Radian measure and angular displacement

In circular motion, it is more convenient to measure in **radians** rather than degrees.

> **One radian (rad) is defined as the angle subtended at the center of the circle by an arc equal in length to the radius of the circle.**

So, to find the angle, we divide the arc length by the radius.

$$\theta = \frac{arc\,length}{radius} \rightarrow \theta = \frac{s}{r}$$

So the angle of a full circle would be -

$$\theta = \frac{2\pi r}{r} = 2\pi\ rad$$

So we can conclude that $2\pi\ rad = 360^\circ$

$$1\ rad = 57.3^\circ$$

Angular speed

For an object moving in a circle,

> **The angular speed is defined as the angle swept out by the radius of the circle per unit time.**

The **angular velocity** is the angular speed in a specific direction (e.g., clockwise).

$$angular\ speed\ \omega = \frac{\Delta\theta}{\Delta t}$$

Since $\theta = \frac{s}{r}$, we can substitute $\Delta\theta = \frac{\Delta s}{r}$ to get $\omega = \frac{\Delta s}{r \cdot \Delta t}$ which becomes

$r\omega = \frac{\Delta s}{\Delta t}$.

The formula of velocity is $v = \frac{\Delta s}{\Delta t}$.

So we can see that we get a new equation $v = r\omega$

Another equation we have is when an object has traveled through 1 revolution in time T, it will have rotated through 2π radians. So we get $\omega = \frac{2\pi}{T}$.

Formula list

> 💡 θ is the angle in radians, s is the arc length, r is the radius, t/T is the time taken, ω is angular velocity, and v is velocity.

$\theta = \frac{s}{r}$

$\Delta\theta = \frac{\Delta s}{r}$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\omega = \frac{\Delta s}{r \cdot \Delta t}$

$\omega = \dfrac{2\pi}{T}$ (only when the object has made a full revolution in time T)

$$v = \frac{\Delta s}{\Delta t}$$

$$v = r\omega$$