Newton–Raphson equations for power flow

Problem Find V, θ at all system buses, given P, V at generators, and P, Q at load buses.

Bookkeeping total # of buses $= N$; # generators $= N_G$
Bus 1 = slack bus (known V_1, θ_1)
Buses $2, 3, \ldots, N_G$ = generators (known P_k, V_k)
Buses $N_G + 1, \ldots, N$ = loads (known P_k, Q_k)

Unknowns $\mathbf{x} = \begin{bmatrix} \theta_2 & \ldots & \theta_N & \vdots & V_{N_G+1} & \ldots & V_N \end{bmatrix}^T$;

$$\# \text{ of unknowns} = \underbrace{(N-1)}_{\theta\text{'s}} + \underbrace{(N-N_G)}_{V\text{'s}} = 2N - N_G - 1$$

Power Injections $P_k(\mathbf{x}) = \sum_{m=1}^{N} V_k V_m \left[G_{km}\cos(\theta_k - \theta_m) + B_{km}\sin(\theta_k - \theta_m)\right]$;
for $k = 2, \ldots, N$

$Q_k(\mathbf{x}) = \sum_{m=1}^{N} V_k V_m \left[G_{km}\sin(\theta_k - \theta_m) - B_{km}\cos(\theta_k - \theta_m)\right]$;
for $k = N_G + 1, \ldots, N$

Equations $\mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_2(\mathbf{x}) - P_2 \\ \vdots \\ P_N(\mathbf{x}) - P_N \\ Q_{N_G+1}(\mathbf{x}) - Q_{N_G+1} \\ \vdots \\ Q_N(\mathbf{x}) - Q_N \end{bmatrix} = \mathbf{0}$; # of eqns. $= 2N - N_G - 1$

Update formula $\boxed{\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} - [\mathbf{J}(\mathbf{x}^{(i)})]^{-1}\mathbf{f}(\mathbf{x}^{(i)})}$

Jacobian $\mathbf{J} = \begin{bmatrix} \left[\frac{\partial P}{\partial \theta}\right]^{(N-1)\times(N-1)} & \left[\frac{\partial P}{\partial V}\right]^{(N-1)\times(N-N_G)} \\ \left[\frac{\partial Q}{\partial \theta}\right]^{(N-N_G)\times(N-1)} & \left[\frac{\partial Q}{\partial V}\right]^{(N-N_G)\times(N-N_G)} \end{bmatrix}$;

dimension $= (2N - N_G - 1) \times (2N - N_G - 1)$

Elements of J $\partial P_k/\partial\theta_m = V_k V_m \left[G_{km}\sin(\theta_k - \theta_m) - B_{km}\cos(\theta_k - \theta_m)\right]$; $k \neq m$

$\partial P_k/\partial\theta_k = -Q_k(\mathbf{x}) - V_k^2 B_{kk}$

$\partial Q_k/\partial\theta_m = -V_k V_m \left[G_{km}\cos(\theta_k - \theta_m) + B_{km}\sin(\theta_k - \theta_m)\right]$; $k \neq m$

$\partial Q_k/\partial\theta_k = P_k(\mathbf{x}) - V_k^2 G_{kk}$

$\partial P_k/\partial V_m = V_k \left[G_{km}\cos(\theta_k - \theta_m) + B_{km}\sin(\theta_k - \theta_m)\right]$; $k \neq m$

$\partial P_k/\partial V_k = P_k(\mathbf{x})/V_k + V_k G_{kk}$

$\partial Q_k/\partial V_m = V_k \left[G_{km}\sin(\theta_k - \theta_m) - B_{km}\cos(\theta_k - \theta_m)\right]$; $k \neq m$

$\partial Q_k/\partial V_k = Q_k(\mathbf{x})/V_k - V_k B_{kk}$