COMPUTATIONAL ALGORITHM FOR ANALYSIS OF COMPLEX ELECTRICAL NETWORKS

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Abstract – This paper is concerned about the algorithms to compute the solution of electrical networks using computational techniques – Gauss elimination and Gauss Jordan. This work is also extended to extract the comparison among both methodologies. The comparison is done on basis of computation time and number of operations involved in the course. The analysis proved that both the processes tend to give same result for any kind of linear system of equations. For complex analysis and simulations, the computations are performed using MATLAB programming. This paper explicitly reveals that these techniques can also be applied to different streams with more complex postulates like physics, mechanical, economics, etc.

I. INTRODUCTION

Circuit theory and analysis is very important aspect of electrical and electronics engineering. In fact, the circuit solution reveals the performance of the circuit theoretically and the improvements can be made using the suitable substitutes for corresponding network. The term "solution of circuit" refers to determination of voltages and current in specified nodes and loops respectively. The circuit solution is a difficult task for more complex networks and they can be solved if they are implemented using computer aided analysis. The computer aided analysis and design preliminarily begins with designing an appropriate algorithm to solve the problem and extensively this algorithm is implemented using any of the convenient programming languages or platforms like FORTAN, C, C++, JAVA, PYTHON, MATLAB, MAPLR, R-Programming, etc. [1] Due to zero error analysis the computer aided analysis became more popular and used extensively to solve the networks.

The advantages of these computer aided techniques over the conventional manual solution are high computation speed, ability to solve more complex networks, less prone to error, memory efficient and workspace is stored for future analysis. [2]

However, these computer cannot explicitly program themselves and built certain algorithms to solve any kind of circuit. So, in order to utilize these software packages to solve the electrical networks efficiently one must be able to build the algorithm according to the requirement and implement the same using any convenient software packages.

Such algorithms are built by many researchers to solve the circuit at less computation time, those are Gauss Elimination and Gauss Jordan methods, these both are efficient methods to solve any kind of linear system of equations. So, in order to implement these two techniques one must frame the circuit in terms of system of linear equations. Those are fed an input to the system and it is evaluated for voltages or currents according to the program.

The modelling of the electrical networks in terms of linear system of equations is done using generalized laws, those are Kirchhoff's Current law and Kirchhoff's Voltage law. By using these two laws, the nodal and mesh analysis is done to frame equations that can be solved using the programming interface.

The further sections discuss the features of the modelling and implementing the raised methodologies via programming language MATLAB.

II. METHODOLOGY

The methodology begins with formulating the circuit to system of equations using mesh and nodal analysis.

The better understanding of Nodal Analysis and Mesh Analysis is done considering it's procedure in framing the network.

Procedure to construct Nodal Analysis:

1. Identify the principal nodes and reference node.
2. Label the node voltages with respect to reference node.
3. Frame nodal equations by applying KCL at principal nodes.
4. Solution of nodal equations obtained in step 3.

Consider the following circuit to solve for voltages using Nodal Analysis.

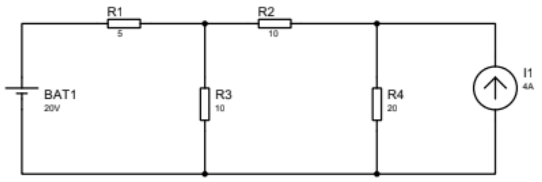

Figure 1: Circuit Illustration

Following the procedure mentioned above, equations are formulated as follows:

$$V_2 = 4V_1 - 40$$

$$3V_2 - 2V_1 = 80$$

$$V_1 = 20V$$
$$V_2 = 40V$$

Procedure to construct Mesh Analysis:

1. Identify the meshes in the network and label the currents in clock-wise or counter clock-wise direction.
2. Frame Mesh equations by applying KVL at all meshes.
3. Solution of mesh equations obtained in step 2.

Consider the following circuit to solve for currents using Mesh Analysis.

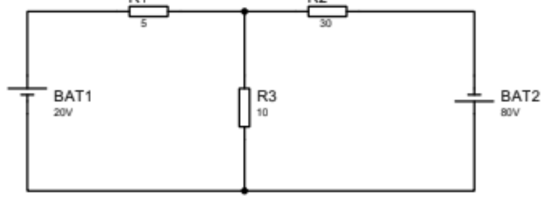

Figure 2: Circuit Illustration

Following the procedure mentioned above, equations are formulated as follows:

$$4I_2 = 6I_1 - 8$$

$$4I_2 = I_1 + 8$$

$$I_1 = \frac{16}{5}A$$
$$I_2 = \frac{14}{5}A$$

Gauss Elimination Method:

Gauss Elimination is the sequential process which eliminates the variables in its set of equations.

Let the whole procedure be consecutively separated into the stages. If we consider the system of equations be of n^{th} order where n number of equations and n number of unknowns are present. To resolve the n unknowns, the Gauss Elimination proceeds with eliminating the variables from equations except from the first equation. The variables are eliminated using preceding equations. Suppose if there are 2 variables and 2 equations to be solved then the second equation is made free from one variable using first equation. The process is similar to the 3^{rd} order systems, instead, we eliminate two variables from 3^{rd} equation using first two equations. And we eliminate one variable from 2^{nd} equation using first equation. [3]

Therefore by generalizing the $(n-1)$ variables are eliminated from n^{th} equation using $(n-1)$ equations.

In simplification, the set of equations are represented in matrix and the lower triangle is made to zero. Later the resulting values are substituted in preceding equations to attain all unknowns.

The ultimate solution is obtained using the conventional notation

$$AX = B$$

Example:
Consider the following equations to solve variables x and y.

$$x + y = 1$$

$$x + 2y = 2$$

The matrix is framed as follows:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Following the sequential process discussed earlier, the matrix is re-written as

$$R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$y = 1$$

$$x + 1 = 1$$

$$x = 0$$

The augmented matrix is given by

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Generalized formula for 2nd order is given by

$$a_{11}x + a_{12}y = b_1$$

$$a_{21}x + a_{22}y = b_2$$

$$\begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & b_1 \\ 0 & a'_{22} & b'_2 \end{bmatrix}$$

$$a'_{22} y = b'_2$$

$$y = \frac{b'_2}{a'_{22}}$$

$$x = \frac{(b_1 - a_{12}\frac{b'_2}{a'_{22}})}{a_{11}}$$

Since, the obtained solution is substituted in preceding equations, this method is known as back substitution method.

Gauss Jordan Method:

Gauss Jordan method is the extension of Gauss elimination method where the back substitution is avoided and the solution is formulated using the property of identity matrix.

$$AI = A$$

Therefore, the augmented matrix which is formulated using the set of equations is converted to identity matrix, the resulting solution gives the value to unknowns specified.

Here, unlike in Gauss elimination method both the upper and lower triangular elements are made to zero. The number of operations are increased compared to Gauss elimination but ease of solution is done in Gauss Jordan. [4] [5]

Example:
Consider the following equations to solve variables x and y.

$$x + y = 1$$

$$x + 2y = 2$$

The matrix is framed as follows:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The augmented matrix is given by

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, performing the row operations to translate the augmented matrix into identity matrix.

$$R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$y = 1$$
$$x = 0$$

Generalized formula for 2nd order is given by

$$a_{11}x + a_{12}y = b_1$$

$$a_{21}x + a_{22}y = b_2$$

$$\begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & b_1 \\ 0 & a'_{22} & b'_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & b'_1 \\ 0 & 1 & b'_2 \end{bmatrix}$$

$$y = b'_2$$
$$x = b'_1$$

III. IMPLEMENTATION USING MATLAB

Flow Chart :

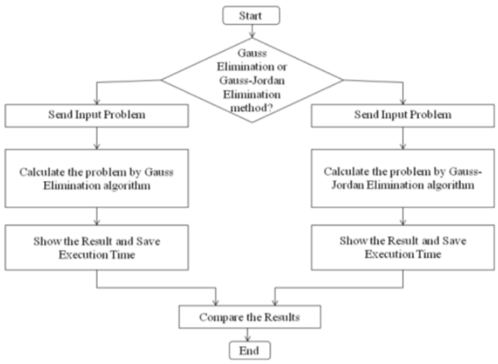

Figure 3: Overall implementation

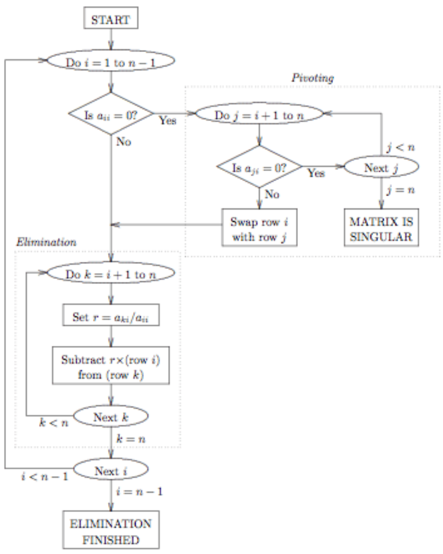

Figure 4: Flow chart of Gauss Elimination

Algorithm:

1. Gauss Elimination Method:

1. Start
2. Initialize A and B
3. Evaluate number of rows (nr) and number of columns (nc).
4. Generate Augmented matrix using A and B, X = [A B]
5. Initialize col = 1
6. Evaluate row = col+1
7. Evaluate fraction = X(row,col)/X(col,col)
8. Perform X(row,:) = X(row,:) - fraction*X(col,:)
9. row = row+1
10. repeat steps 6-9 till row<=nr
11. col = col+1
12. repeat steps 5-11 till col<=nc
13. Stop

2. Gauss Jordan Method:

1. Start
2. Initialize A and B
3. Evaluate number of rows (nr) and number of columns (nc).
4. Generate Augmented matrix using A and B, X = [A B]
5. Initialize col = 1
6. Evaluate row = col+1
7. Evaluate fraction = X(row,col)/X(col,col)
8. Perform X(row,:) = X(row,:) - fraction*X(col,:)
9. row = row+1
10. repeat steps 6-9 till row<=nr
11. col = col+1
12. repeat steps 5-11 till col<=nc
13. Initialize row = 1
14. Evaluate col = row+1
15. Evaluate fraction = X(row,col)/X(col,col)
16. Perform X(row,:) = X(row,:) - fraction*X(col,:)
17. col = col+1
18. repeat steps 6-9 till col<=nr
19. row = row+1
20. repeat steps 5-11 till col<=nr
21. Stop

Program:

1. Gauss Elimination Method:

```
function [v] = guass_elim(A,B)
    nr = length(B); %number of
rows
    nc = nr+1; %number of
columns of augmented matrix
```

```
    X = [A B]; %Augmented
Matrix
    for col = 1:nc
        for row = col+1:nr
            fraction =
X(row,col)/X(col,col);%ratio
between attributes
            X(row,:) = X(row,:)
- fraction*X(col,:);%Row
operation
        end
    end
end
```

2. Gauss Jordan Method:

```
function [v] =
guass_jordan(A,B)
    nr = length(B);
    nc = nr+1;
    X = [A B];
    for col = 1:nc
        for row = col+1:nr
            fraction =
X(row,col)/X(col,col);
            X(row,:) = X(row,:)
- fraction*X(col,:);
        end
    end
    for row = 1:nr
        for col = row+1:nr
            fraction =
X(row,col)/X(col,col);
            X(row,:) = -
X(row,:) + fraction*X(col,:);
        end
        X(row,:) =
X(row,:)/X(row,row);
    end
    v = X(:,nc);
end
```

IV. RESULTS AND DISCUSSIONS

To analyze the performance of both the methods, let us consider an complex electrical circuit with determining four nodal voltages with respect to reference voltages provided as per figure:

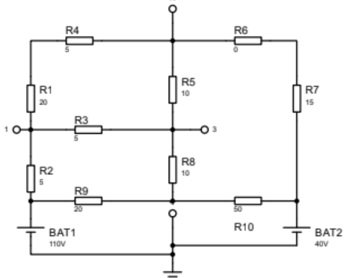

Figure 5: Circuit Illustration

The equations obtained in the process are as follows:

$$11v_1 - v_2 - 5v_3 = 550$$

$$-6v_1 + 31v_2 - 15v_3 = 400$$

$$4v_3 - 2v_1 - v_2 - v_4 = 0$$

$$17v_4 - 10v_3 = 630$$

The matrix representation is given by

$$\begin{bmatrix} 11 & -1 & -5 & 0 \\ -6 & 31 & -15 & 0 \\ -2 & -1 & 4 & -1 \\ 0 & 0 & -10 & 17 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 550 \\ 400 \\ 0 \\ 630 \end{bmatrix}$$

The solution obtained using Gauss Jordan and Gauss elimination are given in Table 1

Method	V_1	V_2	V_3	V_4
Gauss Elimination	98.011	75.66	90.49	90.29
Gauss Jordan Method	98.011	75.66	90.49	90.29

Table 1: Result comparison between Gauss Jordan and Gauss Elimination

Similarly, the time consumption is measured for different number of variables in the set of equations. The comparison is represented in Table 2

Number of Unknowns	Gauss Elimination Time Elapsed (ms)	Gauss Jordan Time Elapsed (ms)
2	14	25
3	16	31
4	20	36
5	26	39
6	29	56
7	46	76

Table 2: Time comparison between Gauss Jordan and Gauss Elimination.

The same difference is characterized using figure 6.

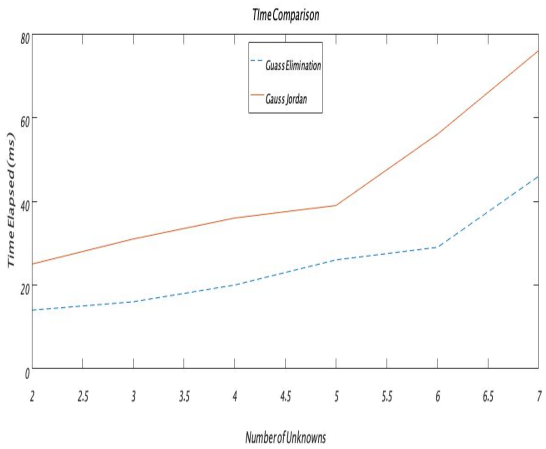

Figure 6: Time comparison

V. CONCLUSIONS

Many scientific and engineering problems can take the form of a system of linear equations. These equations may contain thousands of variables, so it is important to solve them as efficiently as possible. In this paper, the unknown variables in linear system are carried out by using Gauss Elimination Method and Gauss-Jordan Elimination Method. And the experiment is demonstrated with the help of java programming language. The results are carried out by solving 2, 3, 4, 5, 6 and 7 unknown variables. According to these experimental results, Gauss Elimination Method is faster than the Gauss Jordan Elimination method. Therefore Gauss Elimination Method is more efficient than the Gauss Jordan Elimination method.

VI. REFERENCES:

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