University of Toronto
Department of Electrical and Computer Engineering
ECE411S – Real-time Computer Control
LAB 2
CONTROL OF A MAGNETICALLY LEVITATED BALL

1 Purpose

The purpose of this experiment is to familiarize you with modelling nonlinear systems using Simulink, performing linearization, designing and evaluating various controllers, and verifying your design on the nonlinear system.

2 Introduction

We are most familiar with control design tools developed for linear systems. Many industrial systems contain nonlinearities—for example, saturation. It is therefore important to understand limitations to design procedures based on linear systems. A magnetic levitation system to suspend a ball is often used as an illustrative model for control design of a nonlinear system. You can read more about a fully operational maglev train connecting Shanghai Pudong International Airport to Pudong using the link http://en.wikipedia.org/wiki/Shanghai_Maglev_Train.

The simplified differential equation for the system is

$$\ddot{y} = g - k\left(\frac{u^2}{y^2}\right),$$

where u, the control input, is the current through the electromagnet, y is the vertical displacement of the ball from the magnet, g is the gravitational acceleration constant, and k is a constant determined by the physical dimensions and material of the system. For the purposes of this experiment, $g = 9.8$ and $k = 1$. The design specification is to achieve stability by keeping the ball suspended at some fixed distance from the electromagnet.

3 Preparation

The majority of control system designs are based on linear models. We shall first build a Simulink model of the nonlinear system, then linearize the model you have built to design a controller.

Complete the following preparation exercises and hand in your answers, **on a separate sheet**, to your T.A. at the beginning of the lab. Every student **must hand in his/her own work**. There is space provided below to record your preparation work for the experiment.

3.1 Building the Nonlinear Model

1. Take the state vector $x = (y, \dot{y})$, corresponding to the position and velocity of the ball. Write down the nonlinear differential equation for x in the form

$$\dot{x} = f(x, u)$$

$$y = h(x, u),$$

where $f(x, u) = (f_1(x, u), f_2(x, u))$ is a 2-dimensional vector with component functions $f_1(x, u)$ and $f_2(x, u)$.

2. Set up your Simulink model, defining a suitable nonlinear function in your model. Include, in your model, the **In1** block, under the **Sources** library. This represents your input. Similarily, include the **Out1** block to represent the output, y. (These will be important for the linmod command you will use in Section 3.2.) Save, print and label your model.

3.2 Linearizing the System

1. Let the desired equilibrium position be $\bar{y} = 0.5$. The equilibrium position is maintained by a steady-state input current $\bar{u}$. Thus at equilibrium, $y = \bar{y}$ and $u = \bar{u}$, and there should be no change in the ball's position. Determine and record $\bar{u}$, taking it to be positive. For your own record, write down $f(x, u)$, $h(x, u)$, and $\bar{u}$ in the space below.

2. Linearize the system around the equilibrium state $\bar{x} = (\bar{y}, 0)$ analytically. Consider the system of differential equations in the following form:

$$\dot{x} = f(x, u)$$
$$y = h(x, u).$$

Given this nonlinear system and an equilibrium condition, its linearization around $(\bar{x}, \bar{u})$ is given by

$$\delta\dot{x} = \left[\ \frac{\partial f}{\partial x}\ \right]_{\bar{x},\bar{u}} \delta x + \left[\ \frac{\partial f}{\partial u}\ \right]_{\bar{x},\bar{u}} \delta u$$
$$\delta y = \left[\ \frac{\partial h}{\partial x}\ \right]_{\bar{x},\bar{u}} \delta x + \left[\ \frac{\partial h}{\partial u}\ \right]_{\bar{x},\bar{u}} \delta u,$$

where $\delta x = x - \bar{x}$, $\delta u = u - \bar{u}$ and $\delta y = y - h(\bar{x}, \bar{u})$ denote the deviations of x, u and y from equilibrium. The notation $\left[\ \frac{\partial f}{\partial x}\ \right]_{\bar{x},\bar{u}}$ denotes the constant matrix having the i–j-th element $\frac{\partial f_i}{\partial x_j}(\bar{x}, \bar{u})$. Let $A = \left[\ \frac{\partial f}{\partial x}\ \right]_{\bar{x},\bar{u}}$ and so on. Determine the matrices A, B, C and D

and the transfer function $G(s)$ from δu to δy. Provide the matrices A, B, and C as well as the transfer function $G(s)$ in the space below.

3. Use the MATLAB command `linmod` to linearize your system at the same equilibrium points. (Use the name of the Simulink model you created in Section 3.2 in the command.) Check that the same results are obtained, and print your code.

4 Experiment

4.1 Continuous Control Design Using Linearized Model

1. Note that when $\delta x = 0$, $y = \bar{y}$ and $x_2 = \dot{y} = 0$, the ball is balanced at the desired setpoint. When the ball is initially placed under the electromagnet at $t = 0$, $\delta x(0) \neq 0$. The control objective can now be recast as finding a controller so that $\delta x(t) \underset{t\to\infty}{\longrightarrow} 0$ when initially $\delta x(0)$ is nonzero. Let $\delta x(0) = (0.8, 0.5)$. Consider a control law of the form $\delta u = F\delta x$. The closed-loop system is
$$\frac{d}{dt}\delta x = (A + BF)\delta x.$$
We know that if the eigenvalues of $(A + BF)$ have negative real parts, we will have $\delta x(t) \underset{t\to\infty}{\longrightarrow} 0$, achieving the design objective.

   Find $F = [f_1 \;\; f_2]$ so that the eigenvalues of $(A + BF)$ have negative real parts, and try to tune it so that the convergence to 0 is reasonably "fast". In terms of the original control signal u, the feedback law is
$$u = \bar{u} + F\delta x = \bar{u} + f_1(y - \bar{y}) + f_2\dot{y}. \tag{1}$$
Such a controller is called a PD (proportional derivative) controller. In using this control law, we are assuming that both the position and velocity of the ball can be measured. We call the control law a **state feedback control law**. Add the controller to the Simulink diagram for the original **nonlinear** system. Note that for the simulation of the nonlinear system, you need to set the initial condition to $x(0) = \bar{x} + \delta x(0)$. Tune the parameters f_1 and f_2 if necessary, and check that the design specs are satisfied on the Simulink scope. Print the model and response of your closed-loop, state feedback system. Save your work for demonstration to the TA later.

2. Since we cannot usually measure the velocity, we can use a feedback law of the form
$$u(s) = \bar{u} + f_1(y(s) - \bar{y}) + f_2\frac{\alpha s}{s + \alpha}y(s), \tag{2}$$

where the second term approximates differentiation for α large, say ≥ 10. This control law is equivalent to a control law of the form

$$u(s) = \bar{u} - f_1 \bar{y} + \frac{cs + d}{s + \alpha} y(s). \tag{3}$$

We call it an **output feedback control law**. Add this controller to the Simulink diagram for the original **nonlinear** system. Tune the parameters f_1, f_2 and α if necessary, and show that the design specs are satisfied on the Simulink scope. Put your final continuous time control law in the space below. Comment also on whether there are any differences in the response under state feedback control compared to that under output feedback control.

Print and label the Simulink model and simulation of your output feedback controller. Save your work.

4.2 Digital Control Design for the Nonlinear System

The previous steps can be thought of as intermediate steps to the final digital control design for the original nonlinear system. In this section, you apply the sampled-data version of your design to the nonlinear system, and study how the nonlinearity and the choice of parameters affect the performance of the closed-loop system.

1. In your m-file, set the sampling period to $T = 0.05$. Implement the state feedback control law in digital control form with zero-order holds in appropriate spots in the Simulink diagram for the original nonlinear system. Does the closed-loop nonlinear system still satisfy the specifications? If not, tune your controller gains and try again. Print and label the final model and response of this system.

2. If $\dot{y}$ is not available, we cannot implement the state feedback law. One way of approximating the state feedback law is to note that it is a PD control law and we can directly approximate

the derivative term by a finite difference. Define $d = f_2 \frac{\alpha s}{s+\alpha} y$. Then d satisfies the differential equation

$$\dot{d} + \alpha d = f_2 \alpha \dot{y} \tag{4}$$

Let us use a backward finite difference to approximate a derivative, i.e, approximate

$$\dot{d}(kT) \approx \frac{d(kT) - d(kT - T)}{T}$$

Applying this to (4), we get, at the sampling instants,

$$\frac{d(kT) - d(kT - T)}{T} + \alpha d(kT) = f_2 \alpha \frac{d(kT) - d(kT - T)}{T}$$

Simplifying, we get the following equation for d:

$$d(kT) = \frac{1}{1 + \alpha T} d(kT - T) + \frac{f_2 \alpha}{1 + \alpha T} [y(kT) - y(kT - T)]$$

On substituting into (2), we obtain the following discretized controller:

$$u(kT) = \bar{u} + f_1(y(kT) - \bar{y}) + d(kT) \tag{5}$$

Implement this controller on the nonlinear system. Tune the control parameters, if necessary, to achieve the control specs. Print and label your Simulink diagram for this digital controller.

3. Now discretize your first order output feedback controller (3) using `c2d(system,T,'tustin')` (type "help tustin" to get more information about the tustin or bilinear transformation). Implement this controller on the nonlinear system. Use the **Discrete Transfer Fn** block. Again tune the control parameters, if necessary, to achieve the control specs. Print and label your Simulink diagram for this digital controller. Compare the performance of the 2 discretized output feedback controllers as well as compare them to the state feedback one. Discuss briefly your observations and conclusions in the space below.

4. We have designed controllers based on the linear approximation to the original nonlinear system. If the system were linear, the design specs would be satisfied regardless of the size of the initial deviation from the equilibrium. Here you study the effects of the nonlinearity by increasing the size of $\delta x(0)$. In particular, determine the value of $y(0) = x_1(0)$ for which the specs can no longer be satisfied. Record your observations in the space below.

5. Finally, study the effects of the choice of sampling period by increasing T. Do your designs still work? Does the allowable range of initial deviation become smaller? What is the value of T for which your designs no longer work? Record your observations in the space below.

Finally, demonstrate to the TA all your results for Sections 4.1 and 4.2 using your Simulink diagrams and the Simulink scope.

- *Name:*
- *Student No.:*
- *Name:*
- *Student No.:*
- *Lab Section, Date, Time:*