Fourier Series of Pulse Train and multi – source electrical network solution using Fourier series

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Abstract – The Fourier series, a basic mathematical tool which has wide applications in the field of signal analysis is formulated for the pulse train. Along with this I modulated the superposition theorem used in electrical networks using Fourier series to identify the output of provided electrical network. The magnitude and phase plots are plotted for reference and verified with actual output.

Keywords – Fourier Series, Signal analysis

INTRODUCTION

Fourier Series deals with the expressing any form of signal with weighted sum of cosine and sine functions. It is evaluated in both continuous and discrete sequences.

The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an *arbitrary* periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical.

Series can be expressed as follows

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos cos\ (wn)\ + b_n sin(wn)$$

Where,

$$a_0 = \frac{1}{2T} \int_{-T}^{T} f(t)\, dt$$

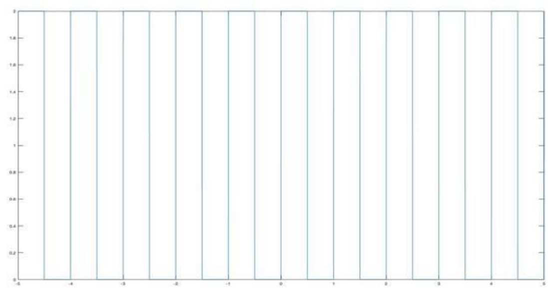
$$a_n = \frac{1}{T} \int_{-T}^{T} f(t) * cos(nt)dt$$

$$b_n = \frac{1}{T} \int_{-T}^{T} f(t) * sin(nt)dt$$

Many problems in physics involve vibrations and oscillations. Often the oscillatory motion is simple (e.g. weights on springs, pendulums, harmonic waves etc.) and can be represented as single sine or cosine functions. However, in many cases, (electromagnetism, heat conduction, quantum theory, etc.) the wave forms are not simple and, unlike sines and cosines, can be difficult to treat analytically. Fourier methods give us a set of powerful tools for representing any periodic function as a sum of sines and cosines

METHODOLOGY

A pulse wave is expressed as follows:



From the theory of Fourier series the coefficients of Fourier series is evaluated as

$$a_0 = \frac{1}{3}$$

$$a_n = \frac{2}{n\pi} sin(\frac{n\pi}{3})$$

$$b_n = 0$$

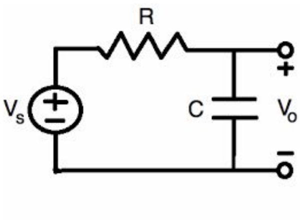
The Fourier series is evaluated as follows:

$$f(t) = 1 + \sum_{n=1}^{\infty} \frac{2}{n\pi} sin(\frac{n\pi}{3}) \cos cos \ (wn)$$

If series is truncated up to 3 terms

$$f(t) = 1 + \frac{2}{\pi} sin(\frac{\pi}{3}) \cos cos \ (w) + \frac{2}{2\pi} sin(\frac{2\pi}{3}) \cos cos \ (2n)$$

Similarly, consider the electrical network as follows:



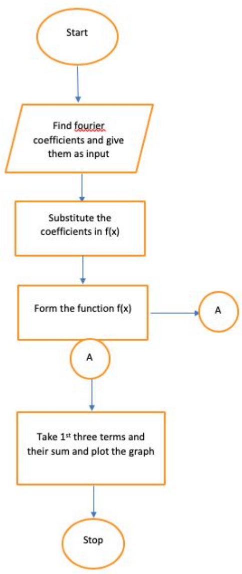
The output can be expressed as follows:

$$V_0 = (V_1 + V_2 + V_3 + V_4) * \frac{SCR+1}{SC}$$

Where V_i is i^{th} term in Fourier Series

IMPLEMENTATION USING MATLAB

Flow chart:



Algorithm:

1. Start
2. Find Fourier Coefficients and give them as input
3. Substitute the coefficients in f(x)
4. Form the function f(x)
5. Truncate the series to 3 terms
6. Plot the graph
7. Stop

Program:

1. Fourier Series of Pulse Wave Form:

```
clc;
close all;
clear all;
t = 0:0.01:3;
term1 = (1/3)
term2 =
(1/(2*pi))*sin(pi/3)*cos(pi*t);
term3 =
(1/4*pi)*sin(2*pi/3)*cos(2*pi*t);
sum = term1+term2+term3;
subplot(3,2,2);
plot(t,term1,'k');
grid on;
subplot(3,2,3);
plot(t,term2,'g');
grid on;
subplot(3,2,4);
plot(t,term3,'b');
grid on;
subplot(3,2,[5,6])
plot(t,sum,'r');
grid on;
x = sawtooth(2*pi*t);
subplot(3,2,1);
plot(t,x,'g');
grid on;
```

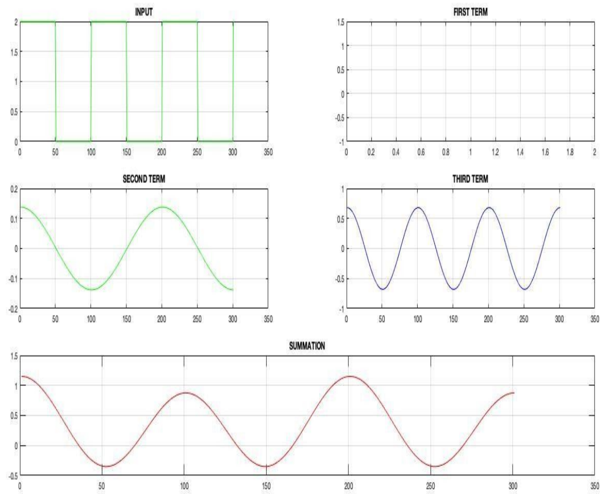
2. Multi – Source Fourier Solution:

```
clc;
clear all;
close all;
t = 0:0.01:5;
term1 = 1/2;
term2 = (2/pi)*sin(pi/2)*cos(2*t);
term3 =
(2/3*pi)*sin(3*pi/2)*cos(6*t);
```

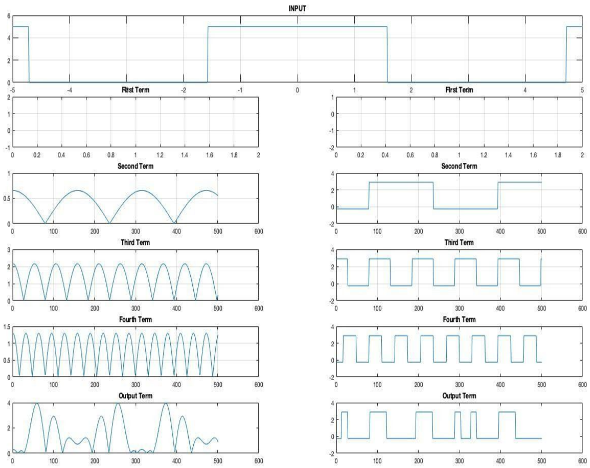
```
term4 =
(2/5*pi)*sin(5*pi/2)*cos(10*t);
sum = term1+term2+term3+term4;
vout1 =
term1*((1+(2*1j*2))/(2*1j*2));
mag1 = abs(vout1);
theta1 = angle(vout1);
vout2 =
term2*((1+(2*1j*2))/(2*1j*2));
vout3 =
term3*((1+(2*1j*2))/(2*1j*2));
vout4 =
term4*((1+(2*1j*2))/(2*1j*2));
mag2 = abs(vout2);
theta2 = angle(vout2);
mag3 = abs(vout3);
theta3 = angle(vout3);
mag4 = abs(vout4);
theta4 = angle(vout4);
vout = vout1+vout2+vout3+vout4;
mag5 = abs(vout);
theta5 = angle(vout);
voutfs =
sum*((1+(2*1j*2))/(2*1j*2));
t1 = -5:0.01:5;
x = 2.5+2.5*square((t1+(pi/2)));
subplot(5,2,[1,2])
plot(t1,x);
grid on;
subplot(6,2,3);
plot(mag1);
grid on;
subplot(6,2,4)
plot(theta1);
grid on;
subplot(6,2,5)
plot(mag2);
grid on;
subplot(6,2,6)
plot(theta2);
grid on;
subplot(6,2,7)
plot(mag3);
grid on
subplot(6,2,8)
plot(theta3);
grid on;
subplot(6,2,9)
plot(mag4);
grid on;
subplot(6,2,10)
plot(theta4);
subplot(6,2,11)
plot(mag5);
subplot(6,2,12)
plot(theta5);
figure;
subplot(2,1,1)
plot(t,voutfs)
grid on;
subplot(2,1,2)
plot(t,vout);
grid on;
```

RESULTS AND DISCUSSIONS

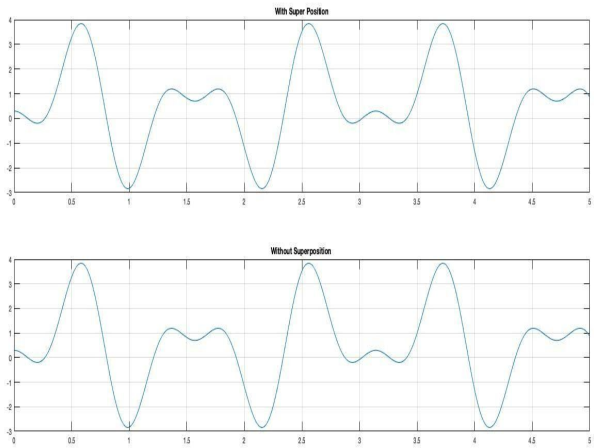
The results obtained by determining Fourier approximation of pulse wave which is truncated up to three terms is given as follows:



The output of multi-source electrical network solution is given by:



The comparison between super-position and without super position is given as



CONCLUSIONS

The Fourier series approximation is formulated for a pulse train wave form up to three terms using program-1 and plotted corresponding sinusoidal series obtained. The similar approach is used to determine the output of an electric network specified using program-2 and plotted corresponding magnitude and phase plots. The figures depicted prove that the circuit with superposition and without superposition yield the same result.

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