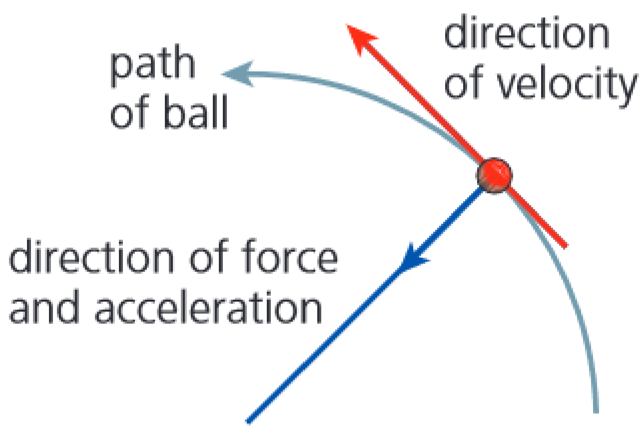
Centripetal acceleration and centripetal force

An object traveling in a circle may have a constant speed, but not constant velocity. This is because while the magnitude is the same, the direction is constantly changing.

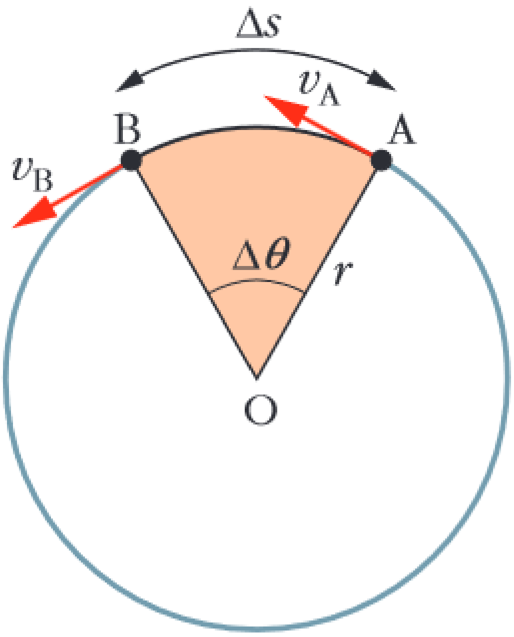
If the velocity is not constant, we know that the object is accelerating and has a force acting upon it. This acceleration and force are called the **centripetal acceleration** and **centripetal force**.

Both the centripetal force and acceleration have the same direction: towards the center of the circle. This means that at any instant, the force acts at a right angle to the velocity of the object.

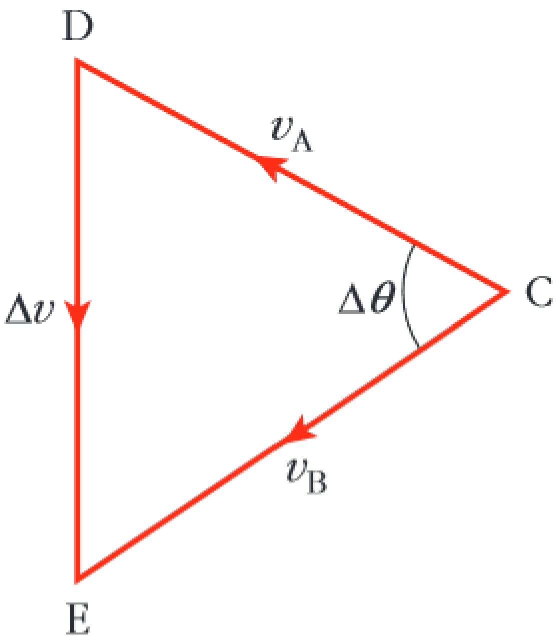


A ball spinning on a string has a force acting to the center due to the tension of the string. This tension is the centripetal force. If the string breaks, the ball will fly off in the direction of its velocity in that instant.

Finding the acceleration and force



At point A, the object has a velocity v_A and at point B, v_B. They are both vectors. The arc length is Δs and the radius is r. $v_A = v_B$ in terms of magnitude.



If we add another vector Δv and connect v_A and v_B, we can see that it forms a triangle $\triangle CDE$ with the same angle $\Delta\theta$.

Now, if we shrink the first circle until the arc Δs can be considered a straight line, we will get a triangle similar to $\triangle CDE$. Similar triangles can be ratioed to get -

$$\frac{\Delta v}{v_A} = \frac{\Delta s}{r}$$

We can find Δv to get

$$\Delta v = \Delta s \cdot \frac{v_A}{r}$$

If we divide both sides by Δt we get -

$$\frac{\Delta v}{\Delta t} = \frac{\Delta s}{\Delta t} \cdot \frac{v_A}{r}$$

Now, knowing that $a = \frac{\Delta v}{\Delta t}$, $v = \frac{\Delta s}{\Delta t}$, and $v_A = v_B = v$,

$$a = v \cdot \frac{v}{r}$$

Or

$$a = \frac{v^2}{r}$$

In angular speed terms, $v = r\omega$, so

$$a = \frac{r^2\omega^2}{r}$$

$$a = r\omega^2$$

Force is defined as the product of the mass and its acceleration.

So $F = \frac{mv^2}{r} = mr\omega^2$

Examples of circular motion

A ball whirled around on a string has centripetal force because of the tension in the string.

A satellite orbiting Earth has a centripetal force because of gravity.

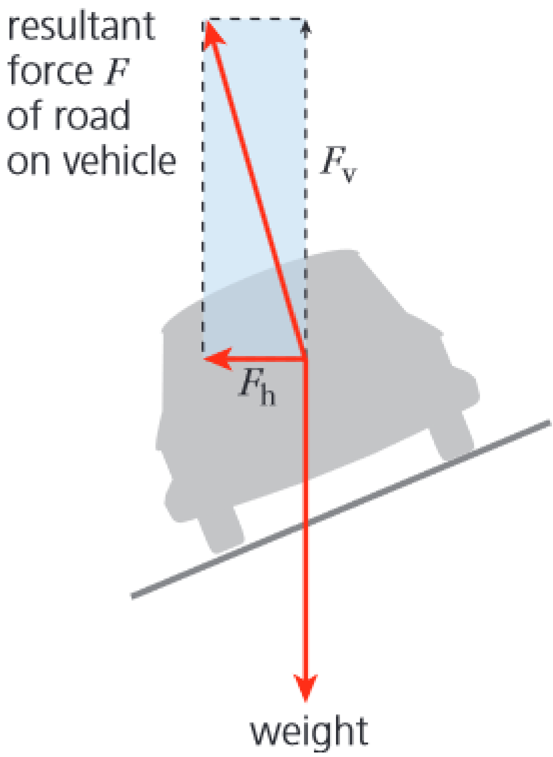
A charged particle moving in a magnetic field arcs towards its negative charge.

A car traveling in a curved path has a centripetal force from the friction between the tires and the road. If there was no friction, i.e., it has no centripetal force, it skids across in a straight line.

Passengers in a car seem to be flung away when the car turns around a corner. This is because while the seat provides centripetal force, the force is below the passenger's center of mass, causing rotation at that point. The passenger's upper body will move outward unless there is another force stopping the rotation.

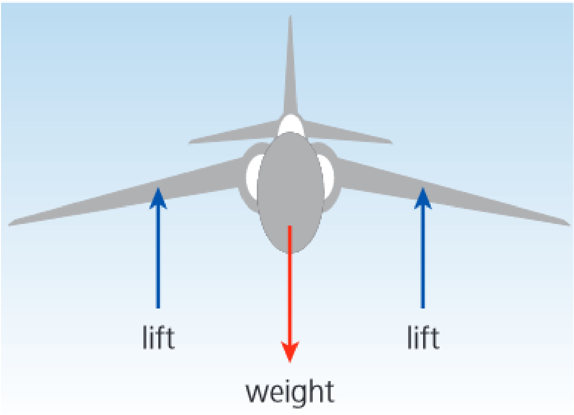
Banking

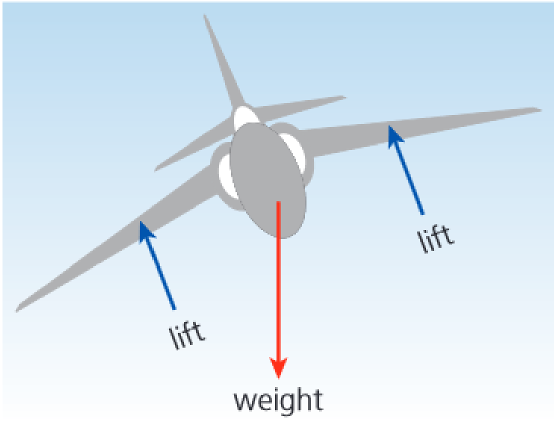
For cornering which does not rely on friction, banking can be used.



The road provides a normal force which is at an angle to the vertical, and can be resolved into components F_v and F_h. F_v is equal to the weight of the car, which maintains equilibrium while F_h provides a centripetal force to the side. Many roads are banked to increase safety and also passenger comfort.

Aircrafts use banking because there is no friction in the air to provide a centripetal force when turning.





Aircrafts have a lift force keeping the plane up so when it banks to a side, the lift provides a horizontal force, which is the centripetal force.

Continuing examples of circular motion

A centrifuge is a device used to spin objects at high speeds about an axis. They separate particles within a mixture. Bigger particles require more centripetal force to maintain a circular motion. And so, the particles are separated. Bigger centrifuges are used on human to test the effect of large accelerations on their bodies.

Left: separating a solid from a liquid in a lab, Right: centrifuge used to test acceleration on humans

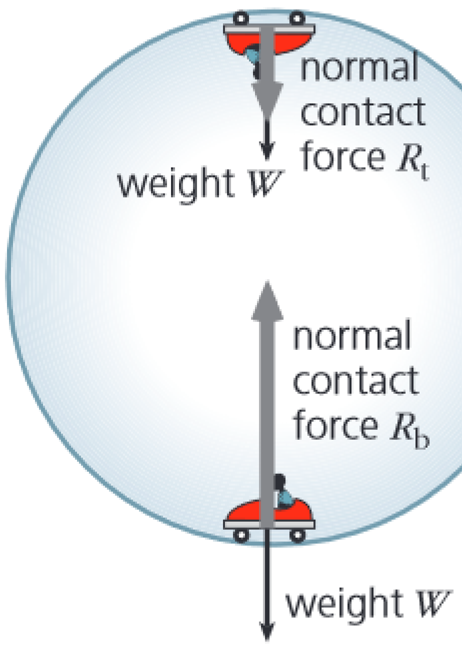
Motion in a vertical circle

Some theme parks have rides where a person moves in a vertical circle (ferris wheels). A person on this ride must have a resultant force acting towards the center of the circle.

The forces acting on the person are the person’s weight, which is always downwards, and the normal contact force of the seat, which acts at right angles to the seat.

If the person is moving at velocity v,

1. At the bottom of the ride, the normal contact force must provide the centripetal force, as well as overcome the weight of the person.
   a. The centripetal force is given by $\frac{mv^2}{r} = normal_{bottom} - W$
2. At the top of the ride, the normal contact force is in the same direction as the weight of the person.
   a. The centripetal force is given by $\frac{mv^2}{r} = normal_{top} + W$



This means the normal force at the top is less than the normal force at the bottom. If the speed v is not large enough, the weight might be larger than the centripetal force and the person would fall inwards.

Formula list

$a = \frac{v^2}{r}$

$a = r\omega^2$

$F = \frac{mv^2}{r}$

$F = mr\omega^2$

$F = normal_{bottom} - W$ (for vertical motion at the bottom)

$F = normal_{top} + W$ (for vertical motion at the top)