ECE320: Fields and Waves

Lab 3 Report: Design of a Double Stub Matching Network

PRA103

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1 Introduction

This laboratory session deals with designing a double stub matching network for a given load and frequency. The theoretical circuit looks as follows:

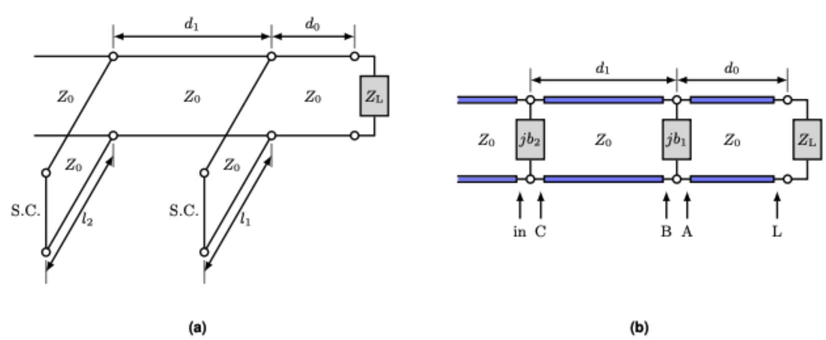


Figure 1: Schematics for a double stub tuner: **(a)** transmission-line schematic, and **(b)** equivalent circuit. A match is achieved by varying the stub length l_1 and l_2 whereas the distances between the stubs d_1 and d_0 load are fixed.

2 Unknown Load Impedance

In this laboratory session, an unknown load has to be matched using the double stub. However, before determining the needed double stub lengths, it is necessary to first directly connect the load to the VNA and measure its impedance. The measured Smith Chart in 2 shows the load impedance across a range of frequencies, 300 MHz to 1.3 GHz, with the marker placed in the middle, 800 MHz, which is the given frequency to match the load at.

2.1 Measurement Smith Chart Plot of Load Impedance versus Frequency

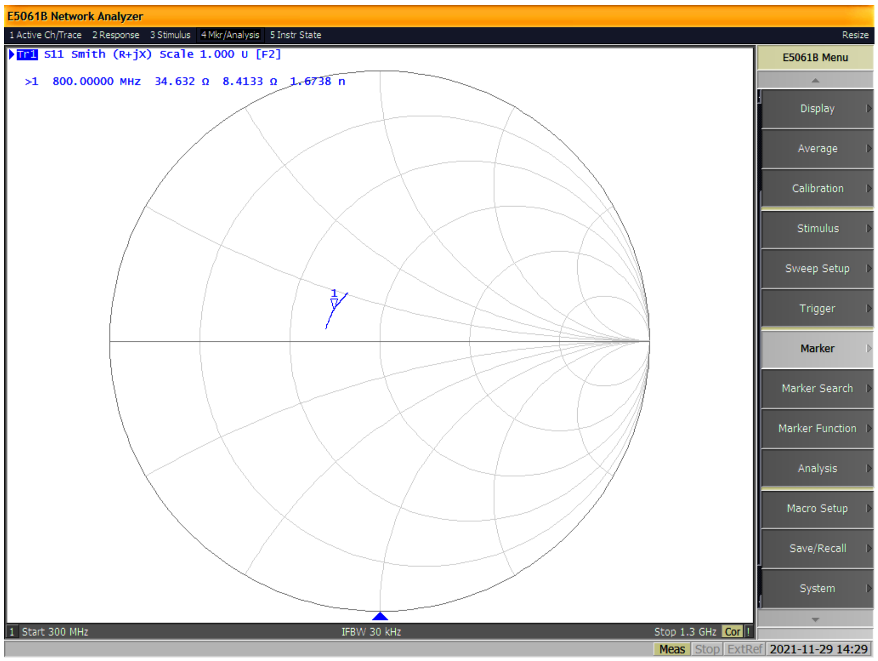


Figure 2: Image of Smith Chart plot of load impedance versus frequency for a range of 0.3 to 1.3 GHz, with the marker set at 800 MHz, which reads $Z_L = 34.632 + j8.4133\Omega$

2.2 Measurement Smith chart plot of load impedance versus frequency, de-embedded by 0.2 ns

Vector Network Analyzer (VNA) measurements are made at the calibration plane, which is typically the end of the cables attached to the VNA ports. Sometimes, there is an additional section of transmission line (e.g an extra cable) between the ends of the cables and the output ports of the device under test (DUT), and as a result the VNA measurement data includes the effects of the connecting feed lines. The concept of de-embedding undoes this extra "rotation". The figure 3 shows the Smith Chart Plot de-embedded by 0.2 ns.

De-embedding is typically achieved by rotating Z_L counter-clockwise (away from generator/towards the load) by the electrical length of the cable. Essentially, a virtual extra length is added to the line.

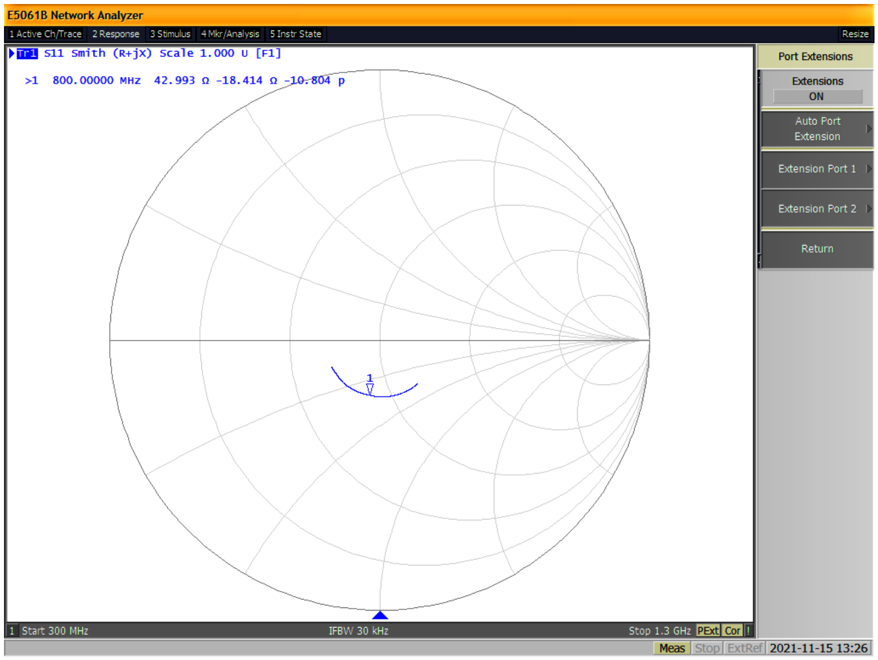


Figure 3: Smith Chart that is de-embedded by 0.2 ns, with the same frequency range and load as fig 2. This marker reads a value $Z_L^{'} = 42.993 - j18.414\Omega$

2.3 Equivalent electrical distance at 800 MHz associated with the 0.2 ns port extension

For this lab, $u_p = 3 \times 10^8$ m/s since the stub consists of air-filled coaxial lines ($\epsilon_r = 1$) and the frequency is $f = 800\text{MHz} = 8 \times 10^8\text{Hz}$. This allows us to compute the wavelength λ:

$$\lambda = \frac{u_p}{f} = \frac{3 \times 10^8 \text{ m/s}}{8 \times 10^8 \text{ Hz}} = 3.75 \times 10^{-1}\text{m} \tag{1}$$

De-embedding with $T = 0.2$ns means an additional physical distance of $cT = 6$ cm and hence an electrical distance of $\frac{cT}{\lambda} = \frac{cT}{c/f} = Tf = 0.2 \times 10^{-9} \times 8 \times 10^8 = 0.16\lambda$.

This means a de-embedding of $T = 0.2$ns rotates z_L anti-clockwise on the VSWR circle, towards the load, by 0.16λ. This operation can be performed on the Smith Chart, as seen in fig 4. $z_L^{'} = 0.92 - j0.4$ is the normalized impedance and can be multiplied by the characteristic impedance to obtain its value in Ω. $Z_L^{'} = 50(0.92 - j0.4) = 46 - j20\Omega$, which is close to the value obtained through the VNA, $42.983 - j18.41\Omega$.

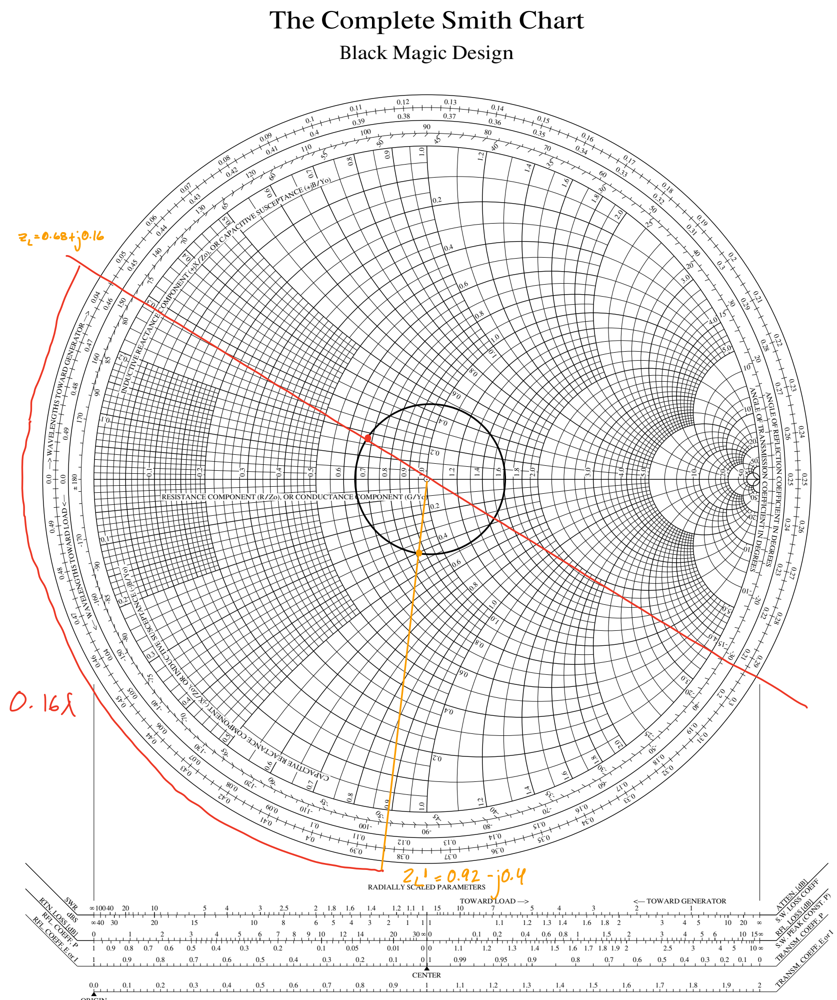


Figure 4: Effect on the Smith Chart by introducing a de-embedding of $T = 0.2$ns. The rotated (de-embedded) value of z_L is $z_L^{'} = 0.92 - j0.4$

3 Smith Chart Operations

All Smith Chart operations/results will be summarized in the following subsections, but the Smith chart can be found in Appendix A, in section 7.1.

3.1 Determination of z_A by rotating z_L by $d_0 = 3.4$ cm on the Smith chart, and transformation of z_A to y_A on the Smith chart.

We know $Z_L = 34.6 + j8.41$. In order to find z_A, we must normalize the impedance of the load, which is done by $z_L = \frac{Z_L}{Z_0}$ therefore:

$$z_L = \frac{34.6 + j8.41}{50\Omega} = 0.6868 + j0.1638 \tag{2}$$

In order z_A, we need to rotate z_L by an electrical length, $\hat{d_0}$. $\hat{d_0}$ can be computed using λ, and the physical distance corresponding to $\hat{d_0}$, which is $d_0 = 3.4$ cm:

$$\hat{d_0} = \frac{d_0}{\lambda} = 0.0907\lambda$$

With the known z_A, we can find y_A by rotating z_A by 0.25λ. This results in $y_A = 0.95 - j0.5$

Note that the order of these operations does not affect the final value of y_A, i.e., flipping (rotation of 0.25λ) and then rotation, or rotation and then flipping.

4 Double Stub Design Procedure

All Smith Chart operations/results will be summarized in the following subsections, but the Smith chart can be found in Appendix A, in section 7.1.

Looking at fig 1, we know that d_1 and d_0 are fixed. To match, we need $y_{\text{in}} = 1$, meaning at point C we wish to be on the $g = 1$ circle, and use a short-circuit stub of length l_2 to match whatever susceptance is present at point C. Since d_1 is fixed, all possible normalized admittances at point C ($g = 1$ circle) map to all possible normalized admittances at point B (rotated $g = 1$ circle). Hence, we have a circle of all possible values we must be on at point B such that the load L is matched in this network.

Now, approaching the problem from the other end, y_L is known and so is d_0. y_L can be transformed to y_A on the Smith Chart, using $\hat{d_0}$. Now, the only difference between y_A and y_B is another short-circuit stub of length l_1. We know all the possible points y_B can take (the rotated $g = 1$ circle) and we have some y_A whose conductance cannot be changed. Hence, the stub of length l_1 must be used to transform y_A to some point on the rotated $g = 1$ circle to solve our matching problem.

4.1 Design of the double stub matching network for the load provided using a Smith chart

To start we must first obtain the rotated $g = 1$ circle by $\hat{d_1}$:

$$\hat{d_1} = \frac{d_1}{\lambda} = \frac{0.038\text{m}}{0.375\text{m}} = 1.1 \times 10^{-1}$$

With the obtained y_A from the previous subsection, we must now find possible values of y_B. Since the real part of the admittance remains constant, we can move along the conductance circle from y_A until we find the intersections with the rotated $g = 1$ circle. These intersections will tell us the required change in the susceptance, and that helps us determine l_1 and $l_1^{'}$.

Subsequently, we use the VSWR circle from each respective intersection point, to find an intersection with our original $g = 1$ circle. There are two intersection points with the rotated $g = 1$ circle and two intersections with the original $g = 1$ circle, thus there are two possible fundamental solutions.

Using the calculations and operations on our Smith Chart, we were able to calculate the possible solutions:

Solution i	**Electrical length** (λ), $\hat{l_i}$	**Physical length (cm)**, l_i
1	0.375	14.1
2	0.082	3.08
$1^{'}$	0.46	17.3
$2^{'}$	0.297	11.1

Table 1: Corresponds to stub length solutions at 800 MHz in electrical lengths and physical lengths

All fundamental solutions are periodic, as the Smith chart is periodic (0.5λ), thus all general solutions are:

$$\hat{l_1} = 0.375\lambda + \frac{n\lambda}{2} = 14.1 + n \cdot 18.75\text{cm}$$
$$\hat{l_2} = 0.082\lambda + \frac{n\lambda}{2} = 3.08 + n \cdot 18.75\text{cm}$$
$$\hat{l_1'} = 0.460\lambda + \frac{n\lambda}{2} = 17.3 + n \cdot 18.75\text{cm}$$
$$\hat{l_2'} = 0.297\lambda + \frac{n\lambda}{2} = 11.1 + n \cdot 18.75\text{cm}$$

where $n \in Z$. Note that these solutions are designed to match at this frequency only.

5 Experimental Verification

5.1 Experimental determination of the final stub lengths for both fundamental solutions to achieve a match between the load and the line.

In order to find the experimental values, we moved the stubs sliders until we could identify the lengths where the Smith Chart plot was matched (marker at the origin, corresponding to $y_{in} = 1 + j0$). We can see that one set of general solutions can be regenerated by adding another 18.75 cm, which is equal to our $\frac{\lambda}{2}$ value calculated above. This verifies that every $\frac{\lambda}{2}$ there is another solution, as discussed in the previous section.

Solution	**Length (cm)**
l_1	9.5
l_2	5.5
$l_1^{'}$	12.3
$l_2^{'}$	17.4
$l_{1,\text{rep}}$	28.5
$l_{2,\text{rep}}$	24.9

Table 2: Corresponds to stub length solutions found experimentally at 800 MHz in cm

5.2 Measurement Smith chart plots of the matched load for both fundamental solutions

In the figures below, we have our Smith Chart plots of the general solutions calculated theoretically in figure 5, and experimentally in figure 6

5.3 Discussion of obtained results

Our theoretical values were off the matched point, which is the origin point in the Smith chart. There were a potential number of reasons for that:

- Inaccuracies in the Smith chart: The Smith chart does not plot all r/x (or g/b) values. This discretization is not significant but does contribute to the overall error of design.
- The VSWR generated by the intersection of $r = 0.95$ and rotated $g = 1$ circle was extremely small (as it was close to origin), and meant that a small misreading would propagate the error significantly.
- The slider on both the stubs had some length that was not negligible. Once the theoretical values were computed, there was some doubt about where the starting point of the stub slider was. The stub slider was $\approx$ 2cm and that error must be factored into the final error.

- Human error: There was slipping while using the compass and protractor, as well as other human errors such as interpolating g/b values. These errors contributed significantly to the design process.

Having encountered a significant error in all calculated lengths, we decided to revisit the Smith chart operations, but this time on a digital platform, to achieve a higher precision. The corresponding Smith chart can be found in Appendix B, 7.2.

Solution i	**Electrical length** (λ), $\hat{l}_i$	**Physical length (cm)**, l_i
1	0.324	12.2
2	0.234	8.78
$1'$	0.454	17.0
$2'$	0.47	17.6

Table 3: Corresponds to stub length solutions at 800 MHz in electrical lengths and physical lengths

These values are much closer to the experimental values in table 2. Smith charts obtained for these values would have witnessed the marker closer to the origin, and hence would have been better at the matching problem.

6 Discussion of Standing Wave Ratio and Bandwidth

6.1 Measurement plot of the final VSWR and measurement of bandwidth for both fundamental solutions

The VSWR plots can be found in figures 7 and 8.

6.2 Discussion of how the measured results with theoretical values

Recall, the following relationship between VSWR and $|\Gamma|$:

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} \tag{3}$$

and bandwidth here is defined as:

$$\text{BW} = \{ \ \omega \ \exists \ \text{VSWR}(\omega) \leq 2 \ \} \tag{4}$$

Using equation 3, $|\Gamma|$ can be derived. For a VSWR = 2, $|\Gamma| = 1/3$. This has a direct correspondence to the power transmitted to the load, as to achieve maximum power, VSWR = 1, i.e., $|\Gamma| = 0$, which physically makes sense. Recall, the power transmitted to the load can be computed by:

$$P_L = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2) = P_{\max}(1 - |\Gamma|^2) \tag{5}$$

Plugging in a $|\Gamma| = 1/3$ corresponds to $P_L = P_{\max}(1 - \frac{1}{9}) = \frac{8}{9}P_{\max}$, meaning around 90% of the input signal power is transmitted to the load. Now, circling back to the definition of the bandwidth, in equation 4, BW corresponds to a range of frequencies that transfer 90% of the power to the load. Note, at $|\Gamma| = 1/3$, $20\log(1/3) \approx -10dB$, and any value less than 1/3 would further decrease the logarithmic value. Hence, to maintain a power transfer of 90%, $\Gamma_{\mathrm{dB}} = 20\log(\Gamma_L) \leq -10\mathrm{dB}$.

The BW is captured using the VNA, as seen in figures 7 and 8, and is summarized in table 4

Stub Pair	Operating Frequency (MHz)
Short	[744.21, 884.90]
Long	[740.19, 878.88]

Table 4: Bandwidths for stubs designed with different lengths

While both stub lengths do not have a significant difference in their respective bandwidths (operating frequency), the longer stub has a much sharper change to frequencies, whereas the shorter stub does not shoot up after its upper bound mentioned in the table. So, in summary, the shorter stub has a higher bandwidth and should be used in scenarios where the fabricated design (stub) has to be functional in a range of frequencies. On the other hand, the longer stub would be useful if frequencies outside a particular range are desired to be cut off.

To understand why this happens, we can have a look at the input impedance (Z_{in}) of a transmission line with characteristic impedance (Z_0) with a load (Z_L) attached:

$$Z_{\mathrm{in}} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \tag{6}$$

In case of our stubs, $Z_L = 0$ (short circuit) and hence (6) simplifies to:

$$Z_{\mathrm{in}} = Z_0 \frac{jZ_0 \tan(\beta l)}{Z_0} = Z_0 j \tan(\beta l) \tag{7}$$

Where $\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{u_{\mathrm{phase}}}$

Using the expanded version of β, (7) further simplifies to:

$$Z_{\mathrm{in}} = Z_0 j \tan(\beta l) = Z_0 j \tan(\frac{2\pi f l}{u_{\mathrm{phase}}}) \tag{8}$$

Now, we have an expression for Z_{in} as a direct function of the length (l) and frequency (f), which is what we want to discuss in this section. Having a large l leads to larger argument for the tan function, which means a slight change in frequency results in a larger change for Z_{in}. Hence, as seen in the discussion before, a larger stub length has a sharper bandwidth than a short stub length. Note that Y_{in} is the reciprocal of Z_{in}, so undergoes the same effects when l or f are changed.

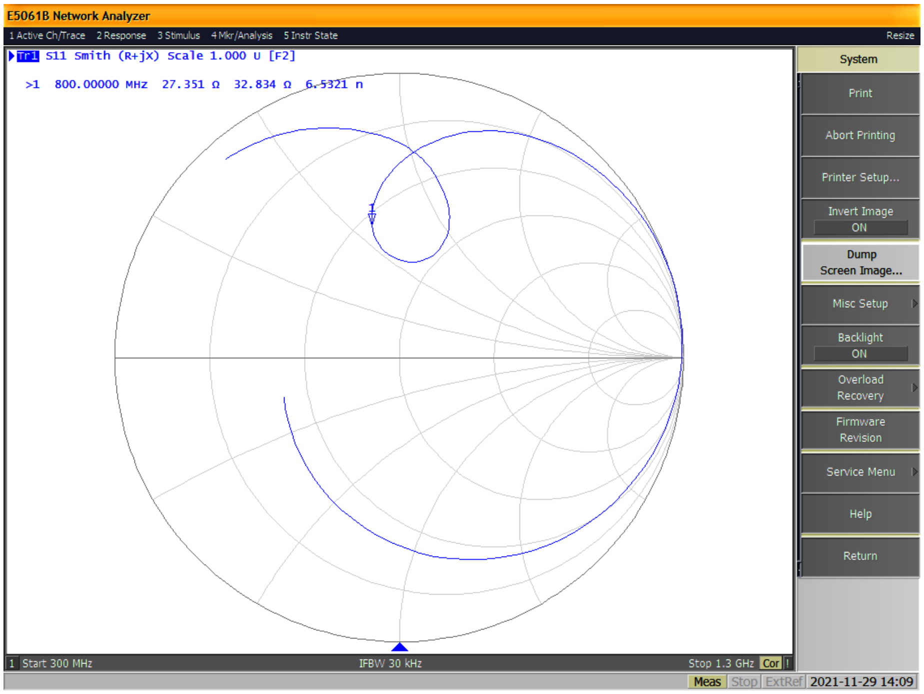
7 Appendices

7.1 Appendix A: Initial Smith Chart

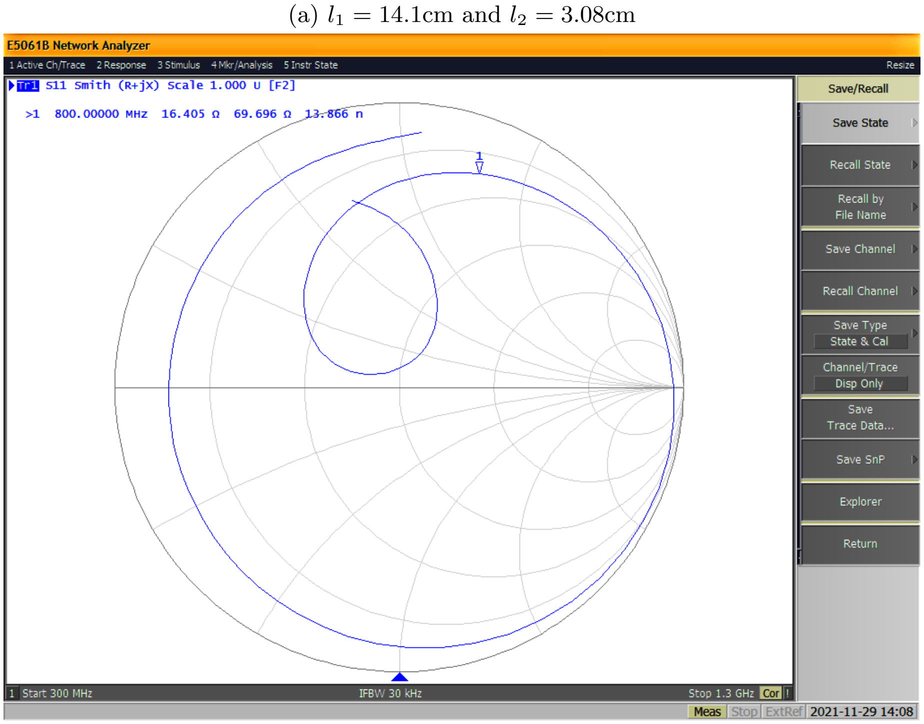
Figure 9 depicts the initial hand-drawn Smith chart.

7.2 Appendix B: Revised Smith Chart

Figure 10 shows the revised digital Smith Chart, with slightly more precise operations.

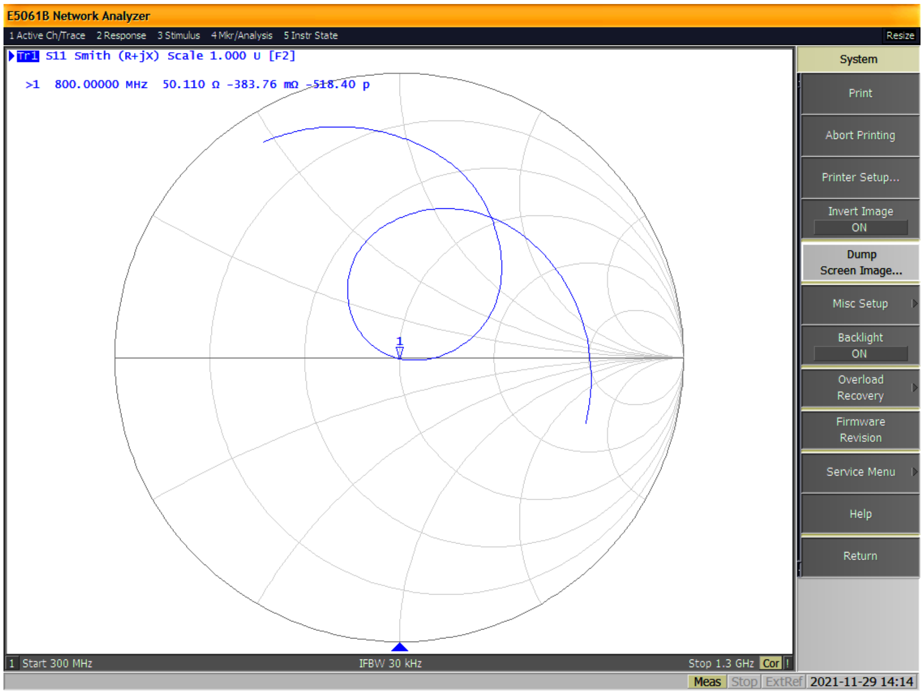


(a) $l_1 = 14.1\text{cm}$ and $l_2 = 3.08\text{cm}$

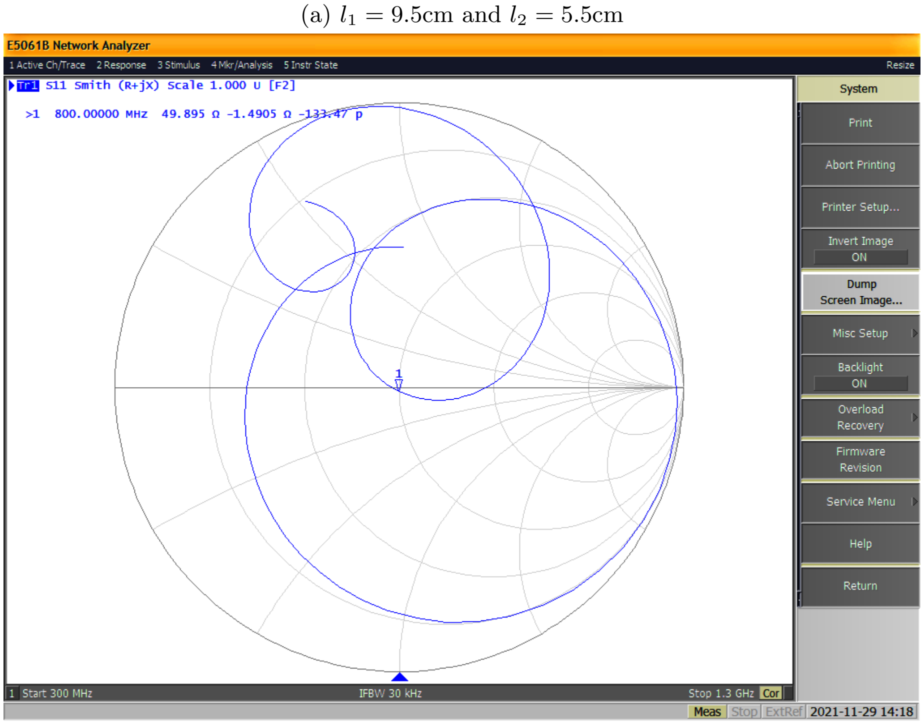


(b) $l_1' = 17.3\text{cm}$ and $l_2' = 11.1\text{cm}$

Figure 5: Measurement Smith Chart plots for determined theoretical values



(a) $l_1 = 9.5$cm and $l_2 = 5.5$cm



(b) $l_1 = 28.5$cm and $l_2 = 5.5$cm

Figure 6: Measurement Smith Chart plots for experimental values

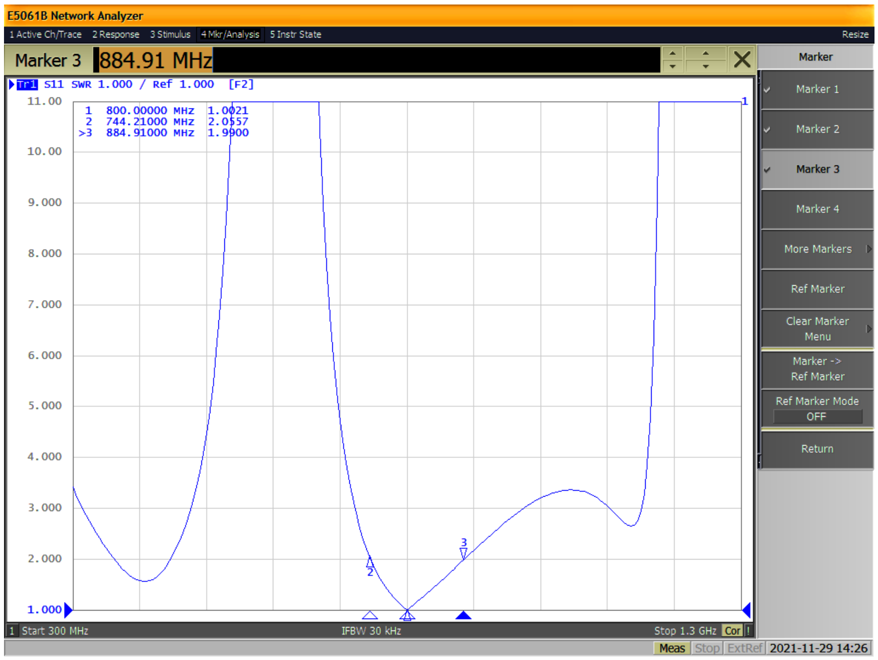


Figure 7: VSWR plot for shorter stub length

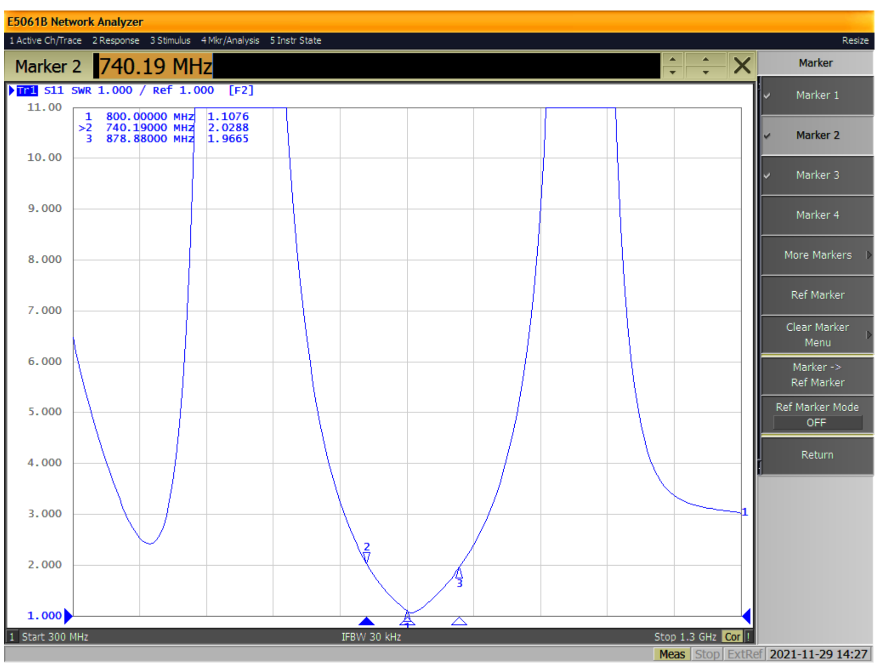


Figure 8: VSWR plot for longer stub length

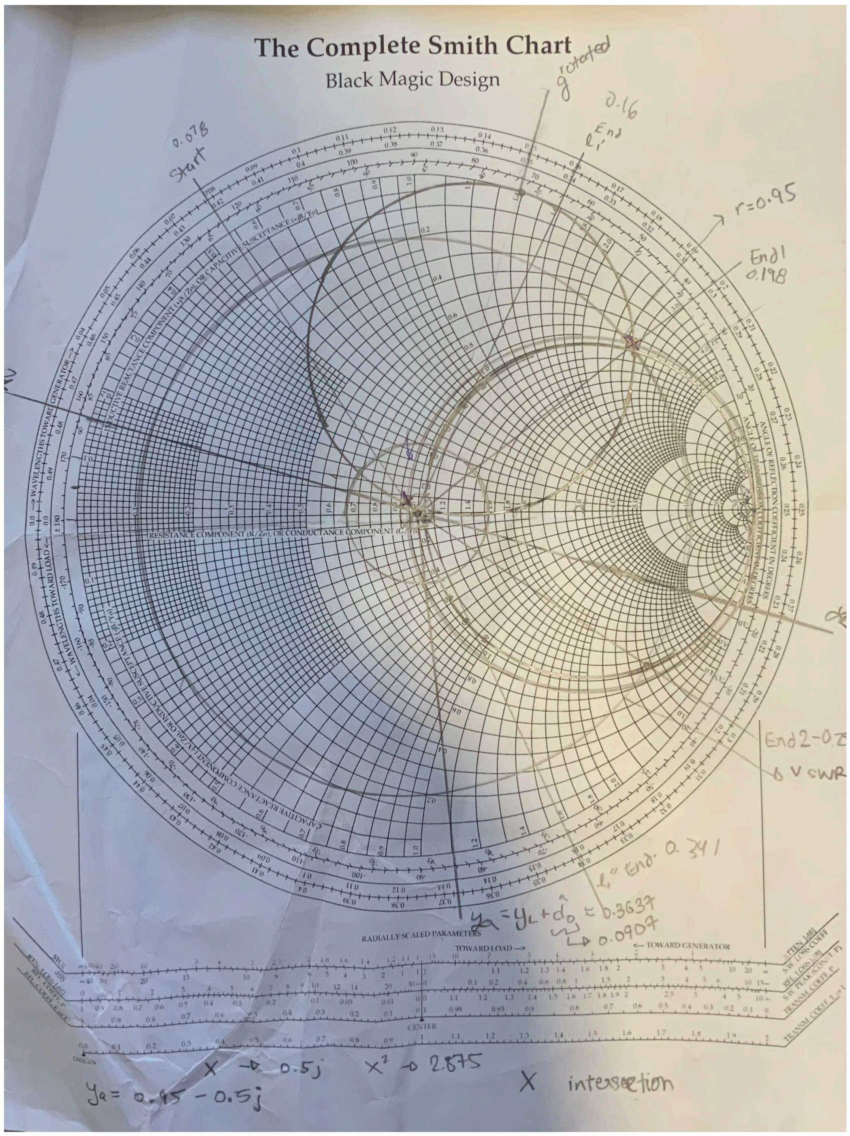


Figure 9: Image of our Smith Chart that was completed during the lab, including calculations

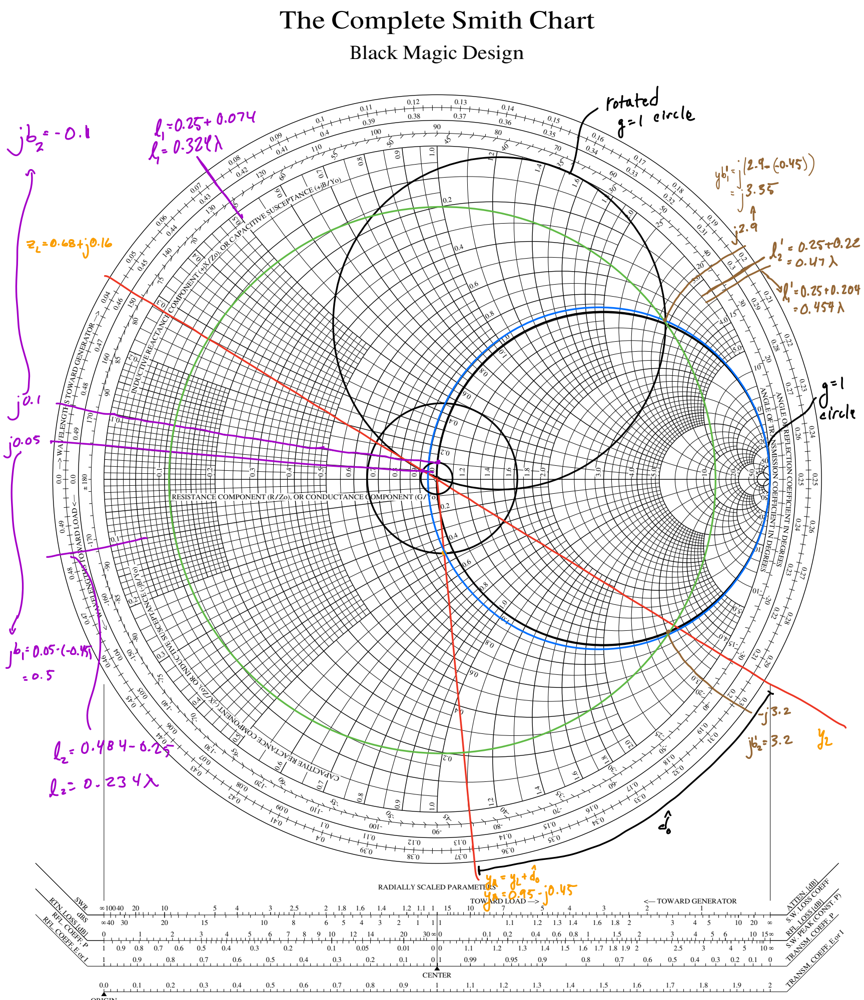


Figure 10: Image of our revised Smith Chart that was completed after the lab, including all calculations