## **Today in Cryptography (5830)**

Digital signatures RSA signatures and full domain hash Schnorr signatures, DSA PKI



Pick random Nc.

# TLS handshake for RSA transport



Check CERT using CA public verification key

Check random PMS C <- E(pk,PMS)

Cert = (pk of bank, signature over it)

ChangeCipherSpec, { Finished, PRF(MS, "Client finished" | | H(transcript)) }

ChangeCipherSpec,

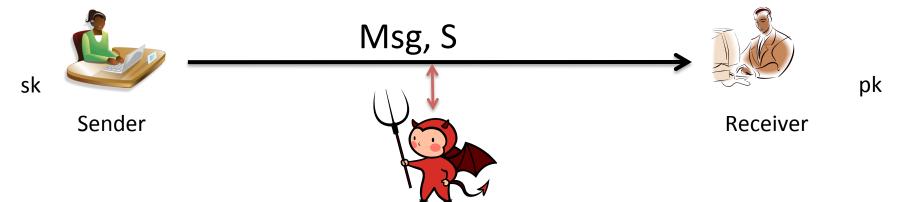
ClientHello, MaxVer, Nc, Ciphers/CompMethods

Bracket notation means contents encrypted

MS <- PRF(PMS, "master secret" | Nc | Ns )

{ Finished, PRF(MS, "Server finished" | | H(transcript')) }

#### **Digital signatures**



#### Two algorithms:

- (1) Key generation outputs (pk,sk)
- (2) Sign (sk, Msg) outputs a signature S (may be randomized)
- (3) Verify(pk,Msg,S) outputs 0/1 (invalid / valid)

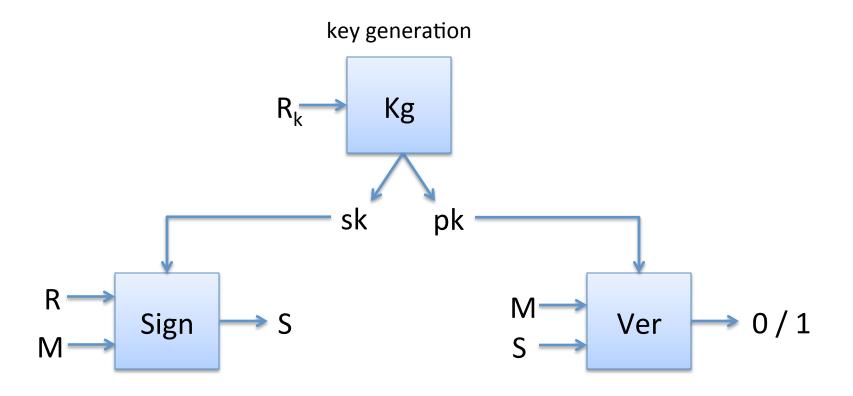
Correctness: Verify(pk,Msg,Sign(sk,Msg)) = 1 always

Security: No computationally efficient attacker can forge signatures for a new message even when attacker gets

$$(Msg_1, S_1), (Msg_2, S_2), ..., (Msg_q, S_q)$$

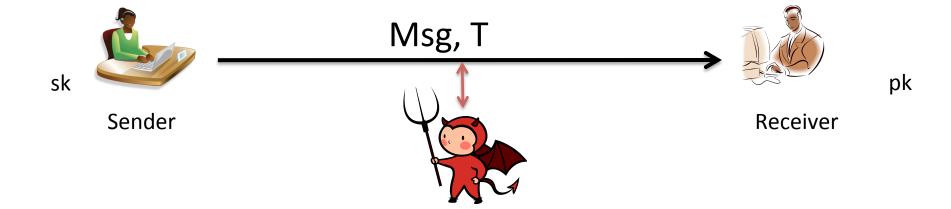
for messages of his choosing and reasonably large q.

## Digital signatures



Anyone with public key can verify a signature Only holder of secret key should be able to generate a signature

### **Digital signatures**



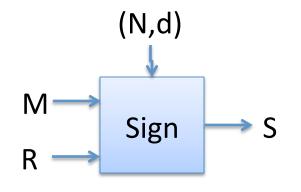
#### "Raw" RSA as a signature scheme:

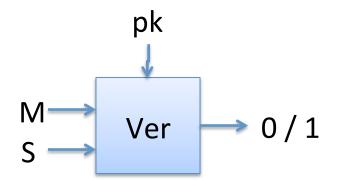
Key generation gives (N,e), (N,d) Sign((N,d),M) = M<sup>d</sup> mod N Verify((N,e),M,S) checks if S<sup>e</sup> mod N = M

Secure? No!

## PKCS #1 RSA signing

Kg outputs (N,e), (N,d) where  $|N|_8 = n$ Let B =  $\{0,1\}^8 / \{00\}$  be set of all bytes except 00 Want to encrypt messages of length  $|M|_8 = m$ 





```
Sign((N,d), M, R)

pad = first n - m - 2 bytes from R that

are in B

Y = 00 || 01 || pad || 00 || H(M)

Return Y<sup>d</sup> mod N
```

```
Verify((N,e), M, S)

Y = Ce mod N ; aa||bb||w = Y

If (aa ≠ 00) or (bb ≠ 01) or (00\notin w)

Return error

pad || 00 || h = w

Return H(M) = h
```

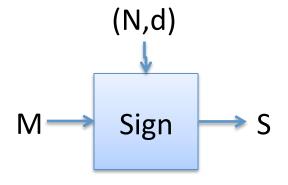
## Digital signature security

 Padding oracle attacks that work against RSA PKCS#1 v1.5 decryption work against similar implementations of signing

## **Full Domain Hash RSA**

Kg outputs pk = (N,e), sk = (N,d)

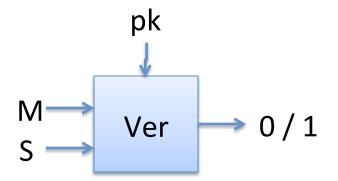
H is a hash function



X = 00 || H(1||M) || ... || H(k||M)

 $S = X^d \mod N$ 

Return S



```
Ver((N,e), M, S)
```

 $X = S^e \mod N$ 

 $X' = 00 \mid | H(1||M) \mid | ... \mid | H(k||M)$ 

If X = X' then

Return 1

Return 0

## **Schnorr signatures**

Choose prime q and we'll work in multiplicative group  $\mathbf{Z}_{q}^{*}$  sk = k chosen in  $\mathbf{Z}_{q}$  pk = g<sup>k</sup>

```
\frac{Sign(k, M)}{R = g^r}; e = H(M \mid\mid R); s = r - xe
Return(s,e)
```

```
\frac{\text{Ver}(pk = g^k, M, (s,e))}{R_v = g^s * pk^e ; e_v = H(M \mid \mid R_v)}
If e_v = e then Return 1
Return 0
```

# DSA (digital signature algorithm)

Choose prime q and p s.t. p-1 | q . Set  $g = h^{(p-1)/q} \mod p$  sk = k chosen in  $\mathbf{Z}_q$  pk =  $g^k$ 

```
Sign(k, M)

r <- $ \mathbf{Z}_q until R = (g^r \mod p) mod q \neq 0

s <- k^{-1}(H(M) + k R) \mod q (start over if s = 0)

Return (R,s)
```

```
\begin{split} & \underline{\text{Ver}(pk = g^k, M, (R,s))} \\ & \text{If R,s not in } \textbf{Z}_q \\ & \text{w} <- s^{-1} \text{ mod } q \; ; \; \text{u1} = \text{H(m)} * \text{w mod } q \\ & \text{u2} = \text{R*w mod } q \; ; \; \text{v} = (g^{u1} \, \text{pk}^{u2} \, \text{mod p}) \, \text{mod } q \\ & \text{If v} = \text{R then Return 1} \\ & \text{Return 0} \end{split}
```



# TLS handshake for RSA transport



Pick random Ns

 $PMS \leftarrow D(sk,C)$ 

Pick random Nc

ClientHello, MaxVer, Nc, Ciphers/CompMethods

ServerHello, Ver, Ns, SessionID, Cipher/CompMethod

CERT = (pk of server, signature over it)

Check CERT using CA public verification key

Pick random PMS C <- E(pk,PMS)

Bracket notation means contents encrypted

C

ChangeCipherSpec,
{ Finished, PRF(MS, "Client finished" || H(transcript)) }

ChangeCipherSpec, { Finished, PRF(MS, "Server finished" || H(transcript')) }

MS <- PRF(PMS, "master secret" || Nc || Ns )

# Certificate Authorities and Public-key Infrastructure





M = (pk', data)

S = Sign(sk,M)

Give me a certificate for pk', please

http://amazon.com



pk', data, S



M = (pk',data)

If Ver(pk,M,S) then

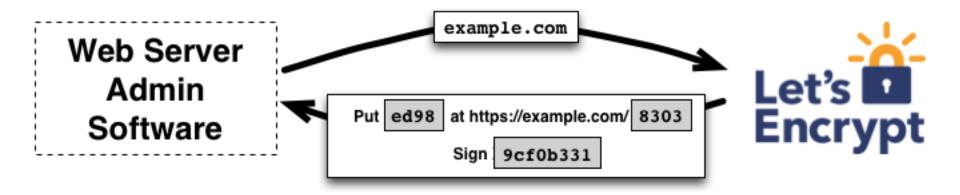
trust pk'

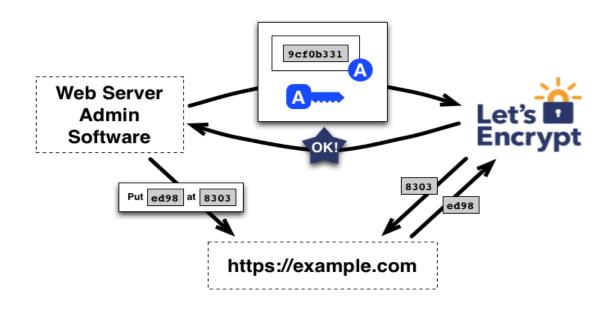
(pk',sk')

This prevents man-in-the-middle (MitM) attacks

```
Certificate:
  Data:
      Version: 1 (0x0)
       Serial Number: 7829 (0x1e95)
      Signature Algorithm: md5WithRSAEncryption
       Issuer: C=ZA, ST=Western Cape, L=Cape Town, O=Thawte Consulting cc,
               OU=Certification Services Division,
              CN=Thawte Server CA/emailAddress=server-certs@thawte.com
      Validity
          Not Before: Jul 9 16:04:02 1998 GMT
          Not After: Jul 9 16:04:02 1999 GMT
       Subject: C=US, ST=Maryland, L=Pasadena, O=Brent Baccala,
               OU=FreeSoft, CN=www.freesoft.org/emailAddress=baccala@freesoft.org
       Subject Public Key Info:
          Public Key Algorithm: rsaEncryption
          RSA Public Key: (1024 bit)
               Modulus (1024 bit):
                   00:b4:31:98:0a:c4:bc:62:c1:88:aa:dc:b0:c8:bb:
                   33:35:19:d5:0c:64:b9:3d:41:b2:96:fc:f3:31:e1:
                   66:36:d0:8e:56:12:44:ba:75:eb:e8:1c:9c:5b:66:
                   70:33:52:14:c9:ec:4f:91:51:70:39:de:53:85:17:
                   16:94:6e:ee:f4:d5:6f:d5:ca:b3:47:5e:1b:0c:7b:
                  c5:cc:2b:6b:c1:90:c3:16:31:0d:bf:7a:c7:47:77:
                   8f:a0:21:c7:4c:d0:16:65:00:c1:0f:d7:b8:80:e3:
                   d2:75:6b:c1:ea:9e:5c:5c:ea:7d:c1:a1:10:bc:b8:
                  e8:35:1c:9e:27:52:7e:41:8f
               Exponent: 65537 (0x10001)
  Signature Algorithm: md5WithRSAEncryption
       93:5f:8f:5f:c5:af:bf:0a:ab:a5:6d:fb:24:5f:b6:59:5d:9d:
       92:2e:4a:1b:8b:ac:7d:99:17:5d:cd:19:f6:ad:ef:63:2f:92:
       ab:2f:4b:cf:0a:13:90:ee:2c:0e:43:03:be:f6:ea:8e:9c:67:
       d0:a2:40:03:f7:ef:6a:15:09:79:a9:46:ed:b7:16:1b:41:72:
       0d:19:aa:ad:dd:9a:df:ab:97:50:65:f5:5e:85:a6:ef:19:d1:
       5a:de:9d:ea:63:cd:cb:cc:6d:5d:01:85:b5:6d:c8:f3:d9:f7:
       8f:0e:fc:ba:1f:34:e9:96:6e:6c:cf:f2:ef:9b:bf:de:b5:22:
       68:9f
```

### **Free CAs**





## Revocation

Certificates must often be revoked

Short expirations

– CRLs (Certificate revocation lists)

OCSP (online certificate status protocol)

## The Web PKI Ecosystem

 http://conferences.sigcomm.org/imc/2013/ papers/imc257-durumericAemb.pdf

 ~1800 CAs that can sign any domain controlled by 683 organizations

SSL Certificates

Signing Service

SIM-ID

DocProof

Cooperation Dutch government

> DigiNotar reports security incident

Read the press release >>

Read the press release >>



DigiNotar focuses on ensuring the integrity of

information exchange. More information >>

information flow, and legal guarantees for all online