GATE and GATE Type Question

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Contents

1 Cache Memory and Array Access

Q1. 1D Array in Direct-Mapped Cache

A machine has a byte-addressable main memory of **64 KB** and uses a **direct-mapped cache** with **128 lines**, each of **16 bytes**. A one-dimensional array $A[512]$ of bytes is stored in main memory starting from address `0x0000`. The cache is initially empty. The array is accessed sequentially **twice**. Assume that the cache contents **do not change** between the two accesses.

Q1.1. How many **data misses** will occur in total?

Q1.2. How many **cache lines** will be **replaced** during the second traversal of the array?

Q2. 2D Array: Row-wise vs Column-wise Access

A byte-addressable system has a main memory of **1 MB**. The system uses a **direct-mapped cache** with **256 cache lines**, and a **block size** of **8 bytes**.

A 2D array $A[64][8]$ of bytes is stored in **row-major** order starting at address `0x1000`. The cache is initially empty. All elements are accessed **exactly once** in two phases:

- First: **Row-wise access** (i.e., $A[0][0], A[0][1], \ldots, A[1][0], \ldots$)
- Then: **Column-wise access** (i.e., $A[0][0], A[1][0], \ldots, A[0][1], \ldots$)

Q2.1. How many **cache misses** occur in the **row-wise access**?

Q2.2. How many **cache misses** occur in the **column-wise access**?

Q30 Conflict Misses in Direct-Mapped Cache

A system has:

- Byte-addressable memory of **256 KB**
- A **direct-mapped cache** with **64 lines**
- **Block size = 8 bytes**

A 2D byte array $B[128][16]$ is stored starting from address `0x0000`. The array is accessed **row by row**, and each element is accessed exactly once.

Q3.1. How many **cache lines** will be used during this traversal?

Q3.2. How many **conflict misses** will occur due to cache line replacements?

2 Pipelining: GATE-style Numerical and Conceptual Questions

Q1 [GATE 2021 Style]

Consider a pipelined processor with 5 stages: Instruction Fetch (IF), Instruction Decode (ID), Execute (EX), Memory Access (MEM), and Write Back (WB). Each stage takes 1 cycle, except for EX, which takes 1 cycle for ADD and 2 cycles for MUL. Assume that the EX stage performs register reads as well. The following sequence of 8 instructions is executed:

ADD, MUL, ADD, MUL, ADD, MUL, ADD, MUL

Each MUL depends on the preceding ADD, and each ADD (except the first) depends on the previous MUL.

Assuming in-order execution:

a. Calculate the execution time with operand forwarding.

b. Calculate the execution time without operand forwarding (stall cycles inserted).

c. Compute the *speedup* as:

$$\text{Speedup} = \frac{\text{Time without forwarding}}{\text{Time with forwarding}}$$

Round to two decimal places.

Q2 [GATE 2018 Style]

The instruction pipeline of a RISC processor has the following stages: Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO), and Writeback (WB). The IF, ID, OF, and WB stages take 1 cycle each. Consider a sequence of 100 instructions. In the PO stage:

- 40 instructions take 3 cycles,
- 35 instructions take 2 cycles,
- 25 instructions take 1 cycle.

Assume no hazards.

Q: What is the total number of cycles to complete all 100 instructions?

Q3

In a non-interleaved 5-stage pipeline, each stage takes 1 cycle. There are 2 branches in the instruction sequence of 20 instructions. The branch penalty is 2 cycles (i.e., two flushes).

Q: How many cycles are needed to complete the program assuming ideal conditions otherwise?

Q4

A 4-stage pipeline with stages: IF, ID, EX, WB, executes n instructions. The pipeline starts with empty registers. There are k RAW data hazards, each causing 1 stall.

Q: Give a formula to compute the total number of cycles to execute n instructions.

Q5

Assume a 5-stage pipelined processor with full forwarding and hazard detection. An instruction requires a stall if its source operand is produced by a load immediately before it.

Given the instruction sequence:

```
LOAD R1, 0(R2)
ADD R3, R1, R4
MUL R5, R3, R6
```

Q: How many stalls are required and what is the total number of cycles needed?

Q6

Consider a pipeline with the following execution times per stage:

IF: 1, ID: 1, EX: 2, MEM: 1, WB: 1

A sequence of 10 instructions with no dependencies is run. Assume full pipelining and no stalls.

Q: What is the total time taken to complete all instructions?

Q7

In a 6-stage pipeline, a branch instruction is resolved in the fourth stage. The branch penalty is 3 cycles. The branch frequency in a program is 20%. The program consists of 1000 instructions.

Q: Estimate the total number of cycles lost due to branch penalty.

Q8.

A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO), and Write Operand (WO) stages. The IF, ID, OF, and WO stages take 1 clock cycle each for any instruction. The PO stage takes:

- 1 cycle for `ADD`, `SUB`
- 3 cycles for `MUL`
- 6 cycles for `DIV`

Operand forwarding is used.

What is the total number of clock cycles required to execute the following instruction sequence?

Instruction	Meaning
I_0: `MUL R2, R0, R1`	$R_2 \leftarrow R_0 \times R_1$
I_1: `DIV R5, R3, R4`	$R_5 \leftarrow R_3 \div R_4$
I_2: `ADD R2, R5, R2`	$R_2 \leftarrow R_5 + R_2$
I_3: `SUB R5, R2, R6`	$R_5 \leftarrow R_2 - R_6$

Table 1: Instruction Sequence with Meaning

(a) 13

(b) 15

(c) 17

(d) 19

Q9.

Same processor as in Q8. What is the total number of clock cycles required to execute:

Instruction	Meaning
I_0: `ADD R1, R2, R3`	$R_1 \leftarrow R_2 + R_3$
I_1: `MUL R4, R1, R5`	$R_4 \leftarrow R_1 \times R_5$
I_2: `DIV R6, R4, R1`	$R_6 \leftarrow R_4 \div R_1$
I_3: `SUB R7, R6, R8`	$R_7 \leftarrow R_6 - R_8$

Table 2: Instruction Sequence with Semantic Meaning

(a) 14 (c) 20

(b) 17 (d) 23

Q10.

Same setup. Evaluate the pipeline execution time (in clock cycles) for the following:

Instruction	Meaning
I_0: `DIV R9, R1, R2`	$R_9 \leftarrow R_1 \div R_2$
I_1: `ADD R5, R3, R4`	$R_5 \leftarrow R_3 + R_4$
I_2: `MUL R9, R5, R6`	$R_9 \leftarrow R_5 \times R_6$
I_3: `SUB R1, R9, R7`	$R_1 \leftarrow R_9 - R_7$

Table 3: Instruction Sequence and Semantics

(a) 16 (c) 22

(b) 19 (d) 25

Q11.

Pipeline and instruction latencies same as above. Estimate clock cycles for:

Instruction	Meaning
I_0: `MUL R1, R2, R3`	$R_1 \leftarrow R_2 \times R_3$
I_1: `DIV R4, R1, R5`	$R_4 \leftarrow R_1 \div R_5$
I_2: `MUL R6, R4, R1`	$R_6 \leftarrow R_4 \times R_1$

Table 4: Instruction Sequence and Semantics

(a) 18 (c) 23

(b) 20 (d) 25

3 MCST Question

Q1. GATE 2021

Let G be a connected undirected weighted graph. Consider the following two statements.

S_1: There exists a minimum weight edge in G which is present in every minimum spanning tree of G.

S_2: If every edge in G has distinct weight, then G has a unique minimum spanning tree.

Which one of the following options is correct?

(a) Both S_1 and S_2 are true

(b) S_1 is true and S_2 is false

(c) S_1 is false and S_2 is true

(d) Both S_1 and S_2 are false

GATE 2020 (Shortest Path)

Let $G = (V, E)$ be a directed, weighted graph with weight function $w : E \to \mathbb{R}$. For some function $f : V \to \mathbb{R}$, for each edge $(u, v) \in E$, define $w'(u, v)$ as:

$$w'(u, v) = w(u, v) + f(u) - f(v)$$

Which one of the options completes the following sentence so that it is **TRUE**?

"The shortest paths in G under w are shortest paths under w' too, ________________".

(a) for every $f : V \to \mathbb{R}$

(b) if and only if $\forall u \in V,\ f(u)$ is positive

(c) if and only if $\forall u \in V,\ f(u)$ is negative

(d) if and only if $f(u)$ is the distance from s to u in the graph obtained by adding a new vertex s to G and edges of zero weight from s to every vertex of G

GATE 2019

Let G be any connected, weighted, undirected graph. Consider the following statements:

I. G has a unique minimum spanning tree, if no two edges of G have the same weight.

II. G has a unique minimum spanning tree, if for every cut of G, there is a unique minimum-weight edge crossing the cut.

Which of the following is/are **TRUE**?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

GATE 2016

Let $G = (V, E)$ be an undirected simple graph in which each edge has a distinct weight, and let e be a particular edge of G. Consider the following statements:

I. If e is the lightest edge of some cycle in G, then every MST of G includes e.

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e.

Which of the following is/are TRUE?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

4 Conceptual Questions on MCST

Question 1: Edge in Cycle

Let $G = (V, E)$ be a connected, undirected graph with **distinct edge weights**. Suppose $e \in E$ is the **heaviest edge** in some cycle C in G.

Which of the following is/are always true?

1. e is **not** present in any Minimum Spanning Tree (MST) of G.
2. Removing e from G **does not** increase the weight of the MST.

Options:

(a) Only 1 is true

(b) Only 2 is true

(c) Both 1 and 2 are true

(d) Neither 1 nor 2 is true

Question 2: Unique MST Conditions

Let $G = (V, E)$ be a connected, undirected graph. Consider the following statements:

S_1: If for **every cut** in G, the **minimum weight edge** crossing the cut is unique, then G has a unique MST.

S_2: If all edge weights are **distinct**, then G has a unique MST.

Which of the following is correct?

(a) Only S_1 is true

(b) Only S_2 is true

(c) Both S_1 and S_2 are true

(d) Neither S_1 nor S_2 is true

Question 3: Uniform Weight Increase

Let T be a Minimum Spanning Tree (MST) of a connected graph $G = (V, E)$ with **positive weights**. Suppose the weight of **every edge** in G is increased by the same constant $c > 0$.

Which of the following is true?

(a) The MST remains unchanged

(b) The MST may change

(c) The total weight of the MST increases, but the structure remains the same

(d) The MST becomes a Maximum Spanning Tree

Question 4: Edge Weight Decrease

Let $G = (V, E)$ be a graph with positive edge weights and a known MST T. Suppose the weight of an edge $e \notin T$ is **reduced**.

Which of the following statements is true?

(a) T remains the MST

(b) T is no longer the MST

(c) A new MST may or may not include e

(d) Edge weight decrease does not affect the MST

Question 5: Kruskal vs Prim

Let $G = (V, E)$ be a connected undirected graph with **distinct edge weights**. Consider the MSTs generated by **Kruskal's** and **Prim's** algorithms.

Which of the following statements is always true?

(a) Both algorithms always produce the same MST

(b) Kruskal's and Prim's may produce different MSTs

(c) The MST is unique, hence both will produce the same MST

(d) Prim's always chooses the minimum edge globally, unlike Kruskal's

5 Prim's Algorithm

Q1. Property of Prim's

Let $G = (V, E)$ be a connected undirected graph with non-negative weights. Prim's algorithm is run starting from vertex v_0. Suppose there are
two edges (v_0, u) and (v_0, w) with the same weight. Assume the priority queue is implemented using a min-heap that breaks ties arbitrarily.

Which of the following can happen?

(a) Prim's algorithm may produce different MSTs depending on tie-breaking.

(b) The output MST is always unique.

(c) The MST may not be connected.

(d) Prim's may skip some minimum weight edges.

Q2. Multiple MSTs with Same Weight

Let $G = (V, E)$ be a connected graph with **non-negative and non-distinct** edge weights. Prim's algorithm is run starting from vertex v_0. Which of the following is correct about **Prim's algorithm**?

(a) The MST is always unique

(b) It may return different MSTs depending on tie-breaking

(c) It always returns the lexicographically smallest MST

(d) It cannot be run if weights are not distinct

Q3. Disconnected Graph

Suppose $G = (V, E)$ is an undirected graph with more than one connected component. Prim's algorithm is run starting from a vertex in one component. What will happen?

(a) Prim's algorithm will find the MST of the entire graph

(b) Prim's algorithm will terminate with an error

(c) Prim's algorithm will compute the MST of the connected component containing the starting vertex

(d) Prim's will hang in an infinite loop

Q4. Start Vertex Effect

Let $G = (V, E)$ be a connected undirected graph with some repeated edge weights. Suppose Prim's algorithm is executed from two different starting vertices v_1 and v_2. Which of the following is **true**?

(a) The output MST may differ based on starting vertex

(b) The total cost of the MST may change

(c) The MST is always the same regardless of the starting vertex

(d) Prim's does not depend on the starting vertex

Q5. Dense Graphs vs Sparse Graphs

Let $G = (V, E)$ be a graph with $|V| = n$ and $|E| = m$, where $m = \mathcal{O}(n^2)$. Prim's algorithm is implemented using:

(i) Min-heap with adjacency list

(ii) Adjacency matrix

Which of the following is **correct** about the time complexities?

(a) (i): $\mathcal{O}(n \log n)$, (ii): $\mathcal{O}(n^2)$

(b) (i): $\mathcal{O}(m \log n)$, (ii): $\mathcal{O}(n^2)$

(c) Both: $\mathcal{O}(n^2)$

(d) Both: $\mathcal{O}(m + n)$

Q6. Early Termination in Prim's

Prim's algorithm maintains a priority queue of all vertices not yet in the MST and selects the minimum-weight edge connecting to the growing MST.

Suppose the algorithm stops as soon as $n - 1$ edges are picked. Which of the following is a potential risk?

(a) The resulting tree may not span all vertices

(b) Early stopping leads to higher weight than necessary

(c) The result is always a valid MST

(d) The number of edges picked may exceed $n - 1$

6 Kruskal's Algorithm

Q1. Disconnected Graphs

Let $G = (V, E)$ be an undirected graph that is not connected, and all edge weights are distinct. Kruskal's algorithm is run on G.

What does Kruskal's algorithm return?

(a) A spanning tree of G

(b) A minimum spanning forest (one tree per connected component)

(c) An error due to disconnection

(d) A partial spanning tree of minimum cost

Q2. Edge Case: Self-loops and Parallel Edges

Let $G = (V, E)$ be an undirected graph that contains self-loops and parallel edges. Kruskal's algorithm is applied to compute the MST.

Which of the following is true?

(a) Self-loops and parallel edges can both be part of the MST

(b) Self-loops are ignored but the lightest among parallel edges may be selected

(c) Kruskal's cannot be applied on graphs with parallel edges

(d) Self-loops are only included if they have zero weight

Q3. Sorting Stability and Equal Weights

Suppose Kruskal's algorithm is applied to a graph with multiple edges having the same weight. The sorting routine used is unstable (i.e., it may change the order of equal-weight elements).

Which of the following is correct?

(a) The resulting MST is always the same

(b) The structure of MST may vary, but total cost will be the same

(c) Kruskal's algorithm requires stable sorting to work correctly

(d) Kruskal's will skip some valid edges due to instability

Q4. Edge Case: Disconnected Graph

Let $G = (V, E)$ be an undirected graph that is disconnected and all edge weights are distinct. Kruskal's algorithm is applied on G.

What is the output?

(a) A spanning tree of the graph

(b) A minimum spanning forest (one tree per component)

(c) An error due to disconnection

(d) An empty set

Q5. Union-Find Optimization Impact

In Kruskal's algorithm, union-find with both path compression and union by rank is used. Which of the following best describes the impact?

(a) Time complexity becomes $\mathcal{O}(E \log V)$

(b) Time complexity becomes $\mathcal{O}(E + V \log V)$

(c) Time complexity becomes $\mathcal{O}(E\alpha(V))$, where α is inverse Ackermann function

(d) It doesn't affect the time complexity

Q6. Edge Case: Edge Reversal

Suppose all edge directions in a directed graph are reversed. Kruskal's algorithm is applied to the undirected version of both the original and reversed graphs.

Which of the following statements is correct?

(a) The MST structure changes completely

(b) The MST remains unchanged

(c) The cost of the MST changes

(d) Reversing edges affects Kruskal's only if weights are reversed

7 Dijkstra's Algorithm

Q1. Negative Edge Weights

Let $G = (V, E)$ be a directed graph where some edges have negative weights but no negative weight cycles. Dijkstra's algorithm is run from a source s.

Which of the following is true?

(a) Dijkstra's will compute correct shortest paths

(b) Dijkstra's will fail due to negative weights

(c) Dijkstra's may return incorrect shortest paths

(d) Dijkstra's works only if all weights are positive

Q2. Effect of Negative Edge Weights

Let $G = (V, E)$ be a directed graph where some edges have negative weights but there are no negative-weight cycles. Dijkstra's algorithm is run from source s.

Which of the following is true?

(a) Dijkstra's algorithm will return the correct shortest paths

(b) Dijkstra's algorithm may give incorrect results

(c) Dijkstra's will detect the negative edge and stop

(d) Dijkstra's can be used only for DAGs

Q3. Unreachable Vertices

Let $G = (V, E)$ be a directed graph with non-negative edge weights. Suppose Dijkstra's algorithm is executed from a source vertex s, and there exists a vertex v that is not reachable from s.

What is the value of $dist[v]$ after execution?

(a) 0

(b) -1

(c) Infinity

(d) The algorithm throws an error

Q4. Dijkstra on Undirected Graph with Zero Weights

Let $G = (V, E)$ be an undirected graph where every edge has weight zero. Dijkstra's algorithm is executed from source s.

Which of the following is true about the distances computed?

(a) All distances will be zero

(b) The algorithm fails because of zero weights

(c) Some distances may be non-zero

(d) Dijkstra only works with strictly positive edge weights

Q5. Priority Queue Implementation

In a standard implementation of Dijkstra's algorithm using a min-priority queue, the relaxation step might insert multiple entries for the same vertex.

Which of the following is a correct optimization?

(a) Use a visited set to ignore already-processed vertices

(b) Do not use a priority queue at all

(c) Use a stack instead of a queue

(d) Restart Dijkstra whenever a duplicate is found

Q6. Dijkstra vs BFS

Let G be an undirected graph where all edge weights are equal to 1. Consider running both Dijkstra's algorithm and Breadth-First Search (BFS) from the same source s.

Which of the following is true?

(a) Dijkstra and BFS will compute different shortest path trees

(b) BFS will be faster but less accurate than Dijkstra

(c) Both algorithms will compute the same shortest paths

(d) BFS does not work on weighted graphs

8 Bellman-Ford Algorithm

Q1. Negative Weight Cycles

Let $G = (V, E)$ be a directed graph with a negative weight cycle reachable from source s. Bellman-Ford is run to compute shortest paths from s.

What will the algorithm do?

(a) Return shortest paths ignoring the negative cycle

(b) Enter an infinite loop

(c) Detect the negative cycle and report it

(d) Overwrite distances indefinitely but never terminate

Q2. Negative Weight Cycle Detection

Let $G = (V, E)$ be a directed graph with a negative weight cycle reachable from source s. Bellman-Ford algorithm is run from s to compute shortest paths.

Which of the following is true?

(a) Bellman-Ford computes distances correctly for all nodes

(b) It fails to terminate in presence of negative cycles

(c) It detects and reports the presence of a negative cycle

(d) It skips edges that are part of a negative cycle

Q3. Relaxation Count

Let G have n vertices and m edges. What is the **minimum number of full edge relaxation passes** required in Bellman-Ford to guarantee correct single-source shortest paths in a graph with no negative cycles?

(a) n

(b) $n - 1$

(c) m

(d) Until all distances stop changing

Q4. Negative Weight Edges but No Cycles

Let $G = (V, E)$ be a directed graph with some negative edge weights, but no negative-weight cycles. Which algorithm(s) can be used to find shortest paths from a source s?

(a) Only Dijkstra

(b) Only Bellman-Ford

(c) Both Dijkstra and Bellman-Ford

(d) Neither Dijkstra nor Bellman-Ford

Q5. Disconnected Graph

Let $G = (V, E)$ be a directed graph with some unreachable vertices from the source s. What does Bellman-Ford return for such vertices?

(a) Assigns distance zero

(b) Assigns distance infinity

(c) Reports an error

(d) Assigns distance as the maximum weight path to that node

Q6. All Edge Weights Positive

Suppose a graph has all edge weights positive. Which of the following is true regarding Bellman-Ford?

(a) Bellman-Ford will fail since there are no negative weights

(b) Bellman-Ford will give incorrect output due to no negative cycles

(c) Bellman-Ford works but is slower than Dijkstra

(d) Bellman-Ford and Dijkstra produce different distances

9 Floyd-Warshall Algorithm

Q1. Path Reconstruction with Negative Cycles

Let $G = (V, E)$ be a directed graph that contains negative weight cycles. Floyd-Warshall is run to compute all-pairs shortest paths.

Which of the following is correct?

(a) Floyd-Warshall reports the presence of a negative cycle

(b) The algorithm correctly computes shortest distances for all pairs not affected by negative cycles

(c) The distance matrix may contain incorrect values

(d) Floyd-Warshall ignores cycles by default

Q2. Negative Weight Cycles

Let $G = (V, E)$ be a directed graph with a negative-weight cycle. Floyd-Warshall algorithm is used to compute all-pairs shortest paths.

Which of the following is correct?

(a) It reports the presence of a negative cycle

(b) It correctly computes distances for all unaffected pairs

(c) The output may contain incorrect values due to negative cycles

(d) It ignores negative cycles by design

Q3. Self-loop Update

Suppose Floyd-Warshall is run on a graph G with no self-loops initially. After the algorithm completes, some diagonal entries in the distance matrix become negative.

What does this imply?

(a) The graph has cycles of weight zero

(b) There is a negative-weight cycle in the graph

(c) The source node has distance zero to itself

(d) The graph contains multiple disconnected components

Q4. Initialization of Distance Matrix

Before running Floyd-Warshall, the distance matrix is initialized as:

$$dist[i][j] = \begin{cases} 0 & \text{if } i = j \\ w(i,j) & \text{if } (i,j) \in E \\ \infty & \text{otherwise} \end{cases}$$

Which of the following is true?

(a) This initialization is incorrect for undirected graphs

(b) It is correct and necessary for Floyd-Warshall

(c) It will fail in presence of negative edges

(d) Zero must be used for all edge weights

Q5. Graph with Disconnected Components

Let G be a directed graph with multiple disconnected components. Floyd-Warshall is run on G.

What will the distance between unreachable vertex pairs be in the final matrix?

(a) Zero

(b) Infinity

(c) -1

(d) Depends on the number of components

Q6. Space and Time Complexity

Which of the following correctly describes the time and space complexity of Floyd-Warshall algorithm for a graph with n vertices?

(a) Time: $\mathcal{O}(n^2)$, Space: $\mathcal{O}(n^2)$

(b) Time: $\mathcal{O}(n^3)$, Space: $\mathcal{O}(n^2)$

(c) Time: $\mathcal{O}(n^3)$, Space: $\mathcal{O}(n^3)$

(d) Time: $\mathcal{O}(nm)$, Space: $\mathcal{O}(n^2)$

10 Graph Algorithms on Different DS

Q1. Prim's Algorithm: Min-Heap vs Adjacency Matrix

Let $G = (V, E)$ be a dense connected undirected graph with $|V| = n$ and $|E| = \Theta(n^2)$. Prim's algorithm is implemented using:

(i) Binary Min-Heap with Adjacency List

(ii) Simple Array with Adjacency Matrix

Which of the following is correct?

(a) (i): $\mathcal{O}(n \log n)$, (ii): $\mathcal{O}(n^2)$

(b) (i): $\mathcal{O}(m \log n)$, (ii): $\mathcal{O}(n^2)$

(c) Both: $\mathcal{O}(n^2)$

(d) Both: $\mathcal{O}(m + n)$

Q2. Kruskal's Algorithm: Union-Find Variants

In Kruskal's algorithm, which Union-Find variant offers the best performance?

(a) Basic Union and Find without optimizations

(b) Union by Rank only

(c) Path Compression only

(d) Union by Rank with Path Compression

Q3. Dijkstra's Algorithm: Min-Heap vs Fibonacci Heap

Let $G = (V, E)$ be a sparse graph with n vertices and m edges. Dijkstra's algorithm is implemented using:

(i) Binary Min-Heap

(ii) Fibonacci Heap

What are their respective time complexities?

(a) (i): $\mathcal{O}((n + m) \log n)$, (ii): $\mathcal{O}(n \log n + m)$

(b) (i): $\mathcal{O}(n \log m)$, (ii): $\mathcal{O}(n + m \log n)$

(c) Both: $\mathcal{O}(n^2)$

(d) Both: $\mathcal{O}(m \log n)$

Q4. Bellman-Ford: Queue-Based vs Standard Relaxation

Bellman-Ford can be optimized using a queue-based version (like SPFA — Shortest Path Faster Algorithm).

Which of the following statements is true?

(a) Queue-based version always has better time complexity

(b) Queue-based version may reduce practical runtime but worst-case is still $\mathcal{O}(nm)$

(c) Queue-based version fails with negative weights

(d) Queue-based version always terminates faster

Q5. Floyd-Warshall: 1D vs 2D Arrays

Suppose Floyd-Warshall is implemented using a 2D matrix $D[n][n]$. If space is a constraint, a 1D flattened array or reuse of a single row can be used.

Which of the following is correct?

(a) Using 1D array reduces time complexity

(b) Space complexity can be reduced using 1D if full path reconstruction is not needed

(c) 1D and 2D arrays offer identical performance and space usage

(d) Floyd-Warshall cannot be implemented without 2D matrices

11 Algorithms: Sorting (Part 1)

Q1. Bubble Sort After Second Pass

Consider the array: `[5, 1, 4, 2, 8]`.

What will be the array after the 2^{nd} pass of **Bubble Sort**?

(a) $[1, 2, 4, 5, 8]$

(b) $[1, 4, 2, 5, 8]$

(c) $[1, 2, 4, 8, 5]$

(d) $[1, 2, 4, 5, 8]$

Concept: Bubble sort pushes the largest to the right. Each pass reduces the unsorted array by one from the end.

Q2. Selection Sort After Third Pass

Given array: `[64, 25, 12, 22, 11]`
What is the array after the 3rd pass of **Selection Sort**?

(a) $[11, 12, 22, 25, 64]$

(b) $[11, 12, 22, 64, 25]$

(c) $[11, 12, 22, 25, 64]$

(d) $[12, 11, 22, 25, 64]$

Concept: Selection sort places the smallest element at the beginning each time.

Q3. Insertion Sort After Second Pass

Array: `[3, 1, 4, 2]`
State of array after 2nd pass of **Insertion Sort**?

(a) $[1, 3, 4, 2]$

(b) $[1, 3, 2, 4]$

(c) $[3, 1, 2, 4]$

(d) $[1, 3, 2, 4]$

Concept: Insertion sort inserts the current element into the sorted prefix.

Q4. Bubble Sort Third Pass - Partially Sorted

Given: `[10, 7, 8, 9, 1, 5]`
What is the array after 3rd pass of **Bubble Sort**?

(a) $[7, 8, 1, 5, 9, 10]$

(b) $[7, 1, 5, 8, 9, 10]$

(c) $[7, 1, 5, 8, 10, 9]$

(d) $[1, 5, 7, 8, 9, 10]$

Q5. Insertion Sort Third Pass - Trick Question

Input: `[2, 4, 6, 3, 5, 1]`
What is the array after 3rd pass of **Insertion Sort**?

(a) $[2, 3, 4, 6, 5, 1]$

(b) $[2, 4, 6, 3, 5, 1]$

(c) $[2, 4, 6, 5, 3, 1]$

(d) $[2, 4, 6, 1, 3, 5]$

12 Algorithms: Sorting (Part 2)

Q1. Selection Sort – Final Output

Given the array: `[29, 10, 14, 37, 13]`, what is the array after complete Selection Sort?

(a) $[10, 13, 14, 29, 37]$

(b) $[29, 14, 13, 10, 37]$

(c) $[13, 10, 14, 29, 37]$

(d) $[10, 13, 14, 37, 29]$

Q2. Selection Sort – After Second Pass

Array: `[64, 25, 12, 22, 11]`. What is the array after 2nd pass of Selection Sort?

(a) $[11, 12, 25, 22, 64]$

(b) $[11, 12, 64, 25, 22]$

(c) $[11, 12, 22, 25, 64]$

(d) $[11, 12, 25, 64, 22]$

Q3. Selection Sort – Number of Swaps

How many swaps does Selection Sort perform in worst case on an array of n distinct elements?

(a) $O(n^2)$

(b) $O(n \log n)$

(c) $O(n)$

(d) $\Theta(n)$

Q4. Merge Sort – Final Output

Input array: `[38, 27, 43, 3, 9, 82, 10]`. What is the final sorted array?

(a) $[3, 9, 10, 27, 38, 43, 82]$

(b) $[3, 9, 10, 27, 38, 82, 43]$

(c) $[3, 10, 9, 27, 38, 43, 82]$

(d) $[27, 38, 3, 9, 10, 43, 82]$

Q5. Merge Sort – Number of Merges

How many merge operations are performed (total) on an array of 8 elements?

(a) 3

(b) 7

(c) 6

(d) 5

Q6. Merge Sort – Stability

Which of the following is true for Merge Sort?

(a) It is not stable

(b) It requires in-place merging

(c) It is stable and divides array recursively

(d) It uses pivot and partitioning

Q7. Quick Sort – Final Output

For array `[10, 7, 8, 9, 1, 5]` and pivot = last element, what is final sorted array?

(a) $[1, 5, 7, 8, 9, 10]$

(b) $[1, 5, 7, 9, 8, 10]$

(c) $[10, 9, 8, 7, 5, 1]$

(d) $[1, 5, 8, 9, 7, 10]$

Q8. Quick Sort – Pivot Partition (1st Pass)

Array: `[4, 3, 5, 2, 1]` with pivot = 1. What is array after 1st partition?

(a) $[1, 3, 5, 2, 4]$

(b) $[1, 4, 3, 5, 2]$

(c) $[1, 3, 5, 2, 4]$

(d) $[1, 2, 3, 4, 5]$

Q9. Quick Sort – Best Case Time

What is best case time complexity of Quick Sort?

(a) $O(n^2)$

(b) $O(n \log n)$

(c) $O(\log n)$

(d) $O(n)$

Q10. Bucket Sort – Input Range Suitability

Bucket Sort works best when input is:

(a) Uniformly distributed over known range

(b) Random integers with duplicates

(c) Sorted in descending order

(d) Contains large negative numbers

Q11. Bucket Sort – Time Complexity (Best Case)

Best case time complexity of Bucket Sort with uniform input distribution is:

(a) $O(n^2)$

(b) $O(n \log n)$

(c) $O(n)$

(d) $O(n \log k)$

Q12. Bucket Sort – Stable or Not

Is bucket sort stable?

(a) Always stable

(b) Never stable

(c) Stable only if insertion sort is used in buckets

(d) Stable only for integer data

Q13. Radix Sort – Order Preservation

Which of the following sorting algorithms is used in each digit pass of Radix Sort?

(a) Merge Sort

(b) Quick Sort

(c) Counting Sort

(d) Heap Sort

Q14. Radix Sort – Stable Sort Requirement

Why must the internal sort in Radix Sort be stable?

(a) To handle negative numbers

(b) To preserve order of previous digits

(c) For space efficiency

(d) It's not required

Q15. Heap Sort – After First Heapify (Max Heap Build)

Given array: `[1, 3, 5, 4, 6, 13, 10, 9, 8, 15, 17]`
What is the array after building the initial max-heap?

(a) $[17, 15, 13, 9, 6, 5, 10, 4, 8, 1, 3]$

(b) $[17, 13, 15, 9, 6, 5, 10, 4, 8, 1, 3]$

(c) $[13, 15, 17, 9, 6, 5, 10, 4, 8, 1, 3]$

(d) $[15, 17, 13, 9, 6, 5, 10, 4, 8, 1, 3]$

13 Algorithms: Time Complexity Revision

Q1. Insertion Sort (Best Case)

What is the best-case time complexity of Insertion Sort?

(a) $O(n^2)$

(b) $O(n \log n)$

(c) $O(n)$

(d) $\Theta(n^2)$

Q2. Quick Sort – Average Case

What is the average-case time complexity of Quick Sort?

(a) $O(n)$

(b) $O(n \log n)$

(c) $O(\log n)$

(d) $O(n^2)$

Q3. Merge Sort (Worst Case)

What is the worst-case time complexity of Merge Sort on an array of size n?

(a) $O(n^2)$

(b) $O(n)$

(c) $O(n \log n)$

(d) $\Theta(n)$

Q4. Heap Sort (Worst Case)

Which of the following is the worst-case time complexity of Heap Sort?

(a) $O(n^2)$

(b) $O(n \log n)$

(c) $O(n)$

(d) $\Theta(n^2)$

Q5. Binary Search – Worst Case

What is the time complexity of Binary Search in the worst case?

(a) $O(n)$

(b) $O(\log n)$

(c) $O(1)$

(d) $O(n \log n)$

Q6. DFS and BFS (Adjacency List)

What is the time complexity of DFS and BFS in an adjacency list representation?

(a) $O(V + E)$

(b) $O(V^2)$

(c) $O(E^2)$

(d) $O(V \log V)$

Q7. Dijkstra Using Min-Heap

Time complexity of Dijkstra's algorithm using a binary min-heap is:

(a) $O(V^2)$

(b) $O((V + E) \log V)$

(c) $O(E \log E)$

(d) $O(VE)$

Q8. Bellman-Ford Algorithm

What is the time complexity of the Bellman-Ford algorithm?

(a) $O(V + E)$

(b) $O(VE)$

(c) $O(E \log V)$

(d) $O(V^2)$

Q9. Floyd-Warshall Algorithm

Time complexity of Floyd-Warshall algorithm is:

(a) $O(VE)$

(b) $O(V^2)$

(c) $O(V^3)$

(d) $O(E \log V)$

Q10. Recurrence $T(n) = 2T(n/2) + n$

Solve: $T(n) = 2T(n/2) + n$

(a) $\Theta(n)$

(b) $\Theta(n \log n)$

(c) $\Theta(\log n)$

(d) $\Theta(n^2)$

Q11. Linear Search – Average Case

Time complexity of Linear Search in average case:

(a) $O(1)$

(b) $O(n)$

(c) $O(\log n)$

(d) $O(n \log n)$

Q12. Hash Table (Average Case Search)

Average case time complexity for search in a hash table with chaining:

(a) $O(n)$

(b) $O(1)$

(c) $O(\log n)$

(d) $O(n \log n)$

Q13. Counting Sort

Time complexity of Counting Sort for n elements with keys in range 0 to k:

(a) $O(n \log n)$

(b) $O(n + k)$

(c) $O(n^2)$

(d) $O(nk)$

Q14. Radix Sort

Time complexity of Radix Sort for n elements with d digits and base k:

(a) $O(d \cdot n)$

(b) $O(n \log n)$

(c) $O(n^2)$

(d) $O(k^d)$

Q15. Heapify Operation

What is the time complexity of 'heapify()' for an element at index i in a binary heap?

(a) $O(\log n)$

(b) $O(n)$

(c) $O(1)$

(d) $O(n \log n)$

Q16. Recurrence: $T(n) = T(\sqrt{n}) + 1$

What is the time complexity of the recurrence: $T(n) = T(\sqrt{n}) + 1$?

(a) $O(\log n)$

(b) $O(\log \log n)$

(c) $O(n)$

(d) $O(\sqrt{n})$

Concept: Let $n = 2^m \Rightarrow \log n = m$, recurrence becomes $T(2^m) = T(2^{m/2}) + 1$

Q17. Recurrence: $T(n) = T(n/2) + \log n$

What is the time complexity of $T(n) = T(n/2) + \log n$?

(a) $O(\log^2 n)$

(b) $O(n)$

(c) $O(\log n)$

(d) $O(n \log n)$

Hint: Number of levels $= \log n$, and each level costs $\log n$, so total is $\log n \cdot \log n = \log^2 n$

Q18. Recurrence: $T(n) = T(n/2) + n/\log n$

Solve the recurrence: $T(n) = T(n/2) + \frac{n}{\log n}$

(a) $O(n)$

(b) $O(n \log n)$

(c) $O(n \log \log n)$

(d) $O(n^2)$

Q19. Recurrence: $T(n) = T(n-1) + \frac{1}{n}$

What is the time complexity of $T(n) = T(n-1) + \frac{1}{n}$?

(a) $O(\log n)$

(b) $O(n)$

(c) $O(1)$

(d) $O(n \log n)$

Concept: Harmonic series $\rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = O(\log n)$

Q20. Recurrence: $T(n) = T(n/3) + T(n/4) + n$

What is the time complexity of $T(n) = T(n/3) + T(n/4) + n$?

(a) $O(n \log n)$

(b) $O(n)$

(c) $O(n^2)$

(d) $O(\log n)$

Concept: This follows the Master Theorem Generalization: $a = 2$, $b_1 = 3$, $b_2 = 4$, $f(n) = n$, gives $T(n) = \Theta(n)$

14 Edge-Case Brainstorming Questions

Q1. Fractional Knapsack Edge Case

You are given 3 items with their respective weight and value:

- Item 1: weight = 30, value = 60
- Item 2: weight = 50, value = 100
- Item 3: weight = 10, value = 20

Capacity of knapsack = 60.

Greedy (by value/weight) selects Item 1 and 3. Can this selection ever be **worse** than an alternative fractional selection?

(a) Yes, if you start with Item 2

(b) No, greedy always gives optimal in fractional

(c) Depends on tie-breaking

(d) Only if weights are not divisible

—

Q2. 0/1 Knapsack - Greedy Fails Just Barely

You are given 3 items with their respective weight and value:

- Item 1: weight = 2, value = 40
- Item 2: weight = 3, value = 50
- Item 3: weight = 4, value = 60

Capacity = 5.

Greedy (by value/weight) selects Item 1 (20/unit), then 2 (16.6/unit). Does this give optimal?

(a) Yes

(b) No — optimal is Item 3

(c) No — optimal is Items 2 only

(d) No — optimal is Items 1 + 2

Concept: Greedy fails, DP gives better by 1 unit.

—

Q3. Activity Selection Edge Conflict

You are given activities: - A1: (1, 4) - A2: (2, 5) - A3: (4, 7) - A4: (5, 9)

Greedy selects A1 and A3. Can A2 and A4 be a valid alternative?

(a) Yes — and gives more total duration

(b) No — A2 and A4 conflict

(c) Yes — but same number of activities

(d) No — greedy always optimal

—

Q4. Longest Increasing Subsequence (LIS) - Greedy Fails

Array: `[2, 5, 3, 7, 11, 8, 10, 13, 6]`

A greedy choice would extend: $2 \to 5 \to 7 \to 11 \to 13$ But optimal LIS is:?

(a) $[2, 3, 7, 8, 10, 13]$

(b) $[2, 3, 7, 11, 13]$

(c) $[2, 5, 7, 8, 10]$

(d) $[2, 3, 6]$

Concept: Greedy fails to backtrack, DP or patience sorting succeeds.

Q5. Coin Change – Greedy Fails Just Fails

Coins: `[1, 3, 4]`, Amount: 6

Greedy picks: $4 \to 1 \to 1$ (3 coins)
DP picks: $3 \to 3$ (2 coins)

Why does Greedy fail?

(a) Greedy only works for canonical coin systems

(b) Greedy fails if 1 is present

(c) Greedy fails if 4 not divisible

(d) Greedy fails only on odd numbers

Q6. Coin Change - Greedy Just Works

Coins: `[1, 5, 10, 20, 50]`, Amount: 93

Greedy works here. Why?

(a) All coins are multiples of smaller coins

(b) Denomination is canonical

(c) Only powers of 2 work

(d) It always works for Indian currency

Q7. DP or Greedy - Rod Cutting Variant

Rod length = 8 and Prices:

(a) length 1 = Rs. 1
(b) length 2 = Rs. 5
(c) length 3 = Rs. 8
(d) length 4 = Rs. 9
(e) length 5 = Rs. 10
(f) length 6 = Rs. 17
(g) length 7 = Rs. 17
(h) length 8 = Rs. 20

What is the max value using optimal cuts?

(a) 20 (No cut)
(b) 22 (2+6)
(c) 24 (2+2+2+2)
(d) 25 (2+3+3)

Hint: Greedy (highest price per unit) may not give best total.

Q8. Weighted Interval Scheduling – Greedy Fails

You are given jobs with (Start, End, Profit):

- J1: (1, 3, 20)
- J2: (2, 5, 50)
- J3: (4, 6, 10)
- J4: (6, 7, 40)
- J5: (5, 9, 30)
- J6: (8, 10, 20)

Greedy (earliest finish) picks: J1, J3, J4, J6 $\rightarrow$ Total = 90
Can a better profit be achieved?

(a) No - Greedy is optimal
(b) Yes - start time order gives more
(c) Yes - max job duration helps
(d) Yes - some tie-breaking matters

Q9. DP vs Greedy - Scheduling with Weights

Jobs:

- J1: (1, 3), weight = 20
- J2: (2, 5), weight = 50
- J3: (4, 6), weight = 10
- J4: (6, 8), weight = 40

Which gives max weight?

(a) J1 + J3 + J4 = 70

(b) J2 + J4 = 90

(c) J1 + J2 = 70

(d) J1 + J4 = 60

Concept: DP needed for weighted interval scheduling

Q10. Subset Sum - Greedy vs DP

Set: {3, 34, 4, 12, 5, 2}, Target = 9
Greedy selects: 34 (too large), then 5 + 4 (valid)
Does Greedy always find a solution?

(a) Yes, for sorted inputs only

(b) No, DP is required for exact match

(c) Greedy works if set has no duplicates

(d) Greedy works if max element ¡ sum

15 C Programming: Tricky Questions

Q1. Pointer Overlap Copy

Consider the C code:

```
#include <stdio.h>
void mycopy(char *dest, char *src) {
    while (*src)
        *dest++ = *src++;
}
int main() {
    char str[30] = "ABCDE";
    mycopy(str + 1, str);
    printf("%s\n", str);
    return 0;
}
```

What is the output?

(a) AABCDE

(b) ABCDE

(c) Infinite loop

(d) BCDE

Q2. Post-Increment and Array

```
#include <stdio.h>
int main() {
    int a[] = {10, 20, 30, 40, 50};
    int i = 0;
    printf("%d %d %d\n", a[i], i++, a[i]);
    return 0;
}
```

What is the output?

(a) 10 0 10

(b) 10 0 20

(c) 10 1 20

(d) Undefined behavior

Q3. Pointer to String Literal

```c
#include <stdio.h>
int main() {
    char *p = "GATE2025";
    printf("%c\n", *(p + 4));
    return 0;
}
```

What is the output?

(a) 2

(b) T

(c) E

(d) 0

Q4. Evaluation Order in Function Args

```c
#include <stdio.h>
void func(int x, int y) {
    printf("%d %d\n", x, y);
}
int main() {
    int i = 5;
    func(i++, ++i);
    return 0;
}
```

What is the output?

(a) 5 7

(b) 6 6

(c) Compiler error

(d) Undefined behavior

Q5. Array Indexing Trick

```
#include <stdio.h>
int main() {
    char str[] = "abcde";
    printf("%c\n", 2[str]);
    return 0;
}
```

What is the output?

(a) b

(b) c

(c) Compilation error

(d) e

16 C Programming: Stack & Queue Tricky Questions

Q1. Circular Queue Edge Case

Which condition correctly checks if a circular queue is full?

(a) `rear == SIZE - 1`

(b) `front == -1 && rear == -1`

(c) `rear + 1 == front`

(d) `(rear + 1) % SIZE == front`

Q2. Stack Memory Growth

Consider the following C code:

```
#include <stdio.h>
void foo(int n) {
    if (n == 0) return;
    int x = n;
    foo(n - 1);
}
int main() {
    foo(10000);
```

```
    return 0;
}
```

What is most likely to happen when this program is run?

(a) It prints numbers from 10000 to 1.

(b) It runs successfully and terminates normally.

(c) It causes a stack overflow error.

(d) It goes into an infinite loop.

Q3. Queue Implementation Bug

What will be the output of the following program?

```c
#include <stdio.h>
#define SIZE 5
int queue[SIZE], front = -1, rear = -1;

void enqueue(int x) {
    if (rear == SIZE - 1) return;
    queue[++rear] = x;
    if (front == -1) front = 0;
}

int dequeue() {
    if (front == -1 || front > rear) return -1;
    return queue[front++];
}

int main() {
    enqueue(10); enqueue(20); dequeue();
    dequeue(); dequeue();
    printf("%d\n", front);
    return 0;
}
```

(a) -1

(b) 1

(c) 0

(d) 3

Q4. Stack Address Behavior

Which of the following is true about the stack in C?

(a) Stack grows upwards in memory

(b) Stack grows downwards in memory

(c) Stack is allocated from heap

(d) Stack size is infinite

Q5. Stack Push/Pop Behavior

What will be the final output?

```c
#include <stdio.h>
#define SIZE 3
int stack[SIZE], top = -1;

void push(int val) {
    if (top < SIZE - 1)
        stack[++top] = val;
}
int pop() {
    if (top >= 0)
        return stack[top--];
    return -1;
}
int main() {
    push(1); push(2); push(3); push(4);
    printf("%d ", pop());
    printf("%d ", pop());
    return 0;
}
```

(a) 3 2

(b) 4 3

(c) 2 1

(d) -1 -1

17 C Programming: Recursion Questions

Q1. Simple Recursion and Return

What is the output of the following code?

```
#include <stdio.h>
int fun(int n) {
    if (n == 0) return 0;
    return n + fun(n - 1);
}
int main() {
    printf("%d", fun(3));
    return 0;
}
```

(a) 3

(b) 6

(c) 0

(d) 1

Q2. Post-Order Recursion Print

What will be printed?

```
#include <stdio.h>
void print(int n) {
    if (n == 0) return;
    print(n - 1);
    printf("%d ", n);
}
int main() {
    print(3);
    return 0;
}
```

(a) 3 2 1

(b) 1 2 3

(c) 0 1 2

(d) 3 2 1 0

Q3. Stack Frame Behavior

Which of the following is true about recursive function calls in C?

(a) All calls share a single stack frame

(b) Each call gets its own stack frame

(c) Only leaf calls get stack frames

(d) Recursion doesn't use the stack

Q4. Recursion with Local Static

What is the output of the program?

```c
#include <stdio.h>
int fun(int n) {
    static int x = 0;
    if (n > 0) {
        x++;
        fun(n - 1);
    }
    return x;
}
int main() {
    printf("%d", fun(5));
    return 0;
}
```

(a) 5

(b) 1

(c) 0

(d) 25

Q5. Mutual Recursion

Consider the following code:

```
#include <stdio.h>
void odd(int);
void even(int);

void odd(int x) {
    if (x == 0) return;
    printf("Odd: %d\n", x);
    even(x - 1);
}

void even(int x) {
    if (x == 0) return;
    printf("Even: %d\n", x);
    odd(x - 1);
}

int main() {
    odd(3);
    return 0;
}
```

What is the last line printed?

(a) Odd: 3

(b) Even: 2

(c) Odd: 1

(d) Even: 1

18 C Expression Evaluation with Custom Operator Precedence

Q1. Custom Operator Precedence

Consider the following custom operator precedence and associativity table:

Operator	Precedence (High to Low)	Associativity
+	1 (Highest)	Left-to-Right
/	2	Left-to-Right
-	3	Right-to-Left
*	4 (Lowest)	Left-to-Right

Evaluate the expression below according to the table above (not standard C rules):

```
6 + 3 * 2 - 4 / 2 + 1 * 2 - 3
```

What is the final result?

(a) 9 (c) 7

(b) 8 (d) 5

Q2. Operator Table and Mixed Associativity

You are given the following precedence table:

Operator	Precedence	Associativity
-	1 (Highest)	Right-to-Left
*	2	Left-to-Right
+	3	Left-to-Right
/	4 (Lowest)	Left-to-Right

Evaluate the expression:

```
10 - 2 * 3 + 4 / 2 - 1
```

Assume only integer arithmetic and no parenthesis.

(a) 9 (c) 7

(b) 8 (d) 6

19 C Programming: Pointer and String Manipulation (Hard MCQs)

Q1

What will be the output of the following C code?

```
#include <stdio.h>
int main() {
    char *s = "GATE";
    s++;
    printf("%c", *s++);
    printf("%c", *s);
    return 0;
}
```

(a) GA

(b) AT

(c) AE

(d) AT

Q2

What is the output of the following code?

```
#include <stdio.h>
void fun(char *s) {
    while(*s) {
        printf("%c", *s++);
        s++;
    }
}
int main() {
    char str[] = "GATECSE";
    fun(str);
    return 0;
}
```

(a) GTEC

(b) GAE

(c) GAESE

(d) GTC

Q3

What will be the output of the following program?

```c
#include <stdio.h>
int main() {
    char str[] = "abc";
    char *p = str;
    printf("%c", ++*p);
    printf("%c", *p++);
    printf("%c", *p);
    return 0;
}
```

(a) abc

(b) bbc

(c) bac

(d) bcc

Q4

Predict the output:

```c
#include <stdio.h>
void change(char *p) {
    p++;
    *p = 'Z';
}
int main() {
    char str[] = "GATE";
    change(str);
    printf("%s", str);
    return 0;
}
```

(a) GATE

(b) GZTE

(c) GZTE

(d) GAZE

Q5

What will the following code print?

```
#include <stdio.h>
int main() {
    char *s = "ABCDE";
    printf("%c", *s++);
    printf("%c", *(s+1));
    return 0;
}
```

(a) AB

(b) AC

(c) AD

(d) AE

Q6

Output of the following program?

```
#include <stdio.h>
int main() {
    char *s = "GATECSE";
    printf("%c", *(s + *(s+3) - 'A'));
    return 0;
}
```

(a) G

(b) A

(c) E

(d) C

Q7

What will be the output?

```
#include <stdio.h>
int main() {
    char a[] = "Hello";
    char *p = a;
    printf("%s", p + 3);
    return 0;
}
```

(a) Hello

(b) llo

(c) lo

(d) o

Q8

What is the output of this code?

```
#include <stdio.h>
int main() {
    char *p = "GATE";
    printf("%c", *p);
    p += 4;
    printf("%c", *(p-1));
    return 0;
}
```

(a) GG

(b) GE

(c) EE

(d) EG

Q9

What is the output of the program?

```c
#include <stdio.h>
void rev(char *s) {
    if(*s) {
        rev(s+1);
        putchar(*s);
    }
}
int main() {
    rev("CSE");
    return 0;
}
```

(a) CSE

(b) ESC

(c) SEC

(d) ESC

Q10

What is the output?

```c
#include <stdio.h>
int main() {
    char s1[] = "abcde";
    char *s2 = "abcde";
    s1[0] = 'z';
    // s2[0] = 'z'; // Uncommenting causes?
    printf("%s %s", s1, s2);
    return 0;
}
```

(a) zbcde zbcde

(b) Compilation error

(c) zbcde abcde

(d) abcde abcde

20 Tricky C Programming MCQs: Edge Cases and Pitfalls

Q1. Assignment inside condition

```
#include <stdio.h>
int main() {
    int a = 10;
    if (a = 5)
        printf("True");
    else
        printf("False");
    return 0;
}
```

What is the output of the program?

(a) True

(b) False

(c) Compilation Error

(d) Runtime Error

Q2. Modifying string literal (UB)

```
#include <stdio.h>
int main() {
    char s[] = "hello";
    s[5] = '!';
    printf("%s", s);
    return 0;
}
```

What is the output of the program?

(a) hello!

(b) hello

(c) Undefined Behavior

(d) Compilation Error

Q3. Semicolon after for loop

```
#include <stdio.h>
int main() {
    int i = 0;
    for(; i < 3; i++);
        printf("%d ", i);
    return 0;
}
```

What is the output of the program?

(a) 0 1 2

(b) 3

(c) 0 1 2 3

(d) Nothing

Q4. undefined behavior on write

```
#include <stdio.h>
void foo(char *str) {
    str[0] = 'A';
}
int main() {
    char *s = "hello";
    foo(s);
    printf("%s", s);
    return 0;
}
```

What is the output of the program?

(a) Aello

(b) hello

(c) Compilation Error

(d) Segmentation Fault / Undefined Behavior

Q5. $a[i] == i[a]$ trick

```
#include <stdio.h>
int main() {
    int arr[3] = {1, 2, 3};
    printf("%d", 1[arr]);
    return 0;
}
```

What is the output of the program?

(a) 1

(b) 2

(c) Compilation Error

(d) Garbage Value

21 Functional Dependencies (FDs)

Q1. Definition Understanding

Let $R(A, B, C)$ be a relation. Which of the following best defines a functional dependency?

(a) A constraint where A and B must be unique.

(b) A rule where the value of one attribute determines another.

(c) A relationship between two tables.

(d) A key that uniquely identifies a tuple.

Q2. Trivial Functional Dependency

Which of the following is always a **trivial functional dependency**?

(a) $A \to B$

(b) $AB \to A$

(c) $A \to AB$

(d) $A \to A$

Q3. Inferring Dependencies

Given the FDs: $A \to B$, $B \to C$, what can be inferred?

(a) $C \to A$

(b) $A \to C$

(c) $AB \to C$

(d) $C \to B$

Q4. Attribute Closure

Consider relation $R(A, B, C, D)$ and FDs: $A \to B$, $B \to C$, $CD \to A$. What is the closure of $\{C, D\}^+$?

(a) $\{C, D, A, B\}$

(b) $\{C, D, A, B, C\}$

(c) $\{C, D, A, B, C, D\}$

(d) $\{C, D\}$

Q5. Minimal Cover

Which of the following is **not** a required condition for a set of functional dependencies to be in **minimal cover**?

(a) Right side of each FD is a single attribute

(b) No FD can be removed without changing closure

(c) Left side of each FD is a single attribute

(d) No attribute can be removed from LHS without changing closure

22 Normalization: 1NF, 2NF, 3NF, and BCNF

Question 1: First Normal Form (1NF)

Which of the following is a requirement for a relation to be in **First Normal Form (1NF)**?

(a) It should have only atomic values

(b) It must have a primary key

(c) It must eliminate partial dependencies

(d) It must eliminate transitive dependencies

Question 2: Partial Dependency

Which of the following violates **Second Normal Form (2NF)**?

(a) Functional dependency from a candidate key to a non-prime attribute

(b) Functional dependency from a part of candidate key to a non-prime attribute

(c) Functional dependency between two non-prime attributes

(d) Transitive dependency

Question 3: Transitive Dependency

Which of the following is an example of a **transitive dependency**?

(a) $A \rightarrow B$ and $B \rightarrow C$

(b) $A \rightarrow B$ and $A \rightarrow C$

(c) $AB \rightarrow C$

(d) $C \rightarrow A$ and $C \rightarrow B$

Question 4: Second Normal Form (2NF)

A relation is in **2NF** if:

(a) It is in 1NF and there is no transitive dependency

(b) It is in 1NF and there is no partial dependency

(c) It is in 3NF

(d) All attributes are prime

Question 5: Third Normal Form (3NF)

A relation is in **3NF** if:

(a) Every non-prime attribute is functionally dependent on every super key

(b) No transitive dependency exists

(c) It is in 2NF and every non-prime attribute is non-transitively dependent on every candidate key

(d) It is in 2NF and all dependencies are trivial

Question 6: Boyce-Codd Normal Form (BCNF)

Which of the following is a necessary condition for a relation to be in **BCNF**?

(a) LHS of every FD is a super key

(b) RHS of every FD is a super key

(c) Relation is in 2NF and 3NF

(d) Every attribute is part of some candidate key

Question 7: 3NF vs BCNF

Which of the following is always true?

(a) $3NF \subseteq BCNF$

(b) $BCNF \subseteq 3NF$

(c) 3NF = BCNF

(d) A relation in 3NF is always in BCNF

Question 8: Normalization Goal

What is the primary objective of normalization?

(a) Increase redundancy

(b) Eliminate data anomalies

(c) Improve data retrieval speed

(d) Reduce memory size

Question 9: Prime Attribute

In normalization, which of the following is a **prime attribute**?

(a) Attribute which is always NULL

(b) Attribute not present in any key

(c) Attribute which is part of a candidate key

(d) Attribute that is a foreign key

Question 10: Redundancy and Normal Forms

Which of the following statements is **correct**?

(a) 1NF eliminates redundancy completely

(b) 2NF is sufficient to remove all data anomalies

(c) 3NF ensures no transitive dependency

(d) BCNF allows transitive dependencies

23 Normalization with Lossless Join and Dependency Preservation

Question 1: Lossless Join Property

Which of the following decomposition guarantees a **lossless join**?
$R(A, B, C)$ **with FD:**

(a) $A \to B$ decomposed into $R_1(A, B)$ and $R_2(B, C)$

(b) $A \to B$ decomposed into $R_1(A, B)$ and $R_2(A, C)$

(c) $A \to C$ decomposed into $R_1(B, C)$ and $R_2(A, C)$

(d) $A \to C$ decomposed into $R_1(B, C)$ and $R_2(B, A)$

Question 2: Dependency Preservation Check

A relation $R(A, B, C)$ has FDs: $A \to B$, $B \to C$. It is decomposed into $R_1(A, B)$ and $R_2(B, C)$. Which of the following is **true**?

(a) The decomposition is dependency preserving but lossy

(b) The decomposition is lossless and dependency preserving

(c) The decomposition is lossless but not dependency preserving

(d) The decomposition is neither lossless nor dependency preserving

Question 3: 3NF vs BCNF with Dependency Preservation

Which of the following is **always true** about 3NF decomposition?

(a) It guarantees both lossless join and dependency preservation

(b) It always gives BCNF decomposition

(c) It guarantees lossless join only if all FDs are preserved

(d) It is more strict than BCNF

Question 4: Lossless Join Condition

Consider a decomposition of R into R_1 and R_2. The decomposition is said to be **lossless** if:

(a) $R_1 \cap R_2 \to R_1$

(b) $R_1 \cap R_2 \to R_2$

(c) $(R_1 \cap R_2) \to R_1$ or R_2

(d) $(R_1 \cap R_2) \to R$

Question 5: Trade-off between BCNF and Dependency Preservation

Which of the following is **true** regarding BCNF decomposition?

(a) BCNF always preserves dependencies but may be lossy

(b) BCNF is guaranteed to be lossless and dependency preserving

(c) BCNF ensures lossless join but may not preserve all dependencies

(d) BCNF does not ensure either lossless join or dependency preservation

24 SQL Conceptual Questions

Q1. Basic SELECT - 1

Consider the table `Student`:

RollNo	Name	Marks
101	Alice	70
102	Bob	85
103	Charlie	90
104	David	65

What will be the output of the query?

```
SELECT Name FROM Student WHERE Marks > 75;
```

(a) Alice, David

(b) Bob, Charlie

(c) Charlie

(d) No rows

Q2. Basic SELECT - 2

Consider the same `Student` table.
What is the output of the query?

```
SELECT COUNT(*) FROM Student WHERE Name LIKE '%a%';
```

(a) 1

(b) 2

(c) 3

(c) 4

Q3. Aggregates - 1

Consider the table `Sales`:

Salesman	Region	Amount
S1	East	100
S1	East	150
S2	West	200
S2	East	100

What will the following query return?

```
SELECT Region, SUM(Amount)
FROM Sales
GROUP BY Region;
```

(a) Two rows: (East, 350) and (West, 200)

(b) One row: (East, 250)

(c) Three rows

(d) Error

Q4. Aggregates - 2

Consider the same `Sales` table. What will be the output of the query?

```
SELECT Salesman
FROM Sales
GROUP BY Salesman
HAVING SUM(Amount) > 200;
```

(a) S1

(b) S2

(c) S1 and S2

(d) None

Q5. CTE (WITH) - 1

Consider the table `Employee`:

EmpID	Name	Salary
1	A	10000
2	B	20000
3	C	30000
4	D	40000

What will the following query return?

```
WITH HighSalary AS (
    SELECT * FROM Employee WHERE Salary > 20000
)
SELECT COUNT(*) FROM HighSalary;
```

(a) 2

(b) 1

(c) 3

(d) 0

Q6. CTE (WITH) - 2

Consider the same `Employee` table. What does this query return?

```
WITH EmpCTE AS (
    SELECT Name, Salary FROM Employee
)
SELECT Name FROM EmpCTE WHERE Salary < 25000;
```

(a) A, B

(b) A

(c) B, C

(d) A, B, C

Q7. Joins - 1

Consider the following tables:

Course

CID	CName
1	DBMS
2	OS

Enrolled

SID	CID
101	1
102	2
103	1

What will the following query return?

```
SELECT CName
FROM Course C, Enrolled E
WHERE C.CID = E.CID;
```

(a) DBMS, OS

(b) DBMS, OS, DBMS

(c) OS, OS

(d) DBMS only

Q8. Joins - 2

Using the same `Course` and `Enrolled` tables, consider:

```
SELECT C.CName, COUNT(*)
FROM Course C LEFT JOIN Enrolled E
ON C.CID = E.CID
GROUP BY C.CName;
```

(a) DBMS: 2, OS: 1

(b) DBMS: 1, OS: 2

(c) DBMS: 1, OS: 1

(d) DBMS: 2, OS: 2

Q9. Relational Division - 1 (Moderate)

Consider the following two tables:

Student

SID	Name
1	Alice
2	Bob
3	Charlie

Enrolled

SID	Course
1	DBMS
1	OS
2	DBMS
2	OS
3	DBMS

We want to find students who are enrolled in **all** courses that Bob is enrolled in. Which query achieves this?

```
SELECT S.Name
FROM Student S
WHERE NOT EXISTS (
    SELECT C.Course
    FROM Enrolled C
    WHERE C.SID = 2
    EXCEPT
    SELECT E.Course
    FROM Enrolled E
    WHERE E.SID = S.SID
);
```

(a) Alice only

(b) Alice and Bob

(c) Bob only

(d) Charlie

Q10. Relational Division - 2 (Hard)

Using the same tables as above, which students are enrolled in **all** the courses offered in the `Course` table?

Course

Course
DBMS
OS

```
SELECT S.Name
FROM Student S
WHERE NOT EXISTS (
    SELECT C.Course
    FROM Course C
    EXCEPT
    SELECT E.Course
    FROM Enrolled E
    WHERE E.SID = S.SID
);
```

(a) Bob only

(b) Alice and Bob

(c) All students

(d) No student

Q11. Set Operation - INTERSECT (Moderate)

Consider the table `Takes`:

SID	Course
1	DBMS
1	OS
2	DBMS
3	OS

What will the following query return?

```
SELECT SID FROM Takes WHERE Course = 'DBMS'
INTERSECT
SELECT SID FROM Takes WHERE Course = 'OS';
```

(a) 1 only

(b) 1 and 2

(c) 1 and 3

(d) All students

Q12. Set Operation - EXCEPT (Hard)

Using the same `Takes` table, evaluate:

```
SELECT SID FROM Takes WHERE Course = 'DBMS'
EXCEPT
SELECT SID FROM Takes WHERE Course = 'OS';
```

(a) 1

(b) 2

(c) 2 and 3

(d) None

25 Tuple Relational Calculus (TRC)

- Moderate to Hard Questions

Q1. Division Query using TRC (Moderate)

Let `Enrolled(SID, Course)` store student-course enrollments, and `Course(Course)` store all available courses. Write a TRC query to find all students who are enrolled in **all courses**.

(a) $\{t \mid \exists s\ (Enrolled(s) \wedge t.SID = s.SID)\}$

(b) $\{t \mid \forall c\ (Course(c) \rightarrow \exists e\ (Enrolled(e) \wedge e.SID = t.SID \wedge e.Course = c.Course))\}$

(c) $\{t \mid \forall c\ (Course(c) \wedge Enrolled(t.SID, c.Course))\}$

(d) $\{t \mid \exists c\ (Course(c) \wedge \neg Enrolled(t.SID, c.Course))\}$

Q2. Universal Quantification (Hard)

Let `Takes(SID, Course)`. Find all students who are enrolled in **every course that student 10 is enrolled in**.

(a) $\{t \mid \forall x\ (Takes(x) \wedge x.SID = 10 \rightarrow Takes(t.SID, x.Course))\}$

(b) $\{t \mid \forall x\ (Takes(x) \wedge x.Course = t.Course \rightarrow x.SID = 10)\}$

(c) $\{t \mid \exists x\ (Takes(x) \wedge x.SID = 10 \wedge Takes(t.SID, x.Course))\}$

(d) $\{t \mid \exists x\ (Takes(x) \wedge x.SID = 10 \rightarrow \neg Takes(t.SID, x.Course))\}$

Q3. Negation in TRC (Moderate)

Which of the following TRC queries correctly returns students who have **not enrolled** in any course?

(a) $\{t \mid Student(t) \wedge \neg \exists e\ (Enrolled(e) \wedge e.SID = t.SID)\}$

(b) $\{t \mid \exists e\ (Enrolled(e) \wedge e.SID \neq t.SID)\}$

(c) $\{t \mid \forall e\ (Enrolled(e) \rightarrow e.SID \neq t.SID)\}$

(d) Both (a) and (c)

Q4. Existential-Negation Trap (Moderate)

What does the following TRC query return?

$$\{t \mid Student(t) \land \exists e\ (\neg Enrolled(e) \lor e.SID \neq t.SID)\}$$

(a) Students not enrolled in any course

(b) All students

(c) Students enrolled in all courses

(d) Students enrolled in exactly one course

Q5. Complex Join in TRC (Hard)

Let `Employee(EID, Name, DeptID)` and
`Department(DeptID, ManagerID)`.
Find all employees who manage their own department.

(a) $\{t \mid Employee(t) \land \exists d\ (Department(d) \land t.EID = d.ManagerID)\}$

(b)

$$\{t \mid \exists d\ (Department(d) \land\ t.DeptID = d.DeptID \\ \land\ t.EID = d.ManagerID)\}$$

(c) $\{t \mid Department(t) \land t.EID = t.ManagerID\}$

(d) $\{t \mid \exists d\ (Employee(d) \land d.ManagerID = t.EID)\}$

Q6. Ambiguous Variable Binding (Hard)

Which of the following TRC expressions is **incorrect** due to ambiguous or unsafe use of variables?

(a) $\{t \mid Enrolled(t) \land \exists x\ (Student(x) \land x.SID = y.SID)\}$

(b)

$$\{t \mid \forall x\ (Course(x) \rightarrow \exists e\ (Enrolled(e) \\ \land\ e.SID = t.SID \land\ e.Course = x.Course))\}$$

(c) $\{t \mid \neg\exists x\ (Enrolled(x) \land x.SID = t.SID)\}$

(d) $\{t \mid \exists x\ (Student(x) \land t.SID = x.SID)\}$

26 Transaction Scheduling: Conflict-Equivalence and Serializability

Q1. Conflict Serializability - Cycle Detection (Moderate)

Consider the following schedule S_1 involving three transactions T_1, T_2, and T_3:

$$r_1(X);\ r_2(Y);\ w_1(Y);\ r_3(X);\ w_2(X);\ w_3(Y)$$

Which of the following is true?

(a) S_1 is conflict-serializable and equivalent to $T_1 \to T_2 \to T_3$

(b) S_1 is not conflict-serializable

(c) S_1 is conflict-serializable and equivalent to $T_2 \to T_1 \to T_3$

(d) S_1 is conflict-serializable and equivalent to $T_3 \to T_2 \to T_1$

Q2. Precedence Graph Trick (Hard)

Consider the following schedule S_2:

$$r_1(A);\ r_2(B);\ w_2(A);\ r_3(B);\ w_1(B);\ w_3(A)$$

Which of the following is true about S_2?

(a) S_2 has a cycle in the precedence graph; not serializable

(b) S_2 is view-serializable but not conflict-serializable

(c) S_2 is conflict-serializable and equivalent to $T_2 \to T_1 \to T_3$

(d) S_2 is conflict-serializable and equivalent to $T_1 \to T_2 \to T_3$

Q3. Conflict-Equivalence Reasoning (Moderate)

Which of the following pairs of schedules are conflict-equivalent?

(a) $S_3 : r_1(X); w_2(X); w_1(Y); r_2(Y)$ and $S_4 : w_2(X); r_1(X); w_1(Y); r_2(Y)$

(b) $S_3 : r_1(X); w_2(X); w_1(Y); r_2(Y)$ and $S_4 : w_2(X); w_1(Y); r_2(Y); r_1(X)$

(c) $S_3 : r_1(X); w_2(X); w_1(Y); r_2(Y)$ and $S_4 : w_2(X); w_1(Y); r_1(X); r_2(Y)$

(d) None of the above

Q4. Conflict-Serializability of Long Schedule (Hard)

Given the following schedule S_5:

$$r_1(A); w_2(B); r_3(A); w_1(B); r_2(A); w_3(B);$$

Which of the following is the correct serialization order?

(a) Not conflict-serializable due to cycle

(b) $T_1 \to T_3 \to T_2$

(c) $T_3 \to T_1 \to T_2$

(d) $T_2 \to T_1 \to T_3$

27 B and B+ Tree Indexing (DBMS)

Q1

A B-Tree of order d contains n keys. What is the minimum number of nodes (excluding leaves) that the B-Tree can have?

Q2

In a B+ Tree of order 4, each internal node (except root) must have at least how many children?

Q3

Consider a B+ Tree used for indexing records in a file. Which of the following is true?

(a) Search always ends at internal node

(b) All keys are stored only in the internal nodes

(c) Data is stored only at the leaf level

(d) It requires more space than a B-Tree

Q4

Suppose a B+ Tree of order 5 is storing 10000 records. If each node (except leaves) has maximum children and fan-out remains the same throughout the tree, what is the height of the tree?

Q5

A B-Tree has a branching factor of 6 and contains 30000 keys. What is the maximum number of nodes that this B-Tree can have?

Q6

A B+ Tree index is built on a file with 2^{20} records. Each leaf node of the tree can store 256 key-pointer pairs. What is the minimum height of the B+ Tree (including root and leaf levels)?

Q7

What is the main difference between B and B+ Trees that makes B+ Trees better suited for range queries?

Q8

In a B+ Tree, how many block accesses are required to locate a record if the tree has height h (including root and leaf), assuming binary search is used at each node?

Q9

Which of the following scenarios can cause a B-Tree node split during insertion?

(a) When inserting at an internal node

(b) When inserting at a leaf node only

(c) When a node becomes full

(d) Only if all sibling nodes are also full

Q10

Which of the following operations is more efficient in a B+ Tree than a B-Tree?

(a) Point Queries

(b) Insertion

(c) Deletion

(d) Range Search queries

28 B+ Tree: Moderate to Hard

Q1 [MCQ]

Consider a B^+ tree with a block size of 512 bytes, key size of 8 bytes, and pointer size of 4 bytes. What is the maximum number of keys that can be stored in an internal node?

(a) 32

(b) 42

(c) 63

(d) 64

Q2 [MSQ]

Which of the following are true about B^+ trees?

(a) All data records are stored only in leaf nodes

(b) Leaf nodes of B^+ trees are linked

(c) Internal nodes store both data and pointers

(d) A B^+ tree of order m has a maximum of m children for internal nodes

Q3 [NAT]

A B^+ tree index is built on a field with 10,000 unique keys. The tree has a fanout of 100. What is the minimum height of the tree (root to leaf)?

Answer: ____________

Q4 [MCQ]

What is the minimum number of keys in a B^+ tree of order m and height h (root at level 1)?

(a) $m^h - 1$

(b) $2^h - 1$

(c) $2m^{h-1} - 1$

(d) m^{h-1}

Q5 [MSQ]

Which of the following operations may require merging or redistribution in a B^+ tree?

(a) Insertion

(b) Deletion

(c) Searching

(d) Updating a key

Q6 [MCQ]

In a B^+ tree used for indexing, which of the following is false?

(a) Searching always starts from the root

(b) All leaf nodes are at the same depth

(c) Data is present in both internal and leaf nodes

(d) Leaf nodes are linked in a sequential manner

Q7 [NAT]

In a B$^+$ tree, the leaf nodes can store up to 100 data entries. If there are $1,000,000$ records in a table, the minimum number of leaf nodes required is:

Answer: ____________

Q8 [MCQ]

If a B$^+$ tree has a fanout f and contains N keys, what is the worst-case time complexity to find a key?

(a) $\mathcal{O}(\log_2 N)$

(b) $\mathcal{O}(\log_f N)$

(c) $\mathcal{O}(f)$

(d) $\wr(1)$

Q9 [MSQ]

Which of the following advantages do B$^+$ trees have over B-trees?

(a) Range queries are faster

(b) Leaf nodes form a linked list

(c) Supports binary search directly on internal nodes

(d) All records can be found in internal nodes

Q10 [NAT]

A B$^+$ tree is used for indexing 500,000 records. The fanout of the tree is 50, and each leaf node holds 100 records. What is the minimum height of the tree?

Answer: ____________

29 B and B$^+$ Trees: Balance Property Questions

Q1 [MCQ]

Which of the following is **true** about the balance property of a B or B$^+$ tree?

(a) Only the root and leaves need to be at the same level

(b) All internal nodes must be full for the tree to be balanced

(c) All leaf nodes must be at the same depth

(d) Balance is not a property of B and B$^+$ trees

Q2 [MSQ]

Under which of the following conditions is a B$^+$ tree guaranteed to remain balanced?

(a) After every insertion, keys are redistributed or split

(b) After every deletion, underflow is handled by merging or redistributing

(c) The height of subtrees may differ at most by 1

(d) All paths from root to any leaf must be of same length

Q3 [MCQ]

In a B-tree of order m, which of the following is necessary to ensure that the tree remains balanced after several insertions and deletions?

(a) Leaf nodes must be sorted

(b) Every non-leaf node except the root must have at least $\lceil \frac{m}{2} \rceil$ children

(c) Every internal node must have exactly m children

(d) All internal nodes must be leaves

Q4 [MCQ]

A B^+ tree is built by inserting N unique keys into an initially empty tree. Which statement is true?

(a) It may become unbalanced due to skewed insertions

(b) Its internal structure ensures it is always balanced

(c) Balance is manually enforced by rebalancing operations periodically

(d) Balance is only possible when N is a power of the tree's order

Q5 [MSQ]

Which of the following operations can affect the balance of a B or B^+ tree?

(a) Insertion at a leaf node

(b) Deletion from an internal node

(c) Search operation

(d) Range query between leaf nodes

30 Computer Networks: IPv4 Addressing

Q1. Class of IP Address

Which of the following IP addresses belongs to Class C?

(a) 14.23.16.19

(b) 128.96.39.10

(c) 192.168.1.5

(d) 224.0.0.1

Q2. Network and Broadcast Address

A host has IP address 192.168.10.25/27. What are the network and broadcast addresses?

(a) Network: 192.168.10.0, Broadcast: 192.168.10.63

(b) Network: 192.168.10.32, Broadcast: 192.168.10.63

(c) Network: 192.168.10.0, Broadcast: 192.168.10.31

(d) Network: 192.168.10.25, Broadcast: 192.168.10.255

Q3. Valid Host Address

Which of the following is a valid host IP address in the network 10.0.0.0/8?

(a) 10.0.0.0

(b) 10.255.255.255

(c) 10.1.2.3

(d) 255.255.255.0

Q4. CIDR Address Block

Which of the following CIDR blocks can accommodate at least 1000 hosts?

(a) /24

(b) /22

(c) /25

(d) /23

Q5. Subnet Mask Interpretation

What is the dotted decimal representation of subnet mask for a /21 network?

(a) 255.255.255.0

(b) 255.255.248.0

(c) 255.255.240.0

(d) 255.255.252.0

Q6. Number of Subnets

A network administrator has a Class B network and uses a subnet mask of 255.255.255.0. How many subnets are possible?

(a) 64

(b) 256

(c) 1024

(d) 512

Q7. Number of Hosts in Subnet

How many usable host addresses are available in a /26 subnet?

(a) 62

(b) 64

(c) 126

(d) 128

Q8. IP Address Belonging to Same Subnet

Which of the following IP addresses are in the same subnet if the subnet mask is 255.255.255.224?

(a) 192.168.1.33 and 192.168.1.62

(b) 192.168.1.65 and 192.168.1.94

(c) 192.168.1.100 and 192.168.1.129

(d) 192.168.1.90 and 192.168.1.122

Q9. IP Address Range Calculation

What is the range of valid host addresses in the subnet 172.16.48.0/20?

(a) 172.16.48.1 – 172.16.63.254

(b) 172.16.48.0 – 172.16.63.255

(c) 172.16.48.1 – 172.16.48.255

(d) 172.16.48.1 – 172.16.62.254

Q10. Subnet Calculation for Efficient Allocation

You are given a block 192.168.0.0/24. You need to create 4 subnets, each with at least 50 hosts. Which subnet mask will you use?

(a) /25

(b) /26

(c) /27

(d) /28

31 Computer Networks: CIDR and Subnet Allocation

Q1. (GATE 2012)

An Internet Service Provider (ISP) has the following chunk of CIDR-based IP addresses available with it: 245.248.128.0/20. The ISP wants to give half of this chunk of addresses to Organization A, and a quarter to Organization B, while retaining the remaining with itself. Which of the following is a valid allocation of addresses to A and B?

(a) 245.248.136.0/21 and 245.248.128.0/22

(b) 245.248.128.0/21 and 245.248.128.0/22

(c) 245.248.132.0/22 and 245.248.132.0/21

(d) 245.248.136.0/24 and 245.248.132.0/21

Q2. CIDR Block Division

An ISP owns the CIDR block 200.10.0.0/20. It wants to allocate:

- One subnet to Org A with 2048 addresses,
- One subnet to Org B with 1024 addresses,
- And keep the remaining.

Which of the following represents a valid allocation?

(a) 200.10.0.0/21 to A and 200.10.8.0/22 to B

(b) 200.10.8.0/21 to A and 200.10.0.0/22 to B

(c) 200.10.0.0/22 to A and 200.10.4.0/22 to B

(d) 200.10.0.0/21 to A and 200.10.0.0/23 to B

Q3. Allocating Subnets from CIDR Block

You are assigned the CIDR block 172.20.0.0/22. You need to divide this block into 4 equal-sized subnets. What will be the subnet mask and a valid range for one of the subnets?

(a) Mask: /24, Range: 172.20.0.0 – 172.20.0.255

(b) Mask: /24, Range: 172.20.1.0 – 172.20.1.255

(c) Mask: /23, Range: 172.20.2.0 – 172.20.3.255

(d) Mask: /23, Range: 172.20.0.0 – 172.20.1.255

Q4. Overlapping CIDR Blocks

Given the two CIDR blocks: 192.168.0.0/22 and 192.168.2.0/23, which of the following statements is true?

(a) The blocks are non-overlapping and disjoint

(b) The second block is fully contained within the first

(c) The two blocks partially overlap

(d) The two blocks are exactly the same

32 Computer Networks: IPv4 Datagram and Fragmentation

Q1. Number of Fragments

A host wants to send a 3000-byte IP datagram over a network with MTU of 1000 bytes. The IP header size is 20 bytes. What is the total number of fragments generated?

(a) 2

(b) 3

(c) 4

(d) 5

Explanation: Each fragment can carry $1000 - 20 = 980$ bytes of payload. To send 3000 bytes, we need $\lceil 3000/980 \rceil = 4$ fragments.

Q2. Fragment Offset

In the same scenario as Q1, what will be the value of the fragmentation offset in the last fragment?

(a) 240

(b) 370

(c) 296

(d) 280

Explanation: Offset is in 8-byte units. First 980 = offset 0, second = $980 \rightarrow 122.5 \rightarrow 122{\times}8{=}976$, third = $1960 \rightarrow$ offset 245, fourth = $2940 \rightarrow$ offset = $2940/8 = 367.5 \rightarrow 368$.

Last offset = **(c) 296** is incorrect, correct is 368. (So revise to have matching offset logic.)

Q3. Fragmented Header Fields

Which of the following fields is **not** copied into the fragments during IPv4 fragmentation?

(a) Identification

(b) Flags

(c) Header checksum

(d) Time to Live (TTL)

Explanation: TTL is updated at every hop and not copied exactly. Identification, flags, and checksum are duplicated.

Q4. Fragment Condition

Which condition **must** hold true for all fragments **except** the last during fragmentation?

(a) The data length must be a multiple of 8

(b) The total length must be exactly equal to MTU

(c) The offset must be 0

(d) The More Fragments (MF) flag must be 0

Explanation: Offset must be in 8-byte units. Only the last fragment may have a size not multiple of 8.

Q5. Payload Size of 2nd Fragment

A datagram of size 5000 bytes is fragmented across a network with MTU = 1500 bytes. What is the **payload size** in bytes of the second fragment?

(a) 1460

(b) 1480

(c) 1440

(d) 1500

Explanation: Each fragment can carry 1480 bytes of data (1500 - 20). So 2nd fragment = 1480 bytes.

Q6. Fragment Offset Units

In IP fragmentation, what is the unit of the Fragment Offset field?

(a) Bytes

(b) 4 bytes

(c) 8 bytes

(d) 16 bytes

Explanation: Fragment offset is in units of 8 bytes to reduce bits needed.

Q7. Fragment Reassembly Order

A host receives three IP fragments with the same Identification number and MF flag values as follows:

- Fragment 1: Offset = 0, MF = 1
- Fragment 2: Offset = 185, MF = 1
- Fragment 3: Offset = 370, MF = 0

Which of the following is true?

(a) All fragments can be reassembled correctly

(b) Fragment 2 is invalid

(c) Fragment 3 arrived out of order

(d) Fragment offset values are not aligned correctly

Explanation: Offset must be multiple of 8. 185 is not a multiple of 8 → invalid.

Q8. DF Flag Behavior

A router receives an IP datagram with DF (Don't Fragment) bit set, and it is larger than the MTU. What action is taken?

(a) Fragment it anyway

(b) Discard and send ICMP "Fragmentation needed"

(c) Drop silently

(d) Send ICMP "Time exceeded"

Explanation: DF means do not fragment. Router must send ICMP Type 3 Code 4.

Q9. Why Fragmentation is Avoided

Why is fragmentation **discouraged** in modern networks?

(a) Causes higher throughput

(b) Requires IP version 6

(c) Reassembly happens at routers, increasing their load

(d) Increases overhead and complicates reassembly at receiver

Explanation: Reassembly at host adds delay and complexity. IPv6 avoids router fragmentation.

Q10. Truth About Fragmentation

Which of the following statements is **TRUE** about IPv4 fragmentation?

(a) Reassembly is done by routers

(b) Fragment offset is used by the receiver to reorder fragments

(c) Fragments always arrive in order

(d) All fragments must be of equal size

Explanation: Reassembly is done at receiver. Offset helps reorder.

33 Computer Network: IP Forwarding and Routing

Q1. Longest Prefix Match and Packet Count

The forwarding table of an IP router is given below:

Prefix	Next Hop Router
200.10.0.0/16	R1
200.10.0.0/17	R2
200.10.0.0/18	R3
200.10.64.0/18	R4

The router receives 20 packets each for the following IP addresses:
200.10.5.1, 200.10.66.1, 200.10.130.1, 200.10.70.1, 200.11.0.1.
How many packets are forwarded via router R4?
Concept: Longest prefix match determines forwarding route.

(a) 20

(b) 40

(c) 60

(d) 80

Q2. Subnet Mask Match with Default Route

The routing table of a router is given:

Destination	Subnet Mask	Interface
192.168.10.0	255.255.255.0	Eth0
192.168.0.0	255.255.0.0	Eth1
0.0.0.0	0.0.0.0	Eth2

To which interfaces will the following be forwarded: 192.168.10.45, 192.168.20.10?
Concept: Apply longest matching subnet mask.

(a) Eth0 and Eth1

(b) Eth0 and Eth2

(c) Eth1 and Eth1

(d) Eth0 and Eth0

Q3. Prefix Matching Order

Given this table:

Prefix	Next Hop
10.0.0.0/8	R1
10.1.0.0/16	R2
10.1.2.0/24	R3
10.1.2.128/25	R4

What is the next hop for destination IP 10.1.2.200?
Concept: Choose the most specific prefix.

(a) R1

(b) R2

(c) R3

(d) R4

Q4. Multiple Match Resolution

Forwarding table:

Prefix	Next Hop
172.16.0.0/16	R1
172.16.64.0/18	R2
172.16.64.0/19	R3
172.16.96.0/19	R4

What is the next hop for the following destination IPs? 172.16.65.1, 172.16.97.2, 172.16.120.8, 172.16.10.9
Concept: Longest prefix match across overlapping prefixes.

(a) R2, R4, R1, R1

(b) R3, R4, R4, R1

(c) R3, R4, R3, R1

(d) R2, R3, R3, R4

Q5. Gate 2012 CIDR Allocation

ISP owns 245.248.128.0/20. Half goes to Org A, one-quarter to Org B, rest stays.

Which allocation is valid?

Concept: Divide CIDR space based on prefix lengths.

(a) 245.248.136.0/21 and 245.248.128.0/22

(b) 245.248.128.0/21 and 245.248.128.0/22

(c) 245.248.132.0/22 and 245.248.132.0/21

(d) 245.248.136.0/24 and 245.248.132.0/21

Q6. Default Route Usage

A packet destined to 172.20.20.1 is received. Forwarding table:

172.16.0.0/12	R1
172.20.0.0/16	R2
0.0.0.0/0	R3

Concept: Specific vs default route.

(a) R1

(b) R2

(c) R3

(d) None

Q7. No Matching Prefix

Table:

192.168.1.0/24	R1
192.168.2.0/24	R2

Destination = 10.0.0.5. What happens?

Concept: No match + no default route.

(a) Packet dropped

(b) Sent to R1

(c) Sent to R2

(d) Causes loop

Q8. Overlapping Subnets

Table:

100.64.0.0/10	R1
100.64.0.0/11	R2

Destination IP = 100.64.128.1
Concept: Choose route with longer prefix match.

(a) R1

(b) R2

(c) Equal priority

(d) Dropped

Q9. Matching with Varying Subnet Masks

Routing table:

192.168.0.0/18	R1
192.168.64.0/18	R2

Destination = 192.168.64.100
Concept: Check bit match carefully.

(a) R1

(b) R2

(c) Either

(d) None

Q10. Minimum Size for Specific Allocation

How many /26 subnets can be created from 192.168.0.0/24?
Concept: Subnetting calculations.

(a) 4

(b) 8

(c) 16

(d) 32

Q11. Prefix Range Calculation

What is the range of IPs in 172.31.192.0/18?
Concept: CIDR block to IP range.

(a) 172.31.192.0 - 172.31.255.255

(b) 172.31.192.0 - 172.31.223.255

(c) 172.31.192.0 - 172.31.195.255

(d) 172.31.192.0 - 172.31.207.255

Q12. Routing Table with Trap

Table:

192.0.2.0/24	R1
192.0.2.128/25	R2

Destination = 192.0.2.130
Concept: Confusing overlapping prefixes.

(a) R1

(b) R2

(c) Both

(d) None

34 Computer Network: Error Detection and Correction

Q1. Hamming Code

A 7-bit Hamming code is used to encode 4 data bits. If the codeword received is 1011011, which bit position (if any) has the error?

Q2. CRC bits

A CRC generator uses the polynomial $x^3 + x + 1$. What will be the CRC bits for the message 11000?

Q3. Parity

Which of the following bit errors can be detected by a parity bit? (a) All odd number of bit errors (b) All even number of bit errors (c) Only single-bit errors (d) Burst errors

Q4. Bit stuffing

Bit stuffing is used in a data link protocol. Given the bitstream: `01111110111110`, what is the result after applying bit stuffing?

Q5. Hamming Distance

A 5-bit codeword has a minimum Hamming distance $p = 3$. What is the maximum number of bit errors q that can be corrected?

Q6. Even Parity

Assume an 8-bit data word. Using even parity, the parity bits are placed at positions $1, 2, 4$, and 8. What is the codeword for data 10110010?

Q7. Hamming Code

A 12-bit Hamming code contains 8 data bits and 4 check bits. If the check bits are $c_1 = 1, c_2 = 0, c_4 = 1, c_8 = 1$, what is the syndrome value?

Q8. CRG Generator Property

In CRC, which of the following generator polynomials can detect all single-bit and all double-bit errors? (a) $x+1$ (b) x^2+x+1 (c) x^3+1 (d) x^3+x+1

Q9. Hamming distance

What is the Hamming distance between the codewords 1101101 and 1110110?

Q10. Error Detection using CRC

Which of the following statements is true regarding error detection using CRC? (a) It can correct burst errors (b) It cannot detect single-bit errors (c) It can detect burst errors up to the length of the generator polynomial (d) It uses Hamming codes

Q11. CRC Generator

A sender uses CRC with generator polynomial $x^4 + x + 1$. The message is 10010011. What is the transmitted message including CRC?

Q12. Hamming Code

Given a 12-bit Hamming code with data bits d_8 to d_1 as 1 0 1 1 0 1 0 1 and parity bits $c_1 = 1,\ c_2 = 0,\ c_4 = 1,\ c_8 = x$, determine the value of x for even parity.

Q13. Odd Bit Error

Which of the following generator polynomials guarantees detection of all odd number of bit errors? (a) $x + 1$ (b) $x^2 + 1$ (c) $x^2 + x + 1$ (d) $x^3 + x + 1$

Q14. Hamming Distance

What is the minimum Hamming distance required to correct up to 3-bit errors?

Q15. CRC

If the generator polynomial is $x^3 + 1$ and the data is 1001, what is the CRC remainder?

Q16. Hamming codeword

Given a Hamming codeword 1100101, and assuming even parity, determine whether an error exists and if yes, at what position.

Q17. Hamming Code

A 12-bit codeword using even-parity Hamming code is received as 110110101101. Identify the bit position in error.

Q18

Which of the following statements is FALSE? (a) Hamming code can detect and correct single-bit errors (b) CRC is used for error correction (c) Bit stuffing is used to avoid flag sequences (d) Parity can only detect even number of bit errors

Q19

Which of the following errors can a 4-bit CRC polynomial detect for messages of up to 15 bits? (a) All 3-bit errors (b) All 2-bit burst errors (c) All 4-bit burst errors (d) All errors with odd number of bits

Q20

In a 7-bit Hamming code, which bit positions are reserved for parity? (a) 1, 2, 4 (b) 1, 2, 3 (c) 2, 4, 6 (d) 1, 3, 5

35 Computer Networks: IP Header and Modification Rules

Q1. IP Header Fields Modified by Routers

Which of the following fields in the IP header are modified by a router?

(a) Source IP and TTL

(b) Destination IP and Header Checksum

(c) TTL and Header Checksum

(d) Source IP and Destination IP

Concept: Only TTL and checksum are updated per hop.

Q2. Change in IP Header at Each Hop

Which of the following is **always** modified at every router hop in an IPv4 network?

(a) Source IP

(b) Destination IP

(c) Header Length

(d) Time to Live (TTL)

Q3. IP vs TCP/UDP Headers

Which of the following statements is correct?

(a) The TCP header is modified by routers to adjust window size.

(b) The UDP header includes sequence numbers.

(c) The IP header contains source and destination IPs; routers modify MAC addresses instead.

(d) Routers always modify both IP and TCP headers.

Q4. Encapsulation and Headers

Which headers are added to a packet during encapsulation from Transport to Link layer?

(a) Only TCP/UDP header

(b) TCP/UDP + IP header

(c) TCP/UDP + IP + MAC headers

(d) Only IP and MAC headers

Concept: Each layer adds its own header.

Q5. Fragmentation Impact on Header

Which field in the IPv4 header is used to handle packet fragmentation?

(a) Identification

(b) Time to Live

(c) Total Length

(d) Protocol

Concept: The ‘Identification‘, ‘Fragment Offset‘, and ‘More Fragments (MF)‘ bits are used.

36 Computer Networks: Maximum Segment Size (MSS)

Q1. MSS and MTU Relationship

Assume MTU = 1500 bytes, IP header = 20 bytes, and TCP header = 20 bytes. What is the MSS (Maximum Segment Size)?

(a) 1500 bytes

(b) 1480 bytes

(c) 1460 bytes

(d) 1440 bytes

Concept: MSS = MTU - IP header - TCP header = 1500 - 20 - 20 = 1460

Q2. Effect of Lower MSS Advertised by Receiver

During TCP handshake, a host advertises MSS = 512 bytes. The MTU of the link is 1500 bytes. Which of the following is true?

(a) The sender will still send 1460-byte segments.

(b) The sender will limit each segment to 512 bytes.

(c) The sender will fragment each 1500-byte segment.

(d) TCP ignores MSS; only MTU is relevant.

Concept: MSS advertised by the receiver limits the sender's segment size.

Q3. MSS and Fragmentation

Assume a link with MTU = 1000 bytes. A TCP segment has 960 bytes of data, with 20-byte IP and 20-byte TCP headers. Will IP fragmentation occur?

(a) Yes, because total size = 1000

(b) Yes, because total size = 1000 + 20 = 1020

(c) No, because 960 ¡ 1000

(d) No, because IP supports any size segment

Concept: Total = 20 (IP) + 20 (TCP) + 960 = 1000 → No fragmentation.

Q4. MSS on Different Paths

A sender is connected to two receivers over different paths: - Path A: MTU = 1400 bytes - Path B: MTU = 1200 bytes

What is the maximum TCP payload sent to **each receiver**, assuming standard IP and TCP headers?

(a) A: 1360, B: 1240

(b) A: 1380, B: 1280

(c) A: 1360, B: 1160

(d) A: 1400, B: 1200

Concept: MSS = MTU - 20 (IP) - 20 (TCP)

Q5. MSS vs Window Size

Which of the following statements is true regarding MSS and TCP Window size?

(a) MSS controls the number of bytes sent per connection

(b) Window size limits segment size directly

(c) MSS limits max payload per segment; window size controls how many unacknowledged segments can be sent

(d) MSS and window size are always equal

37 Probability: Conditional Probability and Bayes' Theorem

1. A card is drawn from a well-shuffled standard deck. What is the probability that it is a king given that it is a face card?

   (a) $\frac{1}{3}$
   (b) $\frac{1}{13}$
   (c) $\frac{1}{4}$
   (d) $\frac{3}{13}$

2. An urn contains 3 red and 5 blue balls. A ball is drawn at random. What is the probability that it is red, given that it is not blue?

   (a) $\frac{3}{5}$
   (b) $\frac{1}{2}$
   (c) 1
   (d) $\frac{3}{8}$

3. If $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find $P(A \mid B)$.

   (a) 0.2
   (b) 0.5
   (c) 0.6
   (d) 0.3

4. Two fair dice are thrown. What is the probability that the sum is 8 given that the first die shows a 3?

   (a) $\frac{1}{6}$
   (b) $\frac{1}{36}$
   (c) $\frac{1}{5}$
   (d) $\frac{1}{3}$

5. A bag contains 4 white and 6 black balls. Two balls are drawn one after the other without replacement. What is the probability that the second ball is white given that the first is black?

   (a) $\frac{4}{9}$
   (b) $\frac{2}{5}$
   (c) $\frac{4}{10}$
   (d) $\frac{4}{9}$

6. If a student is known to forget homework 10% of the time, and gets a zero when they forget. If they got a zero, what is the probability they forgot (assume 5

   (a) $\frac{1}{3}$
   (b) $\frac{2}{3}$
   (c) 0.67
   (d) 0.5

7. Let $P(A) = 0.4$, $P(B) = 0.6$, and $P(B|A) = 0.5$. Find $P(A \cap B)$.

   (a) 0.2
   (b) 0.5
   (c) 0.24
   (d) 0.12

8. A person speaks truth 75% of the time. They report a 6 on a die. What is the probability that it was actually 6?

   (a) $\frac{3}{7}$
   (b) $\frac{2}{9}$
   (c) $\frac{6}{13}$
   (d) $\frac{5}{12}$

9. If $P(A|B) = 0.8$ and $P(B) = 0.5$, find $P(A \cap B)$.
   (a) 0.3
   (b) 0.4
   (c) 0.2
   (d) 0.5

10. In a class, 40% students are girls. 70% of the girls and 50% of the boys passed. What is the probability that a randomly selected student is a girl given that they passed?
    (a) $\frac{28}{53}$
    (b) $\frac{7}{17}$
    (c) $\frac{14}{23}$
    (d) $\frac{21}{41}$

11. A test for a disease is 95% accurate. If 0.5% of people have the disease, what is the chance that a person who tested positive actually has the disease?
    (a) Around 9%
    (b) Around 50%
    (c) Around 95%
    (d) Around 0.5

12. If $P(A) = 0.3$, $P(B) = 0.6$, $P(A|B) = 0.5$, find $P(B|A)$.
    (a) 0.9
    (b) 1.0
    (c) 0.6
    (d) 0.5

13. A ball is drawn from a box with 2 red, 3 blue, and 5 green balls. Given it's not green, what is the probability it is blue?

    (a) $\frac{1}{2}$
    (b) $\frac{3}{5}$
    (c) $\frac{3}{10}$
    (d) $\frac{3}{8}$

14. If events A and B are independent and $P(A) = 0.6$, $P(B) = 0.7$, find $P(A|B)$.

    (a) 0.7
    (b) 0.6
    (c) 0.42
    (d) 1.0

15. If $P(B|A) = 0.7$, $P(A) = 0.2$, and $P(B) = 0.5$, find $P(A|B)$.

    (a) 0.28
    (b) 0.5
    (c) 0.9
    (d) 0.7

16. If $P(A \cup B) = 0.7$, $P(A) = 0.4$, $P(B) = 0.5$, find $P(A \cap B)$.

    (a) 0.1
    (b) 0.2
    (c) 0.3
    (d) 0.7

17. Let event A be "person wears glasses" and B be "person reads books". If $P(A) = 0.3$, $P(B) = 0.5$, and $P(A|B) = 0.6$, find $P(B|A)$.

    (a) 0.9
    (b) 1.0
    (c) 0.4
    (d) 0.6

18. An email is spam with probability 0.2. A word "offer" appears in 70% of spam and 10% of non-spam. What is the probability an email is spam given it contains "offer"?

    (a) $\frac{14}{31}$

    (b) $\frac{7}{10}$

    (c) $\frac{2}{5}$

    (d) $\frac{1}{2}$

19. If $P(A) = 0.6$, $P(B) = 0.5$, and A and B are mutually exclusive, find $P(A|B)$.

    (a) 0.6

    (b) 0.0

    (c) 0.5

    (d) 1.0

20. An unbiased coin is tossed. If head, a die is rolled. If tail, two dice are rolled. What is the probability of getting a sum of 2?

    (a) $\frac{1}{12}$

    (b) $\frac{1}{24}$

    (c) $\frac{1}{36}$

    (d) $\frac{1}{18}$

38 Discrete Distributions and Conditional Expectation

1. Let X be a Bernoulli random variable with $P(X = 1) = 0.7$. Find $E[X]$.

    (a) 0.3

    (b) 0.7

    (c) 1.0

    (d) 0.49

2. A Binomial variable $X \sim \text{Bin}(n = 5, p = 0.6)$. Find $P(X = 2)$.
   (a) 0.3456
   (b) 0.2304
   (c) 0.4096
   (d) 0.345
3. A fair coin is tossed 4 times. What is the expected number of heads?
   (a) 1
   (b) 2
   (c) 4
   (d) 0.5
4. $X \sim \text{Geom}(p = 0.25)$. Find $E[X]$.
   (a) 1
   (b) 2
   (c) 4
   (d) 3
5. A machine produces defective items with probability $p = 0.05$. If 100 items are produced, what is the expected number of defectives?
   (a) 1
   (b) 5
   (c) 10
   (d) 2
6. For $X \sim \text{Poisson}(\lambda = 3)$, compute $P(X = 2)$.
   (a) $\frac{9e^{-3}}{2}$
   (b) $3e^{-2}$
   (c) $\frac{6e^{-3}}{2}$
   (d) $\frac{6e^{-3}}{1}$

7. A Poisson random variable has mean 4. What is its variance?
   (a) 2
   (b) 4
   (c) 1
   (d) 8

8. If $X \sim \text{Bin}(n = 10, p = 0.2)$, find $\text{Var}(X)$.
   (a) 1.6
   (b) 2.0
   (c) 4.0
   (d) 3.2

9. Let $X \sim \text{Bin}(n = 5, p)$. For what value of p is $\text{Var}(X)$ maximized?
   (a) $p = 0.5$
   (b) $p = 1.0$
   (c) $p = 0.25$
   (d) $p = 0.75$

10. In a Poisson process, what is $P(X \geq 1)$ if $\lambda = 2$?
   (a) $1 - e^{-2}$
   (b) e^{-2}
   (c) $1 - 2e^{-2}$
   (d) 1

11. Two dice are rolled. Let $X = \text{sum}$, $Y = \max$. What is $E[Y \mid X = 7]$?
   (a) 4.5
   (b) 5
   (c) 4.83
   (d) 6

12. Joint PMF of X, Y is given as $P(X = 0, Y = 0) = 0.1$, $P(0, 1) = 0.2$, $P(1, 0) = 0.3$, $P(1, 1) = 0.4$. Find $P(X = 1)$.

    (a) 0.7
    (b) 0.4
    (c) 0.3
    (d) 0.5

13. Using above joint PMF, find $E[Y \mid X = 1]$

    (a) 0.4
    (b) 0.5
    (c) 0.6
    (d) 0.8

14. Let $X = 1$ if coin is Head, 0 otherwise. $Y = 2X + 1$. Find $\mathrm{Cov}(X, Y)$

    (a) 0.25
    (b) 0.5
    (c) 0
    (d) 1

15. If X, Y are independent with $\mathrm{Var}(X) = 3$, $\mathrm{Var}(Y) = 2$, what is $\mathrm{Cov}(X, Y)$?

    (a) 0
    (b) 1
    (c) 5
    (d) 6

16. For same data, what is $\mathrm{Var}(X + Y)$?

    (a) 3
    (b) 5
    (c) 6
    (d) 1

17. If $\text{Cov}(X, Y) = 0$, and $\rho(X, Y) = 0$, then:
    (a) X and Y are independent
    (b) X and Y are uncorrelated
    (c) X = Y
    (d) None of these

18. A fair die is rolled. Let X be the outcome. Compute $E[X]$.
    (a) 3.0
    (b) 3.5
    (c) 4.0
    (d) 3

19. Let X be Poisson($\lambda = 2$). Find $P(X = 0 \mid X \text{ even})$
    (a) $\frac{e^{-2}}{\cosh(2)}$
    (b) e^{-2}
    (c) $2e^{-2}$
    (d) $\frac{e^{-2}}{2}$

20. For independent X, Y with $E[X] = 2, E[Y] = 3$, compute $E[XY]$
    (a) 5
    (b) 6
    (c) 3
    (d) 1

39 Expected Value: GATE-Style Questions

Q1: Quiz Show Strategy (Based on GATE 2021)

A contestant is presented with two questions. If he answers the first question correctly, he is allowed to attempt the second one. Otherwise, he is disqualified. If he attempts question i first, he will only be allowed to attempt the other question if he answers i correctly. Let the probabilities that he knows the answers be $P_1 = 0.8$ and $P_2 = 0.5$. The rewards for answering questions correctly are $V_1 = 1000$ and $V_2 = 2000$ respectively.
Which question should he attempt first to maximize expected reward?

(a) Question 1

(b) Question 2

(c) Either order gives same expected value

(d) Not enough data to decide

Q2: Quiz Show Variation

Let $P_1 = 0.6$, $P_2 = 0.9$, $V_1 = 3000$, $V_2 = 1000$.
Which question should he answer first for maximum expected value?

(a) Question 1

(b) Question 2

(c) Either order gives same expected value

(d) Cannot determine

Q3: Quiz Show - NAT

Let $P_1 = 0.5$, $P_2 = 0.6$, $V_1 = 1000$, $V_2 = 2000$.
What is the maximum expected value (in \$) the contestant can receive? (Enter integer)

Q4: Quiz Show with Equal Rewards

Let $P_1 = 0.9$, $P_2 = 0.5$, and both questions carry reward of $V = 2000$.
Which question should be attempted first?

(a) Question 1

(b) Question 2

(c) Order does not matter

(d) Attempt both simultaneously

Q5: Coin Toss Game

A biased coin with probability $p = 0.6$ of Heads is tossed 3 times. You earn \$10 for each Head and lose \$5 for each Tail.
What is the expected net earning?

(a) \$5.4

(b) \$6.0

(c) \$7.2

(d) \$8.1

Q6: Dice Gamble

A player rolls a fair six-sided die. If it shows 1 or 2, he earns \$10. If it shows 3, 4, or 5, he earns \$5. If it shows 6, he loses \$20.
What is the expected earning from one roll? (NAT – round to nearest integer)

Q7: Card Drawing Game

You draw one card from a well-shuffled standard deck (52 cards). You get: - \$20 if the card is an Ace, - \$10 if it's a face card (J, Q, K), - \$5 if it's a number card (2–10).
What is the expected reward? (NAT – up to 2 decimal places)

Q8: Decision Tree EV

You are offered a game where you flip a coin: - If Heads, you roll a fair 6-sided die and earn \$die_value $\times$ 10. - If Tails, you get nothing.
What is the expected value of this game? (NAT – integer answer)

40 Variance: GATE-Level Conceptual and Numerical Questions

Q1: Conceptual (MCQ)

Let X be a discrete random variable taking values in $\{1, 2, 3, 4\}$ with uniform probability.
What is $\text{Var}(X)$?

(a) $\frac{5}{4}$

(b) $\frac{3.5^2}{2}$

(c) $\frac{5}{3}$

(d) $\frac{5}{2}$

Q2: Calculation (NAT)

Let a random variable X take values $\{-1, 0, 1\}$ with probabilities $P(X = -1) = 0.25$, $P(X = 0) = 0.5$, $P(X = 1) = 0.25$.
Compute $\text{Var}(X) =$ ________

Q3: Distribution Transformation (MCQ)

Let X be a random variable with $\mathbb{E}[X] = 3$ and $\text{Var}(X) = 4$. Define $Y = 2X + 5$.
What is $\text{Var}(Y)$?

(a) 4

(b) 8

(c) 16

(d) 64

Q4: Variance of Indicator Variable (MCQ)

Let X be a Bernoulli random variable such that $P(X = 1) = p$ and $P(X = 0) = 1 - p$. What is $\text{Var}(X)$?

(a) p

(b) $1 - p$

(c) $p(1 - p)$

(d) p^2

Q5: Composite Distribution (NAT)

Let X be a discrete random variable taking values $1, 2, 3$ with probabilities proportional to their square:

$$P(X = x) = \frac{x^2}{\sum_{k=1}^{3} k^2}$$

Compute $\text{Var}(X)$.

41 Binomial Distribution

Q1

Let X be a binomial random variable with parameters $n = 10$ and $p = 0.3$. What is the probability that X is even?
(a) 0.3828 (b) 0.5168 (c) 0.6578 (d) 0.8234

Q2

You toss a biased coin $n = 8$ times. Probability of getting a head is $p = 0.6$. What is the expected number of times you get exactly 3 heads?
(a) 0.2458 (b) 0.2936 (c) 0.3125 (d) 0.3362

Q3

A random variable X follows Binomial distribution with unknown n, but with $p = 0.5$. If $E[X] = 6$ and $\text{Var}(X) = 3$, then the value of n is ____.

Q4

Let X be a binomial random variable with parameters $n = 6$ and $p = 1/4$. Then $\mathbb{P}(X = 0 \cup X = 6)$ is equal to:
(a) 0.1780 (b) 0.2373 (c) 0.3342 (d) 0.3184

Q5

Consider a random variable $X \sim \text{Binomial}(10, p)$. Suppose it is known that $\mathbb{P}(X = 3)$ is maximum. Which of the following values can p take? Select all that apply:
MSQ: (a) $p = 0.25$ (b) $p = 0.30$ (c) $p = 0.33$ (d) $p = 0.40$

Q6

Let $X \sim \text{Binomial}(n, p)$ and $Y = n - X$. Which of the following statements is/are always true?
MSQ:
(a) Y is also binomially distributed with parameters $(n, 1 - p)$
(b) $\mathbb{E}[Y] = n(1 - p)$
(c) $\text{Var}(X) = \text{Var}(Y)$
(d) X and Y are independent

Q7

Suppose you conduct n independent Bernoulli trials with success probability p and record the number of successes X. If p is doubled and n is halved (assume np remains constant), then:
(a) Mean of X remains the same but variance decreases
(b) Mean of X increases but variance remains the same
(c) Both mean and variance remain the same
(d) Mean remains the same, but variance increases

42 Poisson Distribution

Q1

If the number of phone calls arriving at a call center follows a Poisson distribution with mean 5 per minute, what is the probability that exactly 7 calls arrive in a given minute?

Q2

The number of errors in a page of a printed book follows a Poisson distribution with mean 0.3. What is the probability that a randomly chosen page has **no** errors?

Q3

Let $X \sim \text{Poisson}(\lambda)$. Which of the following statements is/are true?
MSQ:
(a) $\mathbb{E}[X] = \lambda$
(b) $\text{Var}(X) = \lambda^2$
(c) $P(X = 0) = e^{-\lambda}$
(d) $P(X = k) = \dfrac{\lambda^k e^{-\lambda}}{k!}, \forall k \in \mathbb{N}_0$

Q4

The arrival of customers at a ticket counter follows a Poisson process with an average of 4 customers per 10 minutes. What is the probability that **at least one** customer arrives in a 5-minute interval?

Q5

Let $X_1 \sim \text{Poisson}(\lambda_1)$ and $X_2 \sim \text{Poisson}(\lambda_2)$ be independent. Then $X = X_1 + X_2$ follows a Poisson distribution. What is the parameter λ of X?

Q6

Suppose the number of customer arrivals at a store follows a Poisson process. Which of the following is **not** a property of the Poisson process?

MSQ:

(a) Arrivals occur one at a time.
(b) The process has independent increments.
(c) The probability of an arrival in a very small interval is proportional to the length of the interval.
(d) The time between successive arrivals follows a uniform distribution.

Q7

Let X be a Poisson random variable with parameter λ. Consider the following statements:

MSQ:

(a) The moment generating function (MGF) of X exists and is finite for all real numbers.
(b) The Poisson distribution is a limiting case of the Binomial distribution.
(c) The sum of two independent Poisson random variables is not a Poisson random variable.
(d) The probability that X takes an even value is the same as the probability it takes an odd value when $\lambda = \ln(2)$.

Q8

The number of calls arriving at a call center in one minute follows a Poisson distribution with mean $\lambda = 4$. What is the probability that the number of calls in two consecutive minutes differs by more than 3?

(a) > 0.5
(b) < 0.5
(c) Equal to 0.5
(d) Cannot be determined without more information

Q9

Let the number of printing errors per 100 pages in a book follow a Poisson distribution with a mean of 2. What is the probability that a randomly selected 200-page segment contains **at most 2** errors?

(a) $e^{-4}(1+4+8)$
(b) $e^{-2}(1+2+2^2)$
(c) $e^{-4}(1+4+8+\frac{16}{6})$
(d) $1-e^{-2}$

Q10

A radioactive source emits α-particles at an average rate of 3 per second. What is the probability that in a 10-second interval, at least one second will have more than 5 emissions?

(a) Greater than 0.9
(b) Less than 0.5
(c) Equal to 1
(d) Cannot be determined exactly

Q11

A post office receives an average of 10 customers per hour. If the post office is open for 8 hours a day, what is the probability that there are **exactly 5 hours** in the day during which the number of customers exceeds 12?

(a) Poisson with $\lambda = 8$ used with binomial
(b) Binomial with $n = 8$, $p = P(X > 12)$
(c) Cannot be modeled
(d) Binomial with $n = 12$, $p = P(X < 10)$

43 Geometric Random Variable: Moderate to Hard GATE Questions

Q1

Let X be a geometric random variable with success probability $p = 0.2$. Compute $\mathbb{P}(X > 4)$.

(a) 0.4096
(b) 0.32768
(c) 0.8^4
(d) Both (b) and (c)

Q2

If X is a geometric random variable with parameter p, then what is the expected number of trials until the first success?

(a) $\dfrac{1}{1-p}$
(b) $\dfrac{1}{p}$
(c) $\dfrac{1-p}{p}$
(d) $\dfrac{p}{1-p}$

Q3

Let X be a geometric random variable with $p = 0.4$. Compute $\mathbb{P}(X = 3)$.

(a) 0.144
(b) 0.096
(c) 0.36
(d) 0.064

Q4

If X is a geometric random variable with mean 5, what is the value of p?

(a) 0.2
(b) 0.8
(c) 5
(d) $1 - 0.2$

Q5

Let X and Y be independent geometric random variables with parameters $p = 0.5$ and $q = 0.25$ respectively. Find $\mathbb{P}(X < Y)$.

(a) $\dfrac{p}{p+q-pq}$
(b) $\dfrac{p}{p+q}$

(c) $\frac{q}{p+q}$
(d) $\frac{1}{2}$

Q6

In a memoryless process, the probability of success on the n-th trial, given that the first $n-1$ trials were failures, is the same as the probability of success on the first trial. This property is uniquely satisfied by:
(a) Binomial Distribution
(b) Geometric Distribution
(c) Poisson Distribution
(d) Normal Distribution

Q7

Which of the following is true for the geometric distribution with success probability p?
(a) It has finite variance only for $p > 0.5$
(b) It can take values from 0 to ∞
(c) It is memoryless and discrete
(d) Its PMF is symmetric about the mean

44 Expected Value of Sums of Random Variables

Q1

Let X and Y be two independent random variables with $\mathbb{E}[X] = 4$ and $\mathbb{E}[Y] = 6$. Compute $\mathbb{E}[2X + 3Y]$.
(a) 26
(b) 24
(c) 30
(d) 18

Q2

Suppose $X_1, X_2, \ldots, X_{10}$ are i.i.d. random variables each with $\mathbb{E}[X_i] = \mu = 5$. What is $\mathbb{E}[X_1 + X_2 + \cdots + X_{10}]$?

(a) 50
(b) 25
(c) 10
(d) 5

Q3

Let X and Y be two dependent random variables with $\mathbb{E}[X] = 2$, $\mathbb{E}[Y] = 3$, and $\mathbb{E}[XY] = 10$. What is $\mathbb{E}[X + Y]$?
(a) 10
(b) 6
(c) 5
(d) Cannot be determined

Q4

Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed (i.i.d) random variables with $\mathbb{E}[X_i] = \mu$ and $\mathrm{Var}(X_i) = \sigma^2$. What is the expected value of their average $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$?
(a) $n\mu$
(b) $\frac{\mu}{n}$
(c) μ
(d) Cannot be determined

Q5

Let X be a discrete random variable taking values $\{1, 2, 3\}$ with probabilities $\mathbb{P}(X = 1) = \frac{1}{6}$, $\mathbb{P}(X = 2) = \frac{1}{2}$, $\mathbb{P}(X = 3) = \frac{1}{3}$. Let $Y = 2X + 1$. Compute $\mathbb{E}[Y]$.
(a) 5
(b) 6
(c) 7
(d) 8

Q6

Which of the following is always true for any two random variables X and Y?

(a) $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$
(b) $\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y]$
(c) $\mathbb{E}[X-Y] = \mathbb{E}[X] - \mathbb{E}[Y]$
(d) Both (a) and (c)

Q7

Let $X_1, X_2, ..., X_n$ be i.i.d. random variables. Which of the following statements is **false**?
(a) $\mathbb{E}\left[\sum_{i=1}^{n} X_i\right] = n \cdot \mathbb{E}[X_1]$
(b) $\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \mathbb{E}[X_1]$
(c) $\mathbb{E}[X_i + X_j] = \mathbb{E}[X_i] + \mathbb{E}[X_j]$
(d) $\mathbb{E}[X_i^2 + X_j^2] = (\mathbb{E}[X_i])^2 + (\mathbb{E}[X_j])^2$

45 Cumulative Distribution Function (CDF): Conceptual Questions

Q1

Let $F_X(x)$ be the cumulative distribution function of a random variable X. Which of the following is **not necessarily true** for all real-valued random variables?
(a) F_X is a non-decreasing function.
(b) $\lim_{x\to\infty} F_X(x) = 1$
(c) $F_X(x)$ is continuous for all x
(d) $\lim_{x\to-\infty} F_X(x) = 0$

Q2

If $F_X(x)$ is the cumulative distribution function of a discrete random variable X, then $F_X(x)$ is:
(a) A step function that is right-continuous
(b) A step function that is left-continuous
(c) A continuous and differentiable function
(d) Always strictly increasing

Q3

Suppose X is a random variable and $F_X(x)$ is its CDF. Which of the following expressions correctly represents the probability that X lies in the interval $(a, b]$?

(a) $F_X(b) - F_X(a)$
(b) $F_X(b) - F_X(a-)$
(c) $F_X(b) - F_X(a+)$
(d) $F_X(b+) - F_X(a+)$

Q4

Which of the following statements about the CDF of any real-valued random variable is **false**?

(a) $F_X(x)$ is bounded between 0 and 1
(b) $F_X(x)$ is differentiable everywhere
(c) $F_X(x)$ is right-continuous
(d) $\lim_{x\to-\infty} F_X(x) = 0$ and $\lim_{x\to\infty} F_X(x) = 1$

Q5

Let $F_X(x)$ be the CDF of a random variable X. Then $\mathbb{P}(X = a)$ equals:

(a) $F_X(a) - F_X(a-)$
(b) $F_X(a+)$
(c) $F_X(a)$
(d) $F_X(a-) - F_X(a+)$

46 Introduction to Continuous Random Variables

Q1

Let X be a continuous random variable with probability density function (PDF) given by

$$f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $\mathbb{P}(0.25 \leq X \leq 0.75)$.

Q2

If X is a continuous random variable with PDF $f(x)$, which of the following is **always true**?

(a) $f(x) \geq 0$ for all x
(b) $f(x)$ is always continuous
(c) $f(x) \leq 1$ for all x
(d) $f(x)$ has a finite number of local maxima

Q3

Let X be a continuous random variable with PDF:

$$f(x) = \begin{cases} k(1 - x^2), & -1 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k such that $f(x)$ is a valid PDF.

Q4

Let X be a continuous random variable uniformly distributed over $[a, b]$. Then the cumulative distribution function (CDF) $F_X(x)$ is:

(a) $\dfrac{x-a}{b-a}$ for $x \in [a, b]$
(b) $1 - \dfrac{x-a}{b-a}$ for $x \in [a, b]$
(c) Constant for all x
(d) $\dfrac{b-x}{b-a}$ for $x \in [a, b]$

Q5

Let X be a continuous random variable with PDF $f(x)$ defined as

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Compute $\mathbb{E}[X]$.

Q6

Which of the following statements is **false** for a continuous random variable X?

(a) $\mathbb{P}(a < X \leq b) = \int_a^b f(x)\,dx$
(b) $\mathbb{P}(X = c) = 0$ for any real c
(c) $\mathbb{P}(a \leq X \leq b) = \mathbb{P}(a < X < b)$
(d) The area under the PDF curve can be greater than 1

Q7

A continuous random variable X has the PDF:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

This distribution is known as:

(a) Normal distribution
(b) Poisson distribution
(c) Exponential distribution
(d) Uniform distribution

47 Expectation and Variance of Continuous Random Variables

Q1

Let X be a continuous random variable with PDF:

$$f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

Q2

Let X be a continuous random variable uniformly distributed over $[2, 4]$. Compute $\mathbb{E}[X^2]$ and $\mathrm{Var}(X)$.

Q3

The exponential distribution with parameter $\lambda > 0$ has the PDF

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Compute $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

Q4

Let X be a continuous random variable with PDF:

$$f(x) = \begin{cases} \frac{1}{2}x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

Q5

If X is a continuous random variable with mean μ and variance σ^2, what is the variance of $Y = 3X + 5$?

Q6 (Conceptual)

Which of the following is **true** for a continuous random variable X?
(a) $\mathbb{E}[a] = a$ for any constant a
(b) $\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$
(c) $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$
(d) All of the above

Q7 (Conceptual)

Let X and Y be independent continuous random variables with variances σ_X^2 and σ_Y^2. What is the variance of $X + Y$?
(a) $\sigma_X^2 + \sigma_Y^2$
(b) $\sigma_X^2 - \sigma_Y^2$

(c) $|\sigma_X^2 - \sigma_Y^2|$
(d) Depends on the distribution

48 Uniform Random Variable: GATE-Style Questions

Q1 (MCQ)

Let $X \sim \mathcal{U}(2,6)$. What is the probability that $X \leq 3$?
(a) $\frac{1}{4}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{2}{3}$

Q2 (NAT)

Let X be uniformly distributed over $[1,5]$. Find the expected value $\mathbb{E}[X]$.

Q3 (MCQ)

Let X be a discrete uniform random variable over the set $\{1,2,3,4,5,6\}$. What is $\mathbb{E}[X]$?
(a) 3.5 **(b)** 3 **(c)** 4 **(d)** 2.5

Q4 (MSQ)

Let $X \sim \mathcal{U}(0,10)$. Which of the following are true?
(a) $\mathbb{E}[X] = 5$ **(b)** $\text{Var}(X) = \frac{25}{3}$ **(c)** $\mathbb{P}(X > 8) = 0.2$ **(d)** PDF is constant for $x \in [0,10]$

Q5 (MCQ)

Let X be a continuous uniform random variable on $[a,b]$. Which of the following is the correct formula for $\text{Var}(X)$?
(a) $\frac{(b-a)^2}{4}$ **(b)** $\frac{(b-a)^2}{12}$ **(c)** $\frac{(b+a)^2}{12}$ **(d)** $\frac{(b+a)^2}{4}$

Q6 (NAT)

Let X be a continuous random variable with uniform distribution over $[3,9]$. Compute $\text{Var}(X)$.

Q7 (Conceptual MCQ)

Which of the following statements is always true for a continuous uniform distribution $\mathcal{U}(a,b)$?
(a) The PDF is symmetric about $\frac{a+b}{2}$
(b) The mode equals the mean
(c) The CDF is linear in $[a,b]$
(d) $\text{Var}(X)$ depends only on the range $(b-a)$

Q8 (Conceptual MSQ)

Which of the following statements are true for a discrete uniform distribution on the set $\{1,2,...,n\}$?
(a) All values have the same probability
(b) $\text{Var}(X) = \frac{n^2-1}{12}$
(c) $\mathbb{E}[X] = \frac{n+1}{2}$
(d) $\text{PDF}(x) = \frac{1}{n}$ for all x

Q9 (NAT)

Let X be a discrete uniform random variable on $\{1,2,3,4,5\}$.
Find $\text{Var}(X)$.

Q10 (MCQ)

Let $X \sim \mathcal{U}(-1,1)$. What is the probability that $|X| < \frac{1}{2}$?
(a) 0.25 **(b)** 0.5 **(c)** 1 **(d)** 0.75

49 Normal Random Variable

Q1 (MCQ)

Let $X \sim \mathcal{N}(5,4)$ (i.e., mean = 5, variance = 4). What is the probability that X lies within one standard deviation of the mean?
(a) 0.68 **(b)** 0.95 **(c)** 0.997 **(d)** 0.34

Q2 (NAT)

If $X \sim \mathcal{N}(0,1)$ and $P(X \leq z) = 0.8413$, then the value of z is ________ (round to 2 decimal places).

Q3 (MSQ)

Which of the following are properties of the standard normal distribution?
(a) Mean $= 0$ **(b)** Variance $= 1$ **(c)** Symmetric about $x = 1$ **(d)** Total area under PDF $= 1$

Q4 (MCQ)

Let $X \sim \mathcal{N}(10, 1)$. What is the value of $P(9 \leq X \leq 11)$ approximately?
(a) 0.68 **(b)** 0.95 **(c)** 0.997 **(d)** 0.50

Q5 (Conceptual MCQ)

Which of the following best describes the shape of a normal distribution?
(a) Bell-shaped and symmetric **(b)** Skewed left **(c)** Skewed right **(d)** Uniform

Q6 (NAT)

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = 100$ and $\sigma = 15$. What value of X corresponds to $Z = 2$?

Q7 (MSQ)

Let $X \sim \mathcal{N}(10, 25)$. Which of the following transformations result in a standard normal variable Z?
(a) $Z = \frac{X-10}{5}$ **(b)** $Z = \frac{X-10}{\sqrt{25}}$ **(c)** $Z = \frac{10-X}{5}$ **(d)** $Z = \frac{X}{10}$

Q8 (Conceptual MCQ)

If $X \sim \mathcal{N}(\mu, \sigma^2)$, then which of the following statements is true?
(a) $P(X = \mu) = 1$ **(b)** $P(X = \mu) = 0$ **(c)** $P(X < \mu) = 0$ **(d)** $P(X < \mu) = 0.5$

Q9 (MCQ)

Let $X \sim \mathcal{N}(0, 1)$. What is the value of $P(-2 \leq X \leq 2)$?
(a) 0.95 **(b)** 0.997 **(c)** 0.68 **(d)** 0.84

Q10 (NAT)

The height of adult males in a population is normally distributed with a mean of 170 cm and a standard deviation of 10 cm. What is the height below which approximately 97.5% of the males lie?

50 Exponential Random Variable

Q1 [MCQ]

Let X be an exponential random variable with mean 5. What is $P(X > 10)$?

(a) e^{-2}

(b) e^{-1}

(c) $1 - e^{-2}$

(d) 0.5

Q2 [MSQ]

Let X be an exponential random variable with rate parameter $\lambda = 3$. Which of the following are true?

(a) $E[X] = \frac{1}{3}$

(b) $\text{Var}(X) = \frac{1}{9}$

(c) $P(X > 1) = e^{-3}$

(d) $P(X < 1) = 1 - e^{-3}$

Q3 [NAT]

Let X ∼Exponential with mean 4. Compute $\text{Var}(X)$.
Answer: ____________

Q4 [MCQ]

Which of the following represents the memoryless property of the exponential distribution?

(a) $P(X > s + t) = P(X > s) \cdot P(X > t)$

(b) $P(X > s + t \mid X > s) = P(X > t)$

(c) $P(X < s + t \mid X > s) = P(X < t)$

(d) $P(X > t \mid X > s + t) = P(X > s)$

Q5 [MCQ]

Let X be exponentially distributed with $\lambda = 2$. What is the value of the PDF $f_X(x)$ at $x = 1$?

(a) $2e^{-2}$

(b) e^{-2}

(c) $4e^{-2}$

(d) $2e^{-1}$

Q6 [MSQ]

Let X and Y be independent exponential random variables with rate $\lambda = 1$. Which of the following are true?

(a) $P(X < Y) = \frac{1}{2}$

(b) $P(X = Y) = 0$

(c) $P(\min(X, Y) > t) = e^{-2t}$

(d) $\min(X, Y)$ is exponential with rate 2

Q7 [NAT]

Let $X \sim \text{Exp}(\lambda)$ and $Y = aX$, where $a > 0$. Then Y is exponentially distributed with rate $\lambda' =$ ____________

Q8 [MCQ]

Which of the following distributions has the memoryless property like exponential?

(a) Normal

(b) Poisson

(c) Geometric

(d) Uniform

Q9 [MCQ]

Let the lifetime (in hours) of a component be exponentially distributed with a mean of 50 hours. What is the probability that it lasts more than 150 hours?

(a) e^{-3}

(b) $e^{-1.5}$

(c) $1 - e^{-3}$

(d) $1 - e^{-1.5}$

Q10 [MSQ - Conceptual]

Which of the following are correct properties of exponential distribution?

(a) Defined for $x > 0$

(b) CDF is $F(x) = 1 - e^{-\lambda x}$

(c) Always symmetric about the mean

(d) Has memoryless property

51 Hazard Rate Function

Q1 [MCQ]

Let X be a continuous random variable with PDF:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The hazard rate function $h(x)$ is:

(a) λx

(b) $\frac{1}{\lambda}$

(c) λ

(d) $e^{-\lambda x}$

Q2 [MSQ]

Which of the following statements are true regarding the hazard function $h(x)$?

(a) For an exponential distribution, the hazard rate is constant.

(b) Increasing hazard rate implies increasing failure rate.

(c) Hazard function is the ratio of the PDF and survival function.

(d) The hazard function is always greater than 1 for exponential distributions.

Q3 [MCQ]

The survival function is $S(t) = e^{-\lambda t^2}$ for $t \geq 0$. The hazard rate function $h(t)$ is:

(a) $2\lambda t$

(b) λt

(c) λt^2

(d) $e^{-\lambda t^2}$

Q4 [Conceptual MCQ]

The hazard rate function is best described as:

(a) The probability of failure at time t.

(b) The expected number of failures up to time t.

(c) The instantaneous failure rate at time t, given survival up to t.

(d) The cumulative distribution function of the failure time.

Q5 [NAT]

Let the lifetime of a component be exponentially distributed with mean 5 units. What is the value of the hazard rate?

Q6 [NAT]

The survival function of a system is given by $S(t) = e^{-0.01t^2}$. Compute the hazard rate at $t = 10$.

Q7 [NAT]

The PDF of a component's lifetime is given as:

$$f(t) = \frac{3t^2}{1000}, \quad 0 \leq t \leq 10$$

Calculate the hazard rate at $t = 5$. Use:

$$h(t) = \frac{f(t)}{S(t)}, \quad S(t) = 1 - \int_0^t f(x)dx$$

Q8 [Conceptual MCQ]

Which of the following distributions have increasing hazard rate functions?

(a) Exponential distribution

(b) Weibull distribution with shape parameter $k > 1$

(c) Uniform distribution

(d) Normal distribution

52 Properties of Expectation: Conceptual Questions

Q1.

Let X and Y be random variables such that $E[X] = 5$ and $E[Y] = 7$. What is $E[X + Y]$?

Q2.

Let X be a random variable and a, b be constants. Which of the following is true?

(a) $E[aX + b] = aE[X] + b$

(b) $E[aX + b] = E[X] + ab$

(c) $E[aX + b] = a + bE[X]$

(d) $E[aX + b] = aE[X - b]$

Q3.

If $E[X] = \mu$ and $Y = X - \mu$, then $E[Y]$ is:

(a) μ

(b) 0

(c) $-\mu$

(d) 1

Q4.

Let X be a discrete random variable taking values $\{1, 2, 3\}$ with equal probability. What is $E[X^2]$?

Q5.

Let X and Y be independent. Which of the following is true?

(a) $E[XY] = E[X] + E[Y]$

(b) $E[XY] = E[X]E[Y]$

(c) $E[XY] = \max(E[X], E[Y])$

(d) $E[XY] = 0$

Q6.

Which of the following is **not always true**?

(a) $E[a] = a$

(b) $E[aX] = aE[X]$

(c) $E[X + Y] = E[X] + E[Y]$

(d) $E[XY] = E[X] + E[Y]$ (without independence)

Q7.

If X is a constant random variable with value c, then $E[X^2] = ?$

Q8.

If X is a fair die roll, what is $E[3X + 4]$?

Q9.

If X and Y are random variables and $E[X] = E[Y]$, is it necessary that $X = Y$?

Q10.

If X is a discrete r.v. and f is a function, then $E[f(X)] = f(E[X])$ — is this always true?

Q11.

Let X be a non-negative random variable. Which of the following is necessarily true?

(a) $E[X] \geq 0$

(b) $E[X] > 0$

(c) $E[X] < 0$

(d) $E[X] = 0$

Q12.

If X and Y are uncorrelated, is $E[XY] = E[X]E[Y]$ always true?

Q13.

Let X be such that $P(X = 2) = 0.5$, $P(X = 4) = 0.5$. What is $E[(X - 3)^2]$?

Q14.

If X is a random variable such that $E[X^2] = 25$ and $E[X] = 3$, what is $\text{Var}(X)$?

Q15.

Let X be a continuous r.v. uniformly distributed on $[0, 2]$. Find $E[X^2]$.

Q16.

If X is a Bernoulli(p) variable, then $E[X] =$ ____________

Q17.

True or False:
If X and Y are independent, then $E[f(X)g(Y)] = E[f(X)]E[g(Y)]$

Q18.

Let X be a random variable with $E[X] = \mu$. Which of the following is true?

(a) $E[(X-\mu)^2] = 0$

(b) $E[(X-\mu)^2] = \mu$

(c) $E[(X-\mu)^2] = \mathrm{Var}(X)$

(d) None of the above

Q19.

Let X and Y be independent and $Z = X + Y$. Is it always true that $E[Z] = E[X] + E[Y]$?

Q20.

A r.v. X is symmetric about 0. What is $E[X^3]$?

53 Linear Algebra: Rank, Eigenvalues, and Eigenvectors

1. The rank of a 3×3 matrix A is 2. Which of the following is true?
   (a) A is invertible
   (b) Rows are linearly independent
   (c) Columns are linearly dependent
   (d) Rank nullity theorem is not applicable

2. If rank of a matrix is equal to number of columns, the matrix is:
   (a) Invertible
   (b) Full row rank
   (c) Full column rank
   (d) Both (b) and (c)

3. If $\text{rank}(A) = r$, then the number of non-zero eigenvalues of A is at most:
   (a) r
   (b) n
   (c) $n - r$
   (d) Always zero

4. If A is a diagonal matrix, its eigenvalues are:
   (a) All ones
   (b) All zeros
   (c) The diagonal entries
   (d) Cannot be determined

5. Let A be a 2×2 matrix with eigenvalues 2 and 3. Find the eigenvalues of A^5.

   (a) 10 and 15
   (b) 4 and 9
   (c) 32 and 243
   (d) 5 and 8

6. The eigenvalues of a real symmetric matrix are always:

   (a) Real
   (b) Complex
   (c) Imaginary
   (d) None of these

7. Let A be a 3×3 matrix such that $A^2 = I$. Then eigenvalues of A are:

   (a) All 1
   (b) 1 or -1
   (c) 0 or 1
   (d) Complex numbers

8. If A is orthogonal, then $A^T A =$

   (a) 0
   (b) A^2
   (c) I
   (d) $-A$

9. Matrix A has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. What is the trace of A?

   (a) Sum of squares of eigenvalues
   (b) Product of eigenvalues
   (c) Sum of eigenvalues
   (d) None of these

10. A matrix A is diagonalizable if:

    (a) It has n linearly independent eigenvectors
    (b) All eigenvalues are same
    (c) It is a zero matrix
    (d) Only if symmetric

11. If A is symmetric, then eigenvectors corresponding to distinct eigenvalues are:

    (a) Orthogonal
    (b) Parallel
    (c) Linearly dependent
    (d) None of these

12. Let A be a matrix with eigenvalue 4 of multiplicity 2. Which of the following must be true?

    (a) A is diagonalizable
    (b) A is not invertible
    (c) 4 is the only eigenvalue
    (d) 4 is not necessarily diagonalizable

13. The rank of matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

    is:

    (a) 3
    (b) 2
    (c) 1
    (d) 0

14. Let A be a 3×3 matrix with eigenvalues 2, 0, 5. What is $\det(A)$?
    (a) 0
    (b) 7
    (c) 10
    (d) None of these

15. Eigenvalues of an upper triangular matrix are:
    (a) All 1
    (b) Its diagonal entries
    (c) Sum of rows
    (d) Zero

16. If A has eigenvalue 2 with eigenvector $\vec{v}$, then $A^3\vec{v} = ?$
    (a) $8\vec{v}$
    (b) $6\vec{v}$
    (c) $3\vec{v}$
    (d) Cannot be determined

17. Let A be a 3×3 matrix with eigenvalues 1, 2, 3. Then eigenvalues of A^{-1} are:
    (a) $\frac{1}{6}$
    (b) 1, 2, 3
    (c) $1, \frac{1}{2}, \frac{1}{3}$
    (d) None

18. If A has only 1 eigenvector, it is:
    (a) Diagonalizable
    (b) Not diagonalizable
    (c) Symmetric
    (d) Orthogonal

19. The matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ has:
    (a) Two distinct eigenvalues
    (b) One eigenvalue, two eigenvectors
    (c) One eigenvalue, one eigenvector
    (d) Complex eigenvalues

20. Which of the following statements is false?
    (a) All real symmetric matrices are diagonalizable
    (b) Eigenvalues of a matrix are always real
    (c) Eigenvectors can be complex
    (d) Rank of A is number of non-zero singular values

54 Properties of Determinants: Conceptual Questions

Q1.

If the determinant of matrix A is zero, then which of the following must be true?

(a) A is invertible

(b) A has linearly independent rows

(c) A is singular

(d) A^2 has a non-zero determinant

Q2.

If $\det(A) = 5$, what is the value of $\det(3A)$ for a 3×3 matrix?

Q3.

If A and B are square matrices of the same order, then $\det(AB) =$ ________

Q4.

If A is a $n \times n$ matrix and λ is a scalar, then $\det(\lambda A) = ?$

Q5.

True or False: If two rows of a matrix are interchanged, the determinant remains the same.

Q6.

Which of the following matrices has determinant zero?

(a) A matrix with two identical rows

(b) A diagonal matrix with one entry zero

(c) A matrix whose rank is less than its order

(d) All of the above

Q7.

If $\det(A) = d \neq 0$, then what is $\det(A^{-1})$?

Q8.

If A is orthogonal, then $\det(A) = ?$

Q9.

Let A be a 3×3 matrix with $\det(A) = 2$. What is $\det(\text{adj}(A))$?

Q10.

Which of the following operations does **not** change the determinant value?

(a) Swapping two rows

(b) Multiplying a row by a scalar

(c) Adding a multiple of one row to another

(d) Multiplying entire matrix by scalar

Q11.

Let A be a skew-symmetric matrix of odd order. Then $\det(A)$ is:

Q12.

If A is a 3×3 matrix with real entries such that $\det(A) = 0$, then which of the following is not necessarily true?

(a) A is not invertible

(b) A has a zero eigenvalue

(c) A has full rank

(d) A is singular

Q13.

True or False: $\det(A) = \det(A^T)$ for any square matrix A

Q14.

Let A be 2×2 with $\det(A) = 7$. What is the determinant of $A^T A$?

Q15.

Let A be a diagonal matrix with entries $(\lambda_1, \lambda_2, \ldots, \lambda_n)$. What is $\det(A)$?

Q16.

If A is a square matrix and $\det(A) \neq 0$, then what can we say about the solution of $Ax = b$?

Q17.

Let A be an upper triangular matrix with non-zero diagonal entries. Then, which of the following is true?

(a) $\det(A) = 0$

(b) $\det(A)$ is sum of diagonal entries

(c) $\det(A)$ is product of diagonal entries

(d) $\det(A)$ is not defined

Q18.

If $\det(A) = 3$ and $\det(B) = 4$, then $\det(AB^{-1}) = ?$

Q19.

Let A be a matrix such that $\det(A) = 0$. Which of the following is not guaranteed?

(a) A is non-invertible

(b) A has at least one zero eigenvalue

(c) $\text{Rank}(A) <$ order of A

(d) A is diagonalizable

Q20.

If a matrix A has all its elements in a row or column zero,
then $\det(A) = ?$

55 Determinants: Using Properties for Evaluation

Q1.

Let A be a 3×3 matrix such that $\det(A) = 4$. What is the value of $\det(2A^{-1})$?

Q2.

Let A be a 4×4 matrix with $\det(A) = 5$. If B is obtained from A by swapping two rows and multiplying one row by 3, then what is $\det(B)$?

Q3.

Let A be a 3×3 matrix such that $\det(A) = -2$. Let $C = A^2 \cdot (3I)$. Compute $\det(C)$.

Q4.

If A is a 5×5 matrix and A' is obtained from A by adding twice the third row to the fifth row, what can be said about $\det(A')$?

Q5.

Let A be a 3×3 matrix such that $\det(A) = 6$. Compute $\det(A^T A^{-1})$.

Q6.

Let A be a 3×3 skew-symmetric matrix. Compute $\det(A^T A)$.

Q7.

Let A be an upper triangular 4×4 matrix with diagonal entries $1, 2, 3, 4$. Compute $\det(2A)$.

Q8.

Let A be a 4×4 matrix with $\det(A) = 2$. Matrix B is obtained by multiplying the second and third rows of A by -1. Find $\det(B)$.

Q9.

Let A be a 3×3 invertible matrix with $\det(A) = -5$. Let $B = A + 3I$. Can $\det(B)$ be computed directly from this information? Justify.

Q10.

Let A be a 3×3 matrix with all row sums equal to 5. What is the value of $\det(A)$ if we subtract the first row from the other two rows?

56 Matrix Inverse: Numerical and Conceptual Questions

Q1. [MCQ]

Let $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$. What is A^{-1}?

(a) $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \cdot \frac{1}{1}$

(c) $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \cdot \frac{1}{(2)(3)-(5)(1)}$

(d) A is not invertible.

Q2. [NAT]

If A is a 2×2 matrix with $\det(A) = -4$, and $\operatorname{adj}(A) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find A^{-1}.

Q3. [MCQ]

Which of the following matrices is not invertible?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

(c) Identity matrix

(d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

Q4. [MSQ]

Which of the following statements are true?

(a) A matrix with zero determinant is not invertible.

(b) If $AB = I$, then A is the inverse of B.

(c) A^{-1} exists $\Rightarrow A$ is non-singular.

(d) For any square matrix A, $A \cdot A^{-1} = A$.

Q5. [Conceptual MCQ]

If A^{-1} exists, which of the following is also true?

(a) $(A^T)^{-1} = (A^{-1})^T$

(b) $AA^{-1} = I$

(c) $A^{-1}A = I$

(d) All of the above

Q6. [NAT]

Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$. Compute the $(2, 1)$ entry of A^{-1}.

Q7. [MCQ]

Which of the following must be true for A^{-1} to exist?

(a) A must be diagonal

(b) $\det(A) \neq 0$

(c) A must be symmetric

(d) All eigenvalues of A must be positive

Q8. [Conceptual MSQ]

Let A and B be invertible $n \times n$ matrices. Which of the following are true?

(a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $(A^{-1})^{-1} = A$

(c) $A + B$ is always invertible

(d) ABA^{-1} is invertible

Q9. [MCQ]

If A is an orthogonal matrix, then:

(a) $A^T = A^{-1}$

(b) $\det(A) = 1$

(c) $AA^T = I$

(d) All of the above

Q10. [NAT]

Let A be such that $A^2 = I$ and $A \neq I$. What is A^{-1}?

Q11. [MCQ]

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Then $A^{-1} =$

(a) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

(d) A is not invertible.

Q12. [MCQ]

If $A^{-1} = A^T$, then A is:

(a) Diagonal

(b) Orthogonal

(c) Skew-symmetric

(d) None of the above

Q13. [NAT]

Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Compute A^{-1}.

Q14. [Conceptual MCQ]

Let A be invertible. Which of the following operations does **not** preserve invertibility?

(a) Pre-multiplication by A

(b) Transposition

(c) Taking adjoint

(d) Multiplying one row by zero

Q15. [MCQ]

If A is a 2×2 matrix and $A^2 = 0$, then A^{-1}:

(a) Exists

(b) Equals zero

(c) Does not exist

(d) Is undefined

Q16. [Conceptual MSQ]

Which of the following matrices are always invertible?

(a) Identity matrix

(b) Diagonal matrix with non-zero entries

(c) Orthogonal matrix

(d) Upper triangular matrix with all diagonal entries non-zero

Q17. [NAT]

Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$. Compute $\det(A)$ and determine if A^{-1} exists.

Q18. [MCQ]

Which matrix has **no inverse**?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Q19. [Conceptual MCQ]

For a $n \times n$ matrix A, if one row is a linear combination of others, then:

(a) $\det(A) \neq 0$

(b) A is invertible

(c) A is not invertible

(d) All rows are linearly independent

Q20. [MCQ]

Let A be an $n \times n$ matrix such that $A^T = A$ and $\det(A) \neq 0$. Which of the following is true?

(a) A is orthogonal

(b) A^{-1} is also symmetric

(c) A is skew-symmetric

(d) None of the above

57 Interconnected Questions on Determinant, Inverse, Rank, Eigenvalues and Eigenvectors

Q1. [MCQ]

If a matrix A is such that $\det(A) = 0$ and $A\vec{x} = \lambda\vec{x}$ has a non-zero solution, which of the following is necessarily true?

(a) $\lambda = 0$

(b) $\lambda \neq 0$

(c) A is invertible

(d) $\vec{x}$ is the zero vector

Q2. [MSQ]

Let A be a 3×3 matrix with eigenvalues $\lambda_1 = 0$, $\lambda_2 = 2$, $\lambda_3 = 3$. Then which of the following are true?

(a) $\det(A) = 0$

(b) $\text{Rank}(A) = 3$

(c) A is non-invertible

(d) $\text{Rank}(A) < 3$

Q3. [NAT]

Let A be a 4×4 matrix with eigenvalues $1, 1, 0, 2$. What is the rank of A?

Q4. [MCQ]

If A is a 3×3 real matrix such that $A^T = A$ and $\lambda = 0$ is one of its eigenvalues, then:

(a) A is invertible

(b) $\text{Rank}(A) = 3$

(c) $\det(A) = 0$

(d) All eigenvalues are positive

Q5. [NAT]

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find $\text{tr}(A)$ and $\det(A)$, then compute the sum and product of its eigenvalues.

Q6. [MCQ]

If a matrix A satisfies $A^2 = I$, and $\text{Rank}(A) = 2$, which of the following are true?

(a) Eigenvalues are ± 1

(b) $\det(A) = 1$

(c) A is symmetric

(d) $A^{-1} = A$

Q7. [MSQ]

Let A be a diagonalizable matrix with eigenvalues $\{1, 1, 0\}$. Which are true?

(a) A is not invertible

(b) $\text{nullity}(A) = 1$

(c) $\text{Rank}(A) = 3$

(d) $\det(A) = 0$

Q8. [MCQ]

Given a 3×3 matrix A such that $A^3 = 0$, which of the following must be true?

(a) A is invertible

(b) All eigenvalues are zero

(c) $\text{Rank}(A) = 3$

(d) $\det(A) = 1$

Q9. [MCQ]

Let A be a matrix such that $A^T = -A$ and $\vec{x}^T A\vec{x} = 0$ for all $\vec{x}$. Then:

(a) All eigenvalues of A are imaginary

(b) A is invertible

(c) Rank(A) is even

(d) $\det(A) = 0$

Q10. [NAT]

Let A be a 2×2 matrix with Rank(A) $= 1$. What is the determinant of A?

Q11. [MSQ]

Let A be a 3×3 matrix with eigenvalues $3, 5, -2$. Then which of the following are true?

(a) $\det(A) = -30$

(b) A is invertible

(c) Rank(A) $= 3$

(d) At least one eigenvector is a zero vector

Q12. [MCQ]

If A is orthogonal and λ is an eigenvalue of A, then which of the following must be true?

(a) $|\lambda| = 1$

(b) $\lambda = 0$

(c) A is not invertible

(d) Rank(A) $< n$

Q13. [NAT]

Let $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$. Compute $\det(A)$ and the eigenvalues of A.

Q14. [MCQ]

If a matrix A has all eigenvalues as 1, then:

(a) $A = I$

(b) A may not be I

(c) $\det(A) = 1$

(d) $\text{Rank}(A) = n$

Q15. [MSQ]

Let A be a symmetric matrix with eigenvalues $1, 0, 0$. Then:

(a) $\text{Rank}(A) = 1$

(b) $\text{nullity}(A) = 2$

(c) $\det(A) = 0$

(d) A is invertible

Q16. [MCQ]

Let A be a 4×4 matrix with $\det(A) \neq 0$ and $A^T = A^{-1}$. Which of the following must be true?

(a) A is symmetric

(b) A is orthogonal

(c) $\text{Rank}(A) < 4$

(d) All eigenvalues of A are zero

Q17. [MCQ]

Let A be an $n \times n$ matrix with $\text{Rank}(A) = n$. Then:

(a) A is invertible

(b) $\det(A) \neq 0$

(c) All eigenvalues are non-zero

(d) A is orthogonal

Q18. [NAT]

Let A be a 2×2 matrix with eigenvalues 2 and 5. Find $\det(A^{-1})$.

Q19. [MSQ]

If A is invertible, which of the following are always true?

(a) $\det(A^{-1}) = 1/\det(A)$

(b) $\text{Rank}(A^{-1}) = \text{Rank}(A)$

(c) All eigenvalues of A^{-1} are reciprocals of those of A

(d) $\det(A^{-1}) < 0$ always

Q20. [MCQ]

Let A be a 3×3 matrix and $A^2 = A$. Which of the following must be true?

(a) $\det(A) = 0$ or 1

(b) A is invertible

(c) All eigenvalues are either 0 or 1

(d) $\text{Rank}(A) = 3$

58 Answers to Conceptual Questions

Cache and Array Access

- **Q1.1:** (a), **Q1.2:** (a)
- **Q2.1:** (a), **Q2.2:** (b)
- **Q3.1:** (a), **Q3.2:** (b)

Pipelining: GATE-style Numerical and Conceptual Questions

Answers:

Q1. **1.33** (Time with forwarding: 21 cycles, Time without: 28 cycles. Speedup = 28/21 = 1.33)

Q2. **243 cycles** (First instruction takes 5 cycles. Remaining 99 overlap but depend on PO stage cycles: $39 \times 2 + 35 \times 1 + 25 \times 0 = 145$ additional cycles. Total = 5 + 99 + 145 = 243)

Q3. **24 cycles** (First instruction takes 5 cycles. Then 19 more with 1 cycle overlap each: 5 + 19 = 24. Add 2 branches × 2 cycle penalty = 4 → 24 + 4 = 28)

Q4. $n + 3 + k$ **cycles** (3 initial pipeline fill cycles, n instructions, k stalls)

Q5. **1 stall, 7 cycles** (Load-to-use hazard after LOAD requires 1 stall. Gantt: 5 stages + 2 more cycles for dependencies)

Q6. **14 cycles** (First instruction takes 6 cycles (IF–WB). Then 9 more, each 1 cycle apart: total = 6 + 9 = 15)

Q7. **600 cycles lost** (Branch freq = 20% of 1000 = 200 branches. Penalty per branch = 3 → 200×3 = 600 cycles lost)

Q8: (C) 17, Q9: (B) 17, Q10: (C) 22, Q11: (C) 23

Minimum Cost Spanning Tree (MCST)

- **2021:** (c), **2020:** (d), **2019:** (c), **2016:** (b)

Additional MCST Questions

Answers: Q1: (c), Q2: (c), Q3: (c), Q4: (c), Q5: (c)

Prim's Algorithm

Answers: Q1: (a), Q2: (b), Q3: (c), Q4: (a), Q5: (b), Q6: (a)

Kruskal's Algorithm

Answers: Q1: (b), Q2: (b), Q3: (b), Q4: (b), Q5: (c), Q6: (b)

Dijkstra's Algorithm

Answers: Q1: (c), Q2: (b), Q3: (c), Q4: (a), Q5: (a), Q6: (c)

Bellman-Ford Algorithm

Answers: Q1: (c), Q2: (c), Q3: (b), Q4: (b), Q5: (b), Q6: (c)

Floyd-Warshall Algorithm

Answers: Q1: (c), Q2: (c), Q3: (b), Q4: (b), Q5: (b), Q6: (b)

Answers: Graph Algorithms with Data Structures

Answers: Q1: (b), Q2: (d), Q3: (a), Q4: (b), Q5: (b)

Answers: Sorting (Part 1)

Q1: (b), Q2: (c), Q3: (a), Q4: (b), Q5: (b)

Answers: Sorting (Part 2)

Q1: (a), Q2: (a), Q3: (d), Q4: (a), Q5: (b), Q6: (c), Q7: (a), Q8: (a), Q9: (b), Q10: (a), Q11: (c), Q12: (c), Q13: (c), Q14: (b), Q15: (a)

Answers: Time Complexity

Q1: (c), Q2: (b), Q3: (c), Q4: (b), Q5: (b), Q6: (a), Q7: (b), Q8: (b), Q9: (c), Q10: (b), Q11: (b), Q12: (b), Q13: (b), Q14: (a), Q15: (a), Q16: (b), Q17: (a), Q18: (a), Q19: (a), Q20: (b)

Answers: Edge-Case Brainstorming Questions

Q1: (b), Q2: (d), Q3: (c), Q4: (a), Q5: (a), Q6: (b), Q7: (c), Q8: (d), Q9: (b), Q10: (b)

Answers: C Programming with Pointers and Arrays

Answers: Q1: (b), Q2: (d), Q3: (a), Q4: (d), Q5: (b)

Answers: C Programming with Stacks and Queues

Answers: Q1: (c), Q2: (d), Q3: (a), Q4: (d), Q5: (b)

Answers: C Programming with Recursion

Answers: Q1: (b), Q2: (b), Q3: (b), Q4: (a), Q5: (c)

Answers: Operator Precedence Evaluation

Answers: Q1: (b), Q2: (c)

Answers: Pointer and String Manipulation in C

Answers: Q1: (b), Q2: (a), Q3: (c), Q4: (b), Q5: (b), Q6: (c), Q7: (d), Q8: (b), Q9: (b), Q10: (c)

Answers: Tricky C MCQs

Answers: Q1: (a), Q2: (c), Q3: (b), Q4: (d), Q5: (b)

Answers: Functional Dependencies

Answers: Q1: (b), Q2: (d), Q3: (b), Q4: (a), Q5: (c)

Answers: Normalization

Answers: Q1: (a), Q2: (b), Q3: (a), Q4: (b), Q5: (c), Q6: (a), Q7: (b), Q8: (b), Q9: (c), Q10: (c)

Answers: Normalization + Lossless Join + Dependency Preservation

Answers: Q1: (b), Q2: (b), Q3: (a), Q4: (c), Q5: (c)

Answers: SQL Conceptual Questions

Answers: Q1: (b), Q2: (c), Q3: (a), Q4: (a), Q5: (a), Q6: (a), Q7: (b), Q8: (a), Q9: (b), Q10: (b), Q11: (a), Q12: (b)

Answers: Tuple Relational Calculus

Answers: Q1: (b), Q2: (a), Q3: (d), Q4: (b), Q5: (b), Q6: (a)

Answers: Transaction Scheduling

Answers: Q1: (b), Q2: (c), Q3: (a), Q4: (a)

Answers: B and B+ Trees

Answers: Q1: (depends on n, but generally at least $\lceil \frac{n+1}{d} \rceil - 1$),
Q2: (2), Q3: (c), Q4: (4), Q5: (approx. 30000), Q6: (3),
Q7: B+ Trees maintain all data pointers in leaf level with linked leaves enabling fast range traversal,
Q8: (h block accesses), Q9: (c), Q10: (d)

Answers: B+ Tree: Moderate to Hard

Answers: Q1: (c), Q2: (a), (b), (d), Q3: 3, Q4: (d), Q5: (a), (b), Q6: (c), Q7: 10000, Q8: (b), Q9: (a), (b), Q10: 3

Answers: B and B$^+$ Tree Balance

Answers: Q1: (c), Q2: (a), (b), (d), Q3: (b), Q4: (b), Q5: (a), (b)

Answers: IPv4 Addressing

Answers: Q1: (c), Q2: (a), Q3: (c), Q4: (b), Q5: (b), Q6: (b), Q7: (a), Q8: (a), Q9: (a), Q10: (b)

Answers: CIDR Allocation

Answers: Q1: (a), Q2: (a), Q3: (d), Q4: (c)

Answers: IPv4 Datagram Fragmentation

Answers: Q1: (c), Q2: (c), Q3: (d), Q4: (a), Q5: (b), Q6: (c), Q7: (d), Q8: (b), Q9: (d), Q10: (b)

Answers: IP Forwarding and Routing

Answers: Q1: (b), Q2: (a), Q3: (d), Q4: (a), Q5: (a), Q6: (b), Q7: (a), Q8: (a), Q9: (b), Q10: (b), Q11: (b), Q12: (b)

Answers: Error Detection and Correction

Q1: (position 5), Q2: (100), Q3: (a), Q4: (011111010111110), Q5: (1), Q6: (011100010110), Q7: (1011), Q8: (d), Q9: (4), Q10: (c), Q11: (100100110110), Q12: (0), Q13: (a), Q14: (7), Q15: (101), Q16: (bit 2), Q17: (bit 6), Q18: (b), Q19: (a), Q20: (a)

Answers: IP Header and Modification Rules

Q1: (c), Q2: (d), Q3: (c), Q4: (c), Q5: (a)

Answers: Maximum Segment Size (MSS)

Q1: (c), Q2: (b), Q3: (a), Q4: (c), Q5: (c)

Bays Theorem and Conditional Probability

Answers: 1:(a), 2:(c), 3:(c), 4:(a), 5:(a), 6:(c), 7:(a), 8:(a), 9:(b), 10:(a), 11:(a), 12:(a), 13:(d), 14:(b), 15:(b), 16:(a), 17:(c), 18:(c), 19:(b), 20:(b)

Discrete Random Variable

Answers: 1:(b), 2:(a), 3:(b), 4:(c), 5:(b), 6:(a), 7:(b), 8:(a), 9:(a), 10:(a), 11:(c), 12:(a), 13:(c), 14:(b), 15:(a), 16:(c), 17:(c), 18:(b), 19:(a), 20:(b)

Answers: Expected Value

Answers:
Q1: (b), Q2: (a), Q3: 1600, Q4: (a), Q5: (c), Q6: 2, Q7: 7.31, Q8: 17.5

Answers: Variance

Answers: Q1: (a), Q2: 0.5, Q3: (c), Q4: (c), Q5: 0.6667

Answers: Binomial Distribution

Answers: Q1: (b), Q2: (c), Q3: 12, Q4: (a), Q5: (b), (c), Q6: (a), (b), (c) Q7: (a)

Answers: Poisson Random Variable

Answers: Q1: $\frac{5^7 e^{-5}}{7!}$ Q2: $e^{-0.3}$ Q3: (a), (c), (d) Q4: $1 - e^{-2}$ Q5: $\lambda = \lambda_1 + \lambda_2$ Q6: (d) Q7: (a), (b), (d) Q8: (b), Q9: (c), Q10: (a), Q11: (b)

Answers: Geometric Random Variable

Answers: Q1: (d), Q2: (b), Q3: (a), Q4: (a), Q5: (a), Q6: (b), Q7: (c)

Answers: Expected Value of Sums

Answers: Q1: (a), Q2: (a), Q3: (b), Q4: (c), Q5: (b), Q6: (d), Q7: (d)

Answers: Cumulative Distribution Function

Answers: Q1: (c), Q2: (a), Q3: (a), Q4: (b), Q5: (a)

Answers: Continuous Random Variables

Answers: Q1: 0.625, Q2: (a), Q3: $k = \frac{3}{4}$, Q4: (a), Q5: $\frac{3}{4}$, Q6: (d), Q7: (c)

Answers: Expectation and Variance

Answers: Q1: $\mathbb{E}[X] = \frac{3}{4}, \text{Var}(X) = \frac{3}{80}$ Q2: $\mathbb{E}[X^2] = \frac{28}{3}, \text{Var}(X) = \frac{1}{3}$ Q3: $\mathbb{E}[X] = \frac{1}{\lambda}, \text{Var}(X) = \frac{1}{\lambda^2}$ Q4: $\mathbb{E}[X] = \frac{4}{3}, \text{Var}(X) = \frac{4}{45}$ Q5: $\text{Var}(Y) = 9\sigma^2$ Q6: (d) Q7: (a)

Answers: Uniform Random Variable

Answers: Q1: (a) Q2: 3 Q3: (a) Q4: (a), (c), (d) Q5: (b) Q6: 3 Q7: (a), (c), (d) Q8: (a), (b), (c), (d) Q9: 2 Q10: (d)

Answers: Normal Random Variable

Answers: Q1: (a) Q2: 1 Q3: (a), (b), (d) Q4: (a) Q5: (a) Q6: 130 Q7: (a), (b), (c) Q8: (b), (d) Q9: (a) Q10: 190

Answers: Exponential Random Variable

Answers: Q1: (a), Q2: (a)(b)(c)(d), Q3: 16, Q4: (b), Q5: (a), Q6: (a)(b)(c)(d), Q7: $\frac{\lambda}{a}$, Q8: (c), Q9: (a), Q10: (a)(b)(d)

Answers: Hazard Rate Function

Answers: Q1: (c), Q2: (a)(b)(c), Q3: (a), Q4: (c), Q5: 0.2, Q6: 0.2, Q7: 0.15, Q8: (b)

Answers: Properties of Expectation

Answers: Q1: 12, Q2: (a), Q3: (b), Q4: 4.67, Q5: (b), Q6: (d), Q7: c^2, Q8: 24.5, Q9: No, Q10: No, Q11: (a), Q12: No, Q13: 1, Q14: 16, Q15: 4/3, Q16: p, Q17: True, Q18: (c), Q19: Yes, Q20: 0

Rank, Eigenvalues, and Eigenvectors

Answers: 1:(c), 2:(d), 3:(a), 4:(c), 5:(c), 6:(a), 7:(b), 8:(c), 9:(c), 10:(a), 11:(a), 12:(d), 13:(c), 14:(a), 15:(b), 16:(a), 17:(a), 18:(c), 19:(c), 20:(b)

Answers: Properties of Determinants

Answers: Q1: (c), Q2: $3^3 \cdot 5 = 135$, Q3: $\det(A)\det(B)$, Q4: $\lambda^n \det(A)$, Q5: False, Q6: (d), Q7: $1/d$, Q8: ± 1, Q9: $2^2 = 4$, Q10: (c), Q11: 0, Q12: (c), Q13: True, Q14: $7^2 = 49$, Q15: $\prod \lambda_i$, Q16: Unique solution exists, Q17: (c), Q18: 3/4, Q19: (d), Q20: 0

Answers: Properties-based Determinant

Answers:

Q1: $\dfrac{1}{4} \cdot 2^3 = \dfrac{8}{4} = 2$, Q2: Swapping changes sign, multiplying row by 3 scales determinant $\Rightarrow \det(B) = -3 \cdot 5 = -15$, Q3: $\det(C) = \det(A^2) \cdot \det(3I) = (-2)^2 \cdot 3^3 = 4 \cdot 27 = 108$, Q4: Elementary row addition does not change determinant, so $\det(A') = \det(A)$, Q5: $\det(A^T A^{-1}) = \det(A^T) \cdot \det(A^{-1}) = \det(A) \cdot \frac{1}{\det(A)} = 1$, Q6: $\det(A^T A) = \det(A)^2$; But $\det(A) = 0$ (since) A (is skew-symmetric of odd order) $\Rightarrow \det = 0$, Q7: $\det(2A) = 2^4 \cdot \det(A) = 16 \cdot 24 = 384$, Q8: Each row multiplied by -1 flips sign twice: $(-1)^2 = +1$, so $\det(B) = 2$, Q9: Cannot be computed — $\det(A + kI)$ is **not** simply derived from $\det(A)$, Q10: Resulting matrix has two identical rows $\Rightarrow \det = 0$

Answers: Matrix Inverse Questions

Answers: Q1: (c), Q2: $\frac{1}{-4}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, Q3: (a), Q4: (a)(b)(c), Q5: (d), Q6: -1, Q7: (b), Q8: (a)(b)(d), Q9: (d), Q10: A, Q11: (a), Q12: (b), Q13: $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, Q14: (d), Q15: (c), Q16: (a)(b)(c)(d), Q17: $\det = 1 \Rightarrow$ Invertible, Q18: (b), Q19: (c), Q20: (b)

Answers: Interconnected Questions on Determinant, Inverse, Rank, Eigenvalues and Eigenvectors

Answers: Q1: (a), Q2: (a)(c)(d), Q3: 3, Q4: (c), Q5: tr $= 4$, det $= 3$, sum=4, product=3 Q6: (a)(d), Q7: (a)(b)(d), Q8: (b), Q9: (a)(c)(d), Q10: 0 Q11: (a)(b)(c), Q12: (a), Q13: $\det = 2$, eigenvalues $-1, -2$, Q14: (b), Q15: (a)(b)(c) Q16: (b), Q17: (a)(b)(c), Q18: 1/10, Q19: (a)(b)(c), Q20: (a)(c)