Cubics

In cubic equation $ax^3 + bx^2 + cx + d = 0$,

$$x^3 + \frac{bx^2}{a} + \frac{cx}{a} + \frac{d}{a} = 0$$

Now, if the equation has roots $\alpha,\,\beta,\,\gamma$, then we get

$$lpha + eta + \gamma = -rac{b}{a}$$
 or $\Sigma lpha$ or S_1

$$\alpha \beta + \alpha \gamma + \beta \gamma = \frac{c}{a}$$
, which is written as $\Sigma \alpha \beta$

$$lphaeta\gamma=-rac{d}{a}$$
 , which is written as $\Sigmalphaeta\gamma$

$$(\alpha + \beta + \gamma)^2 = (\Sigma \alpha)^2$$

If we expand this, we will get

$$\alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\alpha\gamma + 2\beta\gamma$$

$$\Sigma lpha^2 = (\Sigma lpha)^2 - 2(\Sigma lpha eta)$$

$$egin{aligned} &rac{1}{lpha} + rac{1}{eta} + rac{1}{\gamma} = rac{lphaeta + lpha\gamma + eta\gamma}{lphaeta\gamma} \ &= rac{\Sigmalphaeta}{\Sigmalphaeta\gamma} \; ext{or} \; S_{-1} \end{aligned}$$

$$egin{aligned} &rac{1}{lpha^2} + rac{1}{eta^2} + rac{1}{\gamma^2} = rac{lpha^2eta^2 + lpha^2\gamma^2 + eta^2\gamma^2}{lpha^2eta^2\gamma^2} \ &= rac{\Sigma(lphaeta)^2}{(\Sigmalphaeta\gamma)^2} \end{aligned}$$

$$\Sigma lpha^3 = (\Sigma lpha)^2 - 3\Sigma lpha eta \ \Sigma lpha + 3\Sigma lpha eta \gamma$$

It is possible to expand this, but it is very complicated, best to leave it as it is.