Discrete Math

For CSAM. Monsoon 2017

Discrete Mathematics

- Study of mathematical structures that are **finite** instead of **continuous**.
  - Finite number of objects, or countably infinite number of objects.

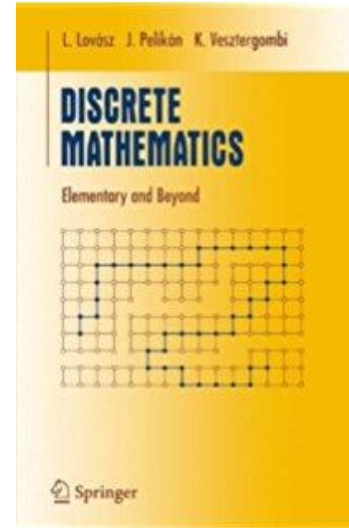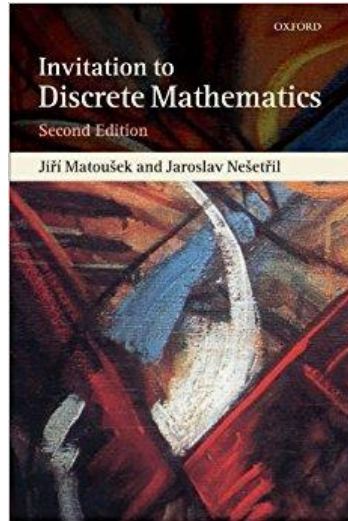- Fundamental to Computer Science as Calculus is to Physics.

Post-conditions

1.   Think in a logical, abstract and formal manner.
     a.   Hone skills in making precise definitions and abstractions.

2.   Apply various proof techniques and learn to write correct proofs.

3.   Learn how Discrete Math is applied in various branches of Mathematics and Computer Science.

Teaching and Learning

1. Attend lectures regularly.
    a. Lectures will consist of questions posed by instructor, which you should attempt to solve in pairs (with the person next to you).
    b. Feel free to discuss and talk - when I am not.
    c. **No laptop or mobile phones in class.**
    d. Written lecture notes or slides will be provided for each lecture.
2. Reading. Some reading assignments will be assigned.
3. Weekly Tutorial: Will consist of problem solving sessions in groups, and sometimes quizzes.
4. Weekly assignments.

Course Material

And additional resources provided.

Suggestions.

- The most essential part of this course are the problems: in the assignments, tutorials, in class, and in the textbooks.

- The more problems you solve, the easier it is to solve more problems.

Grading

- Assignments: 10%
- Quiz : 20%
- Midsem : 30%
- Finals :40%

Topics

- Logic (briefly)
- Sets, functions, relations.
- Basic Counting techniques.
- Asymptotics
- Graph Theory
- Discrete Probability
- Recurrences and generating functions.
- Projective geometry/linear algebraic techniques.
- Ramsey Theory

Plagiarism.

Don't.

Plagiarism

- Plagiarism in assignments, tutorials:
  - First time: 1 letter grade reduction.
  - Second time: **Reported to DAC**.

- Plagiarism in quizzes, exams: **F grade**.

Example 1.

Some students at CSAM form a new club, the secret fist-bumpers club. When the students meet, they do a fist-bump.

Suppose n members of the secret fist-bumpers society meet for dinner at a city restaurant. Of course, they perform their secret routine in front of the resturant manager. How many fist-bumps does he witness ?

Example 2

In an annual meeting of the fist-bumpers society, one of the mathematically minded members observes something surprising and exclaims

``*Hey! At each of our meetings I've observed that there are at least 2 people at the meeting with the identical number of friends!''*.

Could this be a coincidence ?

Example 3

Can you draw these figures on a sheet of paper without lifting your hand off the paper (and not drawing over any of the previously drawn lines) ?

Example 4

In a village are three houses and three wells. Due to frequent quarrels, each household decides to build a path from their house to each of the three wells, with the constraint that the paths do not cross. Help the villagers build their paths.

Example 5

Given a tower of n disks, initially stacked in decreasing size on one of 3 pegs. The goal is to transfer the disks from one peg to the next, with the following rules satisfied:

- The disks can only be moved one at a time.
- A larger disk can never sit atop a smaller disk.

How many moves do we require ?

Example 6

Given n lines in the plane such that no two are parallel, and no three meet at a point, how many regions do they form ?

Example 7

On an island live knights and knaves. The knights always speak the truth, while the knaves always lie. You can not distinguish them by sight. You meet two people on the island - A and B.

- A says: ``I am a knave, but B isn't''

What are A and B ?

Summary

- The preceding examples seem like recreational puzzles.
- Indeed, for a long time people did not believe such questions had deeper mathematical import.
- However, over the last half century, especially with the advent of computers we have seen that these are deep questions with connections not only to computer science, but all parts of mathematics, physics, biology, etc.

- This is a fun course with lots of puzzle-solving type questions. I hope you enjoy the ride.