Oblique asmptotes

When a curve has a quadratic numerator and a linear denominator, it has something called an **oblique asymptote**, which is not horizontal or vertical.

For any curve $y = \frac{ax^2+bx+c}{dx+e}$, split the function into $y = Ax + B + \frac{C}{dx+e}$. The **vertical asymptote** will be $x = -\frac{e}{d}$ and the **oblique asymptote** will be $y = Ax + B \rightarrow A = \frac{a}{d}$.

Example

For the curve $y = \frac{x^2+1}{x-2}$, find the vertical and oblique asymptote.

$x = 2$ is our vertical asymptote because when $x = 2$, the denominator becomes zero.

$\frac{x^2+1}{x-2} = Ax + B + \frac{C}{x-2}$

$A = \frac{a}{d} \rightarrow A = \frac{1}{1} = 1$

Cross multiply.

$x^2 + 1 = (x + B)(x - 2) + C$

Substitute $x = 2$(to make $x - 2 = 0$) and $x = 0$

$x = 2 \rightarrow 5 = (2 + B)(0) + C$

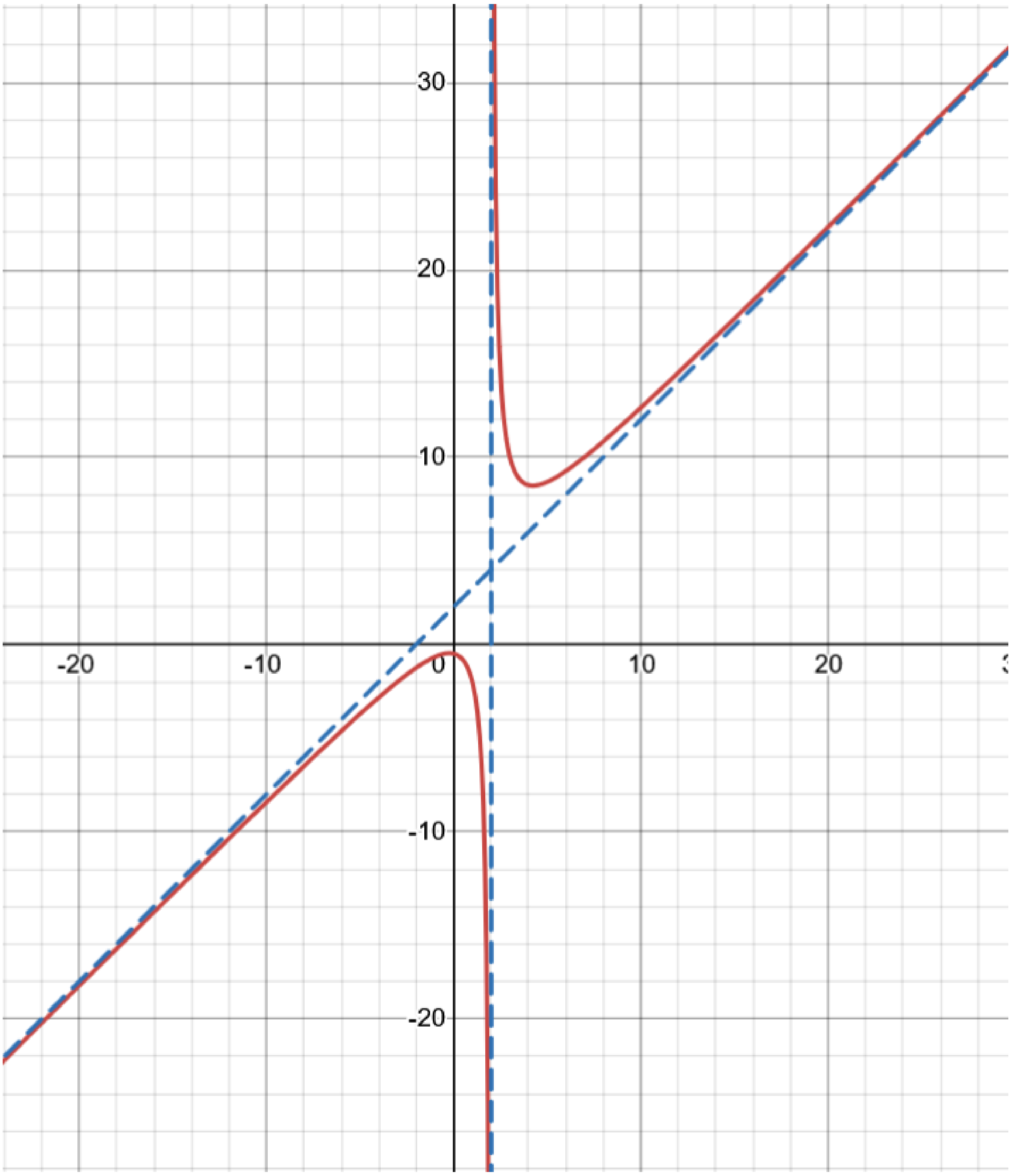
$C = 5$

$x = 0 \rightarrow 1 = (B)(-2) + 5$

$$-2B = -4$$

$$B = 2$$

$\therefore y = Ax + B \rightarrow y = x + 2$ is our oblique asymptote.



https://www.desmos.com/calculator/wmuuow4fuv

In the example sketch, you will notice that **negative x values correspond to points lower than the oblique asymptote**, while **positive x values have points higher than the oblique asymptote**. Also note that there is a turning point.

Let's differentiate now.

For the curve $y = \frac{x^2-9}{1-x}$, find the asymptotes and points of intersection.

Firstly, the vertical asymptote is $x = 1$ as seen from the denominator.

$A = -1$ because $A = \frac{a}{d} = -1$

$\therefore \frac{x^2-9}{1-x} = x + B + \frac{C}{1-x}$

$x^2 - 9 = (x + B)(1 - x) + C$

$x = 1 \rightarrow 1 - 9 = 0 + C$

$-8 = C$

$x = 0 \rightarrow -9 = (B)(1) - 8$

$B = -1$

$\therefore y = Ax + B \rightarrow y = -x - 1$ (Oblique asymptote)

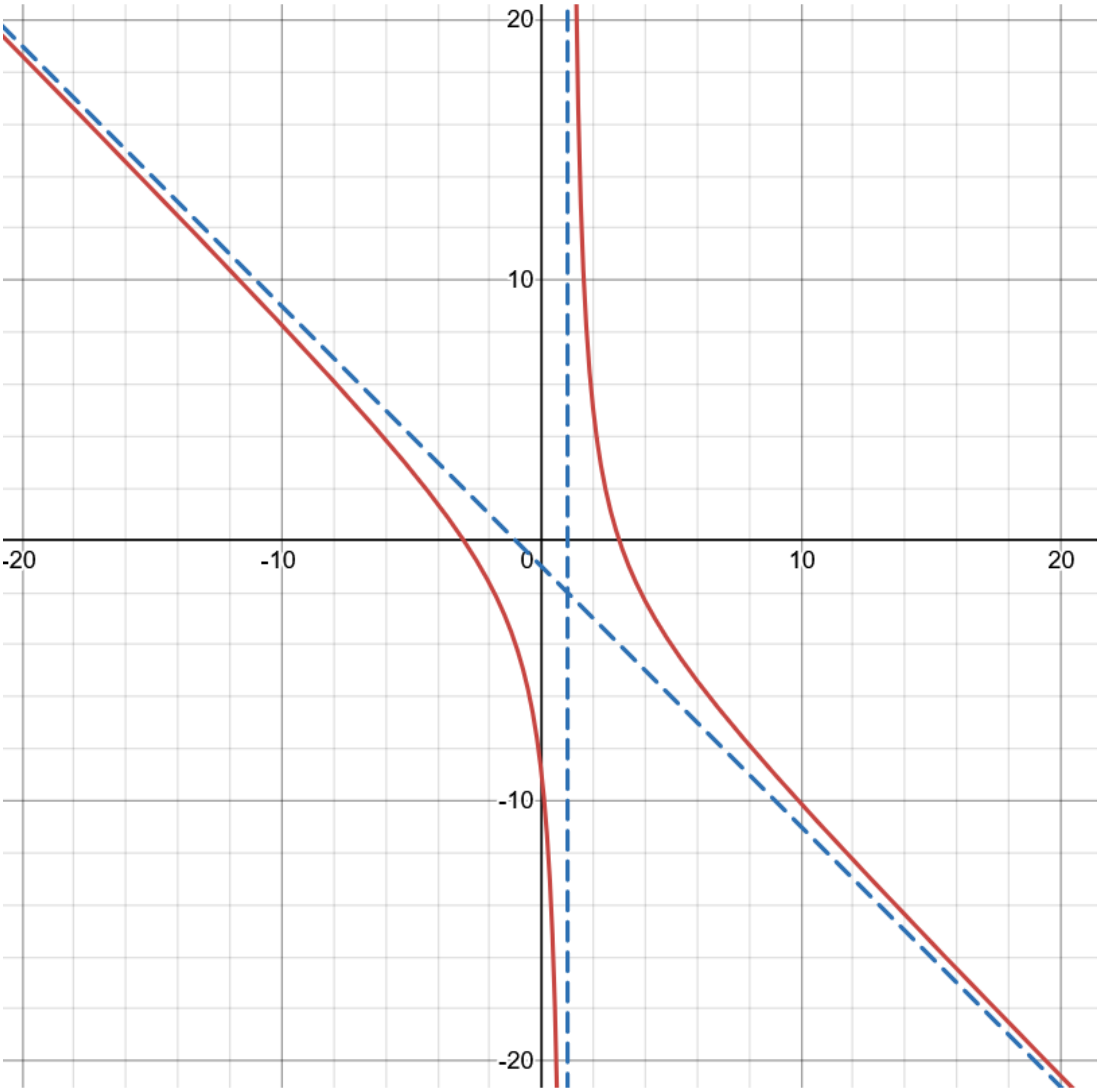
Now, for the intersections.

$x = 0 \rightarrow y = \frac{-9}{1} = -9$

$y = 0 \rightarrow \frac{x^2-9}{1-x} = 0, \; x = \pm 3$

By differentiating, $\frac{dy}{dx} = -1 - \frac{8}{(1-x)^2}$.

$\frac{dy}{dx} = 0 \rightarrow -1 - \frac{8}{(1-x)^2} = 0$ cannot be solved, so there are no turning points for **this function**.



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Because there is no turning point, it looks different from the graph above.

By differentiating, we can draw more accurate graphs without finding every x and y value.