controlled rectifiers (unit 3)

for a single phase half bridge convertor having highly inductive load derive the following.

1] fourier series on supply current.

2] fundamental component of supply current.

3] RMS value of supply current.

1) Fourier series

$$i_s(\omega t) = I_{s(a.v)} + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$$

where $C_n = \sqrt{a_n^2 + b_n^2}$, $\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$

— Here, $a_n = \frac{2}{T}\int_0^T i_s(\omega t) \cos n\omega t \, d\omega t$

$$= \frac{2}{2\pi}\int_0^{2\pi} i_s(\omega t) \cos n\omega t \, d\omega t \quad — (*)$$

from supply current waveform,

$$a_n = \frac{2}{2\pi}\left[\int_\alpha^\pi I_{o(av)} \cos n\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi} (-I_{o(av)}) \cos n\omega t \, d\omega t\right]$$

$$= \frac{I_{o(av)}}{\pi}\left[\int_\alpha^\pi \cos n\omega t \, d\omega t - \int_{\pi+\alpha}^{2\pi} \cos n\omega t \, d\omega t\right]$$

$$= \frac{-I_{o(av)}}{n\pi} \sin n\alpha (1-\cos n\pi)$$

$$a_n = \begin{cases} \dfrac{-2 I_{o(av)}}{n\pi} \sin n\alpha & \text{for } n = 1, 3, 5 \end{cases}$$

(i) Shows that '$a_n = 0$' for Even harmonic of supply current.

$$b_n = \frac{2}{T}\int_0^T i_s(\omega t) \sin n\omega t \, d\omega t \quad (\text{Put } T = 2\pi)$$

$$= \frac{2}{2\pi}\left[\int_\alpha^\pi I_{o(av)} \sin n\omega t \, d\omega t + \int_{\pi+\alpha}^{2\pi} (-I_{o(av)}) \sin n\omega t \, d\omega t\right]$$

$$= \frac{I_{oav}}{\pi}\left[\int_\alpha^\pi \sin n\omega t \, d\omega t - \int_{\pi+\alpha}^{2\pi} \sin n\omega t \, d\omega t\right]$$

$$= \frac{I_{oav}}{n\pi}(1+\cos n\alpha)(1-\cos n\pi)$$

$$b_n = \begin{cases} \dfrac{2 I_{o(av)}}{n\pi}(1+\cos n\pi) & \text{for } n = 1, 3, 5 \\ 0 & \text{for } n = 2, 4, 6 \end{cases}$$

Now $C_n = \sqrt{a_n^2 + b_n^2}$

$$= \left\{ \left[-\frac{2I_{o(av)}}{n\pi} \sin n\alpha \right]^2 + \left[\frac{2I_o}{n\pi}(1+\cos n\alpha) \right]^2 \right\}^{\frac{1}{2}}$$

$$C_n = \frac{4 I_{o(av)}}{n\pi} \cos\left(\frac{n\alpha}{2}\right) \quad \text{for } n = 1, 3, 5$$

— ②.

Now $\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$.

$$= \tan^{-1}\left[\frac{-\frac{2I_o}{n\pi} \sin n\alpha}{\frac{2I_o}{n\pi}(1+\cos n\alpha)} \right]$$

$$\phi_n = -\tan^{-1}\left[\tan \frac{n\alpha}{2} \right]$$

$$\boxed{\phi_n = \frac{-n\alpha}{2}} \quad \text{— ③.}$$

Now $I_{s(av)} = \frac{1}{T}\int_0^T i_s(\omega t)\, d\omega t$ (Put $T = 2\pi$

$$= \frac{1}{2\pi}\left[\int_\alpha^\pi I_{o(av)}\, d\omega t + \int_{\pi+\alpha}^{2\pi} -I_{o(av)}\, d\omega t \right.$$

$$= \frac{I_{o(av)}}{2\pi}\left[\int_{\alpha}^{\pi} d\omega t - \int_{\pi+\alpha}^{2\pi} d\omega t\right]$$

$$I_{1(ON)} = \frac{I_{o(av)}}{2\pi}\left\{ \left(\omega t\right]_{\alpha}^{\pi} - \left[\omega t\right]_{\pi+\alpha}^{2\pi} \right\}$$

$$= \frac{I_{o(ON)}}{2\pi}\left\{ \pi + \alpha - 2\pi - \pi - \alpha \right\}.$$

hence fourier series can be written as

$$\boxed{I_s(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4 I_{o(av)}}{n\pi} \cos\left(\frac{n\alpha}{2}\right) \sin\left(n\omega t - \frac{n\alpha}{2}\right)} \quad — (4).$$

2. fundamental component of supply current:

$$I_{s1} \Rightarrow \frac{C_1}{\sqrt{2}}, \quad C_1 = \frac{4 I_{oavg}}{\pi} \cos\frac{\alpha}{2} \text{ with } n=1$$

$$I_{s1} = \frac{4 I_{o(av)}}{\pi} \cos\frac{\alpha}{2} \times \frac{1}{\sqrt{2}}$$

$$I_{s1} = \frac{2\sqrt{2}\, I_{o(av)}}{\pi} \cos\frac{\alpha}{2}$$

(c) R.M.S value of I_s

$$I_{s(rms)} = \left[\frac{1}{T}\int_0^T i_s^2(\omega t)\, d\omega t\right]^{1/2}$$

$$= \left\{\frac{1}{2\pi}\left[\int_\alpha^\pi I_{o(av)}^2\, d\omega t + \int_{\pi+\alpha}^{2\pi}\left(-I_{o(av)}\right)^2 d\omega t\right\}^{1/2}\right.$$

$$= \left[\frac{I_{o(av)}^2}{2\pi}\left(\int_\alpha^\pi d\omega t + \int_{\pi+\alpha}^{2\pi} d\omega t\right)\right]^{1/2}$$

$$\boxed{I_{s(rms)} = \sqrt{\frac{\pi-\alpha}{\pi}}}$$