Cubics

In cubic equation $ax^3 + bx^2 + cx + d = 0$,

$x^3 + \frac{bx^2}{a} + \frac{cx}{a} + \frac{d}{a} = 0$

Now, if the equation has roots α, β, γ, then we get

$\alpha + \beta + \gamma = -\frac{b}{a}$ or $\Sigma\alpha$ or S_1

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$, which is written as $\Sigma\alpha\beta$

$\alpha\beta\gamma = -\frac{d}{a}$, which is written as $\Sigma\alpha\beta\gamma$

$(\alpha + \beta + \gamma)^2 = (\Sigma\alpha)^2$

If we expand this, we will get

$\alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\alpha\gamma + 2\beta\gamma$

$\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$

$= \frac{\Sigma\alpha\beta}{\Sigma\alpha\beta\gamma}$ or S_{-1}

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{\alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2}{\alpha^2\beta^2\gamma^2}$

$= \frac{\Sigma(\alpha\beta)^2}{(\Sigma\alpha\beta\gamma)^2}$

$\Sigma\alpha^3 = (\Sigma\alpha)^2 - 3\Sigma\alpha\beta\ \Sigma\alpha + 3\Sigma\alpha\beta\gamma$

It is possible to expand this, but it is very complicated, best to leave it as it is.