<u>ECE320: Fields and Waves</u>

Laboratory 1: Waves On Transmission Lines

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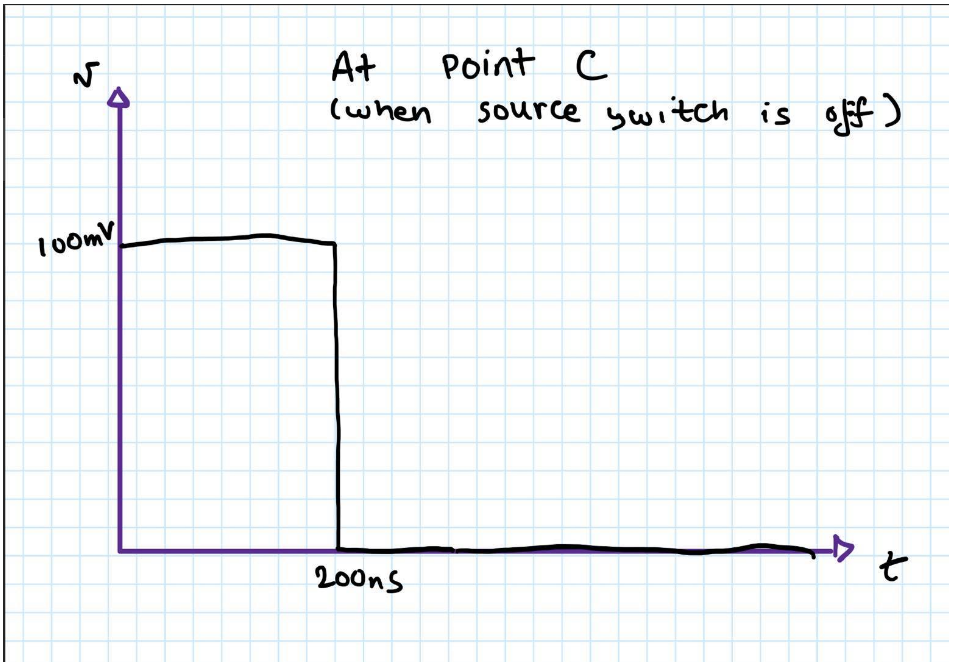
3.2 Determination of the Characteristic Impedance Z_0

I. *Sketch of the waveform at point C when the line is terminated in Z_0.*



II. *Z_0 found using the variable load.*

Z_0 is the point where there are no reflections detected at the source (point B/C) for some Z_L, i.e., the line is matched. For this experiment, this was determined to be $Z_L = 50\Omega$. When the load impedance was set lower to 50 Ω, there was a negative reflection coefficient, and when it was set to be more than 50 Ω, there was a positive reflection.

3.3 Determination of Characteristic Impedance Using $\frac{v_1(t,0)}{i_1(t,0)}$

I. $Z_0 = \frac{v_1(t,0)}{i_1(t,0)}$ *calculated using Ohm's law and measured voltages.*

The question asks to determine the characteristic impedance using Ohm's law, at the start of the transmission line ($z = 0$, at point C), when the resistance between B and C is 100Ω.

Using figure 0, it can be determined that the voltage drop between B and C is 99.375 mV. This means there is a $\frac{99.375\times10^{-3} V}{100\Omega} = 0.99375mA$current flowing into the transmission line (starting at point C, i.e., $i_1(t,0)$). The voltage at point C (which was determined using cursors in figure 1) was determined to be 49.33mV (i.e., $v_1(t, 0)$). Using the equation above, the characteristic impedance is:

$$Z_0 = \frac{49.33mV}{0.99375mA} = 49.64\Omega$$

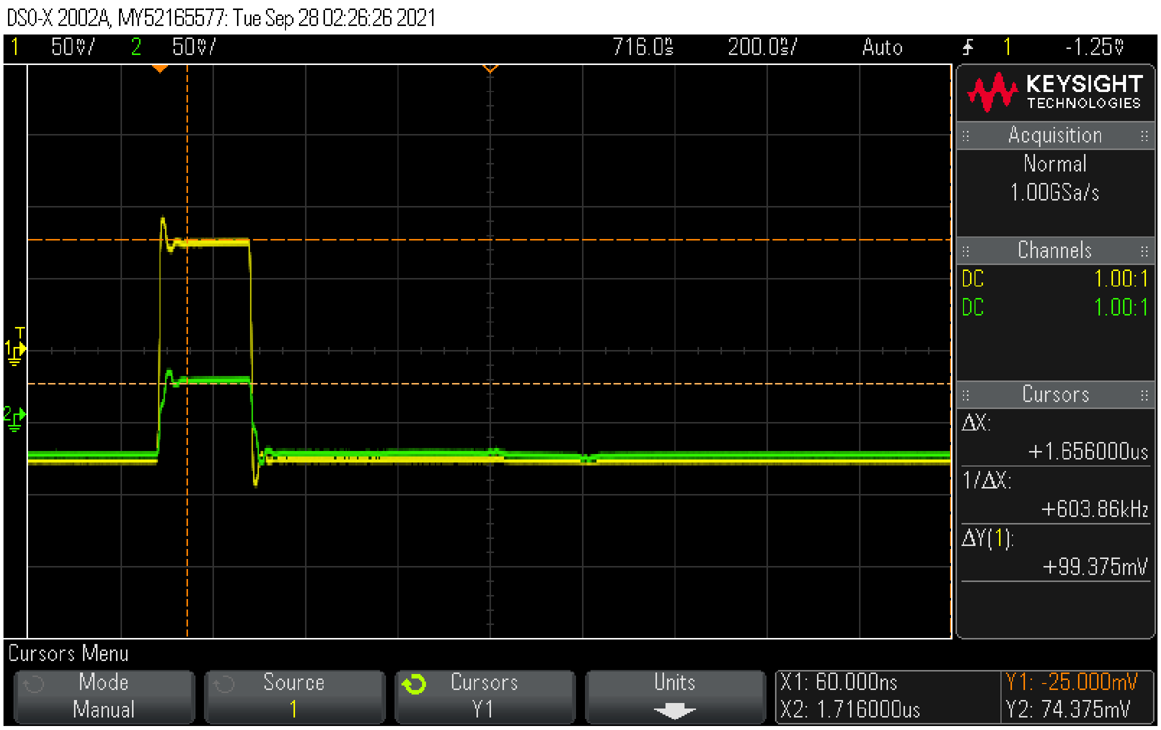


Figure 0: v vs t graph at points B and C for R_L = 50 Ω and R_S =150Ω

3.4 Observation of Travelling Waves

I. *Measurement v vs t graphs at C, D, E, and F for R_L = 50 Ω.*

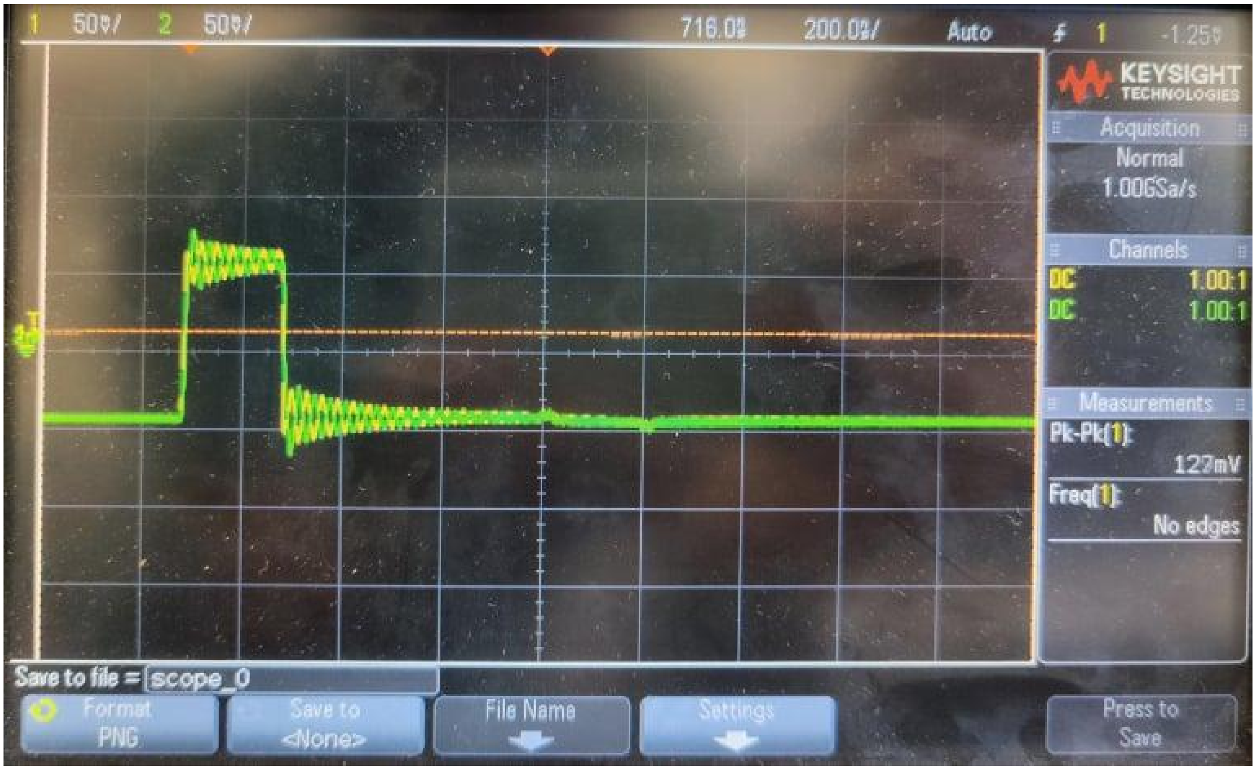


Figure 1: v vs t graph at point C for R_L = 50 Ω

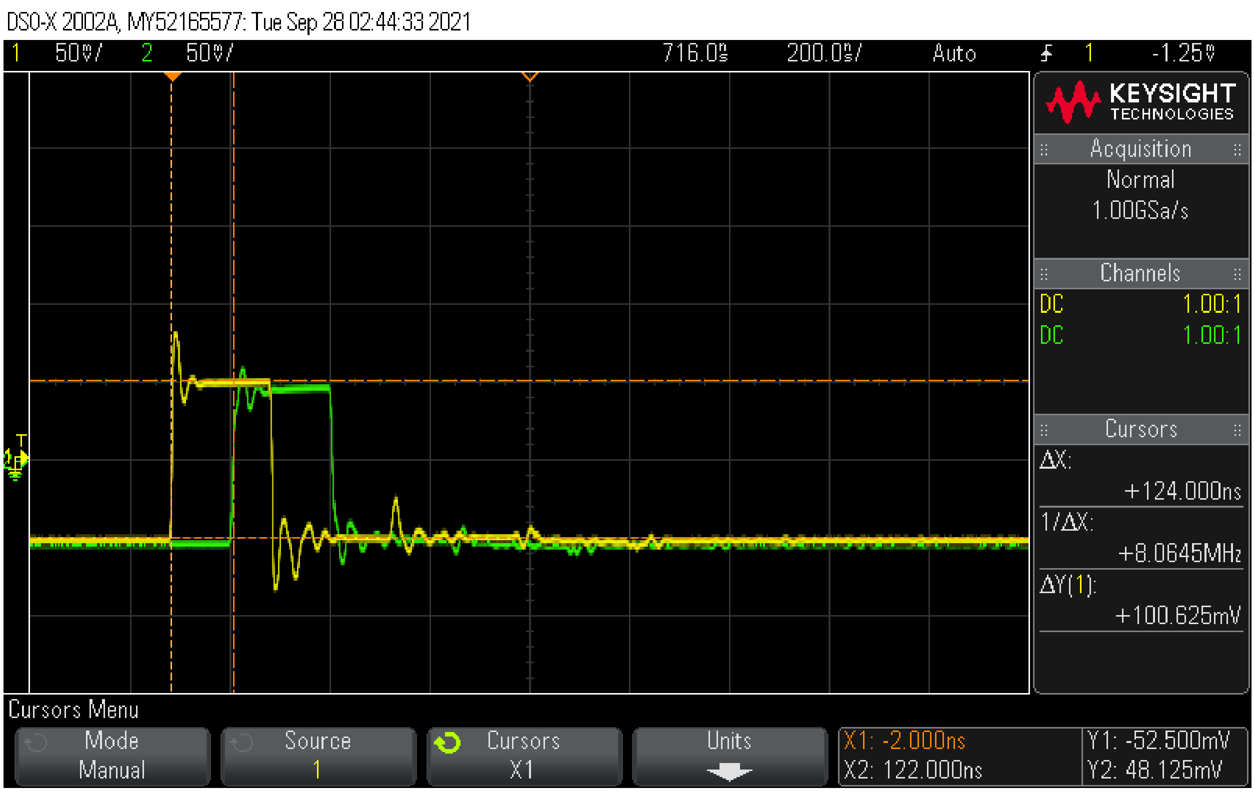


Figure 2: v vs t graph at point D for R_L = 50 Ω

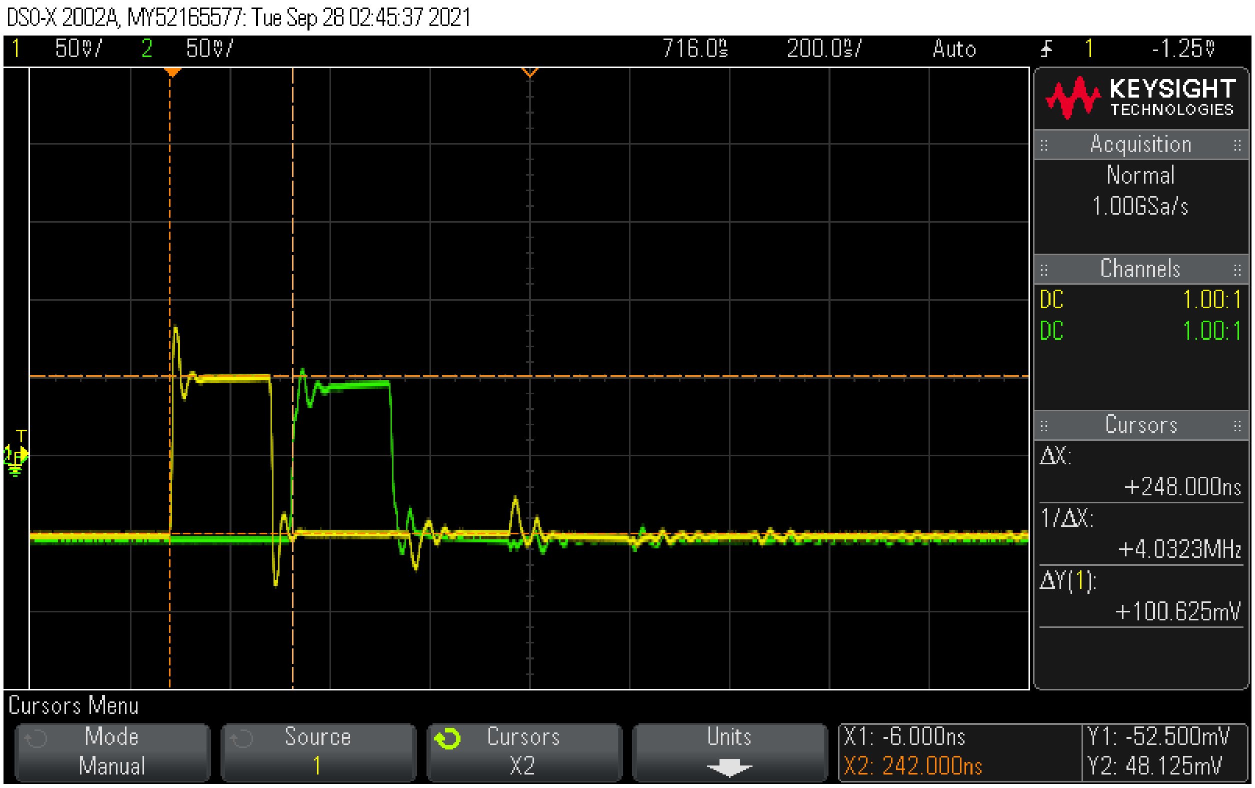


Figure 3: v vs t graph at point E for R_L = 50 Ω

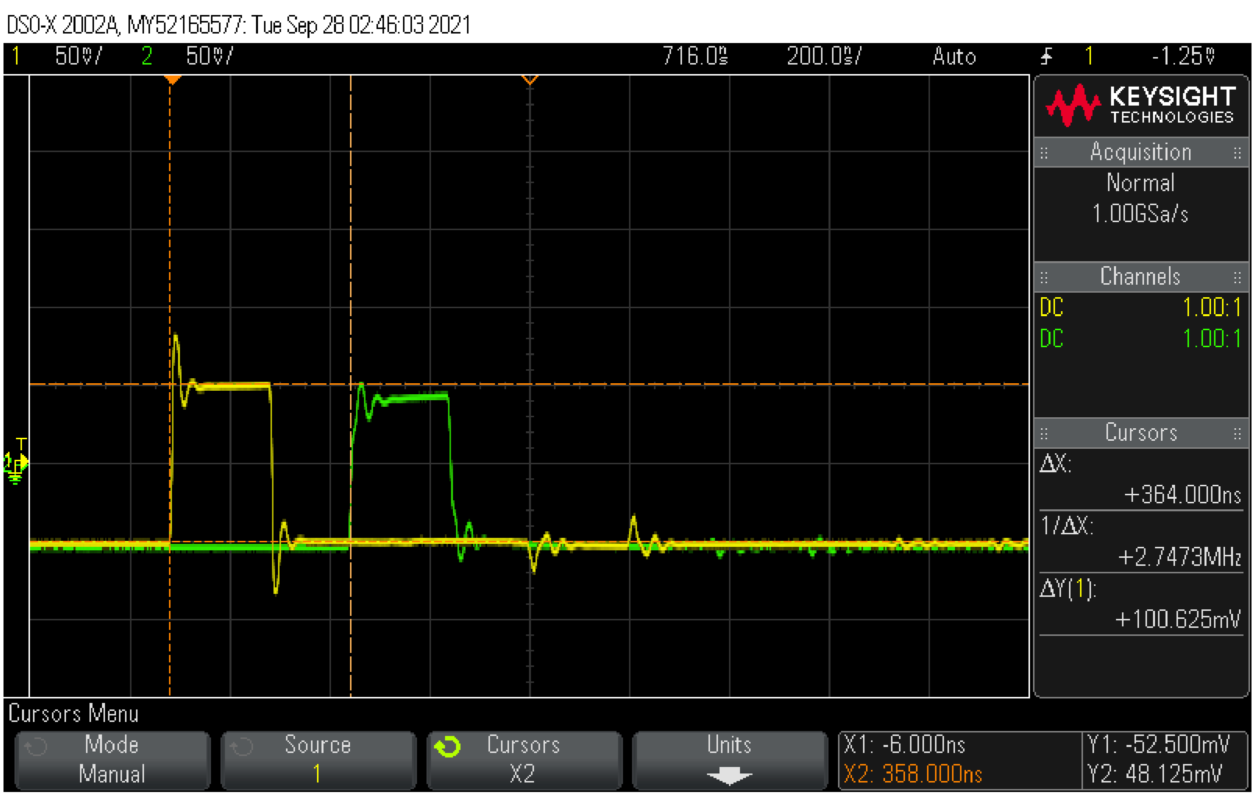


Figure 4: v vs t graph at point F for R_L = 50 Ω

II. *Recorded time delay Δt at points D, E, and F relative to the input signal.*

Using the cursors (x1/x2) values for figures 2-4, the following are the time delays:

Distance	Time Delay
30m (Point D)	124 ns
60m (Point E)	248 ns
90m (Point F)	364 ns

3.5 Determination of Velocity of Propagation

I. *Calculated average velocity of propagation v_{avg} and relative permittivity ε_r.*

Using, $v = displacement/time$:

- $v_{avg} = (v_D + v_E + v_F)/3$
- $v_D = \frac{30m}{124*10^{-9}s} = 241935485\ m/s$
- $v_E = \frac{60m}{248*10^{-9}s} = 241935485\ m/s$
- $v_F = \frac{90m}{364*10^{-9}s} = 247252747\ m/s$
- $v_{avg} = (241935485 + 241935485 + 247252747)/3$
- $v_{avg} = 2.44 * 10^8 m/s$

Relative Permittivity:

- $u_p = \frac{c}{\sqrt{\varepsilon_r}}$. Plugging in the values =>$\varepsilon_r = (\frac{3 \times 10^8 m/s}{2.44 \times 10^8 m/s})^2 = 1.51$

II. *Theoretical bounce diagram and corresponding v vs t graphs at C, D, E, and F. Compare with Section 3.4 measurement results.*

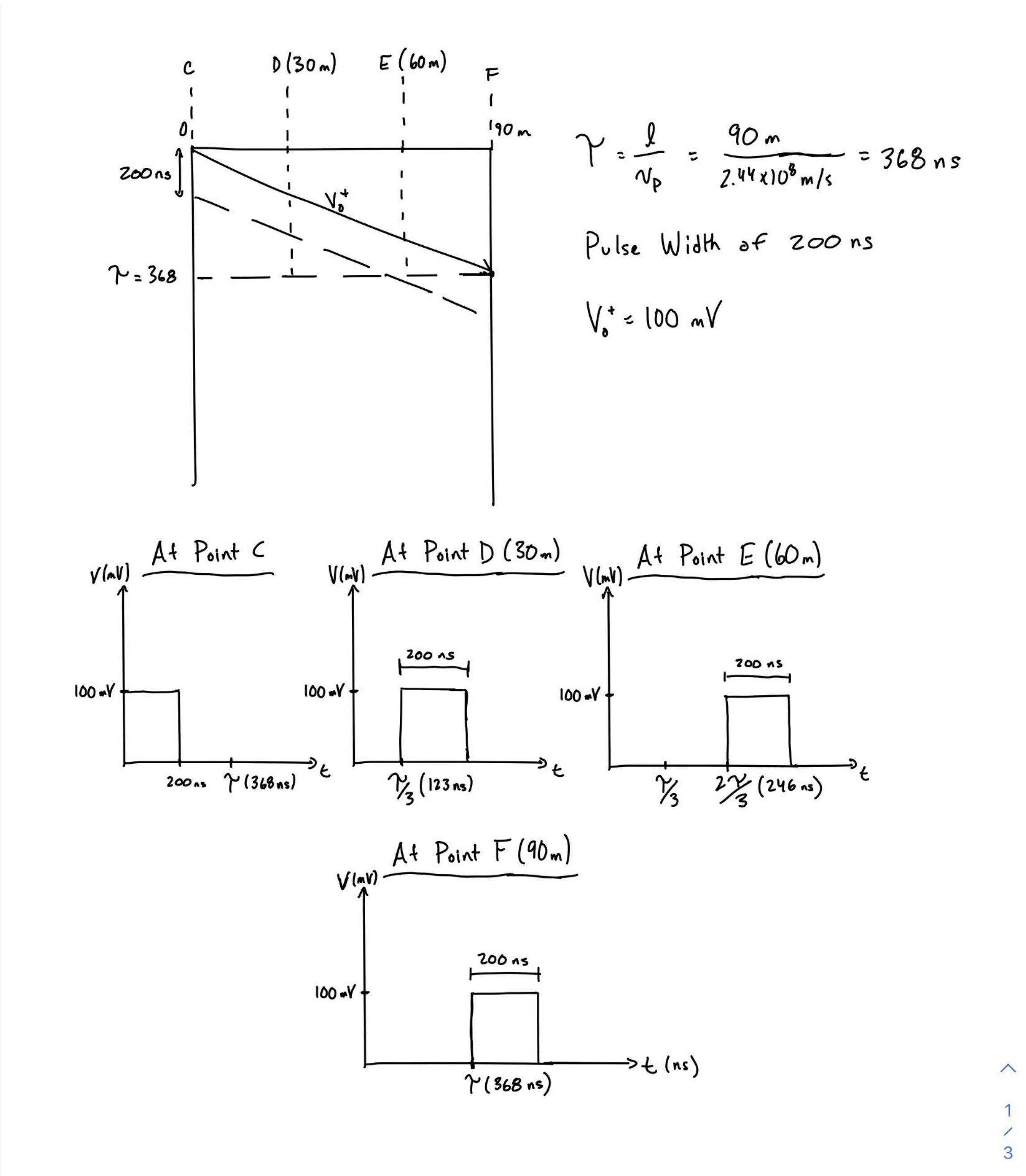


Figure 5: *Theoretical bounce diagram and corresponding v vs t graphs at C, D, E, and F*

3.6 Simple Reflection

I. *Compare calculated and measured Γ_L.*

Theoretical Value:

- $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$
- $\Gamma_L = \frac{100-50}{100+50}$
- $\Gamma_L = 1/3$

Empirical Value:

- Using figure 5, recognize the first pulse is the input wave from the source with an amplitude of 100mV.
- The second pulse is the reflected wave received at the source with an amplitude of 32.5 mV (Δyin figure 5)
- This leads to our reflection coefficient:
  - $\Gamma_L = V_0^- / V_0^+ = 32.5mV/100mV = 0.325$

The theoretical $\Gamma_L$value is ⅓ or 0.33 and the empirical value is 0.325.

II. *Measurement v vs t graphs at C and F for R_L = 100 Ω.*

Note: Channel 1 is the voltage at Point B in the following graphs

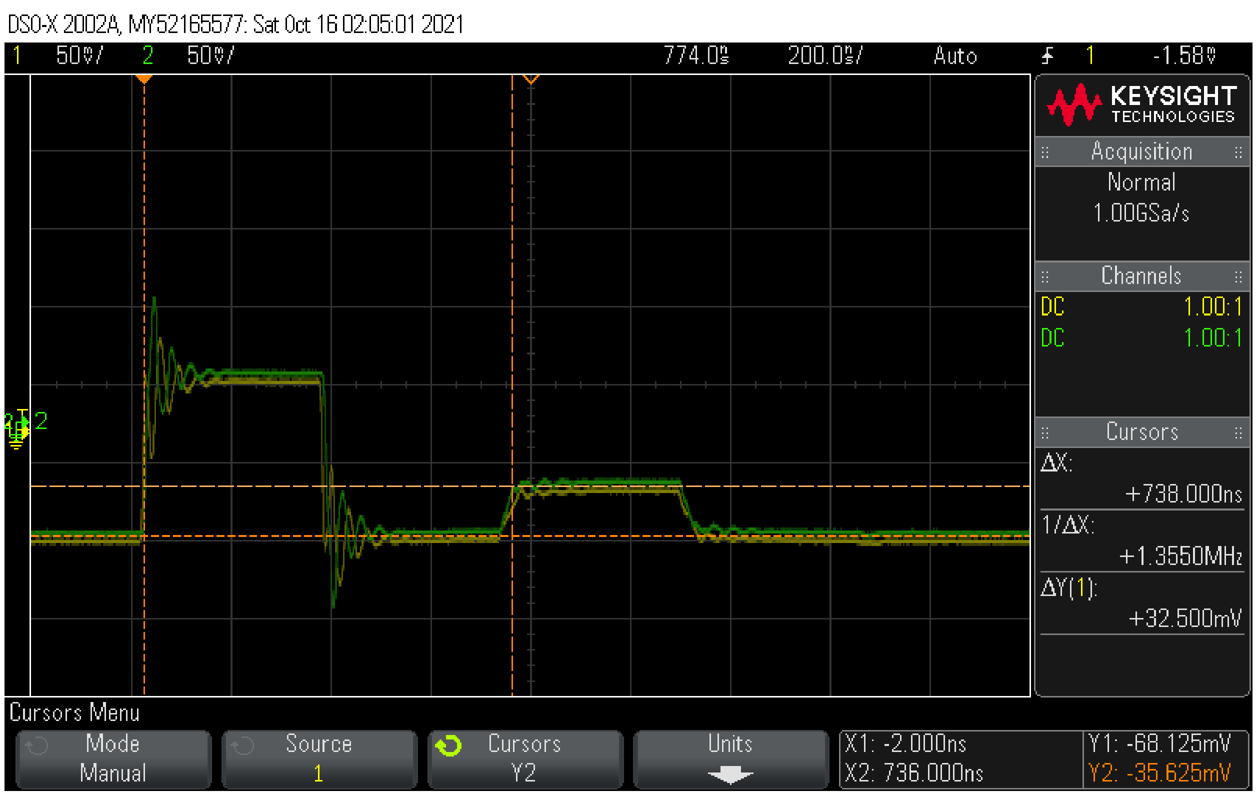


Figure 6: v vs t graph at point C for R_L = 100 Ω

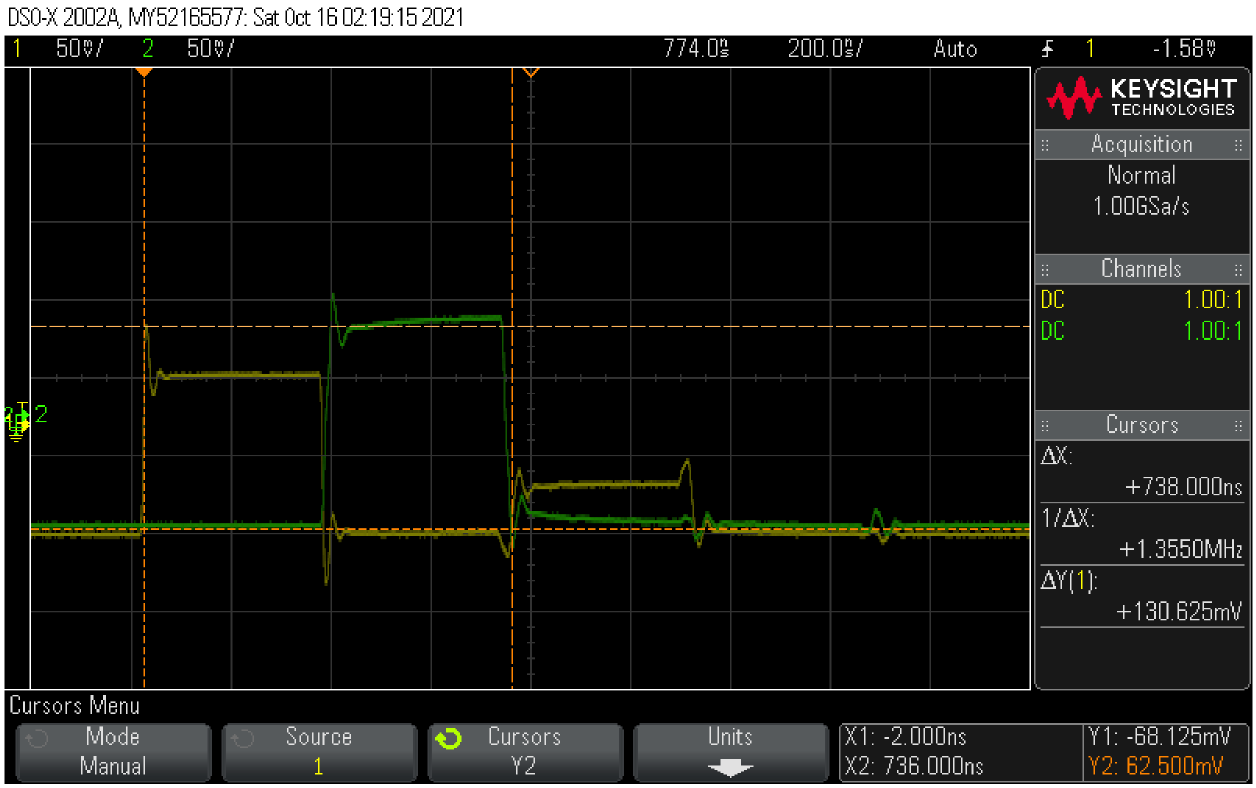


Figure 7: v vs t graph at point F for R_L = 100 Ω

III. *Discuss the relationship between the pulses at C and F.*

Recall, Port C is the start of the transmission line $(d = L = 90m)$ and Port F is the point at the load. While Γ_L was computed in the previous question, $\Gamma_S = 0$ here because the source resistor has been turned off.

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At time t = 0, a pulse of amplitude 100mV is sent out by the wave generator with a pulse width of around 360ns. Since the source resistor has been turned off, there is no voltage drop between ports B and C, as confirmed in figure 5. This pulse is the first pulse witnessed in both figures 5 and 6.

With a $\Gamma_L = 1/3$, the expected amplitude of the reflected wave (which will be detected at port C) is around $1/3 \times 100mV \simeq 33mV$. This is yet again confirmed in figure 5. However, port F can be thought of as what the loads see into the boundary. The load senses a $V_{eff} = 100mV + 33mV$, where the first term comes from the pulse provided, and the second term comes from the reflected component. This justifies why there is a voltage of around $134mV$ at the point F in figure 6.

IV. *Theoretical v vs d graphs at $t = T/2$, T, $3T/2$, and $2T$ where T = pulsewidth.*

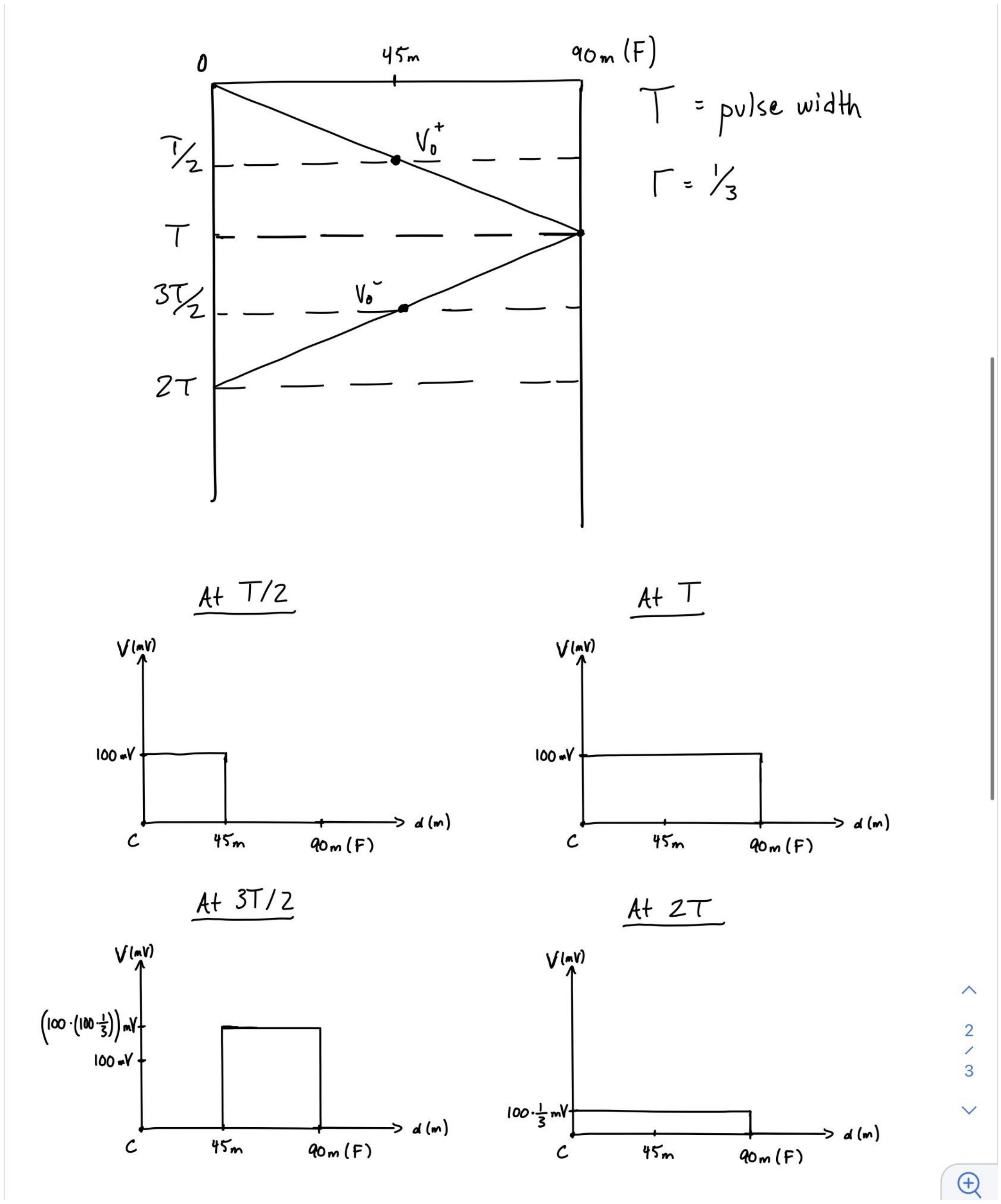


Figure 8: Theoretical v vs d graphs at $t = T/2$, T, $3T/2$, and $2T$ where T = pulsewidth

V. *Discuss the pulse propagation along the line with a mismatch at the load.*

The pulse is sent out at time t = 0s, and propagates towards the load. At the end of the transmission line, because of the mismatch, there will be a reflection from point F (at the load). This is to ensure the boundary conditions are met, as there cannot be an imbalance of voltage which violates Ohm's law, thus reflections take place.

The reflection coefficient is determined in part i of this section. This helps compute how much of the wave is reflected to the source. This phenomenon will repeatedly take place for any signal travelling from the source to the load.

Figure 5 & 6 show a reflection being received at point B/C at t = 738ns ~ 740 ns after the pulse is sent out by the source. Using v_{avg} calculated above and L = 90m, we can confirm that the experimental value matches the theoretical value:

- $velocity = \frac{distance}{time}$ thus $time = \frac{distance}{velocity}$
- $time_{theoretical} = \frac{2*L}{velocity_{avg}} = \frac{180m}{2.44*10^8 m/s} = 737.7ns$

3.7 Multiple Reflections

I. *Measurement v vs t graphs at C and F for R_{SOURCE}= (50 + 100) Ω and R_L = 20 Ω for pulse widths of T and $10T$.*

Note: In the following graphs, channel 1 is point B (which was = to point C so far because the load switch was OFF), the voltage at point C was equal to 50 mV (measured empirically) after the 100Ω the resistor switch is turned on.

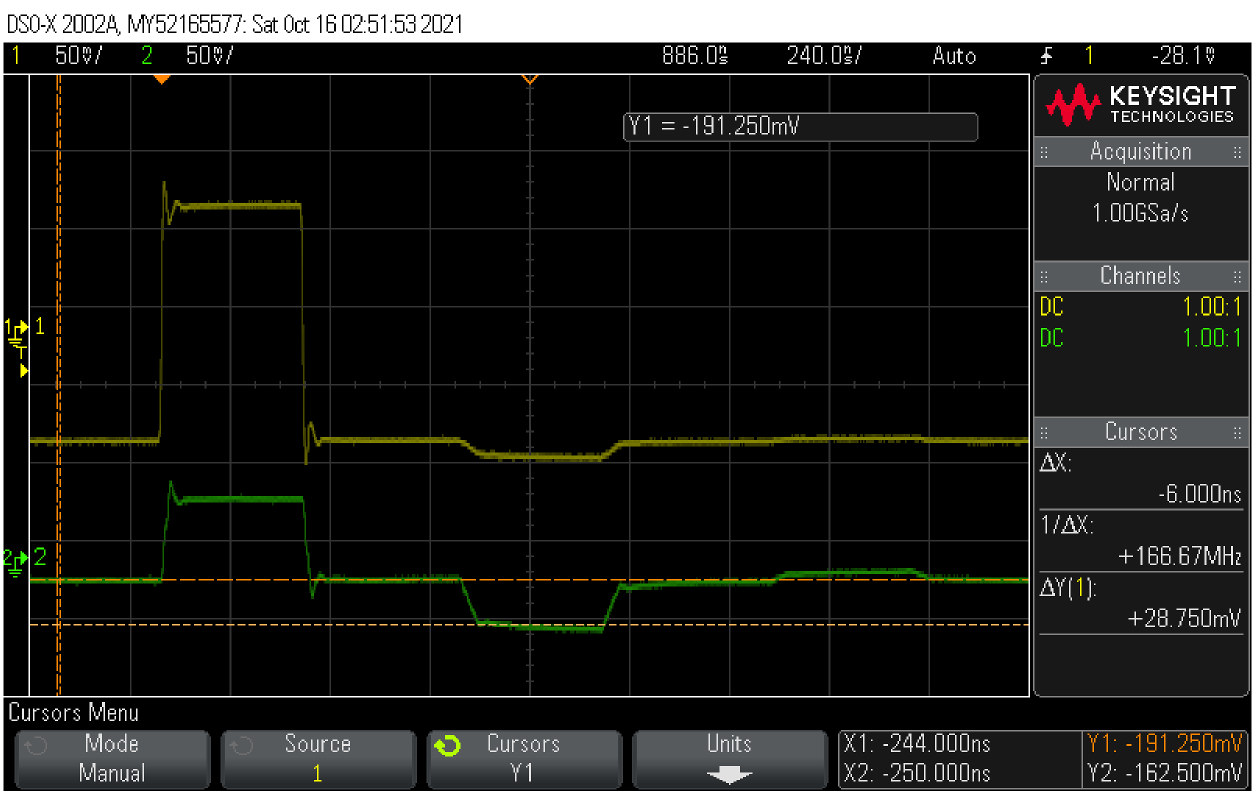


Figure 9: v vs t graph at point C at pulse width of T for R_{SOURCE}= (50 + 100) Ω and R_L = 20 Ω

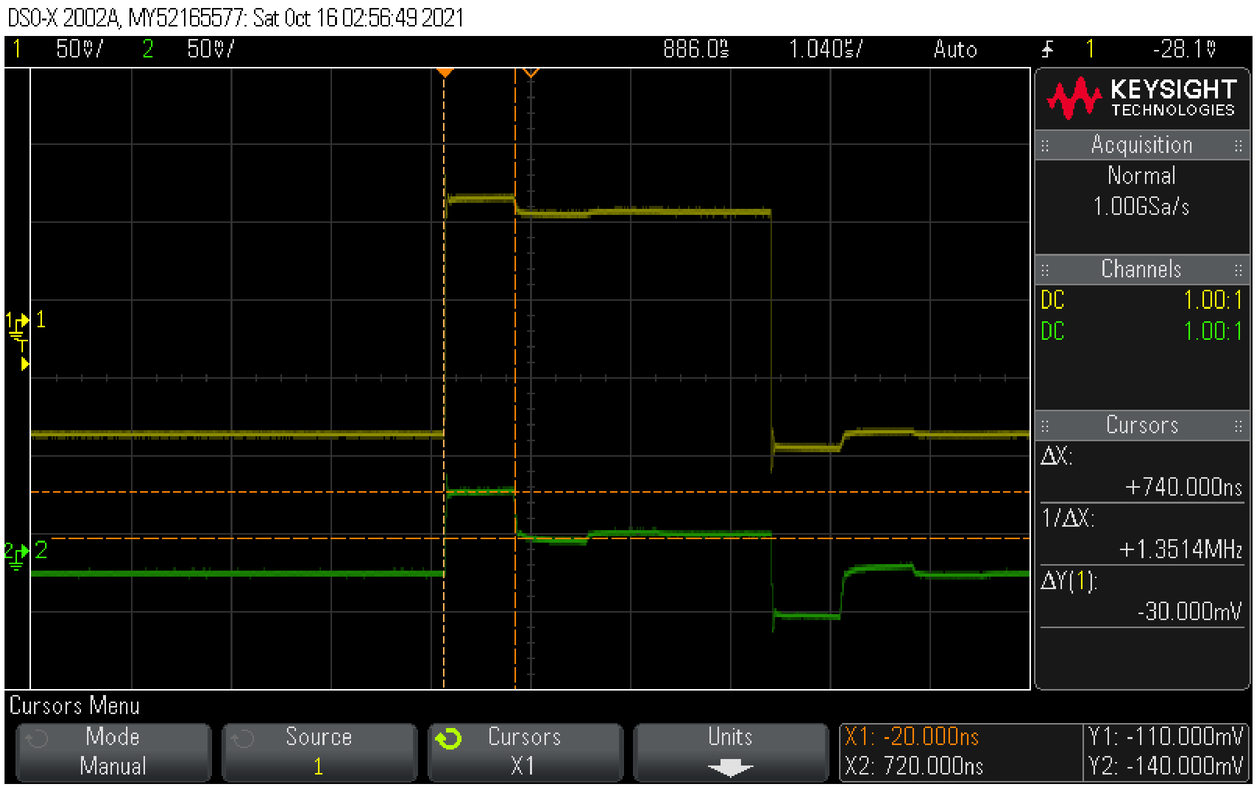


Figure 10: v vs t graph at point C at pulse width of 10T for R_{SOURCE}= (50 + 100) Ω and R_L = 20 Ω

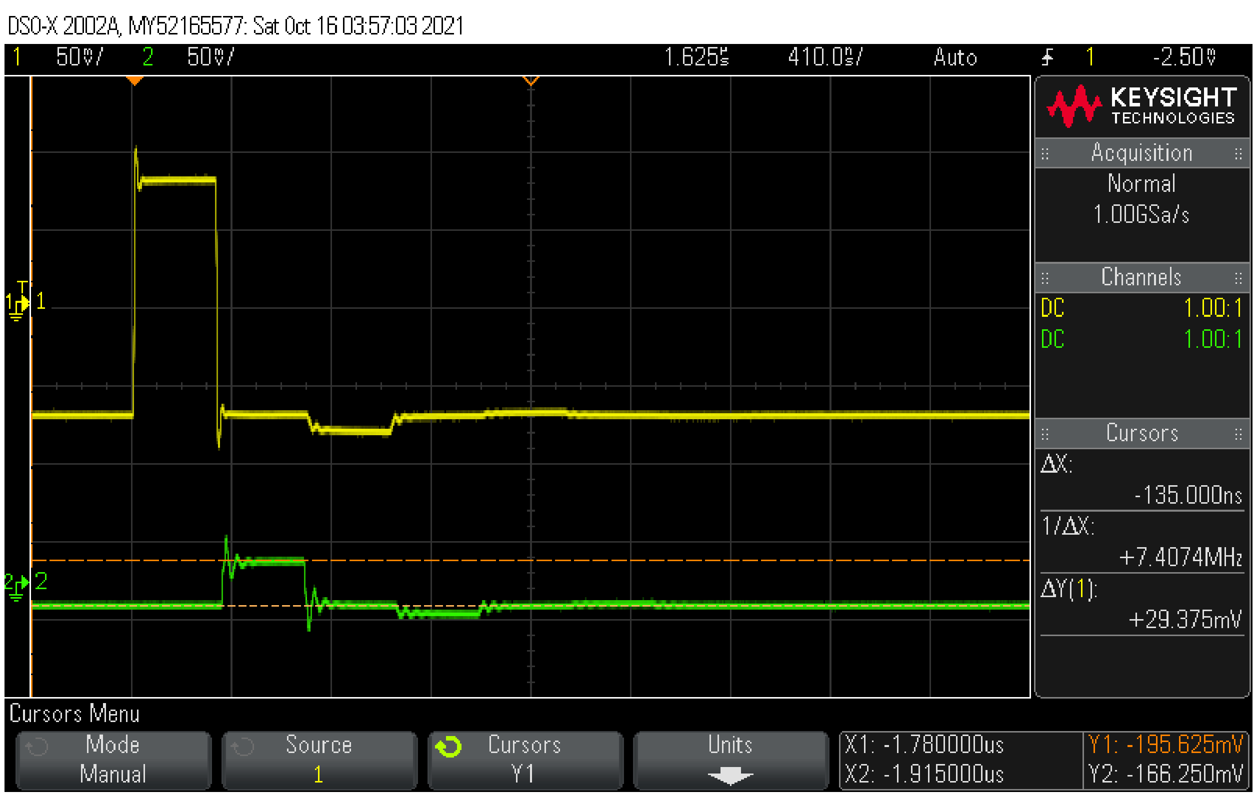


Figure 9: v vs t graph at point F at pulse width of T for R_{SOURCE}= (50 + 100) Ω and R_L = 20 Ω

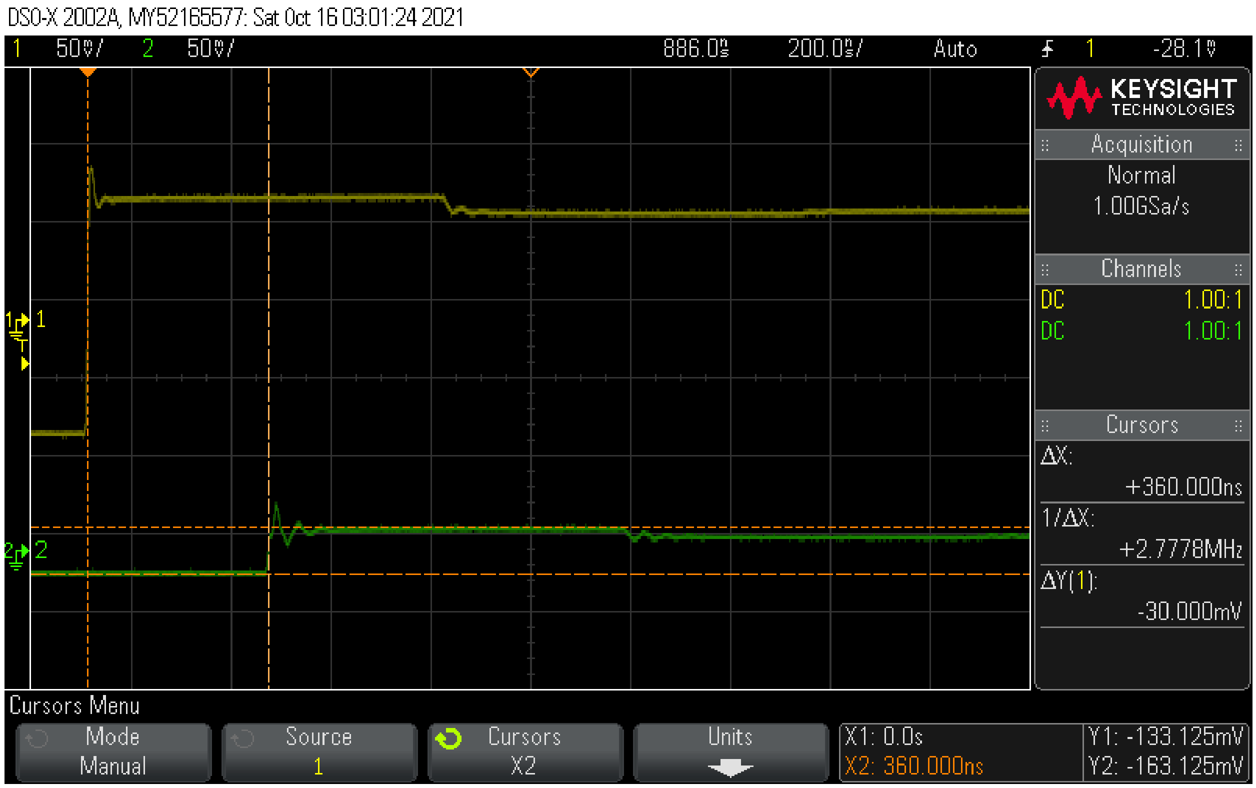


Figure 11: v vs t graph at point F at pulse width of 10T for R_{SOURCE}= (50+100) Ω and R_L = 20 Ω

II. Calculated and measured Γ_S and Γ_L.

Theoretical Values:

1. Reflection coefficient at the source

$$\Gamma_S = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_S = \frac{150-50}{150+50}$$

$$\Gamma_S = 1/2$$

2. Reflection coefficient at the load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{20-50}{20+50}$$

$$\Gamma_L = \ -3/7$$

Measured Value: these were measured by looking at the Δy values in figure 7 and 9.

For point C, theΔyvalue represents the amplitude of the reflected component (from the load to the source) superimposed with another reflected component (from the source to the load).

For point F, the Δyvalue represents the voltage at the load, which should be the superposition of the pulse sent towards the load (50 mV) and the component that did not get reflected at the load due to the boundary condition. The measured values of Γ_L and $\Gamma_S$can be determined through these two equations.

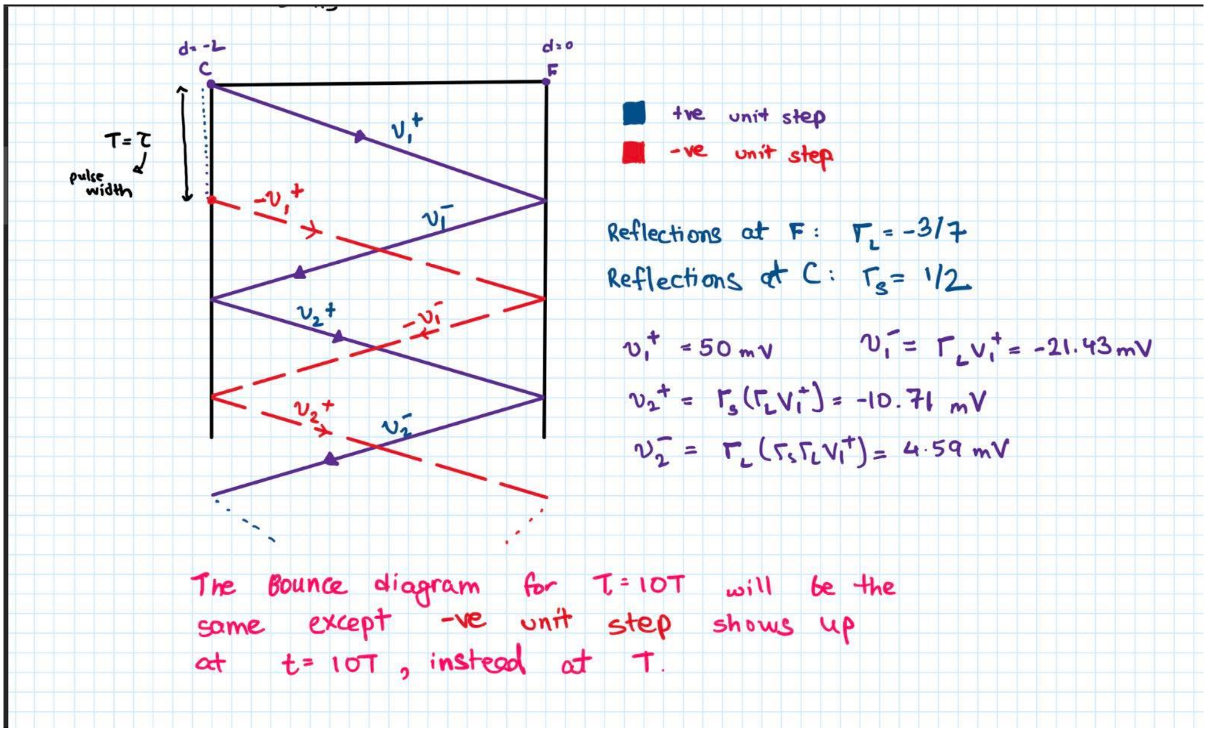
Recall, v_0^+ = 50 mV (this is the wave that is initially travelling, i.e., the pulse sent by the waveform generator)

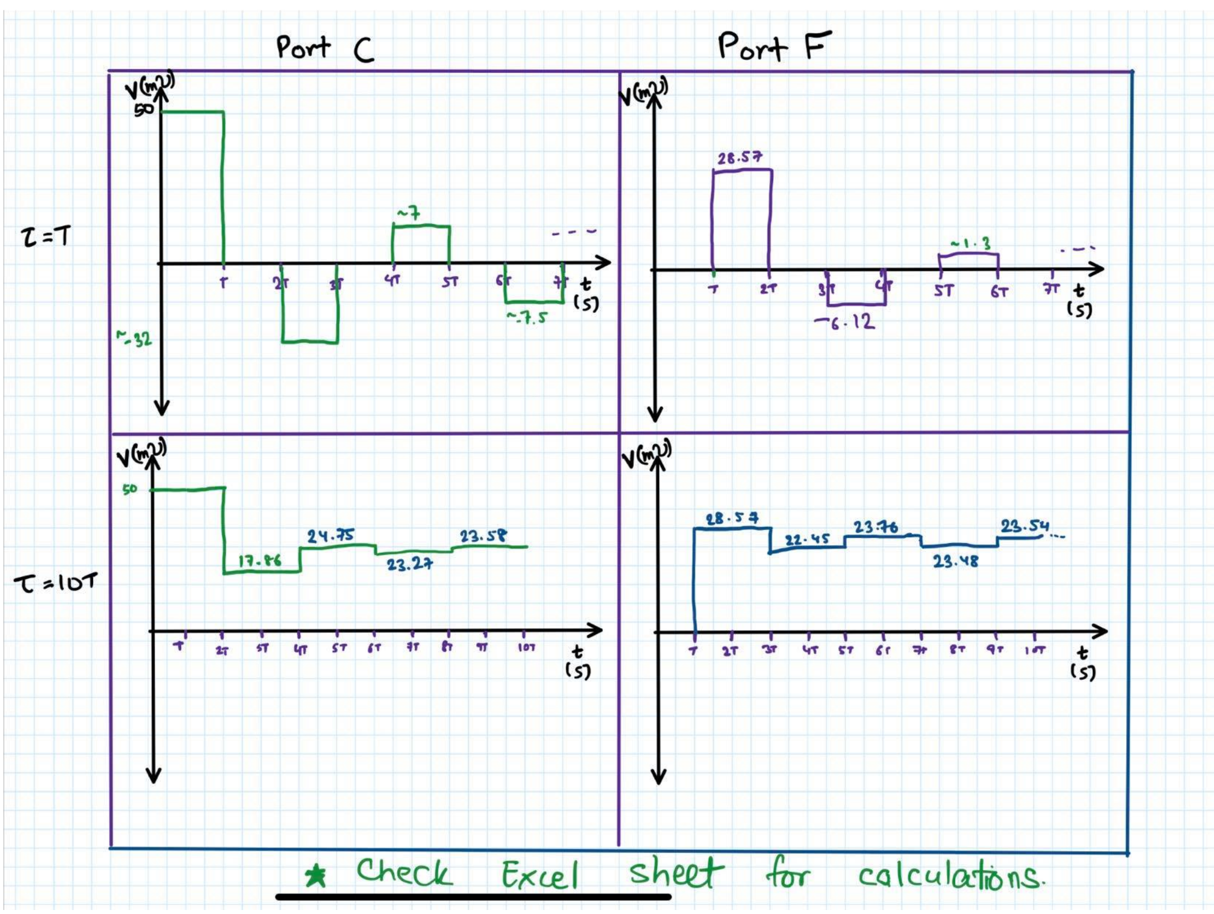
1. At point F (the load):
   a. Measured voltage =$v_L = 29.375mV$(Figure 9)
   b. The boundary conditions imply =>
      $v_L = v_0^+ + \Gamma_L v_0^+$
      $29.375 = 50 + \Gamma_L(50)$
      Solving for Γ_L
      $\Gamma_L = -0.4125$

2. At point C:
   a. Measured voltage = $v_C = -28.750mV$(Figure 7)
   b. $v_C = v_0^- + v_1^+$
      And,
      v_0^-= $\Gamma_L *v_0^+ = -20.625$
      v_1^+ = $\Gamma_S *v_0^-$- =$-20.625\Gamma_S$
      $-28.75 = -20.625 + \Gamma_S * (-20.625)$
      $\Gamma_S = 0.3939$

III. Calculate the corresponding theoretical bounce diagram diagram for pulse widths of T and 10 T and plot the theoretical v vs t graphs for each case at C and F

Bounce Diagrams:


d=-L
C
d=0
F
T=τ
pulse width
v_1^+
$-v_1^+$
v_1^-
v_2^+
$-v_1^-$
v_2^-
+ve unit step
-ve unit step
Reflections at F: $\Gamma_L = -3/7$
Reflections at C: $\Gamma_S = 1/2$
$v_1^+ = 50\ mV$
$v_1^- = \Gamma_L V_1^+ = -21.43\ mV$
$v_2^+ = \Gamma_S(\Gamma_L V_1^+) = -10.71\ mV$
$v_2^- = \Gamma_L(\Gamma_S \Gamma_L V_1^+) = 4.59\ mV$
The Bounce diagram for τ = 10T will be the same except -ve unit step shows up at t = 10T, instead at T.


Port C
Port F
τ=T
τ=10T
V(mV)
t (s)
50
~-32
~7
~-7.5
28.57
-6.12
~1.3
17.86
24.75
23.27
23.58
28.57
22.45
23.76
23.48
23.54
★ Check Excel sheet for calculations.

IV. Discuss how the measured results compare to the theoretically calculated ones.

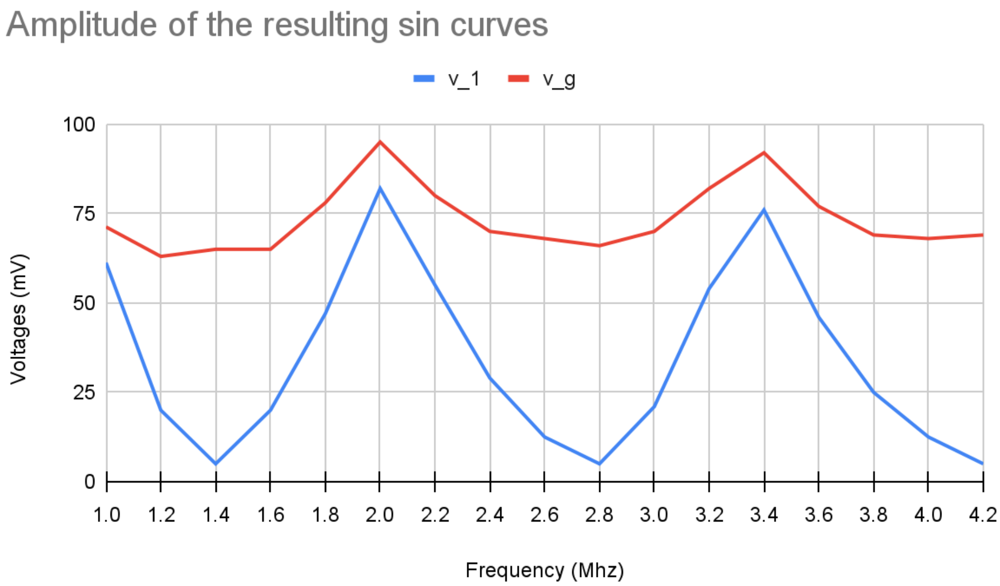
The measured and calculated values are very close for Γ_S(deviation % is 3.75%). However, in terms of Γ_L, the measured value is off from the theoretical value (deviation % is -21.22%).

This deviation can be ascribed to the fact that in figure 7, there is an incorrect cursor value; the cursor measuring the amplitude of the reflected wave is slightly above its proper position. A more accurate value for the reflection coefficient would be derived if the cursor was placed more precisely, resulting in a voltage difference of around -30 - 32mV and hence more accurate Γ_L.

3.8 Input Impedance and Transmission-Line Electrical Length

I. Find three v_1 minimum frequencies for the short circuit load.
Note: The values are not peak to peak

Frequency (Mhz)	v_g (mV)	v_1 (mV)
1	71.25	61.25
1.2	63	20
1.4 **(minimum)**	65	5
1.6	65	20
1.8	78	47
2	95	82
2.2	80	55
2.4	70	29
2.6	68	12.5
2.8 **(minimum)**	66	5
3	70	21
3.2	82	54
3.4	92	76
3.6	77	46
3.8	69	25
4	68	12.5
4. 2**(minimum)**	69	5



II. *Explain why minimum voltages are obtained and discuss the effect on input current.*

When the load is short-circuited, $\Gamma_L = -1 = 1e^{j\pi}$.

Recall, $\overline{V}(d) = V_0^+ e^{j\beta d}(1 + \Gamma_L e^{-j2\beta d})$. Substituting $\Gamma_L = -1$:

$$\overline{V}(d) = V_0^+ e^{j\beta d}(1 - e^{-j2\beta d}) = V_0^+(-2j\,sin(-\beta d))$$

Note taking the modulus here will help evaluate the amplitude determined experimentally in the previous part:

$$|\overline{V}(d)| = \left|V_0^+\right||-2j||sin(-\beta d)| = 2\left|V_0^+\right||sin(-\beta d)|$$

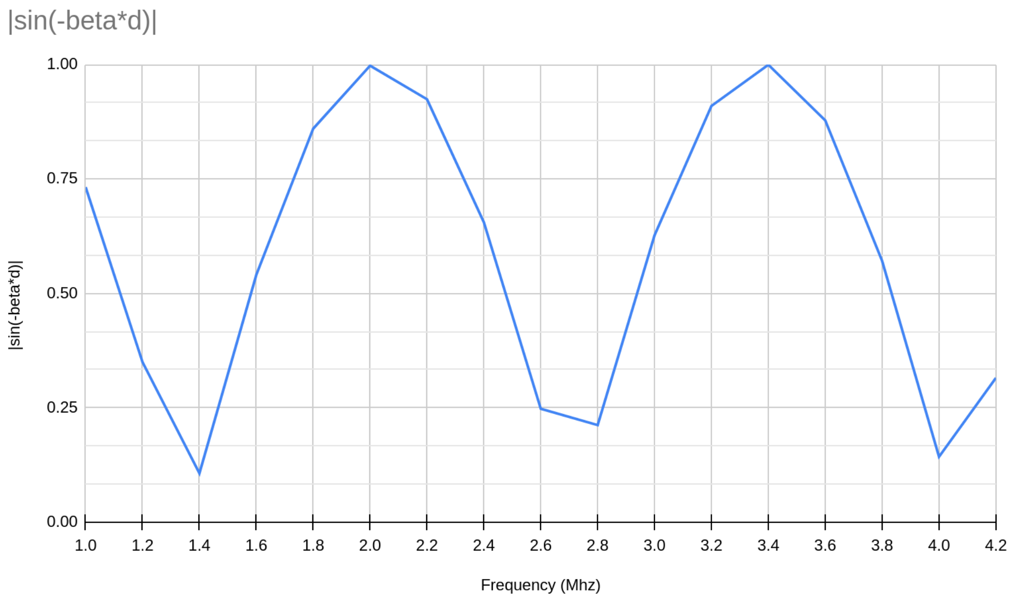
$$\text{Recall } \beta d = \frac{2\pi}{\lambda}d = \frac{2\pi}{v_{avg}/f} \times 90 = \frac{180\pi f}{3.44\times10^8}$$

(Note: $-j$ only causes a phase shift but does not affect the amplitude of the resulting graph, so can be ignored for this analysis)

Since only the frequency is being varied here, we can plug in the expression above to make $\overline{V}(d)$dependent on the frequency:

$$|\overline{V}(f)| = 2\left|V_0^+\right|\left|sin(-\frac{180\pi f}{3.44\times10^8})\right|$$

Plotting the only the sinusoidal expression over the range of frequencies gives the following graph:



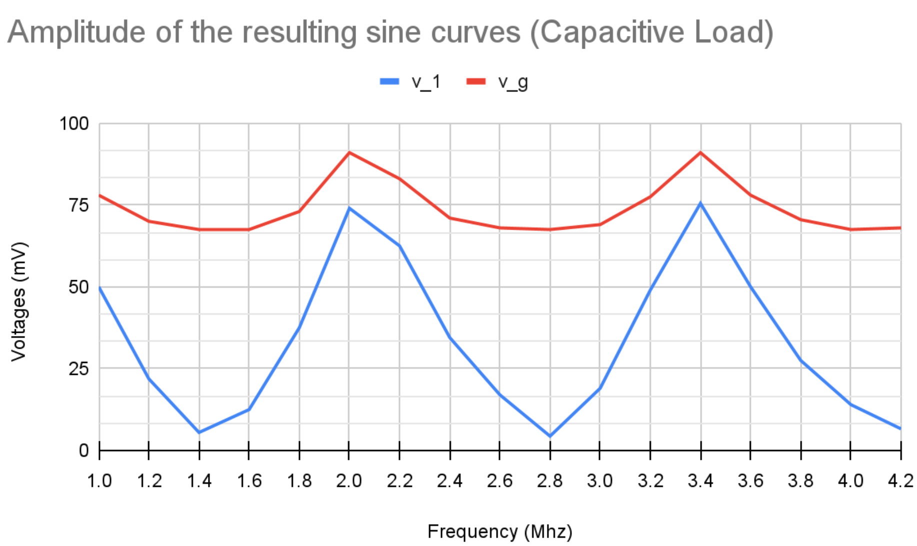
It can be visually interpreted that the minima of the graphs above line up (same frequency). The sine component of $|\overline{V}(f)|$explains the presence of peaks/maxima and minima observed in the experimental values. Underlying this fancy math is a simple explanation in that voltage minimums take place at $\theta_r - 2\beta d = (2k + 1)\pi,\ k \in Z$, where $\theta_r = \pi$ for a short-circuit load.

The current going through a transmission line is given by the following expression: $I(d) = \frac{V_0^+}{Z_0} e^{j\beta d}(1 - \Gamma_L e^{-j2\beta d})$. Looking at this equation, it can be determined there is a current maximum at a voltage minimum (and vice-versa).

III. *Find three v_1 minimum frequencies for the capacitive load.*
Note: These values are not peak-to-peak

Frequency (Mhz)	v_g (mV)	v_1 (mV)
1	78	50
1.2	70	21.87
1.4 **(minimum)**	67.5	5.5
1.6	67.5	12.5
1.8	73	37.5
2	91	74

2.2	83	62.5
2.4	71	34.5
2.6	68	17
2.8**(minimum)**	67.5	4.375
3	69	19
3.2	77.5	49
3.4	91	75.5
3.6	78	50
3.8	70.5	27.5
4 **(minimum)**	67.5	14
4.2	68	6.6



IV. *Discuss how and why the results for the short circuit and the capacitor are different.*
The reflection coefficient for a purely reactive load is given by:

$\Gamma_L = \; -1e^{-j2\theta}$, where $\theta = tan^{-1}(X_L/Z_0)$

In the case of a capacitor, the impedance is given as $Z_c = -\, j/\omega C \; => \; X_L = \frac{-1}{\omega C}$

Hence, we have:

$$\theta = tan^{-1}(\frac{-1}{Z_0\omega C})$$

In this experiment, $Z_0 = 50\Omega$ and $C = 0.01\mu F$=> $Z_0 C = 5 \times 10^{-7}$, making the equation for Γ_L to be:

$$\Gamma_L = \; - \; 1e^{-j2*atan(\frac{-5\times10^7}{\omega})}$$

Note, in when $Z_L = 0$(short circuit), $\Gamma_L = - \; 1$, regardless of the frequency of operation. However for a capacitive load, $\Gamma_L = \; - \; 1e^{-j2\theta}$, where $\theta$is defined as above, and is dependent on the input frequency, which consequently affects the locations of the minima/maxima. What is common between the two loads is that both have a reflection magnitude of 1, meaning the amplitude of the resulting voltages is the same, but the locations of the minima/maxima differ.

Note: All calculations can be found on a Google Sheet [here](#).