

# Today in Cryptography (5830)

Digital signatures

Schnorr signatures, DSA

PKI

Katz-Lindell Chapter 12

Various PKI sources available on web (some references within slides)

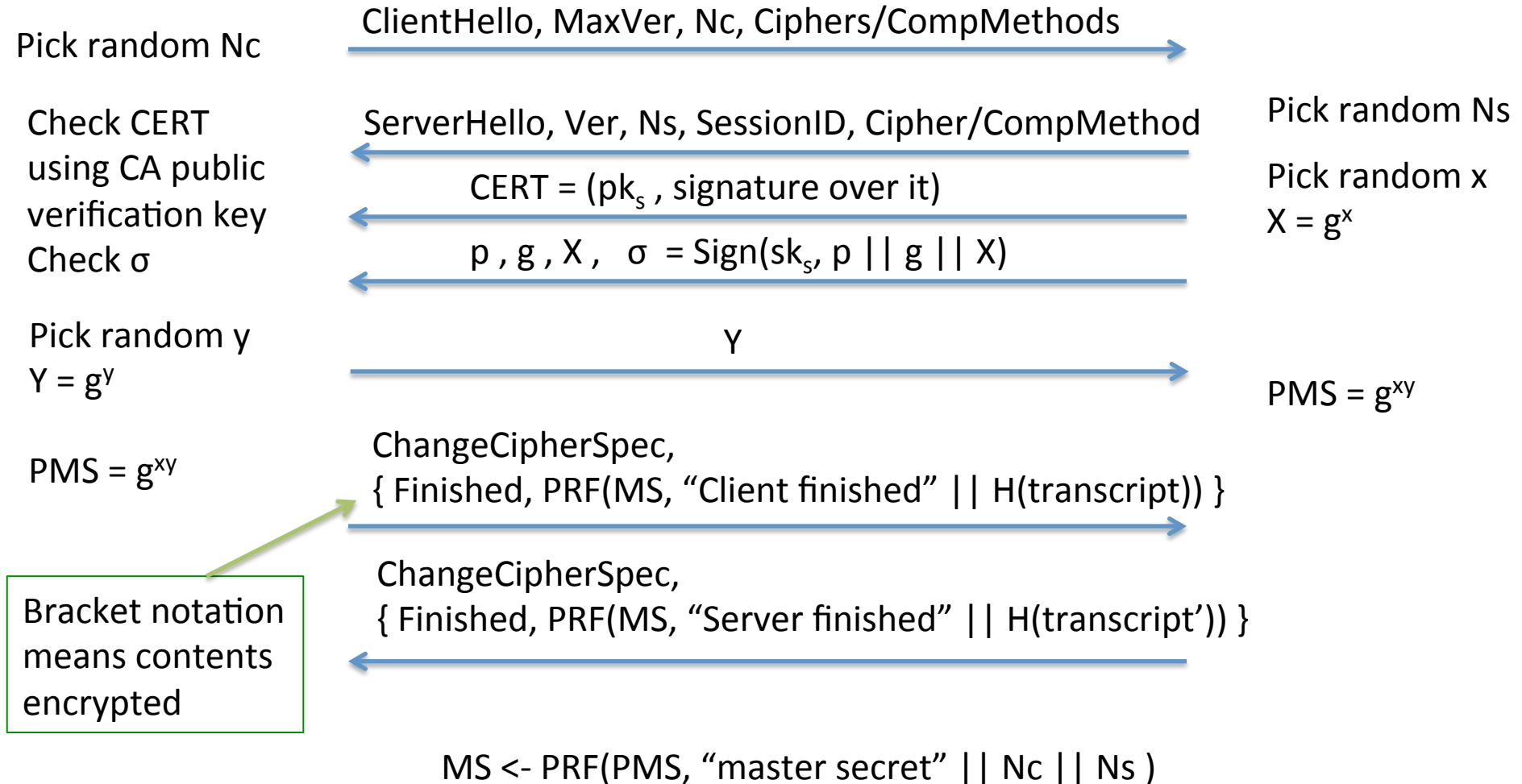


Client

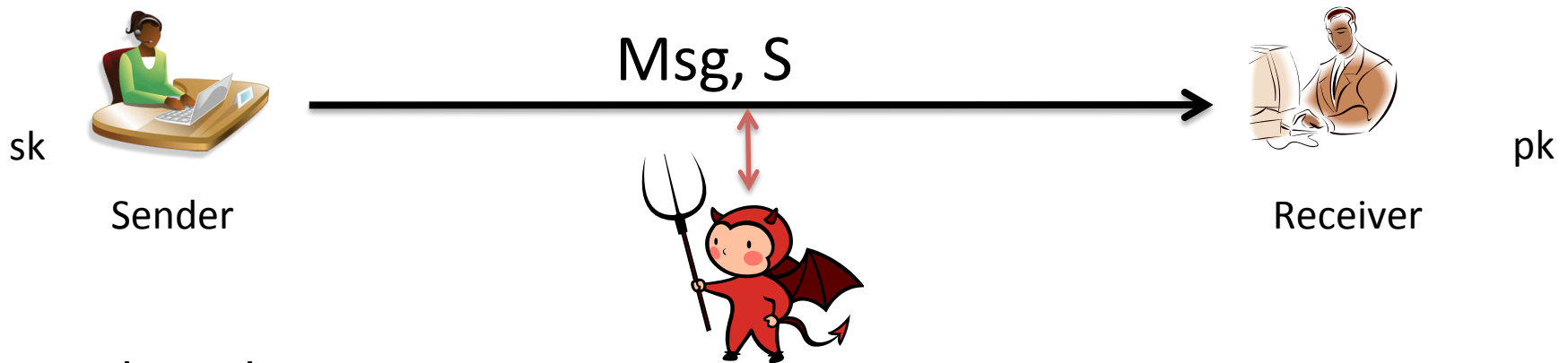


Server

# TLS handshake for Diffie-Hellman Key Exchange



# Digital signatures



## Two algorithms:

- (1) Key generation outputs  $(pk, sk)$
- (2)  $\text{Sign}(sk, \text{Msg})$  outputs a signature  $S$  (may be randomized)
- (3)  $\text{Verify}(pk, \text{Msg}, S)$  outputs 0/1 (invalid / valid)

*Correctness:*  $\text{Verify}(pk, \text{Msg}, \text{Sign}(sk, \text{Msg})) = 1$  always

*Security:* No computationally efficient attacker can forge signatures for a new message even when attacker gets

$(\text{Msg}_1, S_1), (\text{Msg}_2, S_2), \dots, (\text{Msg}_q, S_q)$

for messages of his choosing and reasonably large  $q$ .

# Underlying groups

Let  $p$  be a large prime number

Let  $q$  be a prime such that  $q$  divides  $p-1$

Example:  $p = 2q + 1$  (so-called safe prime  $p$ )

Fix the group  $G = \mathbf{Z}_p^* = \{1, 2, 3, \dots, p-1\}$

Let  $g$  be generator of sub-group of order  $q$ :

$$\{g^0, g^1, g^2, \dots, g^{q-1}\} \text{ subset of } G$$

How to pick  $g$ ?

$g = h^{(p-1)/q} \bmod p$  for some  $h$  and check  $g \neq 1 \bmod p$

If so, try repeat with another  $h$ . Usually use  $h = 2$

# Schnorr signatures

$p, q, g$  specified

$sk = x$  chosen randomly from  $\mathbf{Z}_q$        $pk = X = g^x$

Sign( $x, M$ )

$r \leftarrow \mathbf{Z}_q$

$R = g^r$  ;  $c = H(M \parallel R)$  ;  $z = r + cx \pmod q$

Return ( $R, z$ )

Ver( $X, M, (R, z)$ )

$c = H(M \parallel R)$

If  $g^z = RX^c$  then Return 1

Return 0

Correctness?       $g^z = g^{r + cx} = g^r g^{xc} = RX^c$

# Security of Schnorr signatures



Assume an adversary that can output forgery  $(M, (R, z))$

Then to be valid:

$$g^z = RX^c \quad \text{implies} \quad z = r + cx$$

for  $c = H(M \parallel R)$  .

Assume  $c$  is random ( $H$  is random oracle)

Imagine we can run adversary twice but force forgery to be on same  $R$ , different  $c$  .

In second execution, getting  $(M', (R, z'))$

Then success second time around gives:

$$g^{z'} = RX^{c'} \quad \text{implies} \quad z' = r + c'x$$

But now can compute  $z - z' / (c - c') = x$  the secret key

# Fragility of Schnorr

Repeat randomness failure:

Sign two messages  $M \neq M'$  and reuse random

$\text{Sign}(x, M) \rightarrow (R, z)$

$\text{Sign}(x, M') \rightarrow (R, z')$

Then:  $x = (z - z') / (H(M || R) - H(M' || R))$

If  $r$  is predictable, can recover secret as well

Can improve security by “hedging”:

choose  $r = H(x || M || \text{randomness})$

# DSA (digital signature algorithm)

$p, q, g$  specified

$sk = x$  chosen randomly from  $\mathbf{Z}_q$

$pk = X = g^x$

Sign( $x, M$ )

$r \leftarrow \mathbf{Z}_q$  ;  $R = (g^r \bmod p) \bmod q$

$z = r^{-1} (H(M) + x R) \bmod q$

Return ( $R, z$ )

Ver( $X, M, (R, z)$ )

$w = z^{-1} \bmod q$

$u1 = H(m) * w \bmod q$

$u2 = R * w \bmod q$

If  $R = (g^{u1} X^{u2} \bmod p) \bmod q$

then Return 1

Else Return 0

Correctness?

$$\begin{aligned} g^{u1} X^{u2} &= g^{H(M) w} g^{x R w} = g^{(H(M)+xR) w} \\ &= g^{(H(M)+xR) (H(M)+xR)^{-1} r} = g^r \end{aligned}$$



# Hackers Describe PS3 Security As Epic Fail, Gain Unrestricted Access

BY MIKE BENDEL

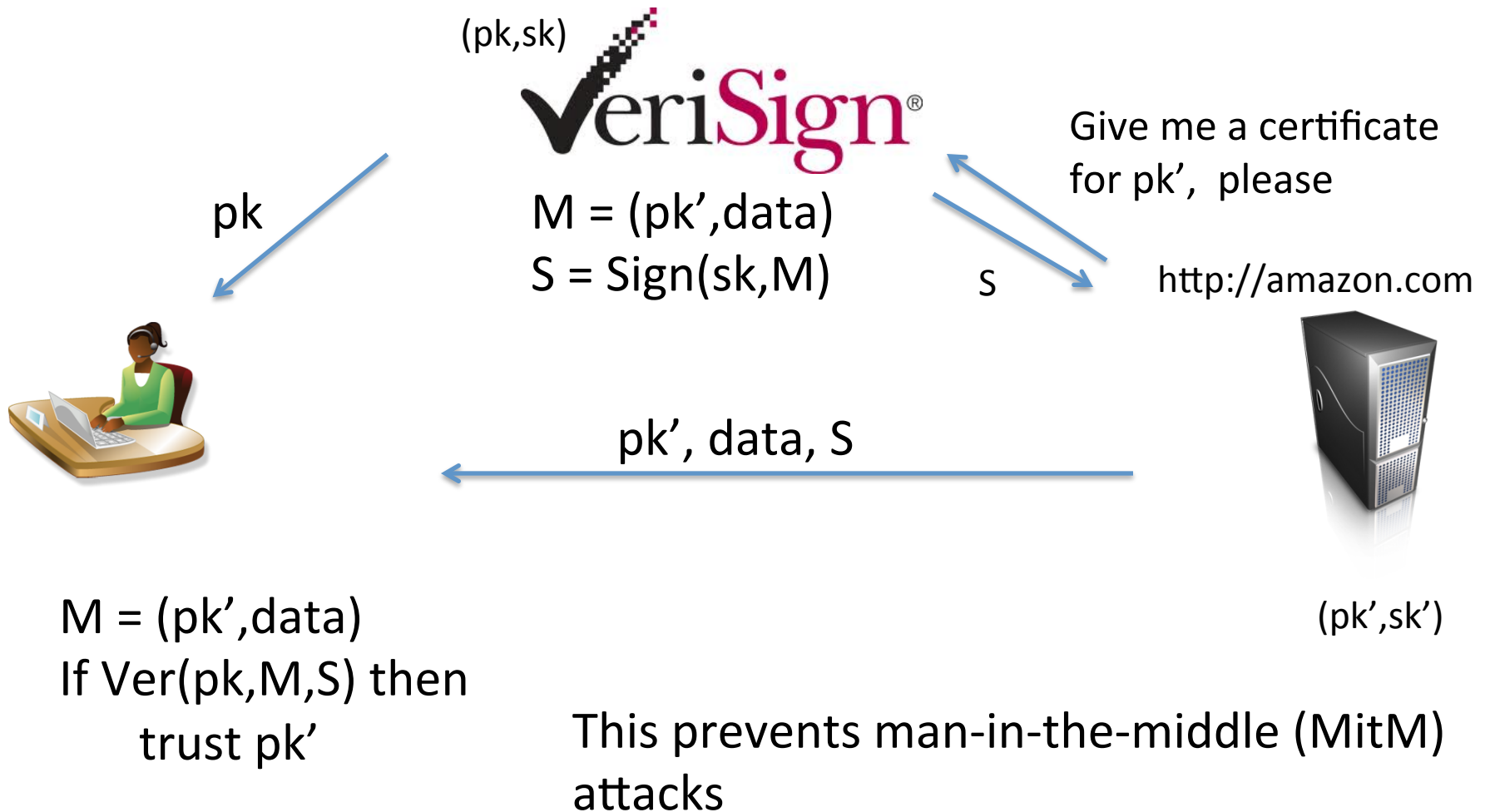
DECEMBER 29, 2010 @ 11:19 AM

The image shows the word "fail0verflow" in a pixelated, monospace font. The characters are white and black, set against a solid black background. The text is contained within a white rectangular box that has a slight drop shadow, giving it a 3D appearance. The font is reminiscent of early computer graphics or digital art.

# Digital signature schemes

|                     | Problems?              | Proofs?                                       | Uses                                       |
|---------------------|------------------------|-----------------------------------------------|--------------------------------------------|
| RSA PKCS#1 v1.5     | Bleichenbacher attacks |                                               | TLS, Certificates, XML                     |
| RSA PSS (PKCS#1 v2) |                        | Security reduces to hardness of inverting RSA |                                            |
| Schnorr             | Randomness fragility   | Security reduces to discrete log problem      |                                            |
| DSA                 | Randomness fragility   | No security reduction                         | Bitcoin (ECC version), TLS, SSH, elsewhere |

# Certificate Authorities and Public-key Infrastructure



Certificate:

Data:

Version: 1 (0x0)

Serial Number: 7829 (0x1e95)

Signature Algorithm: md5WithRSAEncryption

Issuer: C=ZA, ST=Western Cape, L=Cape Town, O=Thawte Consulting cc,  
OU=Certification Services Division,  
CN=Thawte Server CA/emailAddress=server-certs@thawte.com

Validity

Not Before: Jul 9 16:04:02 1998 GMT

Not After : Jul 9 16:04:02 1999 GMT

Subject: C=US, ST=Maryland, L=Pasadena, O=Brent Baccala,  
OU=FreeSoft, CN=www.freesoft.org/emailAddress=baccala@freesoft.org

Subject Public Key Info:

Public Key Algorithm: rsaEncryption

RSA Public Key: (1024 bit)

Modulus (1024 bit):

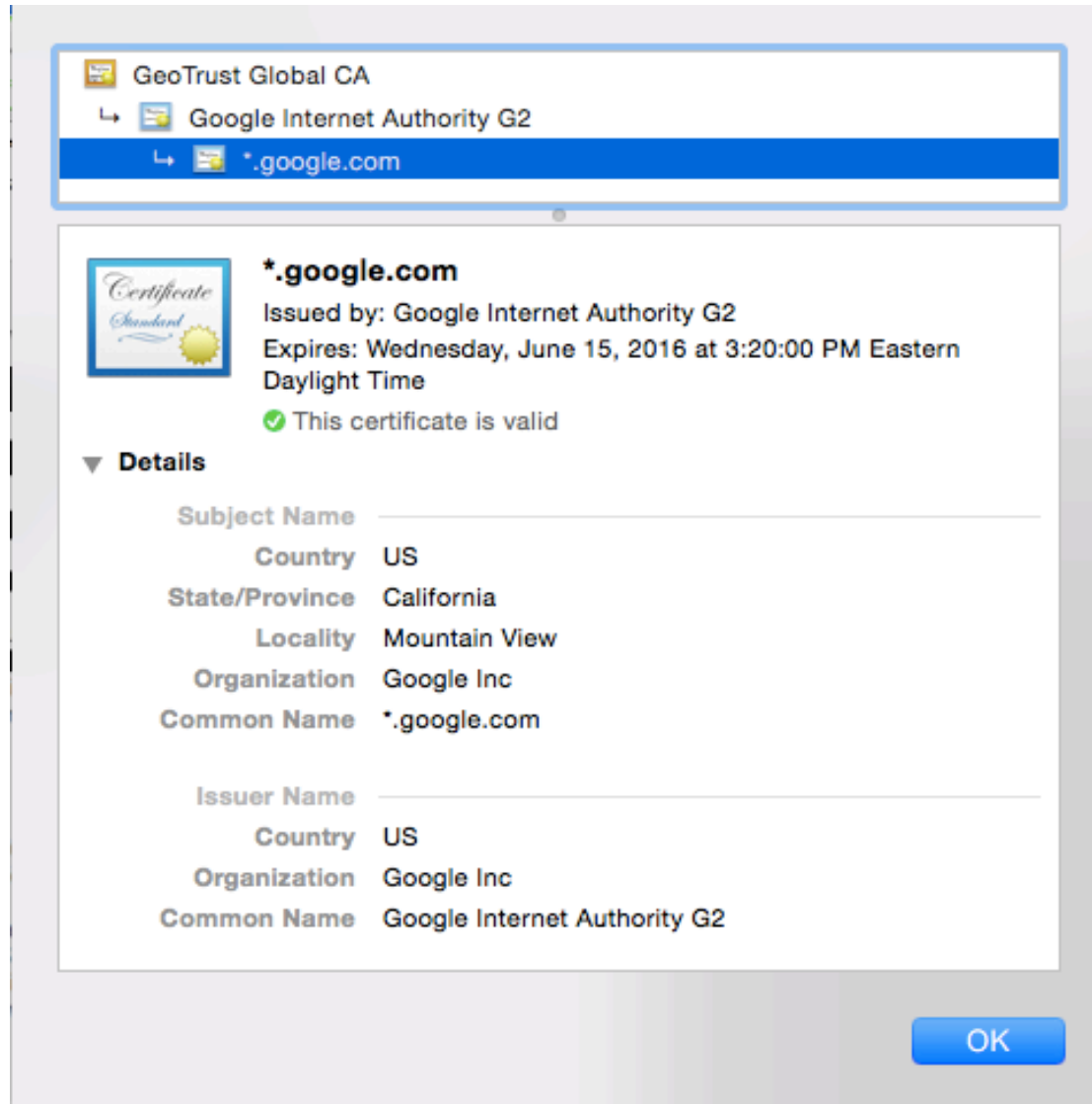
00:b4:31:98:0a:c4:bc:62:c1:88:aa:dc:b0:c8:bb:  
33:35:19:d5:0c:64:b9:3d:41:b2:96:fc:f3:31:e1:  
66:36:d0:8e:56:12:44:ba:75:eb:e8:1c:9c:5b:66:  
70:33:52:14:c9:ec:4f:91:51:70:39:de:53:85:17:  
16:94:6e:ee:f4:d5:6f:d5:ca:b3:47:5e:1b:0c:7b:  
c5:cc:2b:6b:c1:90:c3:16:31:0d:bf:7a:c7:47:77:  
8f:a0:21:c7:4c:d0:16:65:00:c1:0f:d7:b8:80:e3:  
d2:75:6b:c1:ea:9e:5c:5c:ea:7d:c1:a1:10:bc:b8:  
e8:35:1c:9e:27:52:7e:41:8f

Exponent: 65537 (0x10001)

Signature Algorithm: md5WithRSAEncryption

93:5f:8f:5f:c5:af:bf:0a:ab:a5:6d:fb:24:5f:b6:59:5d:9d:  
92:2e:4a:1b:8b:ac:7d:99:17:5d:cd:19:f6:ad:ef:63:2f:92:  
ab:2f:4b:cf:0a:13:90:ee:2c:0e:43:03:be:f6:ea:8e:9c:67:  
d0:a2:40:03:f7:ef:6a:15:09:79:a9:46:ed:b7:16:1b:41:72:  
0d:19:aa:ad:dd:9a:df:ab:97:50:65:f5:5e:85:a6:ef:19:d1:  
5a:de:9d:ea:63:cd:cb:cc:6d:5d:01:85:b5:6d:c8:f3:d9:f7:  
8f:0e:fc:ba:1f:34:e9:96:6e:6c:cf:f2:ef:9b:bf:de:b5:22:  
68:9f

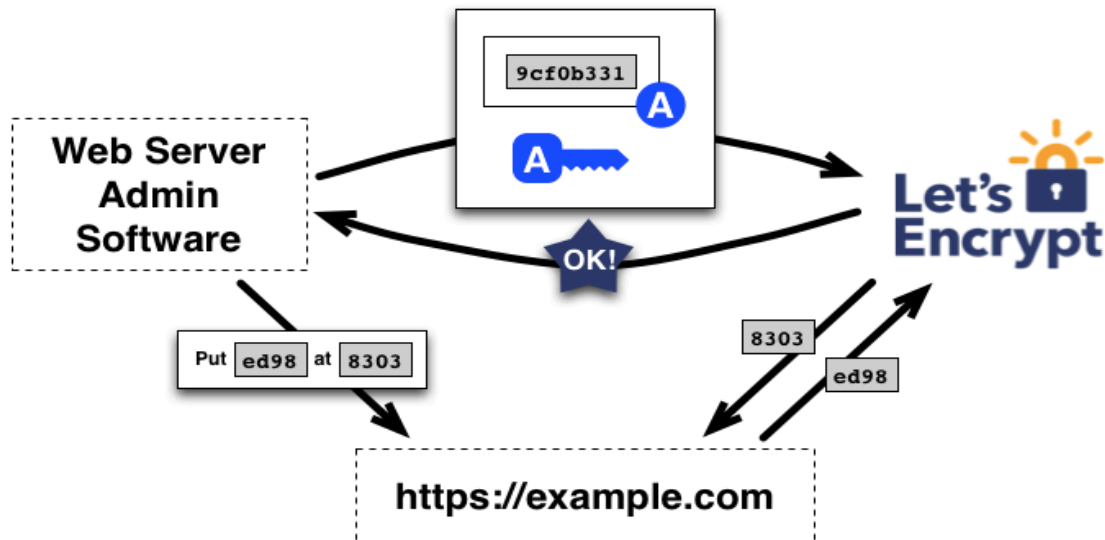
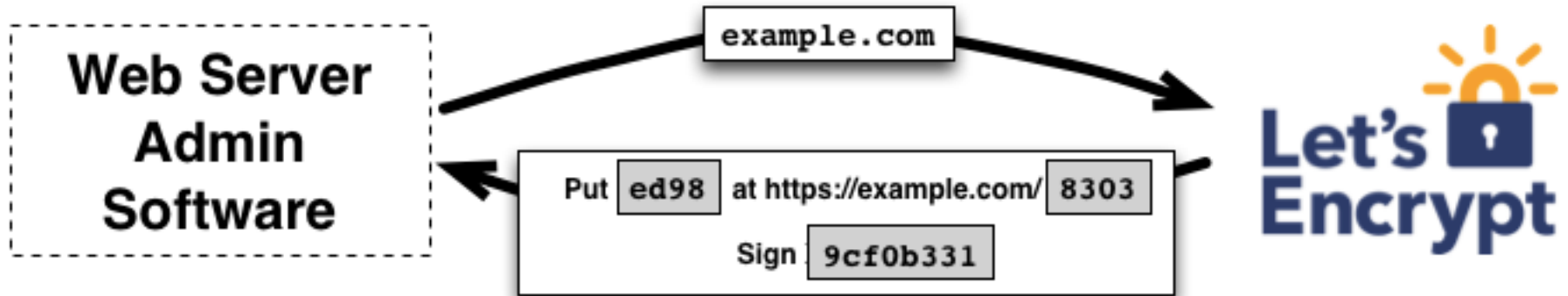
# Certificate chains



# Identity checks?

- CA's must check that requestor of cert is who they say they are
- Domain validated
  - Prove ownership of domain
- Extended validation
  - Establish legal identity of requestor
  - Physical presence of website owner
  - Confirm ownership of domain
  - Etc.

# Free CAs



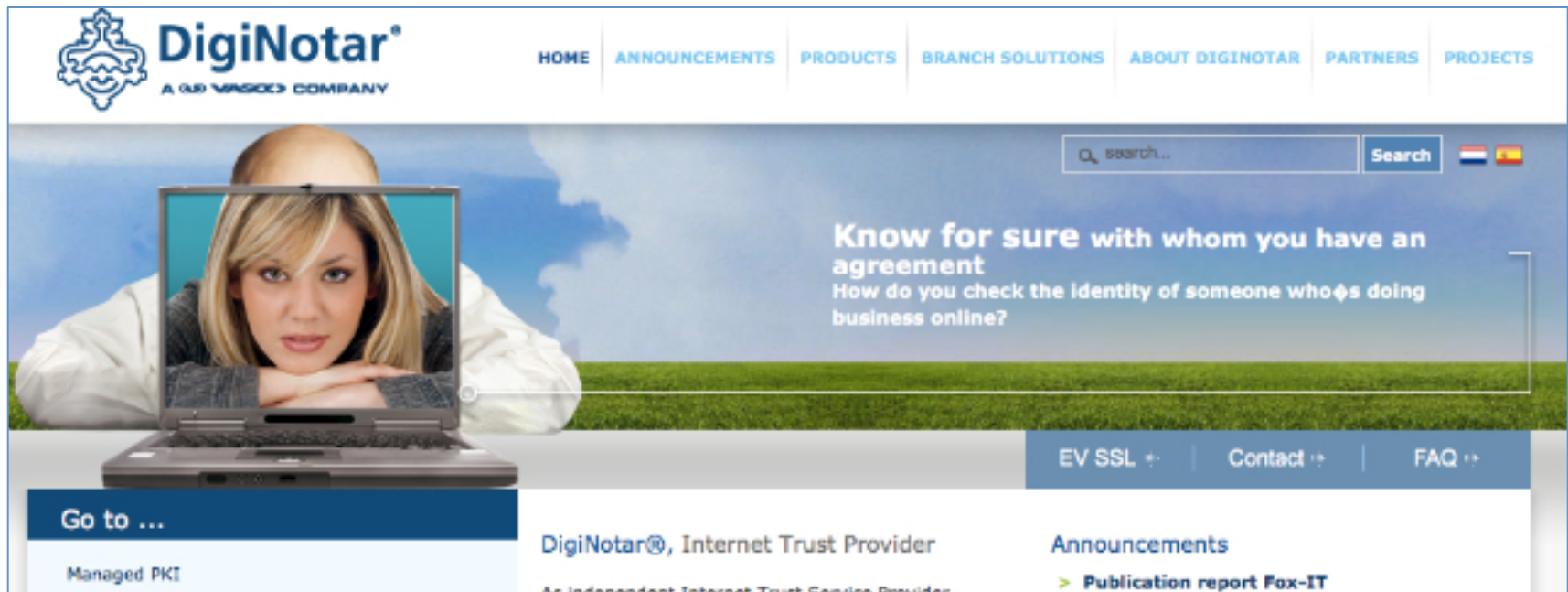
# Revocation

- Certificates must often be revoked
  - Short expirations
  - CRLs (Certificate revocation lists)
  - OCSP (online certificate status protocol)
    - Client queries CA to check on validity of cert
      - privacy concerns, performance / scalability issues
    - Stapling: server periodically gets fresh, time-stamped OCSP signature from CA. Sends to clients



# The Web PKI Ecosystem

- <http://conferences.sigcomm.org/imc/2013/papers/imc257-durumericAemb.pdf>
- ~1800 CAs that can sign *any* domain controlled by 683 organizations



Today, Microsoft issued a [Security Advisory](#) warning that fraudulent digital certificates were issued by the Comodo Certificate Authority. This could allow malicious spoofing of high profile websites, including Google, Yahoo! and Windows Live.

<https://nakedsecurity.sophos.com/2011/03/24/fraudulent-certificates-issued-by-comodo-is-it-time-to-rethink-who-we-trust/>

<https://technet.microsoft.com/library/security/2524375>

# Certificate/public-key pinning

- Client knows what cert/pk to expect, rejects otherwise
  - Pre-install some keys
  - HPKP (HTTP Public Key Pinning)
    - HTTP header that allows servers to set a hash of public key they will use

Public-Key-Pins:

```
pin-sha256="d6qzRu9zOECb90Uez27xWltNsj0e1Md7GkYYkVoZWmM=";  
pin-sha256="LPJNul+wow4m6DsquxbninhsWHlwfp0JecwQzYpOLmCQ=";  
max-age=259200
```

<https://developers.google.com/web/updates/2015/09/HPKP-reporting-with-chrome-46?hl=en>

# Certificate transparency

- Force CAs to log the certificates they sign in a public tamper-evident register
  - Experimental IETF standard
- Google has been pushing this
  - Chrome requires it for “extra validation” certs
  - DigiCert has implemented

# Summary

- Web PKI relies on various trust assumptions
  - Can be undermined in many ways
- Digital signature schemes power PKI and verifying identities:
  - unforgeability under chosen message attack
  - RSA based schemes PKCS#1 1.5 and 2.0
  - Schnorr, DSA based on discrete log problem