provided the matrix $A$ is well conditioned

ANS: - B

**Q.No. 26** Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. One of the eigenvectors of $M$ is

(A) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

ANS: - D

## 2019

Q.16 A $3 \times 3$ matrix has eigenvalues 1, 2 and 5. The determinant of the matrix is ______ .

ANS: -

## 2018

Q.1 Let $N$ be a 3 by 3 matrix with real number entries. The matrix $N$ is such that $N^2 = 0$. The eigen values of $N$ are

(A) 0, 0, 0 (B) 0,0,1 (C) 0,1,1 (D) 1,1,1

ANS: - A

---

Q.28 Consider the following system of linear equations:

$$3x + 2ky = -2$$
$$kx + 6y = 2$$

Here $x$ and $y$ are the unknowns and $k$ is a real constant. The value of $k$ for which there are infinite number of solutions is

(A) 3 (B) 1 (C) $-3$ (D) $-6$

ANS: - C

---

## 2017

**Question Number : 1** **Correct : 1 Wrong : 0**

If $\mathbf{v}$ is a non-zero vector of dimension $3\times 1$, then the matrix $\mathbf{A} = \mathbf{v}\mathbf{v}^T$ has a rank =_______

ANS: - 1

---

**Question Number : 4** **Correct : 1 Wrong : -0.33**

The eigenvalues of the matrix $\mathbf{A} = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}$ are

(A) $-1,\ 5,\ 6$ (B) $1,\ -5 \pm j6$ (C) $1,\ 5 \pm j6$ (D) $1,\ 5,\ 5$

ANS: - C

---

## 2016

Q.28 Consider the matrix $\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ whose eigenvalues are $1, -1$ and $3$. Then Trace of $\left(\mathbf{A}^3 - 3\mathbf{A}^2\right)$ is _______.

ANS: - (-6)

---

# Data Science and Artificial Intelligence

## 2025

Q. 13 The sum of the elements in each row of $A \in \mathbb{R}^{n\times n}$ is 1. If $B = A^3 - 2A^2 + A$, which one of the following statements is correct (for $x \in \mathbb{R}^n$)?

(A) The equation $Bx = 0$ has no solution

(B) The equation $Bx = 0$ has exactly two solutions

(C) The equation $Bx = 0$ has infinitely many solutions

(D) The equation $Bx = 0$ has a unique solution

ANS: - C

---

Q. 25 Which of the following statements is/are correct?

(A) $\mathbb{R}^n$ has a unique set of orthonormal basis vectors

(B) $\mathbb{R}^n$ does not have a unique set of orthonormal basis vectors

(C) Linearly independent vectors in $\mathbb{R}^n$ are orthonormal

(D) Orthonormal vectors $\mathbb{R}^n$ are linearly independent

ANS: - B, D

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Q. 28 Let $A = I_n + xx^\top$, where $I_n$ is the $n \times n$ identity matrix and $x \in \mathbb{R}^n$, $x^\top x = 1$. Which of the following options is/are correct?

(A) Rank of $A$ is $n$

(B) $A$ is invertible

(C) 0 is an eigenvalue of $A$

(D) $A^{-1}$ has a negative eigenvalue

ANS: - A, B

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Q. 37 Let $A \in \mathbb{R}^{n \times n}$ be such that $A^3 = A$. Which one of the following statements is ALWAYS correct?

(A) $A$ is invertible

(B) Determinant of $A$ is 0

(C) The sum of the diagonal elements of $A$ is 1

(D) $A$ and $A^2$ have the same rank

ANS: - D

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Q. 38 Let $\{x_1, x_2, \ldots, x_n\}$ be a set of linearly independent vectors in $\mathbb{R}^n$. Let the $(i, j)$-th element of matrix $A \in \mathbb{R}^{n \times n}$ be given by $A_{ij} = x_i^\top x_j$, $1 \leq i, j \leq n$. Which one of the following statements is correct?

(A) $A$ is invertible

(B) 0 is a singular value of $A$

(C) Determinant of $A$ is 0

(D) $z^\top A z = 0$ for some non-zero $z \in \mathbb{R}^n$

ANS: - A

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Q. 50 Let $x_1, x_2, x_3, x_4, x_5$ be a system of orthonormal vectors in $\mathbb{R}^{10}$. Consider the matrix $A = x_1 x_1^\top + \ldots + x_5 x_5^\top$. Which of the following statements is/are correct?

(A) Singular values of $A$ are also its eigenvalues

(B) Singular values of $A$ are either 0 or 1

(C) Determinant of $A$ is 1

(D) $A$ is invertible

ANS: - A, B

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Q. 52 An $n \times n$ matrix $A$ with real entries satisfies the property: $\|Ax\|^2 = \|x\|^2$, for all $x \in \mathbb{R}^n$, where $\|\cdot\|$ denotes the Euclidean norm. Which of the following statements is/are ALWAYS correct?

(A) $A$ must be orthogonal

(B) $A = I$, where $I$ denotes the identity matrix, is the only solution

(C) The eigenvalues of $A$ are either $+1$ or $-1$

(D) $A$ has full rank

ANS: - A, D

## 2024

| | |
|---|---|
| Q.13 | Consider the matrix $\boldsymbol{M} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$.<br>Which **ONE** of the following statements is **TRUE**? |
| (A) | The eigenvalues of $\boldsymbol{M}$ are non-negative and real. |
| (B) | The eigenvalues of $\boldsymbol{M}$ are complex conjugate pairs. |
| (C) | One eigenvalue of $\boldsymbol{M}$ is positive and real, and another eigenvalue of $\boldsymbol{M}$ is zero. |
| (D) | One eigenvalue of $\boldsymbol{M}$ is non-negative and real, and another eigenvalue of $\boldsymbol{M}$ is negative and real. |

ANS: -

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|---|---|
| Q.35 | Consider the $3 \times 3$ matrix $\boldsymbol{M} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 4 & 3 & 6 \end{bmatrix}$.<br>The determinant of $(\boldsymbol{M}^2 + 12\boldsymbol{M})$ is ______. |

ANS: -

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|---|---|
| Q.47 | Select all choices that are subspaces of $\mathbb{R}^3$.<br><br>**Note**: $\mathbb{R}$ denotes the set of real numbers. |
| (A) | $\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \; \mathbf{x} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \alpha, \beta \in \mathbb{R}\right\}$ |
| (B) | $\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \; \mathbf{x} = \alpha^2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \beta^2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R}\right\}$ |
| (C) | $\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \; 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0\right\}$ |
| (D) | $\left\{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \; 5x_1 + 2x_3 + 4 = 0\right\}$ |

ANS: -

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|---|---|
| Q.48 | Which of the following statements is/are **TRUE**?<br><br>**Note**: $\mathbb{R}$ denotes the set of real numbers. |
| (A) | There exist $\boldsymbol{M} \in \mathbb{R}^{3\times3}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions. |
| (B) | There exist $\boldsymbol{M} \in \mathbb{R}^{3\times3}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has no solutions and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions. |
| (C) | There exist $\boldsymbol{M} \in \mathbb{R}^{2\times3}, \boldsymbol{p} \in \mathbb{R}^2$, and $\boldsymbol{q} \in \mathbb{R}^2$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has infinite solutions. |
| (D) | There exist $\boldsymbol{M} \in \mathbb{R}^{3\times2}, \boldsymbol{p} \in \mathbb{R}^3$, and $\boldsymbol{q} \in \mathbb{R}^3$ such that $\boldsymbol{M}\mathbf{x} = \boldsymbol{p}$ has a unique solution and $\boldsymbol{M}\mathbf{x} = \boldsymbol{q}$ has no solutions. |

ANS: -

| | |
|---|---|
| Q.49 | Let $\mathbb{R}$ be the set of real numbers, $U$ be a subspace of $\mathbb{R}^3$ and $\boldsymbol{M} \in \mathbb{R}^{3\times3}$ be the matrix corresponding to the projection on to the subspace $U$.<br><br>Which of the following statements is/are **TRUE**? |
| (A) | If $U$ is a 1-dimensional subspace of $\mathbb{R}^3$, then the null space of $\boldsymbol{M}$ is a 1-dimensional subspace. |
| (B) | If $U$ is a 2-dimensional subspace of $\mathbb{R}^3$, then the null space of $\boldsymbol{M}$ is a 1-dimensional subspace. |
| (C) | $\boldsymbol{M}^2 = \boldsymbol{M}$ |
| (D) | $\boldsymbol{M}^3 = \boldsymbol{M}$ |

ANS: -

Q.61 Let $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$, and let $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ be the singular values of the matrix

$\boldsymbol{M} = \boldsymbol{u}\boldsymbol{u}^T$ (where $\boldsymbol{u}^T$ is the transpose of $\boldsymbol{u}$). The value of $\sum_{i=1}^{5} \sigma_i$ is ______.

ANS: -