Numerical Analysis of Circuit Theory using Gauss Jordan Method

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***Abstract* –** This paper is concerned about the algorithm to compute the solution of electrical networks using computational technique – Gauss Jordan method. This work is also extended to predict the time of execution using regression. The analysis is done on basis of computation time and number of operations involved in the course. For complex analysis and simulations, the computations are performed using MATLAB programming. This paper explicitly reveals that this technique can also be applied to different streams with more complex postulates like physics, mechanical, economics, etc.

***Keywords* –** circuit analysis, linear solution, gauss jordan, numerical methods

I. INTRODUCTION

Circuit theory and analysis is very important aspect of electrical and electronics engineering. In fact, the circuit solution reveals the performance of the circuit theoretically and the improvements can be made using the suitable substitutes for corresponding network. The term "solution of circuit" refers to determination of voltages and current in specified nodes and loops respectively. The circuit solution is a difficult task for more complex networks and they can be solved if they are implemented using computer aided analysis. The computer aided analysis and design preliminarily begins with designing an appropriate algorithm to solve the problem and extensively this algorithm is implemented using any of the convenient programming languages or platforms like FORTAN, C, C++, JAVA, PYTHON, MATLAB, MAPLR, R-Programming, etc. [1] Due to zero error analysis the computer aided analysis became more popular and used extensively to solve the networks.

The advantages of these computer aided techniques over the conventional manual solution are high computation speed, ability to solve more complex networks, less prone to error, memory efficient and workspace is stored for future analysis. [2]

However, these computer cannot explicitly program themselves and built certain algorithms to solve any kind of circuit. So, in order to utilize these software packages to solve the electrical networks efficiently one must be able to build the algorithm according to the requirement and implement the same using any convenient software packages.

Such algorithms are built by many researchers to solve the circuit at less computation time, those are Gauss Elimination and Gauss Jordan methods, these both are efficient methods to solve any kind of linear system of equations. So, in order to implement these two techniques one must frame the circuit in terms of system of linear equations. Those are fed an input to the system and it is evaluated for voltages or currents according to the program.

The modelling of the electrical networks in terms of linear system of equations is done using generalized laws, those are Kirchhoff's Current law and Kirchhoff's Voltage law. By using these two laws, the nodal and mesh analysis is done to frame equations that can be solved using the programming interface.

The further sections discuss the features of the modelling and implementing the raised methodologies via programming language MATLAB.

II. METHODOLOGY

The methodology begins with formulating the circuit to system of equations using mesh and nodal analysis.

The better understanding of Nodal Analysis and Mesh Analysis is done considering it's procedure in framing the network.

Procedure to construct Nodal Analysis:

1. Identify the principal nodes and reference node.
2. Label the node voltages with respect to reference node.
3. Frame nodal equations by applying KCL at principal nodes.
4. Solution of nodal equations obtained in step 3.

Consider the following circuit to solve for voltages using Nodal Analysis.

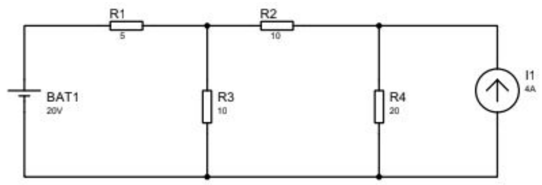


Figure 1: Circuit Illustration

Following the procedure mentioned above, equations are formulated as follows:

$$V_2 = 4V_1 - 40$$

$$3V_2 - 2V_1 = 80$$

$$V_1 = 20V$$
$$V_2 = 40V$$

Procedure to construct Mesh Analysis:

1. Identify the meshes in the network and label the currents in clock-wise or counter clock-wise direction.
2. Frame Mesh equations by applying KVL at all meshes.
3. Solution of mesh equations obtained in step 2.

Consider the following circuit to solve for currents using Mesh Analysis.

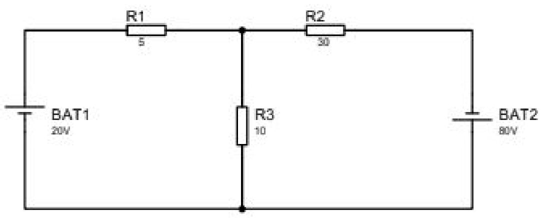


Figure 2: Circuit Illustration

Following the procedure mentioned above, equations are formulated as follows:

$$4I_2 = 6I_1 - 8$$

$$4I_2 = I_1 + 8$$

$$I_1 = \frac{16}{5}A$$
$$I_2 = \frac{14}{5}A$$

Gauss Jordan Method:

Gauss Jordan method is the extension of Gauss elimination method where the back substitution is avoided and the solution is formulated using the property of identity matrix.

$$AI = A$$

Therefore, the augmented matrix which is formulated using the set of equations is converted to identity matrix, the resulting solution gives the value to unknowns specified.

Here, unlike in Gauss elimination method both the upper and lower triangular elements are made to zero. The number of operations are increased compared to Gauss elimination but ease of solution is done in Gauss Jordan. [4] [5]

Example:

Consider the following equations to solve variables x and y.

$$x + y = 1$$

$$x + 2y = 2$$

The matrix is framed as follows:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The augmented matrix is given by

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, performing the row operations to translate the augmented matrix into identity matrix.

$$R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$y = 1$$
$$x = 0$$

Generalized formula for 2^{nd} order is given by

$$a_{11}x + a_{12}y = b_1$$

$$a_{21}x + a_{22}y = b_2$$

The augmented matrix is given by,

$$\begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$$

The modified augmented matrix after stage 1 gives

$$\begin{bmatrix} a_{11} & a_{12} & b_1 \\ 0 & a'_{22} & b'_2 \end{bmatrix}$$

The modified augmented matrix after stage 2 gives

$$\begin{bmatrix} 1 & 0 & b'_1 \\ 0 & 1 & b'_2 \end{bmatrix}$$

The comparison according to matrix gives the solution as

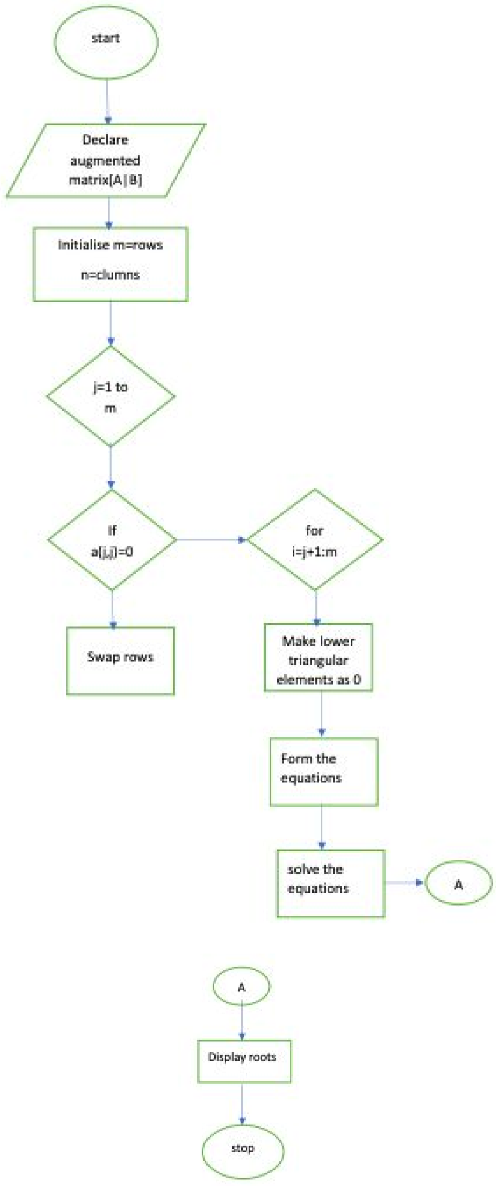
$$y = b'_2$$
$$x = b'_1$$

Since the back substitution is not involved in the process, the gauss Jordan is the most compact and efficient with respect to space complexity and analysis is also easier for higher order equations.

I. IMPLEMENTATION USING MATLAB

The algorithm of Gauss Jordan is implemented using MATLAB Package which has superior capability to perform simulation of matrices.

Flow Chart :



Algorithm:

Gauss Jordan Method:

1. Start
2. Initialize A and B
3. Evaluate number of rows (nr) and number of columns (nc).
4. Generate Augmented matrix using A and B, X = [A B]
5. Initialize col = 1
6. Evaluate row = col+1
7. Evaluate fraction = X(row,col)/X(col,col)
8. Perform X(row,:) = X(row,:) - fraction*X(col,:)
9. row = row+1
10. repeat steps 6-9 till row<=nr
11. col = col+1
12. repeat steps 5-11 till col<=nc
13. Initialize row = 1
14. Evaluate col = row+1
15. Evaluate fraction = X(row,col)/X(col,col)
16. Perform X(row,:) = X(row,:) - fraction*X(col,:)
17. col = col+1
18. repeat steps 6-9 till col<=nr
19. row = row+1
20. repeat steps 5-11 till col<=nr
21. Stop

Program:

```
function [v] =
guass_jordan(A,B)
    nr = length(B); %number of
rows
    nc = nr+1; %number of
columns of augmented matrix
    X = [A B]; %matrix
    for col = 1:nc
        for row = col+1:nr
            fraction =
X(row,col)/X(col,col);
%fraction of element to
principal
            X(row,:) = X(row,:)
- fraction*X(col,:); % lower
triangular to zero
        end
    end
    for row = 1:nr
        for col = row+1:nr
            fraction =
X(row,col)/X(col,col);
%fraction of element to
principal
            X(row,:) =
-X(row,:) + fraction*X(col,:);
%upper triangle to zero
        end
    end
    v = X(:,nc);
end
```

III. RESULTS AND DISCUSSIONS

To analyze the performance of Gauss Jordan method, let us consider an complex electrical circuit with determining four nodal voltages with respect to reference voltages provided as per figure:

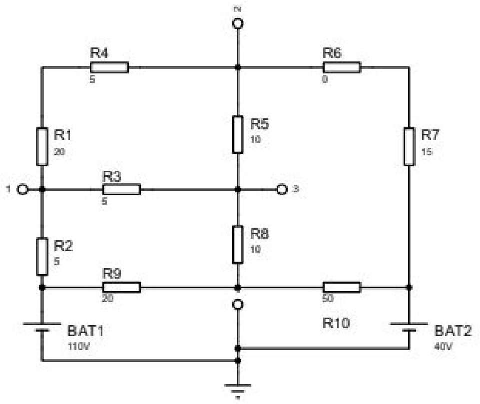


Figure 5: Circuit Illustration

The equations obtained in the process are as follows:

$$11v_1 - v_2 - 5v_3 = 550$$

$$-6v_1 + 31v_2 - 15v_3 = 400$$

$$4v_3 - 2v_1 - v_2 - v_4 = 0$$

$$17v_4 - 10v_3 = 630$$

The matrix representation is given by

$$\begin{bmatrix} 11 & -1 & -5 & 0 \\ -6 & 31 & -15 & 0 \\ -2 & -1 & 4 & -1 \\ 0 & 0 & -10 & 17 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 550 \\ 400 \\ 0 \\ 630 \end{bmatrix}$$

The solution obtained using Gauss Jordan is given in Table 1

Method	V_1	V_2	V_3	V_4
Gauss Jordan	98.011	75.66	90.49	90.29

Table 1: Result analysis of Gauss Jordan Method

Similarly, the time consumption is measured for different number of variables in the set of equations. The comparison is represented in Table 2

Number of Unknowns	Gauss Jordan Time Elapsed (ms)
2	25
3	31
4	36
5	39
6	56
7	76

Table 2: Time elapsed for Gauss Elimination Method.

The same inference is characterized using figure 6.

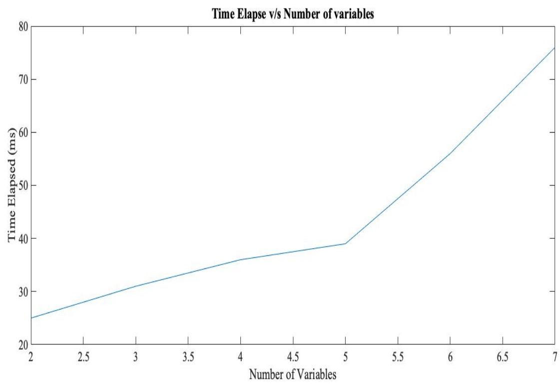


Figure 6: Time Elapsed v/s variable variation graph

Prediction is performed using linear regression concept of machine learning as follows:

Chosen Parameters,

$$\alpha = 0.05$$
$$h(\theta^{(x)}) = \theta(0) + \theta(1) * x$$

The simulation is given according to Table 3 and figures: 7,8.

Number of Unknowns	Predicted Time (ms)
8	81
9	91
10	102
20	205
30	308

Table 3: Predicted output using Regression

Curve Fitting is given by figure 7.

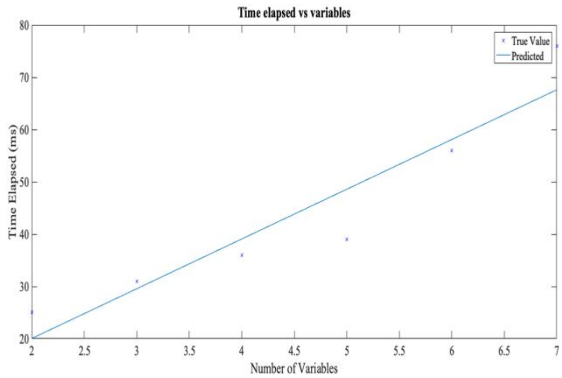


Figure 7: Curve obtained using linear regression

Cost function optimization is given by figure 8.

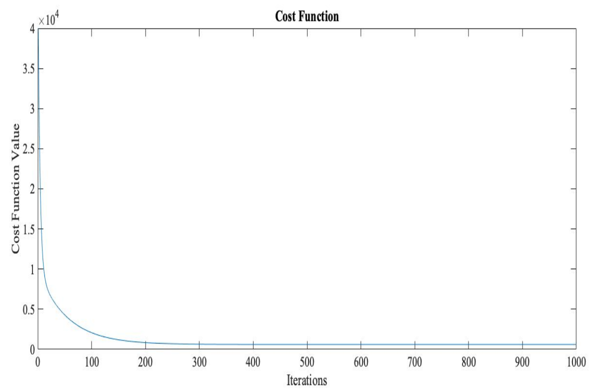


Figure 8: Cost function optimization using linear regression

IV. CONCLUSIONS

Many scientific and engineering problems can take the form of a system of linear equations. These equations may contain thousands of variables, so it is important to solve them as efficiently as possible. In this paper, the unknown variables in linear system are carried out by using Gauss Jordan Method. And the experiment is demonstrated with the help of MATLAB programming language. The results are carried out by solving different unknown variables. The prediction analysis is done using linear regression using cost function optimization. According to these experimental results, Gauss Jordan Method is compact than the Gauss Elimination method. Therefore Gauss Jordan Method is more efficient than the Gauss Elimination method with respect to space complexity.

II. REFERENCES:

[1] T.J. Dekker and W. Hoffmann: "Rehabilitation of the Gauss- Jordan algorithm", Department of Computer Systems, University of Amsterdam, Kruislann 409, 1098 SJ Amsterdam, The Netherlands, 1989

[2] Luke Smith and Joan Powell: "An Alternative Method to Gauss-Jordan Elimination: Minimizing Fraction Arithmetic", TheMathematics Educator, 2011

[3] Adenegan, Kehinde Emmanuel1, Aluko, TopeMoses: “Gauss and Gauss-Jordan elimination methods for solving system of linear equations: comparisons and applications”, Journal of Science and Science Education, Ondo, 19th November, 2012
[4] R. B. Srivastava and Vinod Kumar, “Comparison of Numerical Efficiencies of Gaussian Elimination and Gauss-Jordan Elimination methods for the Solutions of linear Simultaneous Equations”, Department of Mathematics M. L. K. P. G. College, Balrampur, U. P., India, November 2012
[5] K. RAJALAKSHMI, “Parallel Algorithm for Solving Large Sy stem of Simultaneous Linear Equations”, IJCSNS International Journal of Computer Science and Network Security, VOL.9 No.7, July 2009