## **Quadratics**

In  $ax^2+bx+c=0$  with roots lpha and eta, if we change this to  $x^2+rac{b}{a}x+rac{c}{a}=0$ ,

we can conclude that  $lpha+eta=-rac{b}{a}$  and  $lphaeta=rac{c}{a}$  .

In summation notation,  $\Sigma lpha = lpha + eta$  and  $\Sigma lpha eta = lpha eta$ 

$$(lpha+eta)^2=lpha^2+2lphaeta+eta^2 \ lpha^2+eta^2=(lpha+eta)^2-2lphaeta \ lpha^2+eta^2=(\Sigmalpha)^2-2lphaeta \ \Sigmalpha^2=(\Sigmalpha)^2-2\Sigmalphaeta$$

 $lpha^2+eta^2$  can be written as  $\Sigmalpha^2$  in summation notation.

$$(lpha-eta)^2=lpha^2-2lphaeta+eta^2 \ lpha^2+eta^2=(lpha-eta)^2+2lphaeta$$

$$\begin{split} &\frac{1}{\alpha} + \frac{1}{\beta} = \Sigma \frac{1}{\alpha} \\ \downarrow \\ &\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha} + \frac{\alpha}{\beta} \\ &= \frac{\alpha + \beta}{\alpha \beta} \\ &= \frac{\Sigma \alpha}{\Sigma \alpha \beta} \end{split}$$

$$rac{1}{lpha^2} + rac{1}{eta^2} = \Sigma rac{1}{lpha^2}$$
  $\downarrow$ 

$$\begin{aligned}
\frac{1}{\alpha^2} + \frac{1}{\beta^2} &= \frac{\beta^2}{\alpha^2} + \frac{\alpha^2}{\beta^2} \\
&= \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} \\
&= \frac{\Sigma \alpha^2}{(\Sigma \alpha \beta)^2}
\end{aligned}$$

## Finding $lpha^2+eta^2$ from an equation

e.g,  $x^2+5x+7=0$  with roots lpha and eta  $lpha+eta=-5,\ lphaeta=7$ 

$$x=lpha
ightarrowlpha^2+5lpha+7=0 \ x=eta=eta^2+5eta+7=0$$

$$lpha^2 + 5lpha + 7 + eta^2 + 5eta + 7 = 0$$
  $lpha^2 + eta^2 + 5(lpha + eta) + 14 = 0$ 

$$lpha^2 + eta^2 = -5(lpha + eta) - 14$$
  $lpha^2 + eta^2 = -5(-5) - 14$   $lpha^2 + eta^2 = 25 - 14$   $lpha^2 + eta^2 = 11$ 

If  $lpha^2+eta^2<0$ , the roots are called 'complex roots'.