13/01/23

# Unit - 05

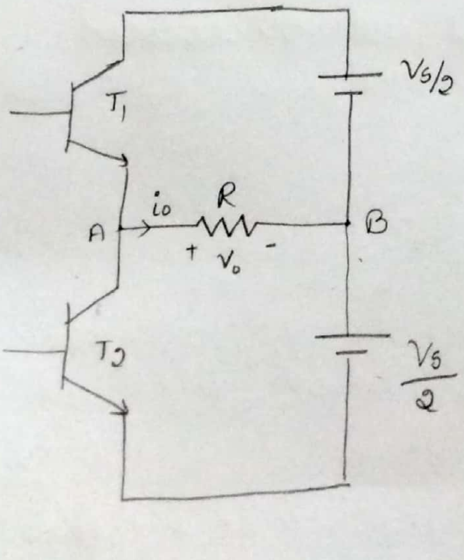
## Inverter

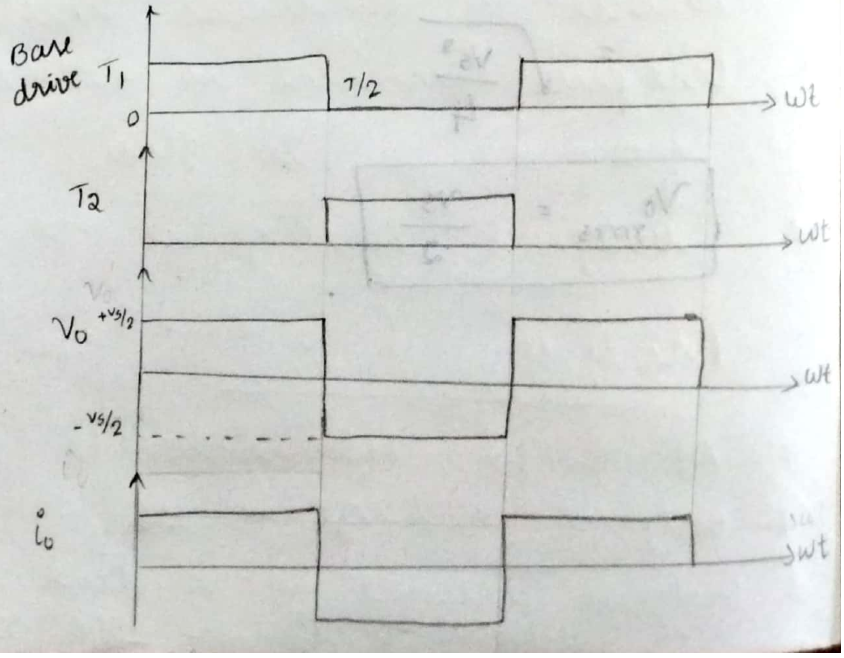
DC Fixed → Inverter → AC (Fixed / variable)

* Inverter → DC to AC converter
* The output voltage could be fixed or variable at a fixed or variable frequency
* For fixed output voltage → gain of an inverter is variable.
* for variable output voltage → gain is constant
* The inverter gain = $\dfrac{\text{Ac output voltage}}{\text{DC input voltage}}$.

* Inverter → Half bridge inverter / Full bridge inverter.

1. Half bridge inverter :



* Half bridge inverter consist of two transistors $T_1$ and $T_2$ which are used as switch.
* During +ve half cycle, transistor $T_1$ conducts from 0 to $T/2$ and current flows from point A to B.
* During -ve half cycle, transistor $T_2$ conducts from $T/2$ to $T$ and current flows from B to A.
* Since the load is resistive, the o/p current is same as voltage waveform.

wave form

* The output vtg = $\pm V_s/2$

The rms value of an output voltage is given by

$$V_{o\,rms} = \sqrt{\frac{1}{T}\int_0^T V_o^2 \, d\omega t}$$

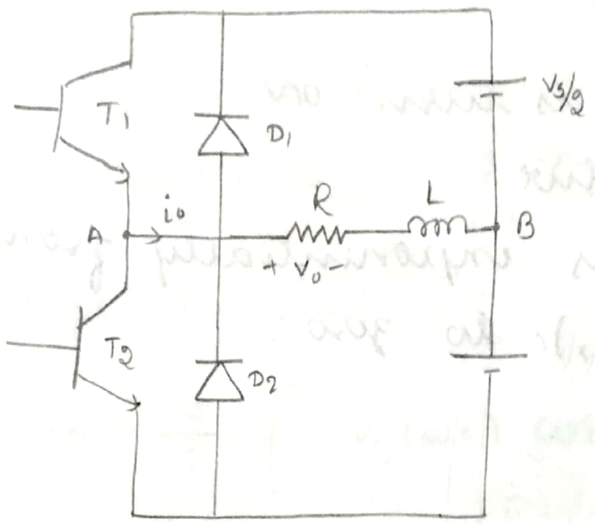
$$= \sqrt{\frac{1}{T}\int_0^T \left[\frac{V_s}{2}\right]^2 d\omega t}$$

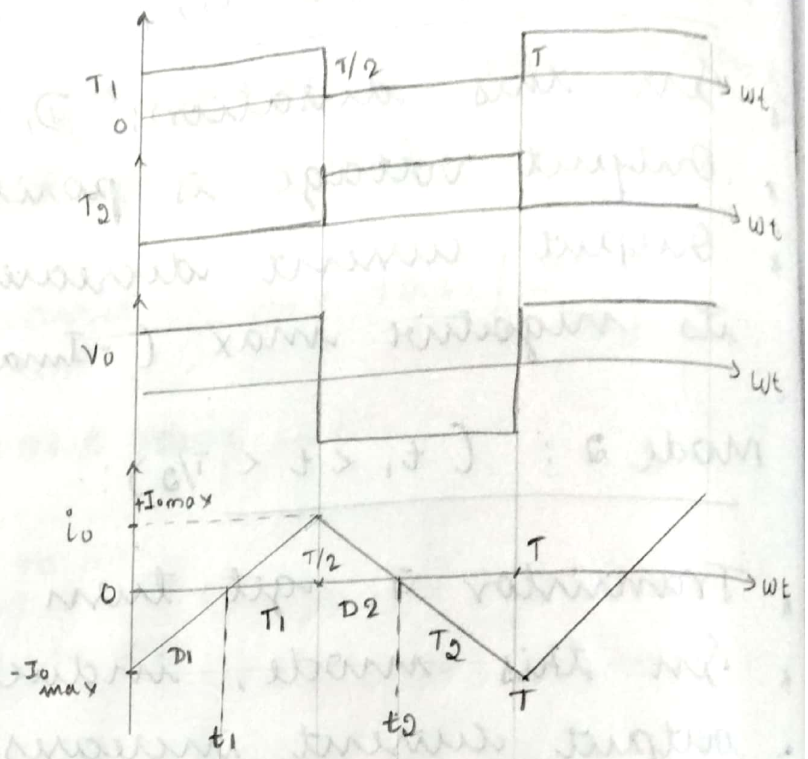
$$= \sqrt{\frac{1}{T} \times \frac{V_s^2}{4}\int_0^T 1 \, d\omega t} = \sqrt{\frac{V_s^2}{4T}\left[T-0\right]}$$

$$= \sqrt{\frac{V_s^2}{4}}$$

$$\boxed{V_{o\,rms} = \frac{V_s}{2}}$$

# Half bridge inverter with RL load:




* A half bridge inverter with RL load consist of two transistors and 2 feedback diodes.
* The load RL is connected between point A & B

* <u>Mode 1:</u> $(t_1 < t < T/2)$:
* At $t = t_1$, transistor $T_1$ get turn ON
*

## Operation:

### Mode 1: $(0 < t < t_1)$:

* In this duration, $D_1$ is turn on.
* Output voltage is positive
* Output current decreases exponentially from its negative max $(-I_{max})$ to zero.

### Mode 2: $(t_1 < t < T/2)$:

* Transistor $T_1$ get turn ON at $t = t_1$.
* In this mode, inductor stores energy and output current increases exponentially from zero to $+I_{max}$.
* The output voltage is +ve and $V_0 = V_s/2$
* At $t = T/2$, output current reaches to its max value and $T_1$ gets turn OFF

### Mode 3: $(T/2 < t < t_2)$:

* Inductor release its energy through $D_2$
* Diode $D_2$ conduct this time.
* Output current is +ve but it decreases from $+I_{o\,max}$ to zero.
* output voltage is -ve.

### Mode 4: $(t_2 < t < T)$:

* At $t = t_2$, transistor $T_2$ get turn ON.
* output voltage is -ve
* Output current is -ve but it increases exponent from 0 to its negative max value.

Obtain the fourier series for the output vtg waveform for half bridge inverter:

The general expression for fourier series is given by

$$v_o(\omega t) = V_{o\,avg} + \sum_{i=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

where $C_n = \sqrt{a_n^2 + b_n^2}$ and $\phi_n = \tan^{-1}\left[\frac{a_n}{b_n}\right]$

$$a_n = \frac{2}{T}\int_0^T v_o(\omega t)\cos n\omega t \cdot d\omega t$$

$T_1 = V_s/2 \longrightarrow 0$ to $\pi$.

$T_2 = -V_s/2 \longrightarrow \pi$ to $2\pi$

$$a_n = \frac{\cancel{2}}{\cancel{2}\pi}\int_0^{2\pi} \frac{V_s}{2}\cos n\omega t \cdot d\omega t.$$

$$= \frac{V_s}{2\pi}\left[\sin n\omega t\right]_0^{2\pi} = \frac{V_s}{2\pi}\left[\overbrace{\sin 2\pi n}^{0} - 0\right]$$

$\boxed{a_n = 0} \rightarrow$ for all values of n.

$$b_n = \frac{2}{T}\int_0^T v_o(\omega t)\sin n\omega t \cdot d\omega t.$$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{V_s}{2}\sin n\omega t \cdot d\omega t.$$

$$= \frac{V_s}{2\pi}\left[-\cos n\omega t\right]_0^{2\pi}$$

$$= \frac{V_s}{2\pi}\left[-\cos n2\pi + \cos 0\right]$$

$\boxed{b_n = \frac{V_s}{n\pi}\left[1 - \cos 2n\pi\right]} \rightarrow$ general eq$^n$.

Here, $\cos n\pi = \begin{cases} 1 & \text{for } n = 2, 4, 6 \ldots \\ -1 & \text{for } n = 1, 3, 5 \ldots \end{cases}$

Hence $b_n = \begin{cases} \dfrac{2V_s}{n\pi} & \text{for odd values} \\ 0 & \text{for even value of } n \end{cases}$

$\therefore$ $C_n = \sqrt{a_n^2 + b_n^2} = b_n = \dfrac{2V_s}{n\pi}$.

$$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) = \underline{\underline{0}}$$

The waveform of Half bridge inverter has symmetric +ve and -ve half cycle, hence avg output voltage is zero.

$$v_o(\omega t) = V_{o_{avg}} + \sum_{i=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

$$v_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t.$$

The output voltage contains only odd harmonics.

* To obtain the rms value of fundamental component by considering the fourier series equation of half bridge inverter, for fundamental component of output voltage can be written as.

$$v_1(\omega t) = \frac{2V_s}{\pi} \sin \omega t \;(\text{for } n=1)$$

It has same frequency as that of square wave and peak value of fundamental component is given by $C_1 = \dfrac{2V_s}{\pi}$

and rms value of fundamental component is given by $\boxed{V_{1\,rms} = \frac{c_1}{\sqrt{2}}}$

$$V_{1\,rms} = \frac{\frac{2V_s}{\pi}}{\sqrt{2}} = \frac{2V_s}{\sqrt{2}\pi} = \frac{\sqrt{2}V_s}{\pi} = \underline{\underline{0.45\,V_s}}$$

Similarly,

rms values of other harmonics can be calculated by $V_{n\,rms} = \frac{2V_s}{\sqrt{2}n\pi} = \underline{\underline{\frac{0.45\,V_s}{n}}}$

14/01/23

1) The single phase half bridge inverter has the DC input of 48V and the load resistance is 4.8V. Determine,

i. rms value of o/p voltage

ii. rms value of fundamental component (n=1)

→ $V_{1\,rms} = 0.45\,V_s = 0.45 \times 48 = \underline{21.6V}$

$\cancel{V_{o\,rms}} = \cancel{0.45V_s}$ $\quad V_{o\,rms} = \frac{V_s}{2} = \frac{48}{2} = \underline{\underline{24V}}$

## Performance parameters of inverter :

o/p generated - Square wave

1. Harmonic factor of $n^{th}$ harmonic ($HF_n$):

It is defined as the ratio of rms value of $n^{th}$ harmonic to rms value of fundamental component.

$$HF_n = \frac{V_n}{V_1}$$

where $V_n$ = rms value of $n^{th}$ harmonic.

$V_1$ = rms value of fundamental component

2. Total harmonic distortion (THD):

It is the ratio of rms values of all the harmonics to rms value of fundamental component

$$THD = \frac{\left[\sum_{n=1,2,3}^{\infty} V_n^2\right]^{1/2}}{V_1}$$

* The total harmonic distortion indicate, the distortion in the waveform
* It is the measure of closeness of the waveform to sine wave.

3. Distortion factor (DF):

It is defined as the measure of harmonic distortion that remain in the particular w/f after filtering. Due to filtering higher order harmonics are eliminated.

The DF of $n^{th}$ harmonic component is given by

$$DF = \frac{1}{V_1}\left[\sum_{n=1,2,3}^{\infty}\left[\frac{V_n}{n^2}\right]^2\right]^{1/2}, \quad DF_n = \frac{V_n}{V_1 n^2} \text{ for } n>1$$

4. Lower order harmonic :

It is the harmonic component which has the nearest frequency to the fundamental and its amplitude within 3% of the fundamental component.

Lower order harmonic should have max. possible frequency so that the harmonic distortion is less.

Problems:

1) The single phase HB inverter has DC input of 70V and the load resistance is $6.4\,\Omega$. calculate rms o/p voltage, rms values of fundamental component and THD.

→ * $V_{o\,rms} = \frac{V_s}{2} = \frac{70}{2} = 35V$

* $THD = \frac{(V_{o\,harmonic})^{1/2}}{V_1}$ $\qquad \boxed{V_{o\,harmonic} = V_{o\,rms}^2 - V_1^2}$

* $V_1 = 0.45 \times V_s$

$V_1 = 0.45 \times 70$

$\boxed{V_1 = 31.5V}$

$THD = \frac{(V_{o\,rms}^2 - V_1^2)^{1/2}}{V_1}$

$= \frac{(35^2 - 31.5^2)^{1/2}}{31.5}$

$THD = 0.4843$

$\boxed{THD = 48.43\%}$

2) DC input vtg is 36V, resistance = 1.2Ω. Calculate THD.

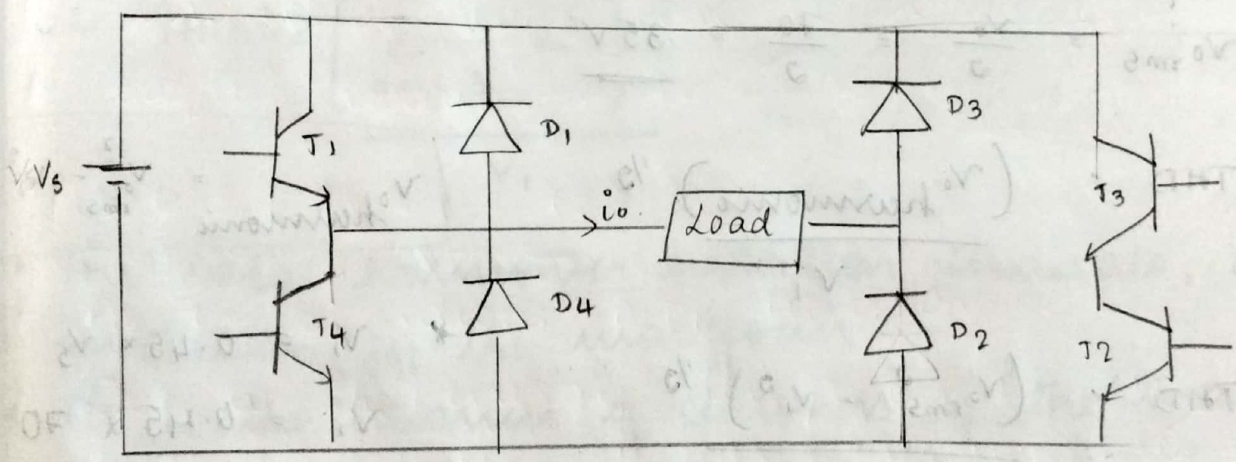
→ $V_{o\,rms} = \frac{V_s}{2} = \frac{36}{2} = 18V$

$V_{1\,rms} = 0.45 \times 36 = 16.2V$

$$THD = \frac{(V_{orms}^2 - V_1^2)^{1/2}}{V_1} = \frac{(18^2 - 16.2^2)^{1/2}}{16.2}$$

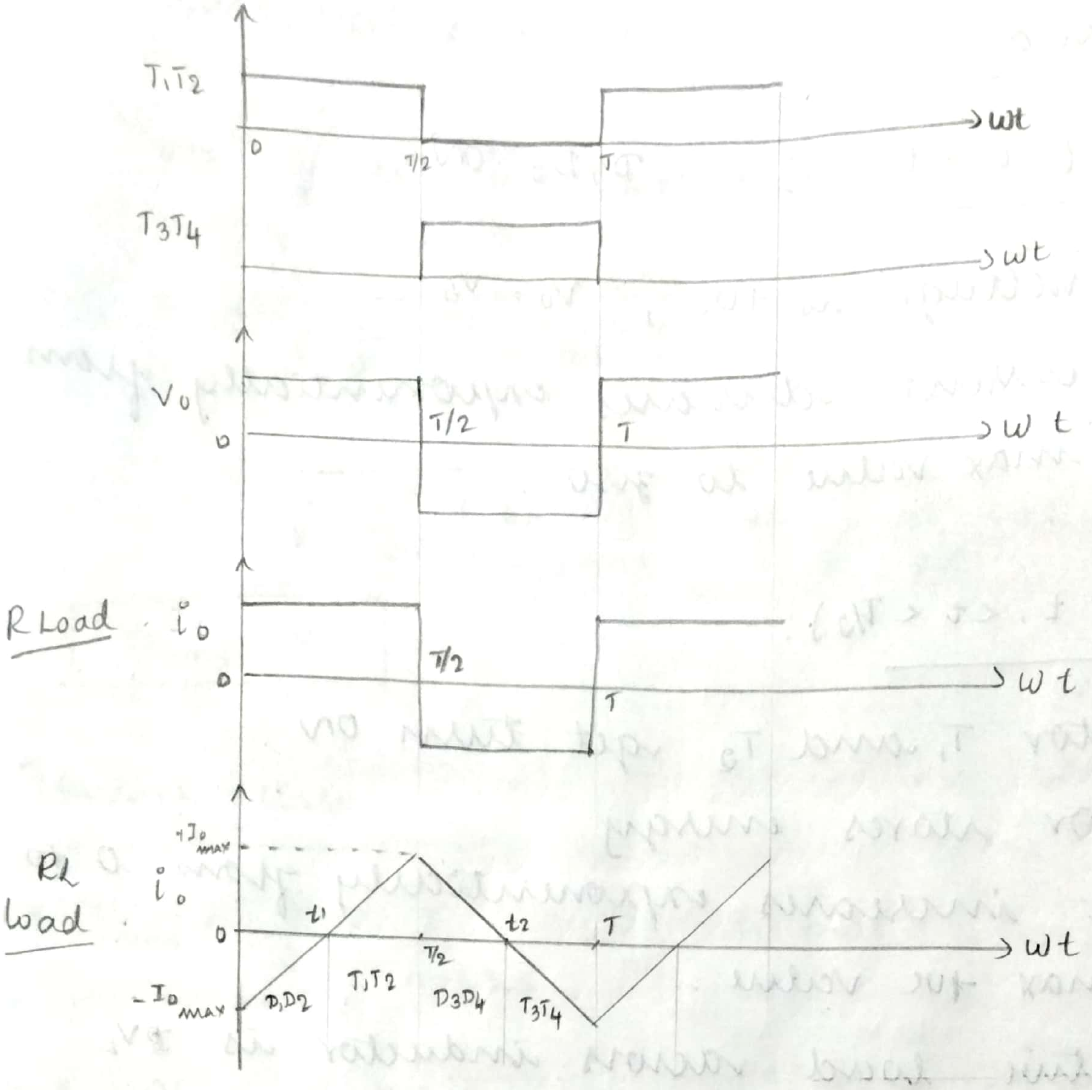
$$\boxed{THD = 48.43\%}$$

## Full bridge inverter :




* A single phase full bridge inverter consist of 4 transistor and 4 feedback diodes.
* Load can be R load or RL load.

Scanned with CamScanner




* ~~A ri~~

For R load:

* During 0 to $T/2$, transistor $T_1$ and $T_2$ get turn ON
* The output voltage is $v_0 = V_S$
* The output current across the load is +ve.
* During ($T/2$ to T):

Transistor $T_3$ and $T_4$ get turn ON ($T_1T_2$ - OFF)

* The current flow across the load is -ve.
* The output voltage is also -ve, $v_0 = -V_S$.

∴ Output voltage $v_0 = \underline{\underline{\pm V_S}}$

For RL load:

* Mode I ( $0 < t < t_1$ ) : $D_1 D_2$ ON.

* Output voltage is +ve , $V_0 = V_S$.
* Output current decreases exponentially from its -ve max value to zero.

Mode 2 ( $t_1 < t < T/2$ ):

* Transistor $T_1$ and $T_2$ get turn ON.
* Inductor stores energy
* Current increases exponentially from 0 to its max +ve value.
* Inductive load across inductor is $\pm V_L$
* Output voltage is +ve, $V_0 = V_S$.

Mode 3 ( $T/2 < t < t_2$ ).

* At $t = T/2$, diode $D_3 D_4$ turn ON.
* Due to the energy dissipated by inductor, output current decreases exponentially from $+I_{max}$ to zero.
* The output voltage is -ve.

Mode 4 ( $t_2 < t < T$ ):

* Thyristor $T_3$ and $T_4$ get turn ON.
* Inductor stores energy in reverse direction
* Output vtg is -ve ( $V_0 = -V_S$ )
* Current increases from 0 to its -ve max value.

The rms value of output vtg is given by

$$V_{o_{rms}} = \sqrt{\frac{1}{T}\int_0^{2\pi} v_o^2(\omega t)\,d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_s^2 \cdot d\omega t}$$

$$= \sqrt{\frac{V_s^2}{2\pi}\left[2\pi - 0\right]} = \sqrt{\frac{V_s^2}{\cancel{2\pi}} \times \cancel{2\pi}}$$

$$\boxed{V_{o_{rms}} = V_s}$$

**Fourier series:**

* $V_{o_{rms}} = \frac{V_s}{2} = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$.

* $V_{1_{rms}} = 0.9V_s$

* $V_{n_{rms}} = \frac{0.9V_s}{n}$.

$C_n = \sqrt{a_n^2 + b_n^2} = \frac{4V_s}{n\pi}$

$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) = 0$.

→ Fundamental component.

$C_1 = \frac{4V_s}{\pi}$

$V_{1\,rms} = \frac{C_1}{\sqrt{2}} = \frac{\frac{4V_s}{\pi}}{\sqrt{2}}$

$V_{1_{rms}} = \frac{4V_s}{\sqrt{2}\pi}$

$$\boxed{V_{1_{rms}} = 0.9V_s}$$

→ $V_{n_{rms}} = \frac{4V_s}{n\sqrt{2}\pi} = \frac{0.9V_s}{n}$

$a_n = \frac{2}{T}\int_0^T v_o(\omega t)\cos n\omega t \cdot d\omega t$

$= \frac{2}{\pi}\int_0^{\pi} V_s \cdot \cos n\omega t \cdot d\omega t$

$= \frac{2V_s}{\pi}\left[\sin n\omega t\right]_0^{\pi}$

$$\boxed{a_n = 0}$$

$b_n = \frac{2}{T}\int_0^T v_o(\omega t)\sin n\omega t \cdot d\omega t$

$= \frac{2}{\pi}\int_0^{\pi} V_s \sin n\omega t \cdot d\omega t$

$= \frac{2V_s}{\pi}\left[-\cos n\omega t\right]_0^{\pi}$

$= \frac{2V_s}{\pi}\left[-\cos n\pi + \cos 0\right]$

$b_n = \frac{2V_s}{\pi}\left[1 - \cos n\pi\right]$ $\quad \cos n\pi = \begin{cases} 1, & n = 2,4,6 \\ -1, & n = 1,3,5 \end{cases}$

$\therefore b_n = \begin{cases} \frac{4V_s}{n\pi} & \text{for odd values} \\ 0 & \text{for even values} \end{cases}$

Scanned with CamScanner

Problems:

Q] A single phase FBI having square wave output has the DC supply of 48 V and o/p resistance of 4.8 Ω. Determine rms o/p vtg & $V_{1\,rms}$.

→ $V_{o\,rms} = V_s = 48V$.

$V_{1\,rms} = 0.9 \times V_s = \underline{43.2V}$

Q] A single phase FBI has resistance load of 2.4 Ω and the DC input vtg is 48V.
Determine rms o/p vtg, o/p power, peak & avg current of each transistor. (k = 50%)

→ $V_{o\,rms} = 48V = V_s$

* o/p Power $= \dfrac{V_{o\,rms}^2}{R} = \dfrac{48^2}{2.4} = \underline{960W}$

* $I_{T\,peak} = \dfrac{V_s}{R} = \dfrac{48}{2.4} = \underline{20A.}$

* $I_{T\,avg} = i_{T\,peak} \times$ Duty cycle.

$= 20 \times 0.5$

$\boxed{I_{T\,avg} = 10A}$

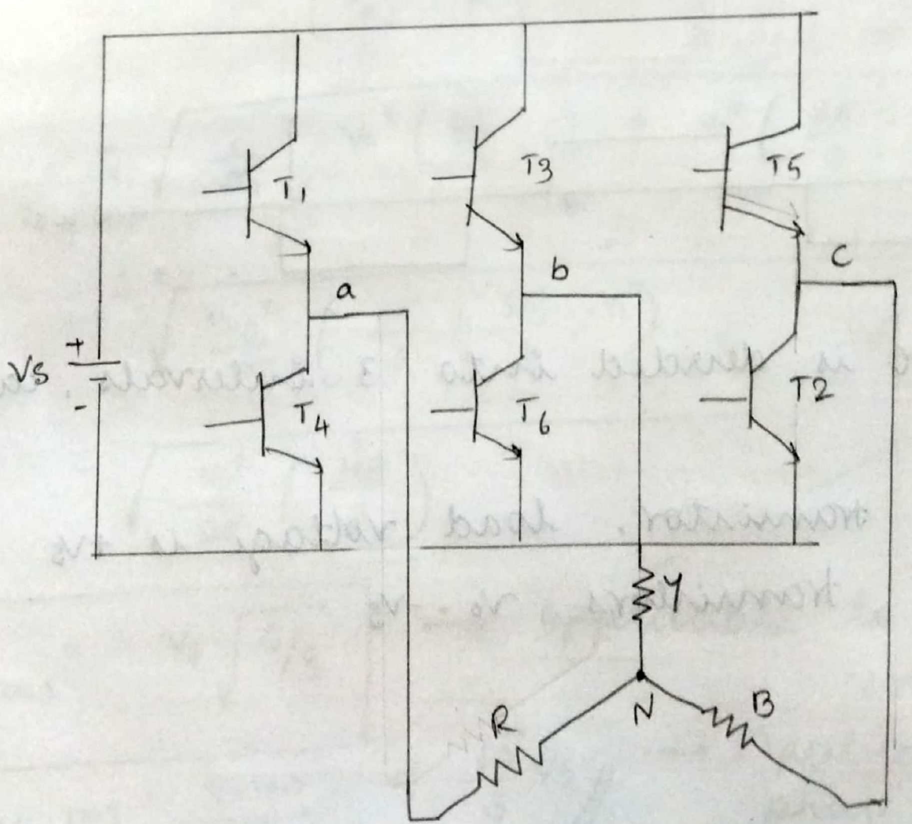
27/01/23

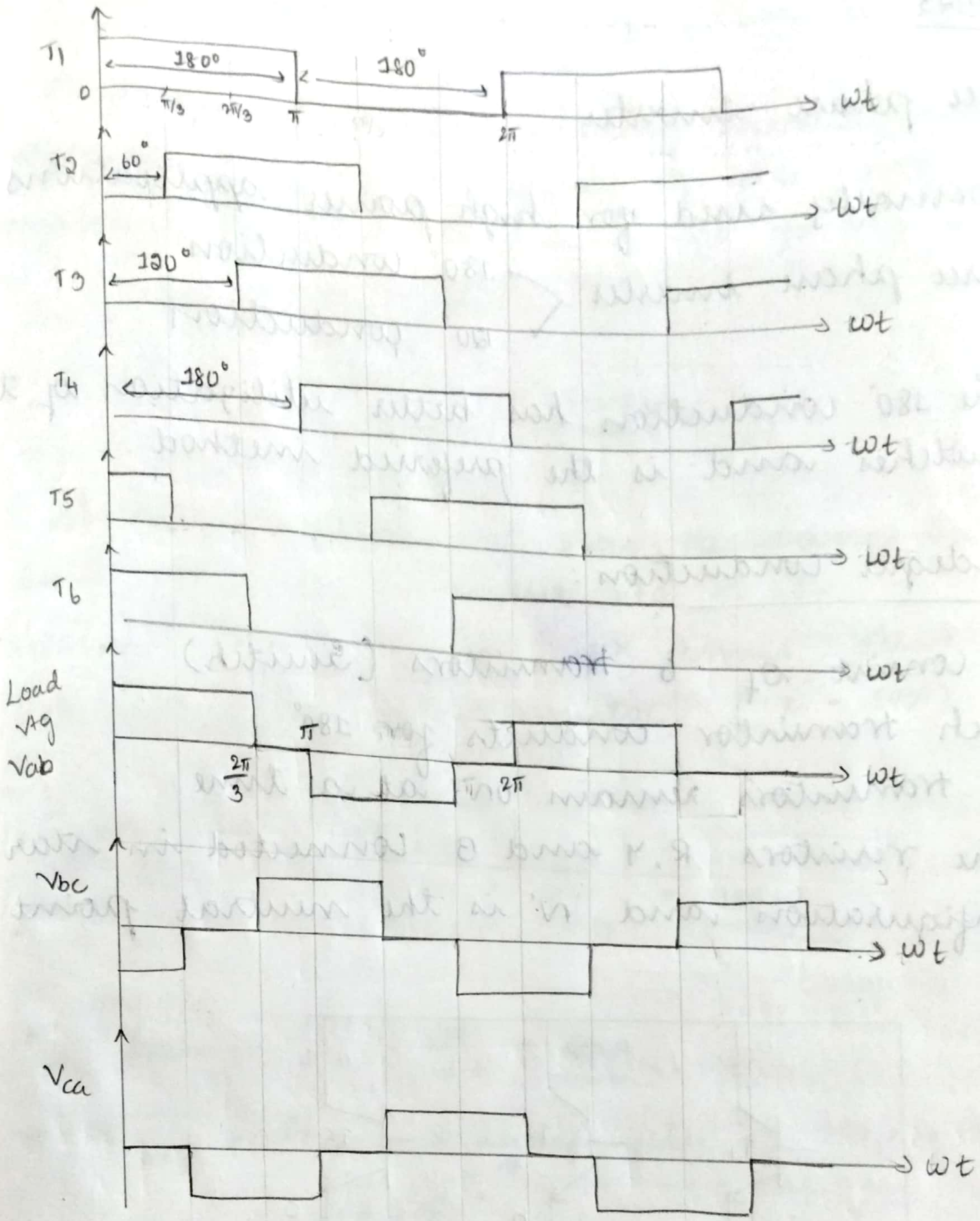
## Three phase inverter:

* Normally used for high power applications.
* Three phase inverter < 180° conduction / 120° conduction
* The 180° conduction has better utilization of the switches and is the preferred method.

### 180° degree conduction:

* It consist of 6 transistors (Switch)
* Each transistor conducts for 180°.
* 3 transistors remain on at a time
* The resistors R, Y and B connected in star configuration and 'N' is the neutral point.






* Half wave is devided into 3 intervals, each of 60°.
* For upper transistor, load voltage is $+V_s$
* For lower transistors $v_0 = -V_s$

| Interval | Conducting transistors | Line v+g $V_{RY}$ | Phase v+g, $V_{RN}$ |
| --- | --- | --- | --- |
| 1 | 5, 6, 1 | $V_S$ | $V_S/3$ |
| 2 | 6, 1, 2 | $V_S$ | $2V_S/3$ |
| 3 | 1, 2, 3 | 0 | $V_S/3$ |
| 4 | 2, 3, 4 | $-V_S$ | $-V_S/3$ |
| 5 | 3, 4, 5 | $-V_S$ | $-2V_S/3$ |
| 6 | 4, 5, 6 | 0 | $-V_S/3$ |

The line voltage is quasi square wave. Its rms value is given by

$$V_{line\ v+g\ rms} = \left[ \frac{1}{2\pi} \int_0^{2\pi} V_{RY}^2 \, d\omega t \right]^{1/2}$$

$$= \sqrt{\frac{1}{2\pi} \left[ \int_0^{2\pi/3} V_S^2 \, d\omega t + \int_{\pi}^{5\pi/3} V_S^2 \, d\omega t \right]}$$

$$= \sqrt{\frac{1}{2\pi} \left[ V_S^2 \left( \frac{2\pi}{3} - 0 \right) + V_S^2 \left( \frac{5\pi}{3} - \pi \right) \right]}$$

$$= \sqrt{\frac{V_S^2}{2\pi} \left( \frac{2\pi}{3} + \frac{5\pi}{3} - \pi \right)}$$

$$= \sqrt{\frac{V_S^2}{\cancel{2\pi}} \left( \frac{\overset{2}{\cancel{4\pi}}}{3} \right)}$$

$$\frac{\frac{V_S\sqrt{2}}{\sqrt{3}}}{\sqrt{3}} = \frac{V_S\sqrt{2}}{3}$$

$$\boxed{V_{L\ rms} = V_S \sqrt{2/3}}$$ → applicable to all 6 line voltage.

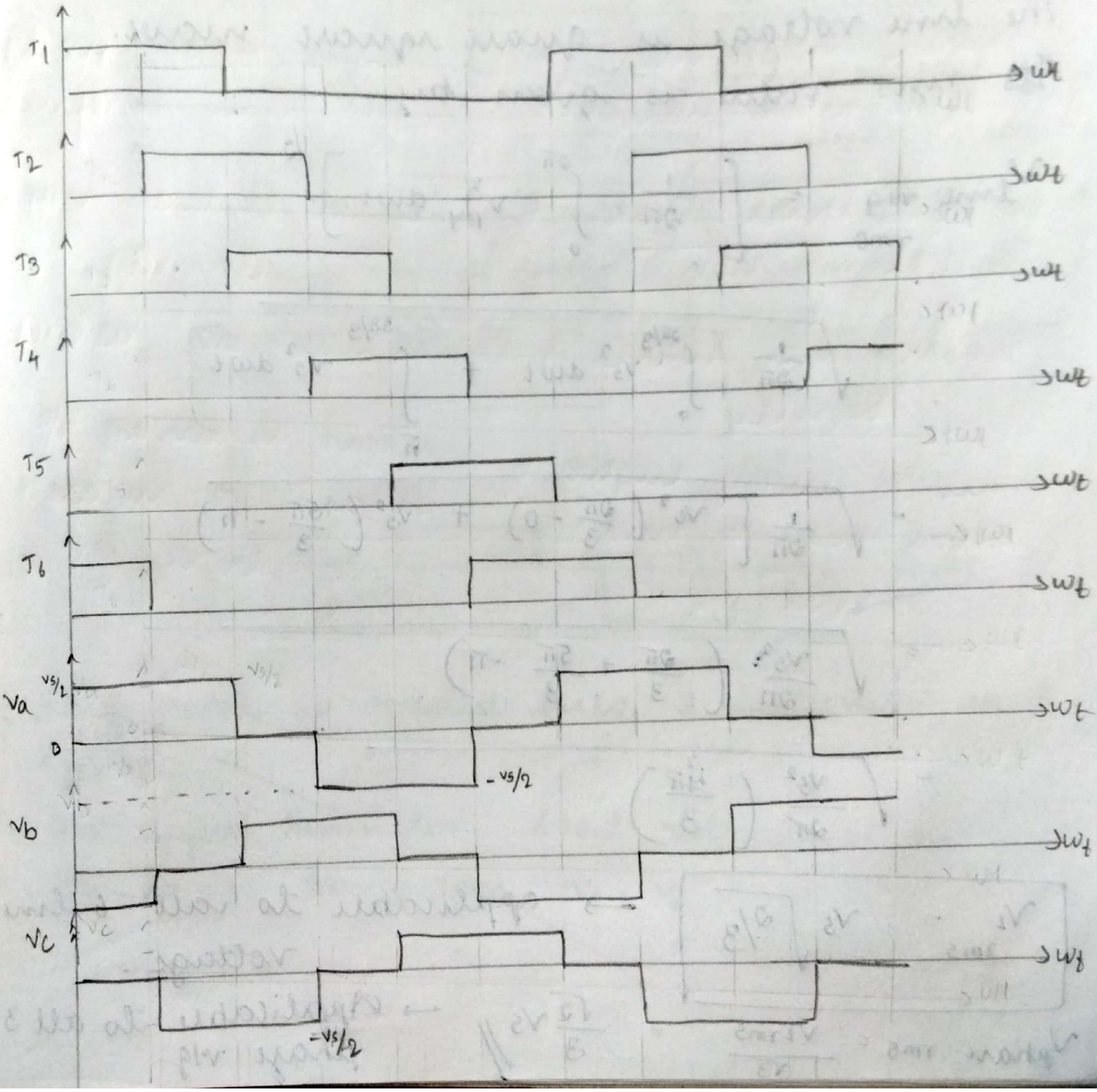
$$V_{phase\ rms} = \frac{V_{L\ rms}}{\sqrt{3}} = \frac{\sqrt{2}}{3} V_S$$ → Applicable to all 3 phase v+g

Scanned with CamScanner

120 degree conduction:

Same ckt and waveform of 180° condution but half wave is divided into 2 interval of 60°.

* Each transistor conducts for 120°.
* Only two transistors remain ON at any instant of time.



| Interval | Conducting transistors | Line vtg $V_{RY}$ | Phase vtg $V_{RN}$ |
|---|---|---|---|
| | | | $V_s/2$ |
| 1 | 6, 1 | $V_s$ | $V_s/2$ |
| 2 | 1, 2 | $V_s/2$ | 0 |
| 3 | 2, 3 | $-V_s/2$ | $-V_s/2$ |
| 4 | 3, 4 | $-V_s$ | $-V_s/2$ |
| 5 | 4, 5 | $-V_s/2$ | 0 |
| 6 | 5, 6 | $V_s/2$ | |

$$V_{phase_{rms}} = \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi/3}\left(\frac{V_s}{2}\right)^2 d\omega t + \int_{\pi}^{5\pi/3}\left(\frac{V_s}{2}\right)^2 d\omega t\right]}$$

$$= \sqrt{\frac{V_s^2}{2\pi \times 4}\left[\frac{2\pi}{3} - 0 + \frac{5\pi}{3} - \pi\right]}$$

$$= \sqrt{\frac{V_s^2}{8\pi} \times \frac{4\pi}{3}}$$

$$\boxed{V_{phase_{rms}} = \frac{V_s}{\sqrt{6}}}$$

$$V_{line\ vtg} = \sqrt{3} \times V_{phase\ rms}$$

$$= \sqrt{3} \times \frac{V_s}{\sqrt{6}} = \frac{\cancel{\sqrt{3}} \times V_s}{\sqrt{2} \times \cancel{\sqrt{3}}} = \frac{V_s}{\sqrt{2}}$$

Comparision between 120° and 180° conduction.

| 180° conduction | 120° conduction |
|---|---|
| * Each device conduct for 180° | * Each device conduct for 120° |
| * 3 device conducts in one interval | * 2 device conducts in one interval. |
| * Line vtg → Quasi square wave with $V_o = \pm V_s$ | * Line vtg → 6 step waveform with $\pm V_s$ & $\pm \frac{V_s}{2}$ |
| * Phase vtg → 6 step waveform with $\pm \frac{2V_s}{3}$ & $\frac{\pm V_s}{3}$ | * Phase vtg → Quasi square wave with $\pm V_s/2$ |
| * Cross conduction is possible | * Cross conduction is not possible. |
| * Devices are better utilized | * Devices are under utilized. |
| * O/P power is higher because of higher vtg level | * Output power is less because of ~~less~~ lower vtg level. |