GATE

Linear Algebra

PYS's and Solution

VARUN KUMAR

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CSE

2025

S1

Q. Consider the given system of linear equations for variables x and y, where k is a real-valued constant. Which of the following option(s) is/are CORRECT?

$$x + ky = 1$$

$$kx + y = -1$$

a. There is exactly one value of k for which the above system of equations has no solution.
b. There exist an infinite number of values of k for which the system of equations has no solution.
c. There exists exactly one value of k for which the system of equations has exactly one solution.
d. There exists exactly one value of k for which the system of equations has an infinite number of solutions.

ANS: - a, d

Q. Let A be a 2 × 2 matrix as given.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

What are the eigenvalues of the matrix A^{13}?

a. 1, −1
b. 2√2, −2√2
c. 4√2, −4√2
d. 64√2, −64√2

ANS: - d

S2

Q. If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ then which ONE of the following is A^8?

a. $\begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}$
b. $\begin{pmatrix} 125 & 0 \\ 0 & 125 \end{pmatrix}$
c. $\begin{pmatrix} 625 & 0 \\ 0 & 625 \end{pmatrix}$
d. $\begin{pmatrix} 3125 & 0 \\ 0 & 3125 \end{pmatrix}$

ANS: - c

Q. Let L, M, and N be non-singular matrices of order 3 satisfying the equations

$$L^2 = L^{-1}, M = L^8, \qquad and \ N = L^2$$

Which ONE of the following is the value of the determinant of $(M - N)$?

a. 0
b. 1
c. 2
d. 3

ANS: - a

Q. Consider a system of linear equations $PX = Q$ where $P \in \mathbb{R}^{3 \, X \, 3}$ and $Q \in \mathbb{R}^{3 \, X \, 1}$. Suppose P has an LU decomposition, $P = LU$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Which of the following statement(s) is/are TRUE?

a. The system $PX = Q$ can be solved by first solving $LY = Q$ and then $UX = Y$.
b. If P is invertible, then both L and U are invertible
c. If P is singular, then at least one of the diagonal elements of U is zero.
d. If P is symmetric, then both L and U are symmetric.

ANS: - a, b, c

2024

Q.12 The product of all eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

(A) -1

(B) 0

(C) 1

(D) 2

Q.49 Let A be any $n \times m$ matrix, where $m > n$. Which of the following statements is/are TRUE about the system of linear equations $Ax = \mathbf{0}$?

(A) There exist at least $m - n$ linearly independent solutions to this system

(B) There exist $m - n$ linearly independent vectors such that every solution is a linear combination of these vectors

(C) There exists a non-zero solution in which at least $m - n$ variables are 0

(D) There exists a solution in which at least n variables are non-zero

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S2

Q.47 Let A be an $n \times n$ matrix over the set of all real numbers $\mathbb{R}$. Let B be a matrix obtained from A by swapping two rows. Which of the following statements is/are TRUE?

(A) The determinant of B is the negative of the determinant of A

(B) If A is invertible, then B is also invertible

(C) If A is symmetric, then B is also symmetric

(D) If the trace of A is zero, then the trace of B is also zero

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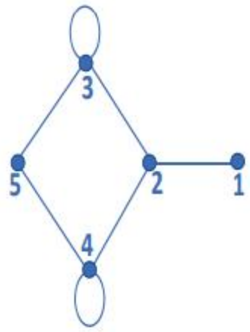
2023

Q.18	Let $$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$ and $$B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}.$$ Let $\texttt{det}(A)$ and $\texttt{det}(B)$ denote the determinants of the matrices A and B, respectively. Which one of the options given below is TRUE?
(A)	$\texttt{det}(A) = \texttt{det}(B)$
(B)	$\texttt{det}(B) = -\texttt{det}(A)$
(C)	$\texttt{det}(A) = 0$
(D)	$\texttt{det}(AB) = \texttt{det}(A) + \texttt{det}(B)$

Q.30	Let A be the adjacency matrix of the graph with vertices $\{1, 2, 3, 4, 5\}$.



Let λ_1, λ_2, λ_3, λ_4, and λ_5 be the five eigenvalues of A. Note that these eigenvalues need not be distinct.
The value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 =$ ___________.

2022

Q.20	Consider the following two statements with respect to the matrices $A_{m\times n}$, $B_{n\times m}$, $C_{n\times n}$ and $D_{n\times n}$. Statement 1: $tr(AB) = tr(BA)$ Statement 2: $tr(CD) = tr(DC)$ where $tr()$ represents the trace of a matrix. Which one of the following holds?
(A)	Statement 1 is correct and Statement 2 is wrong.
(B)	Statement 1 is wrong and Statement 2 is correct.
(C)	Both Statement 1 and Statement 2 are correct.
(D)	Both Statement 1 and Statement 2 are wrong.

Q.37	Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of
(A)	A^3
(B)	A^3 divided by 2
(C)	A^3 divided by 3
(D)	A^3 divided by 6

Q.45 Consider solving the following system of simultaneous equations using LU decomposition.

$$\begin{aligned} x_1 + x_2 - 2x_3 &= 4 \\ x_1 + 3x_2 - x_3 &= 7 \\ 2x_1 + x_2 - 5x_3 &= 7 \end{aligned}$$

where L and U are denoted as

$$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?

(A)	$L_{32} = 2,\ U_{33} = -\frac{1}{2},\ x_1 = -1$
(B)	$L_{32} = 2,\ U_{33} = 2,\ x_1 = -1$
(C)	$L_{32} = -\frac{1}{2},\ U_{33} = 2,\ x_1 = 0$
(D)	$L_{32} = -\frac{1}{2},\ U_{33} = -\frac{1}{2},\ x_1 = 0$

Q.53	Which of the following is/are the eigenvector(s) for the matrix given below? $$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$
(A)	$\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$
(B)	$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$
(C)	$\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$
(D)	$\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

2021, S-1

Q.52	Consider the following matrix. $$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$ The largest eigenvalue of the above matrix is __________.

ANS: - 3

2021, S-2

Q.24 Suppose that P is a 4×5 matrix such that every solution of the equation $P\mathbf{x} = \mathbf{0}$ is a scalar multiple of $[2\ \ 5\ \ 4\ \ 3\ \ 1]^T$. The rank of P is ________.

ANS: - 4

2020

Q.No. 27 Let A and B be two $n \times n$ matrices over real numbers. Let rank(M) and det(M) denote the rank and determinant of a matrix M, respectively. Consider the following statements.

I. $\text{rank}(AB) = \text{rank}(A)\,\text{rank}(B)$

II. $\det(AB) = \det(A)\det(B)$

III. $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$

IV. $\det(A + B) \leq \det(A) + \det(B)$

Which of the above statements are TRUE?

(A) I and II only

(B) I and IV only

(C) II and III only

(D) III and IV only

ANS: - C

2019

Q.9 Let X be a square matrix. Consider the following two statements on X.

I. X is invertible.

II. Determinant of X is non-zero.

Which one of the following is TRUE?

(A) I implies II; II does not imply I.
(B) II implies I; I does not imply II.
(C) I does not imply II; II does not imply I.
(D) I and II are equivalent statements.

ANS: - D

Q.44 Consider the following matrix:

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of R is _________.

ANS: - 12

2018

Q.17 Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that v^T denotes the transpose of v. The largest eigenvalue of A is _____.

ANS: - 3

Q.26 Consider a matrix $\boldsymbol{P}$ whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements.

(I) $\boldsymbol{P}$ does not have an inverse
(II) $\boldsymbol{P}$ has a repeated eigenvalue
(III) $\boldsymbol{P}$ cannot be diagonalized

Which one of the following options is correct?

(A) Only I and III are necessarily true
(B) Only II is necessarily true
(C) Only I and II are necessarily true
(D) Only II and III are necessarily true

ANS: - D

ECE

2025

Q.11 Consider the matrix A below:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

For which of the following combinations of α, β, and γ, is the rank of A at least three?

(i) $\alpha = 0$ and $\beta = \gamma \neq 0$.
(ii) $\alpha = \beta = \gamma = 0$.
(iii) $\beta = \gamma = 0$ and $\alpha \neq 0$.
(iv) $\alpha = \beta = \gamma \neq 0$.

(A) Only (i), (iii), and (iv)

(B) Only (iv)

(C) Only (ii)

(D) Only (i) and (iii)

ANS: - A

2024

Q.30 Let $\mathbb{R}$ and $\mathbb{R}^3$ denote the set of real numbers and the three dimensional vector space over it, respectively. The value of α for which the set of vectors

$$\{[2 \quad -3 \quad \alpha], \quad [3 \quad -1 \quad 3], \quad [1 \quad -5 \quad 7]\}$$

does not form a basis of $\mathbb{R}^3$ is _______.

ANS: - 5 (Check the answer)

Q.55 Consider the matrix $\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$, where k is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(A) $\begin{bmatrix} 1 \\ -\sqrt{2/k} \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ \sqrt{2/k} \end{bmatrix}$

(C) $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

ANS: - A, B (check the answer)

2023

Q.15	Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1 \le k \le n\}$ and $\{\boldsymbol{v}_k \mid 1 \le k \le n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B = P^{-1}AP$, respectively, are
(A)	$\{\lambda_k \det(A) \mid 1 \le k \le n\}$ and $\{P\boldsymbol{v}_k \mid 1 \le k \le n\}$
(B)	$\{\lambda_k \mid 1 \le k \le n\}$ and $\{\boldsymbol{v}_k \mid 1 \le k \le n\}$
(C)	$\{\lambda_k \mid 1 \le k \le n\}$ and $\{P\boldsymbol{v}_k \mid 1 \le k \le n\}$
(D)	$\{\lambda_k \mid 1 \le k \le n\}$ and $\{P^{-1}\boldsymbol{v}_k \mid 1 \le k \le n\}$

ANS: -

Q.38	Let $\boldsymbol{x}$ be an $n \times 1$ real column vector with length $l = \sqrt{\boldsymbol{x}^T \boldsymbol{x}}$. The trace of the matrix $P = \boldsymbol{x}\boldsymbol{x}^T$ is
(A)	l^2
(B)	$\frac{l^2}{4}$
(C)	l
(D)	$\frac{l^2}{2}$

ANS: -

2022

Q.12	Consider a system of linear equations $Ax = b$, where $A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$ This system of equations admits __________.
(A)	a unique solution for x
(B)	infinitely many solutions for x
(C)	no solutions for x
(D)	exactly two solutions for x

ANS: -

2021

Q.16 **If the vectors $(1.0, -1.0, 2.0)$, $(7.0, 3.0, x)$ and $(2.0, 3.0, 1.0)$ in $\mathbb{R}^3$ are linearly dependent, the value of x is __________**

ANS: -

Q.36 **A real 2×2 non-singular matrix A with repeated eigenvalue is given as**

$$\mathbf{A} = \begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

where x is a real positive number. The value of x (*rounded off to one decimal place*) is __________

ANS: -

2020

Q.No. 1 If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_6$ are six vectors in $\mathbb{R}^4$, which one of the following statements is FALSE?

(A) It is not necessary that these vectors span $\mathbb{R}^4$.

(B) These vectors are not linearly independent.

(C) Any four of these vectors form a basis for $\mathbb{R}^4$.

(D) If $\{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_5, \mathbf{v}_6\}$ spans $\mathbb{R}^4$, then it forms a basis for $\mathbb{R}^4$.

ANS: - C

Q.No. 26 Consider the following system of linear equations.

$$x_1 + 2x_2 = b_1 \;;\quad 2x_1 + 4x_2 = b_2 \;;\quad 3x_1 + 7x_2 = b_3 \;;\quad 3x_1 + 9x_2 = b_4$$

Which one of the following conditions ensures that a solution exists for the above system?

(A) $b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

(B) $b_3 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

(C) $b_2 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

(D) $b_3 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

ANS: - A

2019

Q.17 The number of distinct eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to ______.

ANS: - 3

2018

Q.11 Let $\mathbf{M}$ be a real 4×4 matrix. Consider the following statements:

S1: $\mathbf{M}$ has 4 linearly independent eigenvectors.

S2: $\mathbf{M}$ has 4 distinct eigenvalues.

S3: $\mathbf{M}$ is non-singular (invertible).

Which one among the following is TRUE?

(A) S1 implies S2 (B) S1 implies S3

(C) S2 implies S1 (D) S3 implies S2

ANS: - C

Q.22 Consider matrix $\mathbf{A} = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions is _______.

ANS: - 2

EEE

2025

Q.12	Let $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ be the two eigenvectors corresponding to distinct eigenvalues of a 3×3 real symmetric matrix. Which one of the following statements is true?
(A)	$\boldsymbol{v}_1^T\boldsymbol{v}_2 \neq 0$
(B)	$\boldsymbol{v}_1^T\boldsymbol{v}_2 = 0$
(C)	$\boldsymbol{v}_1 + \boldsymbol{v}_2 = \boldsymbol{0}$
(D)	$\boldsymbol{v}_1 - \boldsymbol{v}_2 = \boldsymbol{0}$

ANS: - B

Q.13	Let $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, and $\boldsymbol{b} = \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \end{bmatrix}$. Then, the system of linear equations $\boldsymbol{Ax} = \boldsymbol{b}$ has
(A)	a unique solution.
(B)	infinitely many solutions.
(C)	a finite number of solutions.
(D)	no solution.

ANS: - B

Q.14	Let $P = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and let I be the identity matrix. Then P^2 is equal to
(A)	$2P - I$
(B)	P
(C)	I
(D)	$P + I$

ANS: - A

2024

Q.11	Which one of the following matrices has an inverse?
(A)	$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$
(B)	$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$
(C)	$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$
(D)	$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$

ANS: -

Q.32 The sum of the eigenvalues of the matrix A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is ______ (rounded off to the nearest integer).

ANS: -

2023

Q.11	For a given vector $\mathbf{w} = [1\ 2\ 3]^T$, the vector normal to the plane defined by $\mathbf{w}^T\mathbf{x} = 1$ is
(A)	$[-2 \quad -2 \quad 2]^T$
(B)	$[3 \quad 0 \quad -1]^T$
(C)	$[3 \quad 2 \quad 1]^T$
(D)	$[1 \quad 2 \quad 3]^T$

ANS: -

2022

Q.20 Consider a 3 x 3 matrix A whose (i, j)-th element, $a_{i,j} = (i - j)^3$. Then the matrix A will be

(A) symmetric.

(B) skew-symmetric.

(C) unitary.

(D) null.

ANS: -

Q.42 Consider a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

The matrix A satisfies the equation $6A^{-1} = A^2 + cA + dI$, where c and d are scalars and I is the identity matrix.

Then $(c + d)$ is equal to

(A) 5

(B) 17

(C) -6

(D) 11

ANS: -

2021

Q.1	**Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are**
(A)	1 and 1
(B)	1 and -1
(C)	j and $-j$
(D)	pq and $-pq$

ANS: - B

Q.38 **Let $\mathbf{A}$ be a 10×10 matrix such that $\mathbf{A}^5$ is a null matrix, and let $\mathbf{I}$ be the 10×10 identity matrix. The determinant of $\mathbf{A} + \mathbf{I}$ is ________________.**

ANS: - 1

2020

Q.No. 42 The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is ___________.

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(A) 5
(B) 6
(C) 8
(D) 9

ANS: - MTA

2019

Q.2 M is a 2×2 matrix with eigenvalues 4 and 9. The eigenvalues of M^2 are

(A) 4 and 9 (B) 2 and 3 (C) -2 and -3 (D) 16 and 81

ANS: - D

Q.24 The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, is _________.

ANS: - 3

Q.26 Consider a 2×2 matrix $\boldsymbol{M} = [\boldsymbol{v_1} \quad \boldsymbol{v_2}]$, where, $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are the column vectors. Suppose $\boldsymbol{M^{-1}} = \begin{bmatrix} \boldsymbol{u_1^T} \\ \boldsymbol{u_2^T} \end{bmatrix}$, where $\boldsymbol{u_1^T}$and $\boldsymbol{u_2^T}$ are the row vectors. Consider the following statements:

Statement 1: $\boldsymbol{u_1^T v_1} = \mathbf{1}$ and $\boldsymbol{u_2^T v_2} = \mathbf{1}$
Statement 2: $\boldsymbol{u_1^T v_2} = \mathbf{0}$ and $\boldsymbol{u_2^T v_1} = \mathbf{0}$

Which of the following options is correct?

(A) Statement 1 is true and statement 2 is false
(B) Statement 2 is true and statement 1 is false
(C) Both the statements are true
(D) Both the statements are false

ANS: - C

2018

Q.17 Consider a non-singular 2×2 square matrix $\mathbf{A}$. If $trace(\mathbf{A}) = 4$ and $trace(\mathbf{A}^2) = 5$, the determinant of the matrix $\mathbf{A}$ is _________(up to 1 decimal place).

ANS: - 5.5

Q.44 Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix. The determinant of B is ________ (up to 1 decimal place).

ANS: - 0.9 - 1.1

Civil

2025

CE 1

Q.11	Suppose λ is an eigenvalue of matrix A and x is the corresponding eigenvector. Let x also be an eigenvector of the matrix $B = A - 2I$, where I is the identity matrix. Then, the eigenvalue of B corresponding to the eigenvector x is equal to
(A)	λ
(B)	$\lambda + 2$
(C)	2λ
(D)	$\lambda - 2$

ANS: - D

Q.12	Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ -2 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. For $Ax = b$ to be solvable, which one of the following options is the *correct* condition on b_1, b_2, and b_3:
(A)	$b_1 + b_2 + b_3 = 1$
(B)	$3b_1 + b_2 + 2b_3 = 0$
(C)	$b_1 + 3b_2 + b_3 = 2$
(D)	$b_1 + b_2 + b_3 = 2$

ANS: - B

CE – 2

Q.11	For the matrix $[A]$ given below, the transpose is ______.
	$[A] = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$
(A)	$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$
(B)	$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}$
(C)	$\begin{bmatrix} 4 & 2 & 3 \\ 5 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$
(D)	$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$

ANS: - A

Q.45	Pick the **CORRECT** eigen value(s) of the matrix $[A]$ from the following choices.
	$[A] = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$
(A)	10
(B)	4
(C)	-2
(D)	-10

ANS: - A, C

Mechanical

2025

Q.11	Let **A** and **B** be real symmetric matrices of same size. Which one of the following options is correct?
(A)	$\mathbf{A}^T = \mathbf{A}^{-1}$
(B)	$\mathbf{AB} = \mathbf{BA}$
(C)	$(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$
(D)	$\mathbf{A} = \mathbf{A}^{-1}$

ANS: - C

Instrumentation Engineering

2025

Q.11	A $2n \times 2n$ matrix $A = [a_{ij}]$ has its elements as $$a_{ij} = \begin{cases} \beta & \text{if } (i+j) \text{ is odd,} \\ -\beta & \text{if } (i+j) \text{ is even,} \end{cases}$$ where n is any integer greater than 2 and β is any non-zero real number. The rank of A is
(A)	1
(B)	2
(C)	n
(D)	$2n$

ANS: - A

Q.32	If one of the eigenvectors of the matrix $A = \begin{bmatrix} -1 & -1 \\ x & -4 \end{bmatrix}$ is along the direction of $\begin{bmatrix} \alpha \\ 2\alpha \end{bmatrix}$, where α is any non-zero real number, then the value of x is ______ **(in integer).**

ANS: - 2

Data Science and Artificial Intelligence

2025

Q. 13 The sum of the elements in each row of $A \in \mathbb{R}^{n\times n}$ is 1. If $B = A^3 - 2A^2 + A$, which one of the following statements is correct (for $x \in \mathbb{R}^n$)?

(A) The equation $Bx = 0$ has no solution

(B) The equation $Bx = 0$ has exactly two solutions

(C) The equation $Bx = 0$ has infinitely many solutions

(D) The equation $Bx = 0$ has a unique solution

ANS: - C

Q. 25 Which of the following statements is/are correct?

(A) $\mathbb{R}^n$ has a unique set of orthonormal basis vectors

(B) $\mathbb{R}^n$ does not have a unique set of orthonormal basis vectors

(C) Linearly independent vectors in $\mathbb{R}^n$ are orthonormal

(D) Orthonormal vectors $\mathbb{R}^n$ are linearly independent

ANS: - B, D

Q. 28 Let $A = I_n + xx^\top$, where I_n is the $n \times n$ identity matrix and $x \in \mathbb{R}^n$, $x^\top x = 1$. Which of the following options is/are correct?

(A) Rank of A is n

(B) A is invertible

(C) 0 is an eigenvalue of A

(D) A^{-1} has a negative eigenvalue

ANS: - A, B

Q. 37 Let $A \in \mathbb{R}^{n \times n}$ be such that $A^3 = A$. Which one of the following statements is ALWAYS correct?

(A) A is invertible

(B) Determinant of A is 0

(C) The sum of the diagonal elements of A is 1

(D) A and A^2 have the same rank

ANS: - D

Q. 38 Let $\{x_1, x_2, \ldots, x_n\}$ be a set of linearly independent vectors in $\mathbb{R}^n$. Let the (i, j)-th element of matrix $A \in \mathbb{R}^{n \times n}$ be given by $A_{ij} = x_i^\top x_j$, $1 \leq i, j \leq n$. Which one of the following statements is correct?

(A) A is invertible

(B) 0 is a singular value of A

(C) Determinant of A is 0

(D) $z^\top A z = 0$ for some non-zero $z \in \mathbb{R}^n$

ANS: - A

Q. 50 Let x_1, x_2, x_3, x_4, x_5 be a system of orthonormal vectors in $\mathbb{R}^{10}$. Consider the matrix $A = x_1 x_1^\top + \ldots + x_5 x_5^\top$. Which of the following statements is/are correct?

(A) Singular values of A are also its eigenvalues

(B) Singular values of A are either 0 or 1

(C) Determinant of A is 1

(D) A is invertible

ANS: - A, B

Q. 52 An $n \times n$ matrix A with real entries satisfies the property: $\|Ax\|^2 = \|x\|^2$, for all $x \in \mathbb{R}^n$, where $\|\cdot\|$ denotes the Euclidean norm. Which of the following statements is/are ALWAYS correct?

(A) A must be orthogonal

(B) $A = I$, where I denotes the identity matrix, is the only solution

(C) The eigenvalues of A are either +1 or -1

(D) A has full rank

ANS: - A, D