Quadratics

In $ax^2 + bx + c = 0$ with roots α and β, if we change this to
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$,
we can conclude that $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.
In summation notation, $\Sigma\alpha = \alpha + \beta$ and $\Sigma\alpha\beta = \alpha\beta$

$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\alpha^2 + \beta^2 = (\Sigma\alpha)^2 - 2\alpha\beta$

$\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta$

$\alpha^2 + \beta^2$ can be written as $\Sigma\alpha^2$ in summation notation.

$(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha - \beta)^2 + 2\alpha\beta$

$\frac{1}{\alpha} + \frac{1}{\beta} = \Sigma\frac{1}{\alpha}$

$\downarrow$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

$= \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\Sigma\alpha}{\Sigma\alpha\beta}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \Sigma\frac{1}{\alpha^2}$

$\downarrow$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2}{\alpha^2}+\frac{\alpha^2}{\beta^2}$$
$$=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$
$$=\frac{\Sigma\alpha^2}{(\Sigma\alpha\beta)^2}$$

Finding $\alpha^2+\beta^2$ from an equation

e.g, $x^2+5x+7=0$ with roots α and β

$\alpha+\beta=-5,\ \alpha\beta=7$

$x=\alpha\rightarrow\alpha^2+5\alpha+7=0$

$x=\beta=\beta^2+5\beta+7=0$

$\alpha^2+5\alpha+7+\beta^2+5\beta+7=0$

$\alpha^2+\beta^2+5(\alpha+\beta)+14=0$

$\alpha^2+\beta^2=-5(\alpha+\beta)-14$

$\alpha^2+\beta^2=-5(-5)-14$

$\alpha^2+\beta^2=25-14$

$\alpha^2+\beta^2=11$

If $\alpha^2+\beta^2<0$, the roots are called ‘complex roots’.