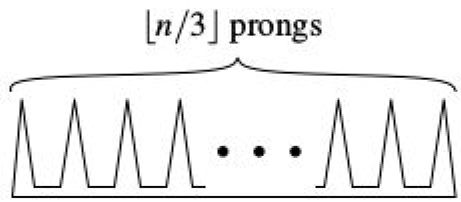
# ZPC
## Solution Set

Total points : 20

Q1. Give an example of a simple polygon that does not have a guarding set of less than [n/3] cameras.

2 points

Solution:




For any n there are simple polygons that require n/3 cameras. An example is a comb-shaped polygon with a long horizontal base edge and n/3 prongs made of two edges each. The prongs are connected by horizontal edges. The construction can be made such that there is no position in the polygon from which a camera can look into two prongs of the comb simultaneously. So we cannot hope for a strategy that always produces less than n/3 cameras. In other words, the 3-coloring approach is optimal in the worst case.

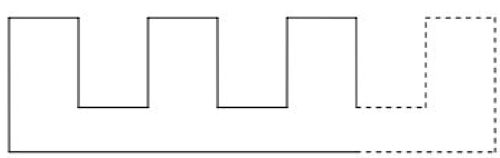
Source [1]

Q2. A *rectilinear polygon* is a simple polygon of which all edges are horizontal or vertical. Let P be a rectilinear polygon with *n* vertices. Give an example to show that [*n*/4] cameras are sometimes necessary to guard it. ( [x] represents the greatest integer function of x. )

2 points

Solution:

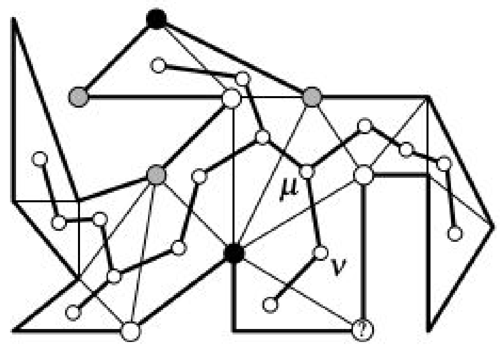
There are a lot of examples to justify that [n/4] cameras are sometimes necessary to guard simple rectilinear polygons. The simplest case is a rectangle. Another typical case is as shown as below:



Each time we add a new corner (shown by dashed line), the number of edges increases by 4, and we need a new camera to monitor it. It's easy to see the number of cameras needed is always [n/4].

Source: [2]

Q3. Give the pseudo-code of the algorithm to compute a 3-coloring of a triangulated simple polygon.
3 points
Solution:




For a graph with a triangulation, we can find a 3-coloring using a simple graph traversal, such as depth first search. Next we describe how to do this. While we do the depth first search, we maintain the following invariant: all vertices of the already encountered triangles have been colored white, gray, or black, and no two connected vertices have received the same color. The invariant implies that we have computed a valid 3-coloring when all triangles have been encountered. The depth first search can be started from any node of $G(T_P)$; the three vertices of the corresponding triangle are colored white, gray, and black. Now suppose that we reach a node ν in G, coming from node μ (as shown in the figure). Hence, t(ν) and t(μ) share a diagonal (where t(x) denotes the triangle corresponding to a node in the dual graph of the triangulation). Since the vertices of t(μ) have already been colored, only one vertex of t(ν) remains to be colored. There is one color left for this vertex, namely the color that is not used for the vertices of the diagonal between t(ν) and t(μ). Because $G(T_P)$ is a tree, the other nodes adjacent to ν have not been visited yet, and we still have the freedom to give the vertex the remaining color.

Source: [1]

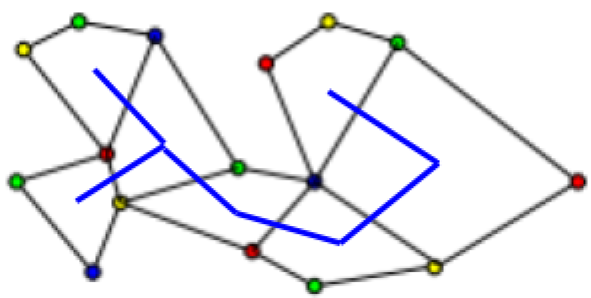
Q4. Given a simple polygon P with *n* vertices and a point *p* inside it, describe a method to compute the region inside P that is visible from *p*.
3 points
Solution:
Any answer that reasonably proposes a method similar to or the same as the methods given in [https://en.wikipedia.org/wiki/Visibility_polygon](https://en.wikipedia.org/wiki/Visibility_polygon) will be awarded points.

Q5. Suppose that a simple polygon P with *n* vertices is given, together with a set of diagonals that partitions P into convex quadrilaterals. How many cameras are sufficient to guard P?

5 points

Solution:

Since the quadrilaterals are convex, each one can be guarded by one guard placed on any of its vertices. Also, the entire polygon can be guarded if each quadrilateral is guarded. Now, we can 4-color P just as we did in the case of a triangulation (see figure). Take the dual graph for the *quadrilateralization* of the polygon(say $D(Q_P)$). Select one node of $D(Q_P)$ and 4-color its corresponding quadrilateral. Then, for every neighbour of the present node in $D(Q_P)$, select the corresponding quadrilateral, colour the two remaining nodes of the quadrilateral in P and repeat the procedure for this node and all subsequent nodes by the way of a graph traversal. Continue until all vertices of P are colored. Colored in this manner, only 4 colors will be needed to color P, and each quadrilateral in P will have one vertex colored with each of the 4 available colors. Hence, P will be guarded if we select all vertices from any of the 4 color classes. By selecting the smallest color class, we will need $\lfloor n/4 \rfloor$ cameras.w



Source : [3]

An AMS Feature Column on orthogonal art gallery :
http://www.ams.org/samplings/feature-column/fcarc-gallery3

Q6. Can you come up with a non-convex polyhedra in 3D space such that even if you place a guard at every vertex there would still be points in the polygon that are not visible to any guard? Do explain your reason.

5 points

This is a fairly weird question to ask. But I am sharing links to some interesting pages that would answer this question. I recommend that you read them in the given order.

1. http://cs.smith.edu/~jorourke/books/ArtGalleryTheorems/Art_Gallery_Chapter_10.pdf
2. https://en.wikipedia.org/wiki/Sch%C3%B6nhardt_polyhedron
3. http://www.ics.uci.edu/~eppstein/junkyard/untetra/

4. https://en.wikipedia.org/wiki/Art_gallery_problem#Three_dimensions - Look at the figure given in this section.

SOURCES

[1] : Prof. Dr. Mark de Berg, Dr. Otfried Cheong, Dr. Marc van Kreveld, Prof. Dr. Mark Overmars (auth.)- Computational geometry - Algorithms and Applications-Springer-Verlag Berlin Heidelberg (2008)
[2] : http://www.ics.uci.edu/~goodrich/teach/cs266/hw/solution3.pdf
[3] : https://parasol.tamu.edu/~amato/Courses/620/hw3-soln.pdf