Inequalities

Consider the curve $y = \frac{x^2+4x-9}{3x-1}$ and let's find the inequality when $\frac{x^2+4x-9}{3x-1} < 0$.

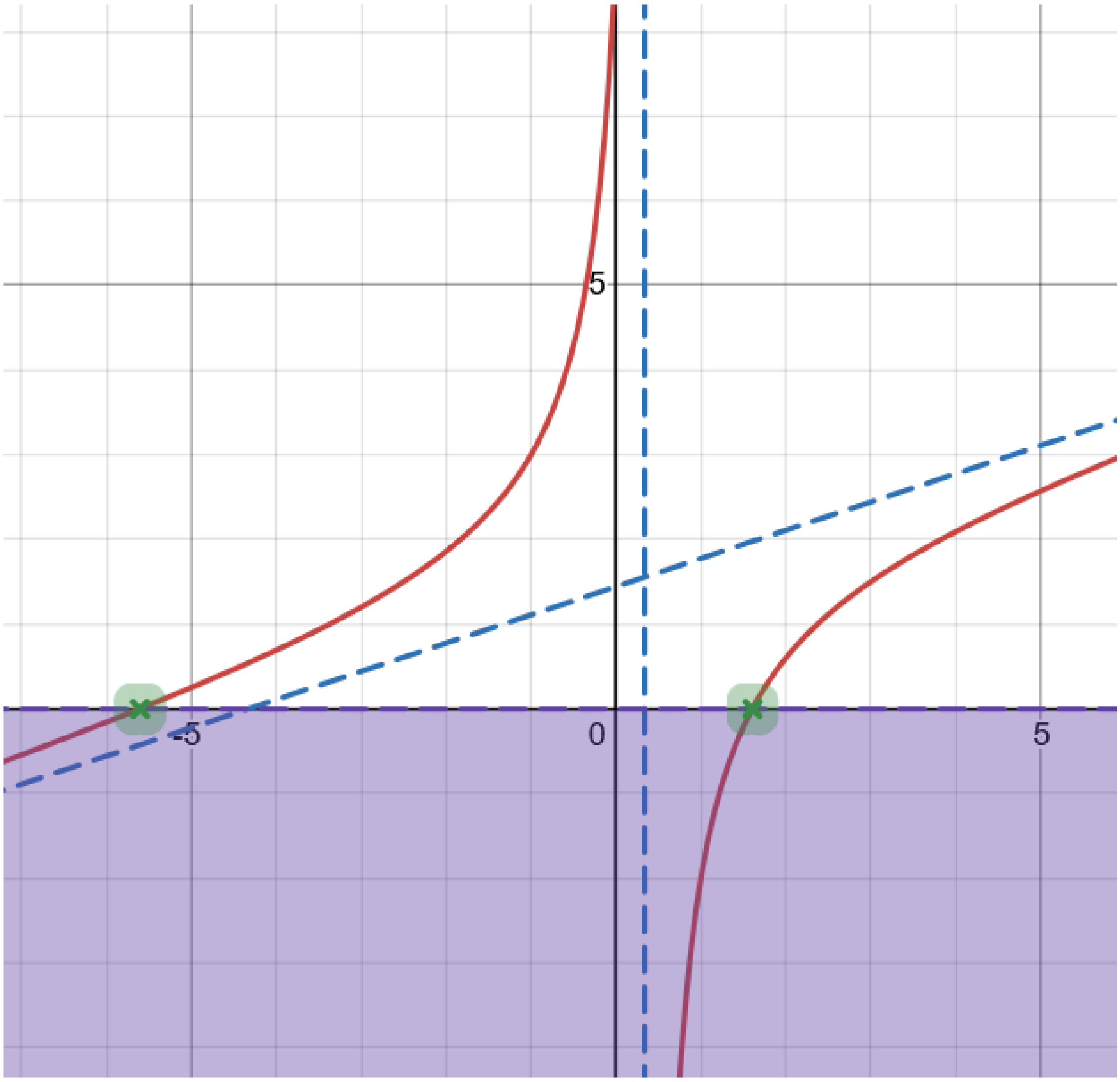
First, solve the inequality.

$\frac{x^2+4x-9}{3x-1} < 0$

$x^2 + 4x - 9 < 0$

$-2 - \sqrt{13} < x < -2 + \sqrt{13}$

Sketch the graph.



https://www.desmos.com/calculator/nrqs3psyjv

Here, we can see that when $y < 0$, there are 2 separate points to consider.

With our original inequality, x cannot be between $-2-\sqrt{13}$ and $-2+\sqrt{13}$, because then y would be above zero.

So, instead we have to take $x < -2-\sqrt{13}$ and $x > -2+\sqrt{13}$ instead.

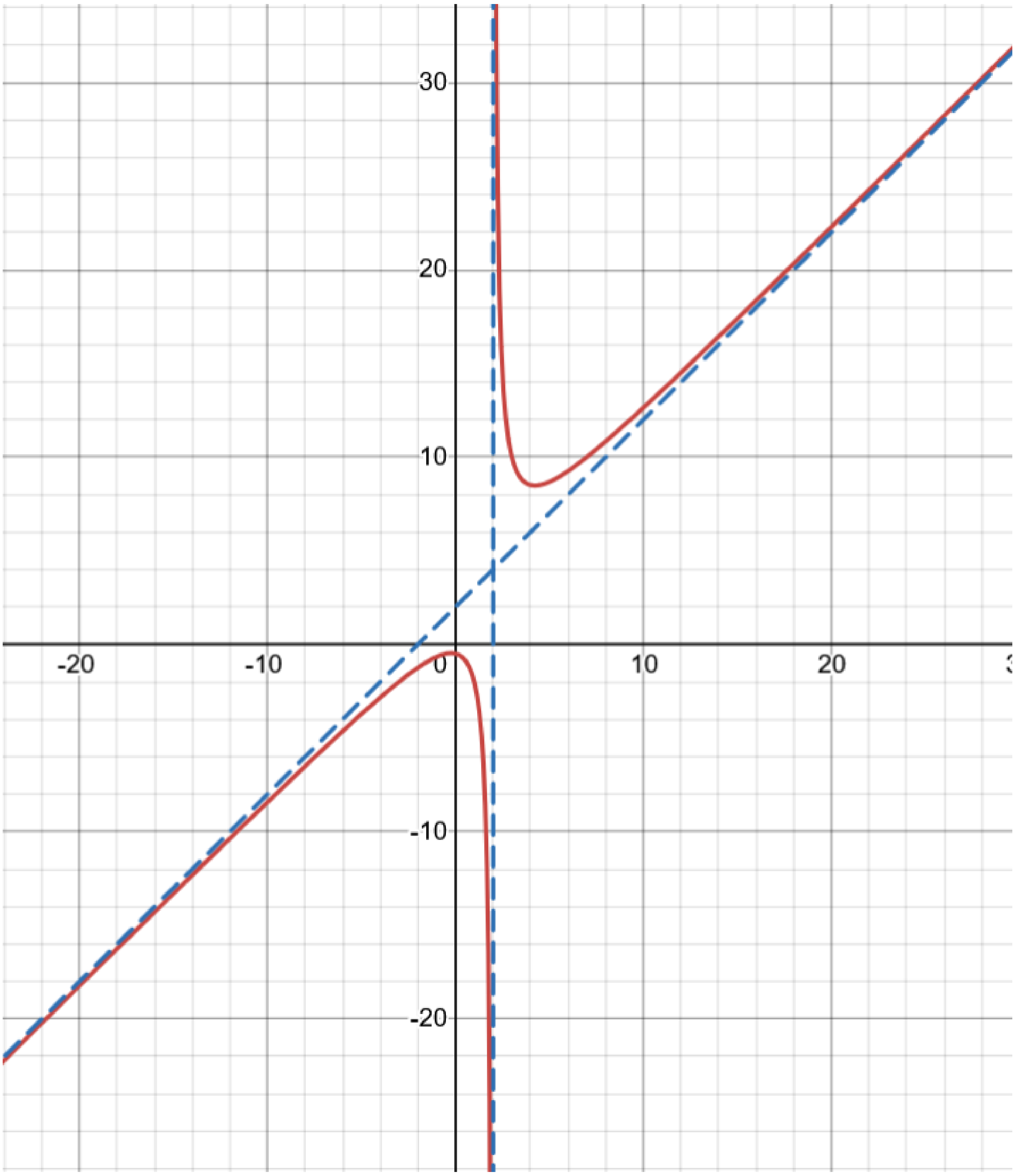
If we instead find $y > 0$, then x must be greater than $-2-\sqrt{13}$ but it cannot be the vertical asymptote(because it doesn't exist), so we get $-2-\sqrt{13} < x < \frac{1}{3}$ and

$x > -2 + \sqrt{13}$. ($\frac{1}{3}$ is the vertical asymptote)

Finding ranges

We can find the range of the curve using 2 methods.

Let's use this function $y = \frac{x^2+1}{x-2}$ with this graph.



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Method 1: Differentiation

$\frac{dy}{dx} = \frac{x^2-4x-1}{(x-2)^2}$

Find the turning point, $\frac{x^2-4x-1}{(x-2)^2} = 0$

$x = 2 \pm \sqrt{5}$ is our 2 turning points.

If you substitute these 2 values into the original equation, you get the range of y.

So, $y < 4 - 2\sqrt{5}$ and $y > 4 + 2\sqrt{5}$ is our range (i.e., y can only exist within that range.)

Method 2: Using discriminants

$y = \frac{x^2+1}{x-2}$

Cross multiply.

$xy - 2y = x^2 + 1$

$x^2 + xy + 2y + 1 = 0$

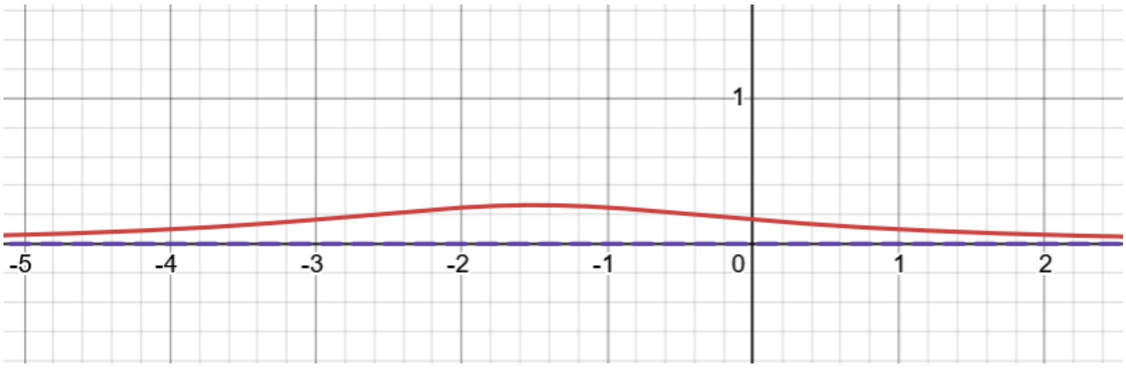
Using $b^2 - 4ac < 0$, we can find the values of which y do not exist.

$y^2 - 4(1)(2y + 1) < 0$

$y^2 - 8y - 4 < 0$

This gives us the same answer $y < 4 - 2\sqrt{5}$ and $y > 4 + 2\sqrt{5}$

Let's look at another type of curve. In $y = \frac{1}{x^2+3x+6}$, it has no vertical asymptotes because the denominator will never be zero($x^2 + 3x + 6 = 0$ has no solutions). Also note that when $|x|$ becomes larger, y approaches 0, so the horizontal asymptote is $y = 0$.



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If we cross multiply the equation, we get

$x^2y + 3xy + 6y - 1 = 0$, from which we can find the discriminant.

$b^2 - 4ac < 0$

$(3y)^2 - 4(y)(6y - 1) < 0$

$9y^2 - 24y^2 + 4y < 0$

$-15y^2 + 4y < 0$

$y(-15y + 4) < 0$

And so we get $y < 0$ and $y > \frac{4}{15}$.

So, the range is $0 < y \leq \frac{4}{15}$ (less than or equal here because it **must** be bigger than $\frac{4}{15}$, but it **can** be $\frac{4}{15}$).