Probability Assignment

Homework 1.1

Jerry and Susan have a joint bank account.

Jerry goes to the bank 20% of the days.

Susan goes there 30% of the days.

Together they are at the bank 8% of the days.

a. Susan was at the bank last Monday. What's the probability that Jerry was there too?

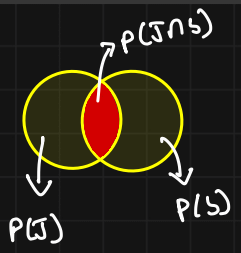
b. Last Friday, Susan wasn't at the bank. What's the probability that Jerry was there?

c. Last Wednesday at least one of them was at the bank. What is the probability that both of them were there?

Let

probability of Jerry at the bank = $P(J)$

probability of Susan at the bank = $P(S)$



Given

$P(J) = 0.2$ $\quad P(S) = 0.3$ $\quad P(J \cap S) = 0.08$

we can derive

$P(J') = 0.8$ $\quad P(S') = 0.7$ $\quad P(J \cup S) = P(J) + P(S) - P(J \cap S)$

$= 0.42$

(a) Probability of Jerry bank while Susan was at the bank

$$P(J/S) = \frac{P(J \cap S)}{P(S)} = \frac{0.08}{0.3} = 0.27$$

(b) Probability of Jerry being back given Susan wasn't at bank.

$$P(J/S') = \frac{P(J \cap S')}{P(S')} = \frac{0.12}{0.7}$$

$$= 0.1714$$

(c) probability that both were there given atleast one of them were there.

$$\frac{P(J \cap S)}{P(J \cup S)} = \frac{0.08}{0.42} = 0.19$$

Homework 1.2

Harold and Sharon are studying for a test.

Harold's chances of getting a "B" are 80%. Sharon's chances of getting a "B" are 90%.

The probability of at least one of them getting a "B" is 91%.

a. **What is the probability that only Harold gets a "B"?**

b. **What is the probability that only Sharon gets a "B"?**

c. **What is the probability that both won't get a "B"?**

Let

probability of Harold getting a B $\Rightarrow P(H)$

probability of Sharon getting a B $\Rightarrow P(S)$

Given

$P(H) = 0.8$ $P(S) = 0.9$ $P(H \cup S) = 0.91$

we can derive $P(H') = 0.2$ $P(S') = 0.1$

$$P(H \cap S) = P(H) + P(S) - P(H \cup S)$$
$$= 0.9 + 0.8 - 0.91$$
$$= 0.79$$

(a) probability of only harold getting a B is $P(H) - P(H \cap S)$

$\Rightarrow 0.8 - 0.79$

$\Rightarrow 0.01$

(b) probability of only sharon getting a B is $P(S) - P(H \cap S)$

$\Rightarrow 0.9 - 0.79$

$\Rightarrow 0.11$

(c) probability that both won't get B $1 - P(H \cup S)$

$\Rightarrow 1 - 0.91$

$\Rightarrow 0.09$

Homework 1.3

Jerry and Susan have a joint bank account.

Jerry goes to the bank 20% of the days.

Susan goes there 30% of the days.

Together they are at the bank 8% of the days.

Are the events "Jerry is at the bank" and "Susan is at the bank" independent?

Let

probability of Jerry going bank be $P(J)$

probability of Susan going bank be $P(S)$

Given

$P(J) = 0.2$ $\quad P(S) = 0.3$ $\quad P(J \cap S) = 0.08$

for an event to be indipendent, it needs to satisfy

$$P(J) \cdot P(S) = P(J \cap S)$$

L.H.S $\Rightarrow$ $0.2 \times 0.3 = 0.06$

R.H.S $\Rightarrow$ 0.08

as $\quad LHS \neq RHS$

The two events are **not** indipendent

Homework 1.4

You roll 2 dice.

a. **Are the events "the sum is 6" and "the second die shows 5" independent?**

b. **Are the events "the sum is 7" and "the first die shows 5" independent?**

Sum of 2 dices table

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(a) $P(\text{Sum of 2 dices is 6}) = 5/36 = P(A)$

$P(\text{second dice showing 5}) = 6/36 = 1/6 = P(B)$

$P(A \cap B) = 1/36$

for the event to be independent

$$P(A \cap B) = P(A) \cdot P(B)$$

R.H.S $\Rightarrow 1/6 \times 5/36$

$\Rightarrow 5/216$

L.H.S $= 1/36$

as L.H.S $\neq$ R.H.S, the event is not indipendent

(b) $P(\text{sum is 7}) = 6/36 = \boxed{1/6 = P(A)}$

$P(\text{first die shows 5}) = 6/36 = \boxed{1/6 = P(B)}$

$P(A \cap B) = 1/36$

for event to be indipendent

$$P(A) \cdot P(B) = P(A \cap B)$$

L.H.S $\Rightarrow P(A) \cdot P(B)$

$\Rightarrow 1/6 \cdot 1/6 = 1/36$

R.H.S $\Rightarrow 1/36$

as L.H.S = R.H.S, the two events are indipendent

Homework 1.5

An oil company is considering drilling in either TX, AK and NJ. The company may operate in only one state. There is 60% chance the company will choose TX and 10% chance – NJ.

There is 30% chance of finding oil in TX, 20% - in AK, and 10% - in NJ.

1. What's the probability of finding oil?
2. The company decided to drill and found oil. What is the probability that they drilled in TX?

Probability for the oil company to consider TX => $\boxed{P(T) = 0.6}$

Probability for the oil company to consider NJ => $\boxed{P(N) = 0.1}$

Probability for the oil company to consider Ak => $P(A) = 1 - P(T) - P(N)$

$= 1 - 0.7$

$\boxed{P(A) = 0.3}$

Given

Probability of finding oil in TX => $P(O/T) = 0.3$

Probability of finding oil in Ak => $P(O/A) = 0.2$

Probability of finding oil in NJ => $P(O/N) = 0.1$

$P(O \cap T) = 0.18$ $\quad P(O \cap A) = 0.06$ $\quad P(O \cap N) = 0.01$

We can derive

(a) Probability of finding the oil.

$$P(O) = P(O \cap A) + P(O \cap T) + P(O \cap N)$$

$$= 0.18 + 0.06 + 0.01$$

$$\boxed{P(O) = 0.25}$$

(b)

$$P(T/O) = \frac{P(T \cap O)}{P(O)}$$

$$= \frac{0.18}{0.25}$$

$$\boxed{P(T/O) = 0.72}$$

Homework 1.6

The following slide shows the survival status of individual passengers on the Titanic. Use this information to answer the following questions

- **What is the probability that a passenger did not survive?**
- **What is the probability that a passenger was staying in the first class?**
- **Given that a passenger survived, what is the probability that the passenger was staying in the first class?**
- **Are survival and staying in the first class independent?**
- **Given that a passenger survived, what is the probability that the passenger was staying in the first class and the passenger was a child?**
- **Given that a passenger survived, what is the probability that the passenger was an adult?**
- **Given that a passenger survived, are age and staying in the first class independent?**

Survived

Cabin

Age	1st	2nd	3rd	Crew	Sub Total
Adult	197	94	151	212	654
Child	6	24	27	-	57
Sub Total	**203**	**118**	**178**	**212**	**711**

Not Survived

Cabin

Age	1st	2nd	3rd	Crew	Sub Total
Adult	122	167	476	673	1,438
Child			52		52
Sub Total	**122**	**167**	**528**	**673**	**1,490**

Total

Cabin

Age	1st	2nd	3rd	Crew	Grand Total
Adult	319	261	627	885	2,092
Child	6	24	79		109
Grand Total	**325**	**285**	**706**	**885**	**2,201**

(a)

probability of passengers did not survive

$$= \frac{\text{passengers did not survive}}{\text{total passengers.}}$$

$$= \frac{1490}{2201} = 0.67$$

(b) probability of passengers in first class

$$= \frac{\text{passengers survived inn first class} + \text{passengers not survived in first class}}{\text{total passengers}}$$

$$= \frac{203 + 102}{2201}$$

$$= 305/2201 = 0.13$$

(c) Given that a passenger survived, probability that he was in the first class.

$$= 203/325 = 0.62$$

(d) For a class to be indipendent

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\Downarrow \qquad \Downarrow$$

$$0.62 \qquad \frac{711}{2201} * 0.14$$

$0.62 \neq 0.04$.

So the events are not indipendent.

(e) $P(\text{Survived} / \text{First class \& Child}) = 6/203 = 0.02$

(f) $P[\text{Survived} / \text{adult}) = 118/711 = 0.16$

(g) Given that the passenger survived

$$P(\text{Adult}) = \frac{654}{711} = 0.91$$

$$P(1^{st}\ \text{Class}) = \frac{197}{203} = 0.97$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\Downarrow \qquad \Downarrow$$

$$\frac{197}{711} \qquad 0.91 \times 0.97$$

$$\Rightarrow \quad 0.27 \neq 0.88$$

The events are not independent.