ECE320: Fields and Waves

Lab 2 Report: Standing Waves and Waveguides

PRA103

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1 Introduction

The objective of this laboratory is to determine some physical properties of a microstrip transmission line, which sits on a substrate with some relative electric permittivity ϵ_r, as shown in figure 1.

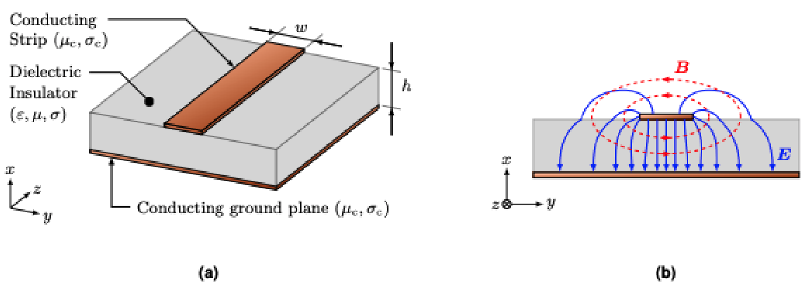

Figure 1: Microstrip line: (a) longitudinal view, and (b) cross-sectional view with **E** and **B** field lines

Notice in 1(b), the **E** and **B** fields exist in both the air and substrate, so the microstrip line has an effective relative electric permittivity ϵ_{eff}, which can be determined using the following equation:

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + (\frac{\epsilon_r - 1}{2})(1 + \frac{10}{s})^{-xy} \tag{1}$$

where

$$s = \frac{w}{h} \quad \text{and}$$

$$x = 0.56(\frac{\epsilon_r - 0.9}{\epsilon_r + 3})^{0.05} \quad \text{and}$$

$$y = 1 + 0.02\ln(\frac{s^4 + 3.7\cdot 10^{-4}s^2}{s^4 + 0.43}) + 0.05\ln(1 + 1.7\cdot 10^{-4}s^3)$$

The microstrip line is placed on a linear translator where a movable probe, connected to a Vector Network Analyser (VNA), is placed above the conducting strip. The probe is used to determine standing wave patterns, which are needed to compute experimental quantities such as the characteristic impedance of the microstrip (Z_0), the wave velocity (u_{phase}) along the line and the effective dielectric constant (ϵ_{eff}).

The characteristic impedance of a microstrip line (Z_0) can be found using:

$$Z_0 = \frac{60}{\sqrt{\epsilon_{\text{eff}}}}\ln(\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}) \tag{2}$$

where

$$t = (\frac{30.65}{s})^{0.75}$$

The theoretical phase velocity u_{phase} is related to the ϵ_{eff} by the following relationship:

$$u_{\text{phase}} = \frac{c_0}{\sqrt{\epsilon_{\text{eff}}}} \tag{3}$$

2 Measurement of Microstrip Line Characteristics

For this section, a short-circuit load was attached to the end of the microstrip line.

2.1 Measured and theoretical values

Before discussing the experimental data, it is essential to compute the theoretical values, for which all the formulae mentioned in `Introduction` were used.

Computations were performed on a `MATLAB` script, which can be found `here`

The following table summarizes all of the given, measured, and calculated theoretical values including:

- Measured width of the transmission line (w)
- Theoretical effective dielectric constant (ϵ_{eff})
- Theoretical phase velocity (u_{phase})
- Theoretical characteristic impedance (Z_0)

Note: Since the **E** and **B** fields are not confined to air or the substrate, $\epsilon_{\text{eff}}\epsilon_0$ is in range of $\epsilon_{\text{air}}\epsilon_0 \approx \epsilon_0$ and $\epsilon_{\text{r}}\epsilon_0$.

Parameter	Value
w	3×10^{-3} m
h	1.5 mm
s	2
x	0.5394
y	0.9995
t	7.7493
Z_0	48.6974Ω
ϵ_{r}	4.4
ϵ_{eff}	3.3470
u_{phase}	1.6398×10^8 m/s

Table 1: Summary of theoretical values

2.2 Experimental VSWR

VSWR stands for Voltage Standing Wave Ratio, and is defined as the ratio between the maximum voltage along the line (V_{max}) and minimum voltage (V_{min}) along the line. VSWR for a transmission line is linked to the reflection coefficient Γ by the following relationship:

$$\mathrm{VSWR} = \frac{V_{\mathrm{max}}}{V_{\mathrm{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{4}$$

This relationship can be utilized to determine the experimental VSWR, once the minima and maxima are identified in the experimental data. However, it is important to note that the collected data is in decibels (dB), so once the minima and maxima are identified, they have to be converted to volts and then 4 can applied. The experimental data can be found in Table 2.

The relationship between v_{dB} in dB and V in volts is given by:

$$v_{\mathrm{dB}} = 20 \times \log_{10}(V) \tag{5}$$

$$V_{\mathrm{min}} = 10^{v_{\mathrm{min}}/20} = 0.0001995262315\mathrm{V}$$

$$V_{\mathrm{max}} = 10^{v_{\mathrm{max}}/20} = 0.008413951416\mathrm{V}$$

Dividing V_{max} by V_{min}:

$$\mathrm{VSWR}_{\mathrm{exp}} = 42.17$$

2.3 Comparing experimental and theoretical VSWR

For this part of the experiment, a short-circuit was attached as the load. Theoretically, the VSWR should be $\frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+|-1|}{1-|-1|} = \infty$. While the experimental value is much less theoretical value, we can compute the experimental $|\Gamma| = \frac{\mathrm{VSWR}-1}{\mathrm{VSWR}+1} = 0.9537$, which is close to the theoretical value that is 1, as we have a shorted load.

Distance d from the load (cm)	**Amplitude of voltage v in dB**
5.9	-48.9
6.4 (1st Min)	-74
6.9	-59
7.4	-51.5
7.9	-47.8
8.4	-45.9
8.9	-44.5
9.4	-43.4
9.9	-42.5
10.4 (1st Max)	-42.2
10.9	-42.4
11.4	-42.9
11.9	-43.4
12.4	-44.3
12.9	-46.3
13.4	-49.6
13.9	-54
14.4 (2nd Min)	-66
14.9	-52.8
15.4	-49.2
15.9	-46.9
16.4	-44.9
16.9	-43.1
17.4 (2nd Max)	-41.5

Table 2: Amplitude of the corresponding waves for a $f = 1\text{GHz}$ sine wave input with a short circuit load attached. Note that d is the distance away from the load, towards the generator

2.4 Experimental wavelength

Any point/value on the Smith chart repeats itself every 0.5λ, whether moving away or toward the load. This is true because 0.5λ is one rotation on the VSWR circle, where λ is the wavelength of the wave propagating along the transmission line.

Inspecting the data in the table, the two minima are $14.4\text{ cm} - 6.4\text{ cm}$ apart. This implies:

$$\begin{aligned} 0.5 \times \lambda &= 14.4 - 6.4 \\ &= 8 \times 10^{-2}\text{ m} \\ \lambda &= 0.16\text{ m} \end{aligned}$$

2.5 Experimental effective dielectric constant

Since the frequency is $f = 1\text{GHz}$, the experimental phase velocity can be computed using the experimental wavelength:

$$u_{\text{phase}} = \lambda f = 0.16 \times 1 \times 10^9 = 1.6 \times 10^8 \text{ m/s}$$

Hence, the effective dielectric constant can be determined by rearranging 3:

$$\epsilon_{\text{eff}} = (\frac{c_0}{u_{\text{phase}}})^2 = 3.515625$$

2.6 Comparison of experimental wavelength and effective dielectric constant to theoretical values

- $\epsilon_{\text{eff, th}} = 3.3470 \approx \epsilon_{\text{eff, exp}} = 3.5162$
- $\lambda_{\text{th}} = 0.163\text{m} \approx \lambda_{\text{ex}} = 0.16\text{m}$

Both these experimental values are close to the theoretical values.

2.7 Plotting the experimental standing wave pattern.

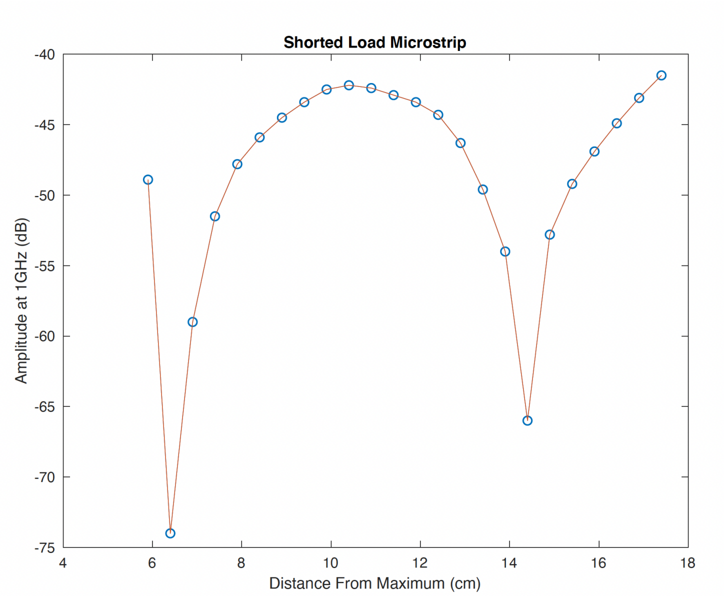

Figure 2: VSW Pattern for a shorted load attached to the end of the microstrip transmission line

3 Using Standing Wave Patterns for Load Calculations

For this section, an unknown load was attached to the end of the microstrip transmission line. Note that changing the load does not change any theoretical values mentioned in Table 1, which makes sense since none of the calculations had any dependence on the load attached to the microstrip transmission line.

3.1 Impedance of the load found from experimental standing wave measurement data

3.1.1 How to compute Z_L

This section is concerned with finding Z_L, which is the impedance of the unknown load attached at the end of the microstrip transmission line. In order to determine this value, it is essential to look at relationship between Z_L and other measurable/calculable values:

The reflection coefficient of a transmission line arises from the need of matching boundary conditions. The leads to the following relationship between Γ, Z_L, Z_0:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

This relationship can be rearranged for our parameter of interest, Z_L:

$$Z_L = -Z_0 \times \frac{\Gamma + 1}{\Gamma - 1} \tag{6}$$

Here, Z_0 is already known. So to compute Z_L, the only required value is Γ. In the previous section, we saw how to compute the VSWR of a transmission line at a particular frequency for a particular load. The VSWR of a transmission line always us to compute the absolute value of a reflection coefficient:

$$|\Gamma| = \frac{\mathrm{VSWR} - 1}{\mathrm{VSWR} + 1} \tag{7}$$

Recall $\Gamma = |\Gamma| \mathrm{e}^{j\theta}$, so the only missing value to compute is the phase difference of the reflection coefficient at the load. θ can be computed using our understanding of voltage standing wave patterns:

$$\theta = \pi + 2\beta d_{min} = \pi + \frac{4\pi d_{\min}}{\lambda} \tag{8}$$

To summarize, computing Z_L involves:

- Identifying voltage minima and maxima in the experimental data
- Computing VSWR
- Computing $|\Gamma|$ using the calculated VSWR
- Computing θ using the identified minima.

3.1.2 Computation of values

Here is the experimental data for an unknown load:

Distance d from the load (cm)	**Amplitude of voltage v in dB**
0	-61.3
0.5	-58
1	-55.8
1.5	-54.6
2	-53.7
2.5	-53.6
3	-53.2
3.5	-51.8
4	-50.6
4.5	-50.1
5	-50
5.5	-50.1
6 (d_{min})	-49.6
6.5	-48.6
7	-47.7
7.5	-46.8
8	-46.7
8.5	-45.8
9	-45.6
9.5 (d_{max})	-45.5
10	-45.6
10.5	-45.9
11	-46.4
11.5	-47
12	-48.3
12.5	-49.1
13	-49.6
13.5 (d_{min})	-49.8
14	-49.6
14.5	-49.1
15	-49.3
15.5	-47.6
16	-47
16.5	-46.6
17 (d_{max})	-46.2

Table 3: Amplitude of the corresponding waves for a $f = 1\text{GHz}$ sine wave input with an unknown load attached. Note that d is the distance away from the load, towards the generator

Looking at our data, there are some discrepancies. In the distances 0 cm-5.5 cm, we see amplitudes (in dB) that are never repeated in the data. This points to some noise or experimental error while collecting the data in that range because the VSW pattern is sinusoidal and hence periodic. The noise in the apparatus could be ascribed to the addition of a load connector that extended the transmission line, leading to noise in the probe. It is assumed that the noisy range of data is similar to the rest of our data. For this reason, we only considered d_{min} starting from 6 cm (from the load).

VSWR was computed using `4`, thus we have the equation:

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

This can be rearranged for $|\Gamma|$:

$$|\Gamma| = \frac{\text{VSWR}-1}{\text{VSWR}+1}$$

VSWR can be computed using a similar procedure seen in the previous section. This leads to VSWR $= 1.64$, therefore we can calculate $|\Gamma|$:

$$|\Gamma| = \frac{1.64-1}{1.64+1}$$

$$|\Gamma| = 0.243$$

Using the distance between two consecutive minimums (7.5 cm) we can compute λ. Note that λ stays unaffected by a change in the load and hence the computed value from the previous section can be used (λ is affected by ϵ_{r} and frequency). However, it is a good idea to compute λ using `Table 3` just to ensure a similar value for λ is obtained:

$$\begin{aligned} 0.5 \times \lambda &= 13.5 - 6 \\ &= 7.5 \times 10^{-2} \text{ m} \\ \lambda &= 0.15 \text{ m} \end{aligned}$$

To find θ, we will use λ calculated above, and use d_{min} which we will determine from Table 3

Looking at the data we can determine $d_{min} = 0.054m$. Now that we have both λ and d_{min} we can solve for phase θ. However, since we excluded the first data points (from 0-5 cm), we have missed our first d_{min} value. Referring to the equation:

$$-2\beta d_{\text{min}} + \theta = (2n+1)\pi$$

Tells us we can obtain θ using another value of n. Hence, instead of $n = 0$ (first minimum), we took $n = 1$ (second minimum). This overall does not affect our computations too much since a complex exponential is periodic in 2π rad.

$$\theta = 3\pi + \frac{4\pi d_{min}}{\lambda} = 3\pi + \frac{4\pi(0.054)}{0.15} = 14.6247 \text{ rad}$$

Using the calculated $|\Gamma|$ and θ, the reflection coefficient at the load can be obtained by computing $|\Gamma|e^{j\theta}$:

$$\Gamma = -0.1136 + \text{j}0.2143$$

Now that we have the value of Γ and using the given $Z_0 = 50\Omega$ (our experimental value was close to this as well), we can calculate for Z_L using `6`:

$$Z_{\text{L, Exp}} = 36.5887 + \text{j}16.6652\Omega$$

3.2 Impedance of the load measured using the vector network analyzer (VNA)

With the Vector Network Analyzer (VNA), the impedance of the load was measured by directly connecting the load to it and obtaining a Smith Chart:

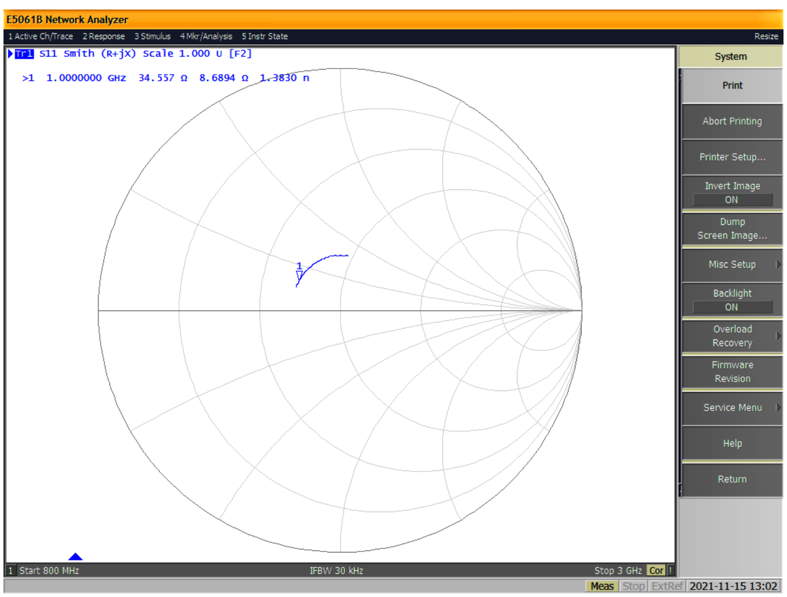

Figure 3: Smith Chart obtained by directly connecting the unknown load to the VNA

From `Figure 3`, it can be seen that the measured impedance ($Z_{\text{L, VNA}}$) is:

$$Z_{\text{L, VNA}} = 34.557 + \text{j}8.6894\Omega$$

3.3 Comparison of results

The signs of the resistance and reactance of $Z_{\text{L, Exp}}$ are the same for that of $Z_{\text{L, VNA}}$. $|Z_{\text{L, Exp}}| = 40.2053\Omega$ whereas $|Z_{\text{L, VNA}}| = 35.6327\Omega$. While the resistances of the theoretical and experimental values are close, there is a significant deviation in the values of the reactances. Hence, on the Smith Chart, the resistances of the experimental and theoretical values will lie almost on the same r (normalized resistance) circle, whereas the reactances will differ significantly.

The deviation in the experimental value can be attributed to the method in which it was computed. $Z_{\text{L, Exp}}$ was dependent on two other experimental values – $|\Gamma|$ and θ. Hence, the error in the final value of $Z_{\text{L, Exp}}$ could have propagated through the intermediate steps. Physically speaking, the error could originate from the adapter and the load connector as it introduces a different material and geometry and hence impacts the characteristic nature of the transmission line.

3.4 Plotting the experimental standing wave pattern

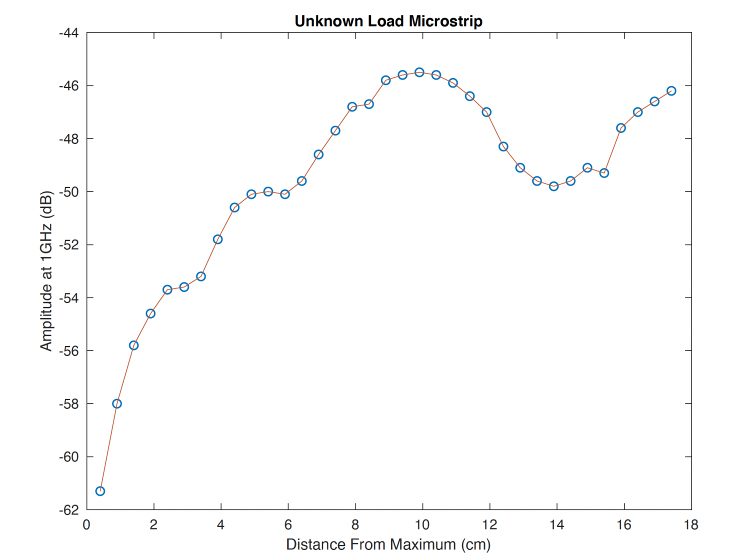

Figure 4: VSW Pattern for an Unknown Load Microstrip

Due to there being noise in the in the beginning of our data, some of it is negligible therefore we decided to remove the data (from 0 to 5.5cm away from the load). The make more sense out of `Figure 4`, we can ignore the first few data points:

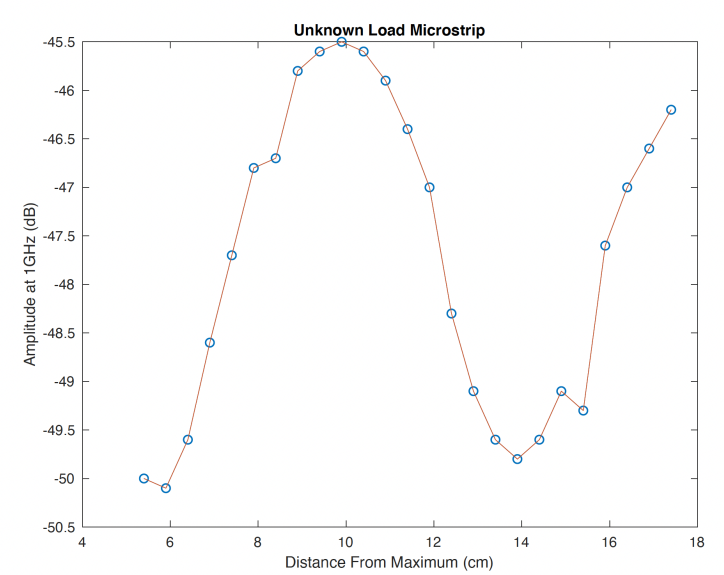

Figure 5: VSW Pattern for an Unknown Load Microstrip Without Noise