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ECE320H1F: Fields and Waves
Laboratory 2: Standing Waves and Waveguides

1 Objective

Two waveguiding structures implemented using planar transmission lines are introduced. The structures are characterized using practical field measurements. Using measured standing wave patterns, properties for both the structures, as well as the loads used to terminated the structures, are experimentally determined.

2 References

[1] F. T. Ulaby and U. Ravaioli, *Fundamentals of Applied Electromagnetics*, 7th ed. Upper Saddle River, NJ: Pearson, 2015.

[2] ——, *Fundamentals of Applied Electromagnetics*, 7th ed. Upper Saddle River, NJ: Pearson, 2015, ch. 1, pp. 60–65.

[3] Agilent Technologies. (2005) Network analyzer basics. [Online]. Available: http://www.keysight.com/upload/cmc_upload/All/BTB_Network_2005-1.pdf

3 Background

3.1 Experiment Setup

The following equipment and components are required to complete this lab:

- Vector network analyzer (VNA)
- Calibration kit
- Two SMA cables
- Linear translator station
- Ruler
- Computer running MATLAB or Microsoft Excel, for recording data

- One short circuit load
- One unknown load

The main component in this lab is a linear translator, which is shown in Fig. 1. It consists of a moveable stage, which is suspended on two rods above a printed circuit board (PCB). On the PCB, there are two waveguiding structures. The first is a microstrip line [2], and the second is a substrate integrated waveguide (SIW), which is a specialized implementation of the rectangular waveguide (used in a later experiment). The translation stage is moved back and forth manually. On the stage, there are two coaxial probes (one over each of the two waveguiding structures), which independently measure the electric fields produced by their respective structures. By measuring these fields as a function of position, useful information about the structures themselves, as well as the loads terminating them, can be determined.

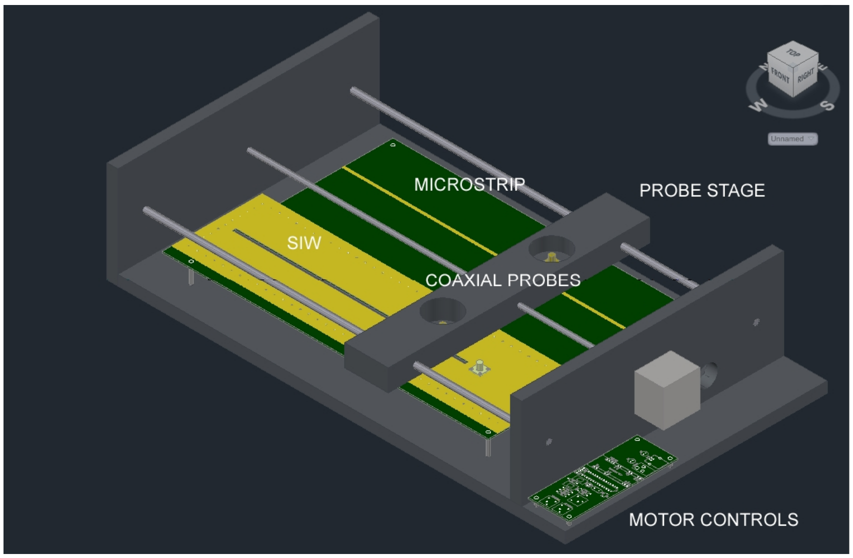

Figure 1 Linear translator.

In this laboratory, a *vector network analyzer* (VNA) is used to measure the *transmission coefficient* from port 1 to port 2. The transmission coefficient is directly proportional to the voltage, and can thus be used to reconstruct the voltage standing wave pattern produced by each of the waveguiding structures. Effectively, the VNA is used as a *tuned receiver*, whereby signals spanning a specified frequency range are transmitted from port 1 and received via port 2. That is, port 1 is used to excite the structure of interest, while port 2 is connected to the coaxial probe in order to measure the fields picked up by the probe.

3.2 Transmission Line Measurement and Voltage Standing Waves

For both coaxial probes mounted above the PCB, there is a short segment of wire extending from the coaxial connector. The short wire effectively forms a small *monopole antenna*, which is sensitive to the electric field oriented on the axis of the antenna (here it is vertically-oriented). When placed in proximity to the microstrip line or the SIW, this probe senses the vertically-oriented fringing field produced by the respective structures in the air region above them. Note that if the microstrip line did not naturally produce any fringing fields, or the SIW did not have a slot cut along its length, this type of measurement would not be possible. Since the electric field is proportional to the voltage produced within the waveguiding structures, the measurement of this field can be used to determine the *voltage standing wave* (VSW) pattern along the line.

The voltage on a transmission line is a combination of a forward traveling wave $V^+(z) = V_0^+ \mathrm{e}^{-j\beta z}$ and a backward traveling wave $V^-(z) = V_0^- \mathrm{e}^{+j\beta z}$. Using the relations $V_0^+ = \Gamma V_0^-$ and $\Gamma = |\Gamma| \mathrm{e}^{j\theta}$, the amplitude of the voltage on the line can be written as:

$$\begin{aligned} |v(z)| &= |V^+(z) + V^-(z)| \\ &= |V_0^+ \mathrm{e}^{-j\beta z} + V_0^- \mathrm{e}^{+j\beta z}| \\ &= |V_0^+||1 + \Gamma \mathrm{e}^{+j2\beta z}| \\ &= |V_0^+||1 + |\Gamma| \mathrm{e}^{j(2\beta z + \theta)}| \,. \end{aligned} \tag{1}$$

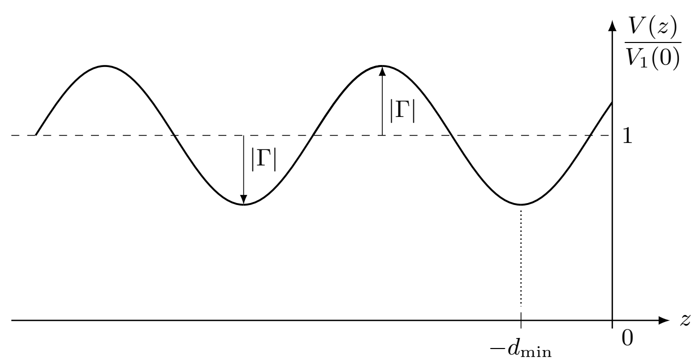

Figure 2 Voltage standing wave on a transmission line.

As shown in Fig. 2 the amplitude of $v(z)$ varies between a maximum value of:

$$|V_{\max}| = |V_0^+| \Big(1 + |\Gamma|\Big) \,, \tag{2}$$

located at $2\beta z_{\max} + \theta = 2n\pi$ (where $\mathrm{e}^{j(2\beta z + \theta)} = +1$), and a minimum value of:

$$|V_{\min}| = |V_0^+| \Big(1 - |\Gamma|\Big) \,, \tag{3}$$

located at $2\beta z_{\min} + \theta = (2n+1)\,\pi$ (where $\mathrm{e}^{j(2\beta z + \theta)} = -1$). The voltage standing wave ratio (VSWR) is defined as:

$$\mathrm{VSWR} = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \,. \tag{4}$$

Rearranging (4) for $|\Gamma|$, it found that:

$$|\Gamma| = \frac{\mathrm{VSWR} - 1}{\mathrm{VSWR} + 1} \,. \tag{5}$$

That is, the magnitude of Γ is easily determined using the VSWR. To determine θ (the phase of Γ) at the load, as shown in Fig. 2, let the first voltage minimum (i.e. $n = 0$) be located at $z_{\text{min}} = -d_{\text{min}}$. Then:

$$\begin{aligned}\theta &= \pi - 2\beta z_{\text{min}} \\ &= \pi + 2\beta d_{\text{min}} \\ &= \pi + \frac{4\pi d_{\text{min}}}{\lambda}\,. \end{aligned} \tag{6}$$

Thus, the complex reflection coefficient $\Gamma = |\Gamma| e^{j\theta}$ at the input port of a transmission line can be determined by measuring the VSWR, as well as the position of a voltage minimum. The input impedance can then be computed from the $\Gamma(z)$ to $Z_{\text{in}}(z)$ relationship.

3.3 Microstrip Line

A microstrip line consists of a narrow strip of conductor suspended over a large conducting ground plane, usually by a dielectric substrate, as shown in Fig. 3a. The geometry of the microstrip line determines its characteristic impedance. By varying the strip width w, substrate height h, and substrate dielectric constant ε_r, a wide range of characteristic impedances can be synthesized.

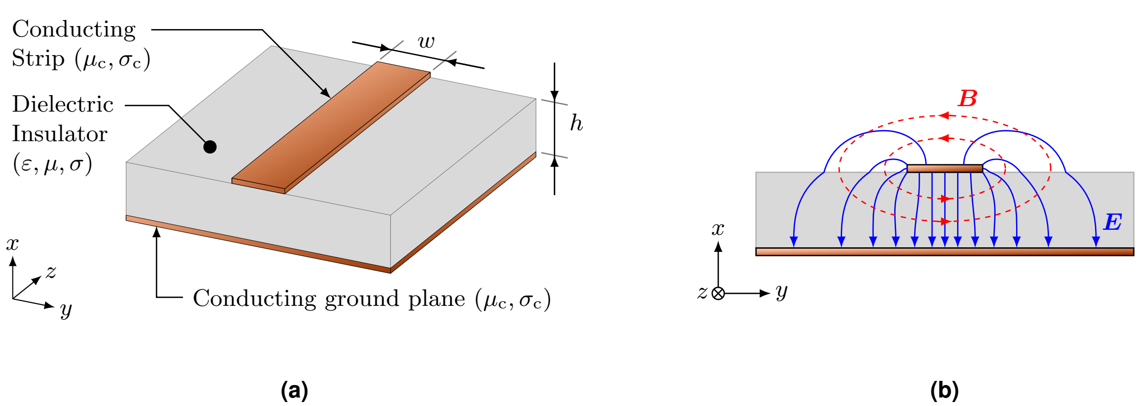

Figure 3 Microstrip line: (a) longitudinal view, and (b) cross-sectional view with $\boldsymbol{E}$ and $\boldsymbol{B}$ field lines.

When the line is excited by placing the strip at an electric potential relative the ground plane, the electric and magnetic field lines shown in Fig. 3b result. In addition to the transverse electromagnetic (TEM) (or "parallel plate" type) field distribution created directly beneath the strip, fringing fields are also created, most of which propagate within the air region above the transmission line. Since the fields are not entirely confined to the air region nor the dielectric region, the wave velocity of the line is somewhere between that of the speed of light in air ($c_0 = 3 \times 10^8\,\text{m/s}$) and that in the substrate ($c_0/\sqrt{\varepsilon_r}$). As a result, the wave velocity is generally defined using an effective dielectric constant ε_{eff}, and the corresponding speed of light along the microstrip line is $c_0/\sqrt{\varepsilon_{\text{eff}}}$. The effective dielectric constant is dependent on the geometry of the line as well as the substrate parameters. It has been the focus of many studies over the years, and has been found to be best described using a combination of conformal mapping and empirical formulas [2]. A common expression for the effective dielectric constant is:

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy} \tag{7}$$

where $s = w/h$ and:

$$x = 0.56\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right)^{0.05} \tag{8a}$$

$$y = 1 + 0.02\ln\left(\frac{s^4 + 3.7\times10^{-4}s^2}{s^4 + 0.43}\right) + 0.05\ln\left(1 + 1.7\times10^{-4}s^3\right) \tag{8b}$$

The characteristic impedance of a microstrip line can be found using empirical formulas as well, such as:

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\text{eff}}}}\ln\left(\frac{6 + (2\pi - 6)\,\mathrm{e}^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right) \tag{9}$$

where

$$t = \left(\frac{30.67}{s}\right)^{0.75} \tag{10}$$

The microstrip line for this laboratory exercise has been fabricated on a substrate known as FR-4, which is a commonly used material for making printed circuit boards. As an interesting historical note, FR-4 was originally developed for aerospace applications, and is so-called because it is flame retardant (FR). FR-4 has a nominal dielectric constant of $\varepsilon_r = 4.4$. The line on the PCB in this lab has been designed on a substrate with a thickness of $h = 1.5\,\text{mm}$, so that it is matched to the system impedance of the VNA, which is $Z_0 = 50\,\Omega$.

4 Microstrip Line Standing Wave Measurements

It is very important to wear a static bracelet when operating the VNA. The VNA is a very sensitive piece of equipment, and can easily be damaged by electrostatic discharge (ESD). When connecting loads and/or calibration standards, use the provided torque wrench. If no torque wrench is provided, tighten the loads or standards *finger tight*. Do NOT overtighten the loads or standards. The calibration kit (seen in Fig. 4) has protective caps on all standards; please ensure that the caps are replaced after the calibration procedure is completed. If any caps are missing, notify the laboratory teaching assistant (TA) and/or the laboratory manager.

Measurements in this experiment are conducted using a vector network analyzer, which is an important piece of test equipment for microwave measurements. As the name suggests, it is used for making measurements of microwave networks, which are circuits with multiple ports (often two). For example, a network analyzer can measure the impedance matrix of a two-port network by injecting suitable test

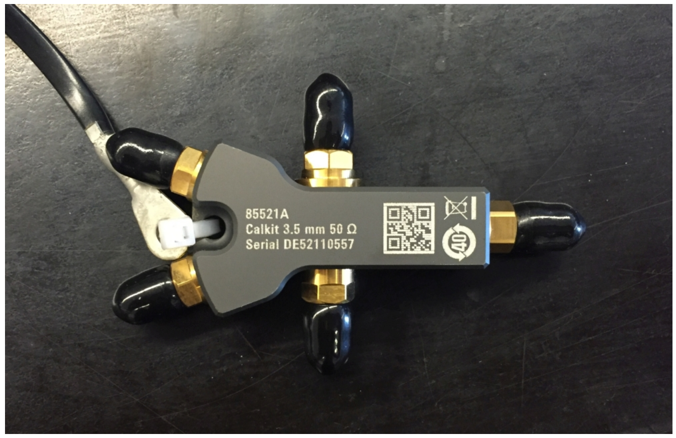

Figure 4 The calibration kit used for ECE320 laboratories. Note the protective caps on all the standards. Please replace the caps after the calibration procedure is completed. If any caps are missing, please notify the teaching assistant and/or laboratory manager.

signals into each port. Usually, however, we use network analyzers to measure *scattering parameters*, or more generally, *reflection and transmission coefficients* of a microwave network. All measurements of microwave networks are based on measuring waves reflected from or travelling through microwave networks [3]. In this experiment, we will use the transmission coefficient measured from the input port on the translator to the probe as a way of measuring the standing wave pattern produced along the microstrip line.

The design frequency for the microstrip line is $f = 1\,\text{GHz}$. Recall that a free-space wavelength λ_0 *cannot* be assumed for the microstrip line configuration.

Notation: In the following instructions, [Command] refers to a softkey on the VNA front panel while *Command* refers to an option appearing on the VNA touchscreen (or monitor if it is hooked up to the VNA), and < Command > refers to something that is typed in using the VNA keypad (or keyboard if it is hooked up to the VNA).

Before measurements are conducted, the VNA must be properly calibrated. In this way, when the two cables from the network analyzer are connected together, a transmission coefficient of unity with zero phase ($T = S_{21} = 1\angle 0°$ will be measured ($|S_{21}| = 0$ dB; $\angle S_{21} = 0°$).

4.1 VNA Setup and Calibration Procedure

1. Set the frequency span over which the measurements and calibration will be performed. On the VNA front panel, press [Start] → < 800 MHz >, and [Stop] → < 1.2 GHz >.
2. Set the number of points to be measured. Press [Sweep Setup] → *Points* → < 801 > → *Return*.
3. Select the calibration kit. Press [Cal] → *Cal Kit* → *85521A*.
4. Initialize the calibration procedure. Press [Cal] → *Calibrate* → *2-Port Cal*.

5. Select *Reflection* in the menu. Begin with Port 1 by connecting the cable attached to port 1 of the VNA to the 'Open' calibration standard. Then press the corresponding *Port1 Open* option on the VNA calibration menu. Repeat for the 'Short' and 'Load' standards. Then repeat the three standard measurements on Port 2. After the six reflection measurements (three standards on each of Ports 1 and 2) are complete, select *Return* in the VNA menu.

6. Now select *Transmission*. Connect Port 1 of the VNA to Port 2 using the 'Through' calibration standard. Select *Port 1-2 Thru* in the VNA menu, then select *Return*.

7. **Make sure you select *Done* at the bottom of the calibration menu after completing the six reflection measurements and the thru measurement. Otherwise the calibration will not be complete, and hence will not be applied. If the *Done* menu item is grayed out, it is because one or more of the calibration steps has been skipped.**

8. Check your calibration by leaving the SMA cables connected end-to-end with the through, and ensure that:

(a) The input reflection coefficient is very low (less than $-20\,\mathrm{dB}$) across the entire frequency band.

(b) The transmission coefficient is nearly unity ($0\,\mathrm{dB}$) across the entire frequency band.

4.2 Measurement of Microstrip Line Characteristics

In this portion of the lab, the linear positioner will be used to measure the fringing fields produced above the microstrip line, the magnitude of which are directly proportional to the voltage along the line. That is, the positioner will be used to plot the voltage standing wave as a function of position.

1. Measure the width w of the transmission line. Determine the effective dielectric constant $\varepsilon_{\mathrm{eff}}$ of the line using (7) (given in Section 3.3). Then calculate the characteristic impedance Z_0 using (9) to confirm it is near $50\,\Omega$. Finally, determine the phase velocity v_{p} of the line, assuming it is constant for all frequencies.

2. Attach the SMA cables and connect port 1 of the network analyzer to the input port of the microstrip line, feeding the SMA cable through the small hole at the end of the linear positioner.

3. Connect port 2 of the network analyzer to the coaxial probe on the microstrip side of the probe stage.

4. Connect a short circuit to the end of the microstrip line.

5. Manually move the stage so that the coaxial probe used to measure the microstrip line is positioned above the load end of the microstrip line.

6. Press [Meas] → S_{21} → [Format] → *Log Mag*.

7. Press [MARKER] → $<1\,\mathrm{GHz}>$. Use [Scale] → *Autoscale* to see the curve.

8. At this point, the transmission coefficient (S_{21}) measured by the network analyzer should be very small (less than $-60\,\mathrm{dB}$).

9. Move the stage away from the load until the first voltage minima is encountered. This is where the measurements for this part of the laboratory begin.

10. Manually move the stage in 5 mm increments away from the load, and plot the standing wave pattern over a minimum one one full wavelength at 1 GHz. Use the ruler supplied to make precise measurements of the probe location relative to the short-circuited end of the PCB.

11. Locate the maxima and minima to obtain more precise locations for the field readings at these positions.

12. Compute the VSWR and compare to that of a short circuit, recalling that the field values reported by the network analyzer are on a power-dB scale.

13. Compute the guide wavelength and corresponding effective dielectric constant of the microstrip line using the measurement data. Compare these to theoretical expectations, and comment on any differences.

4.3 Using Standing Wave Patterns for Load Calculations

In this portion of the lab, the standing wave pattern produced when the microstrip line is terminated in an unknown load is measured. The value of the unknown load can then be determined using analytical/Smith Chart techniques.

1. Disconnect the short circuit used in the previous experiment and connect the unknown load to the end of the microstrip line.

2. Re-position the probe so that it is measuring the field directly above the PCB edge where the load is located.

3. Move the stage in 5 mm increments away from the load, and plot the standing wave pattern over a minimum one full wavelength at 1 GHz. Use the ruler supplied to make precise measurements of the probe location relative to the short-circuited end of the PCB.

4. Locate the maxima and minima to obtain more precise locations for the field readings at these positions.

5. Using the techniques of Section 3.2, compute the impedance of the load based on the standing wave pattern that has been measured. Recall that the field values reported by the network analyzer are on a power-dB scale. The maximum and minimum magnitudes are used to find the VSWR (and hence, the magnitude of the impedance) and d_{min} yields the phase of the load impedance.

6. Disconnect the SMA connectors from the microstrip portion of the translator.

7. Connect the unknown load to port 1 of the network analyzer and measure its impedance.

(a) With the unknown load attached, press [Meas] $\rightarrow S_{11} \rightarrow$ [Format] $\rightarrow$ *Smith* $\rightarrow R + jX$.

(b) Press [Marker] $\rightarrow <1\,\text{GHz}>$ and record the impedance from by the network analyzer. Compare to your computed result and discuss any discrepancies.

Make sure you speak to you TA before leaving the lab, so that you can be assessed your oral grade for this experiment!