completing the square

You can write quadratic equations in a different format by completing the square. $x^2 + bx + c$ is equal to $(x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c$.

How? Let’s find out.

By expanding $(x + \frac{b}{2})^2$, we get $x^2 + bx + \frac{b^2}{4}$, and expanding $-(\frac{b}{2})^2$, we get $-\frac{b^2}{4}$.

So, basically, $(x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c$ can be written as $x^2 + bx + \frac{b^2}{4} - \frac{b^2}{4} + c$ which is just $x^2 + bx + c$.

Sure, this is working backwards, but how do we solve it from the quadratic equation?

1. Make sure the coefficient of x^2 is 1. If there is a coefficient a, divide the whole equation by a. e.g., $ax^2 + bx + c = x^2 + \frac{b}{a}x + \frac{c}{a}$
2. Take bx and divide it by $2x$ (or take $\frac{b}{a}x$ and divide it by $2x$) to get $\frac{b}{2}$ or $\frac{b}{2a}$.
3. Make a square with x and $\frac{b}{2}$ and subtract $(\frac{b}{2})^2$ from the equation.
4. So, it will look like this: $(x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c$
5. And now you’re done!!