11th July 2023 :

Adv. distributions -2

- Poisson db^n.

  ↳ Avg / unit time → Rate (λ).

  λ = # of occurances / time.

  = 2.5 goals / match.

X = # of goals.

R.V

λ = 2.5 goals / 90 min.

· $P(X=0)$ in 10 min.

90 min → 2.5

10 min $= \left(\frac{2.5}{90}\right) \times 10$

$= 0.28$

· λ for 10 min $= ?$

$\lambda = 0.28 = mu$

$P(X=0) = \text{poisson.pmf}(mu = 0.28, k = 0)$

$\simeq 0.75 \simeq \underline{75}\,\%$.

· <u>Assumptions</u> :

★ Counting.   ★ Independent   ★ Const. Rate.

* No simultaneous events.

You receive 240 messages per hour on average - assume Poisson distributed.

Q1) What is the average or expected number of messages in 30 seconds?

60 min $\rightarrow$ 240 msg.

3600 s $\rightarrow$ 240 msg.

1 s $\rightarrow$ $\left(\frac{240}{3600}\right)$ msg.

30 s $\rightarrow$ $\left(\frac{240}{3600}\right) \times 30$ msg = 2 msg.

Q2) What is the probability of one message arriving over a 30 second time interval?

P(X=1) = poisson.pmf (mu = 2, k=1)

= 0.27 ≈ 27%.

$$P[X=k] = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

$k=1,\ \lambda=2$

$$P[X=1] = \frac{(2)^1 \cdot e^{-2}}{1!}$$

$$= 2 \cdot e^{-2}$$

$$= 0.27 \simeq 27\%$$

Q3) What is the probability that there are no messages in 15 seconds?

(0)

240 msgs $\rightarrow$ 60 min

$$\lambda_{1sec} = \frac{240}{3600} \text{ msgs.}$$

$$\lambda_{15sec} = \left(\frac{240}{3600}\right) \times 15 = 1 \text{ msg.}$$

$\rightarrow$ poisson.pmf.

(mu=1, k=0)

$\simeq 0.37 \simeq 37\%$

Q4) What is the probability that there are 3 messages in 20 seconds?

$$\lambda_1 = \frac{240}{3600}$$

$$\lambda_{20} = \left(\frac{240}{3600}\right) \times 20 = 1.34 \text{ msg.}$$

P[X=3] = poisson.pmf(mu = 1.34, k=3)

= 0.104

= 10.4 %

You receive 240 messages per hour on average - assume Poisson distributed.

Q1) What is the average time to wait between two messages? / what is the avg. waiting time for next msg?

240 msg → 3600 s.

1 msg → $\frac{3600}{240}$ = 15 sec.

e.g. 5 msg → 10 sec

1 msg → $\frac{10}{5}$ = 2.

15 s = avg. waiting time for next msg.

Q2) What is the average number of messages per second?

1 msg → 15 sec.

15 sec → 1 msg.

1 sec = $\left(\frac{1}{15}\right)$ msg.

$\boxed{\lambda_1 = (1/15)}$ → (# of msg / sec.)

≃ 0.067 msg/sec.

Q3) What is the probability of having no messages in 10 seconds?

$\lambda_1 = \left(\frac{1}{15}\right)$ msg.

$\lambda_{10} = \left(\frac{1}{15}\right) \cdot 10$ msg $= \left(\frac{10}{15}\right)$

$\lambda_{15} = \left(\frac{1}{15}\right) \cdot 15$

$\lambda_{45} = \left(\frac{1}{15}\right) \cdot 45$

$$\boxed{(\lambda_t)_{sec.} = (\lambda_1) \cdot t} \ *$$

- $\lambda_{10} = (\lambda_1) \cdot 10$

- poisson.pmf$\left(mu = \frac{10}{15}, k=0\right)$

$\hookrightarrow 0.5134$

Q3) What is the probability of having no messages in 10 seconds? , $= 51.34\%$

$\lambda_{10} = \left(\frac{1}{15}\right) \cdot 10$

poisson.pmf(mu = $\frac{10}{15}$, k=0)

Same

$\boxed{10 \text{ sec}}$

0 msg. any general interval.

Q4. What is the probability of waiting for more than 10 seconds for the next message?

$\boxed{10 \text{ sec}}$

0

$P(X=0) = P(T > 10)$

T : time is also Random variable

Prob. of waiting more than 10 sec. for the next/first msg. to arrive.

X: Count of msg.

↓

discrete

T: time.

↓

R.V.

↳ Continuous var.

Cont. dist^n

Q3) What is the probability of having no messages in 10 seconds?

$\lambda_1 = \frac{1}{15}$

$\lambda_{10} = (\lambda_1) \cdot 10$

$\lambda_{10} = \left(\frac{1}{15}\right) \cdot 10$

$P\{X=k\} = \frac{\lambda^k e^{-\lambda}}{k!}$

$P[X=0] = \frac{\left(\frac{1}{15} \times 10\right)^0 \cdot e^{-\left(\frac{1}{15}\right) \cdot 10}}{0!} = e^{-\left(\frac{1}{15}\right) \cdot 10}$

$\boxed{P[X=0] = e^{-\lambda_1 \cdot 10}}$

Q4. What is the probability of waiting for more than 10 seconds for the next message?

$$P[T > 10] = P[X = 0] = e^{-\lambda_1 \cdot 10} = 0.51$$

Q5. What is the prob. Of waiting less than 10 sec for next msg.

$$P[T \leq 10] = 1 - P[T > 10]$$

$$\underbrace{P[T \leq 10] = 1 - e^{-\lambda_1 \cdot 10}}$$

λ_1 : rate/sec

$\lambda_1 = \frac{1}{15}$

Prob. that next msg. will arrive within 10 sec:

$$P[T \leq t] = 1 - e^{-\lambda_1 \cdot t}$$

→ CDF of Expon. Dbn

$T \sim$ Exponential Distribution

$P[X \le 60 \text{ inch}]$ → norm.cdf.

$P[T \le 10]$ → expon.cdf.

$$P[T \le 10] = \left[1 - e^{-\left(\frac{1}{15}\right)\cdot 10}\right] = 0.4865$$

$$= \text{expon.cdf}(\text{scale} = 15, \; x = 10)$$

- Scale → Avg. time I need to wait for next msg to arrive.
- x → time ≡ t.

- Scale = 15 sec. , x = 10 sec.
- λ_1 = # of msg/sec. $= \left(\frac{1}{15}\right)$ msg.

$$\text{Scale} = \frac{1}{\lambda_1} \Rightarrow \lambda_1 = \frac{1}{\text{Scale}}$$

SDE / MLE / DS / developer.

2, 7, 3, 1, 4, 5, 6, ...

avg time to find an error = 5 min

scale = 5

The time taken to debug is exponentially distributed with mean of 5 minutes

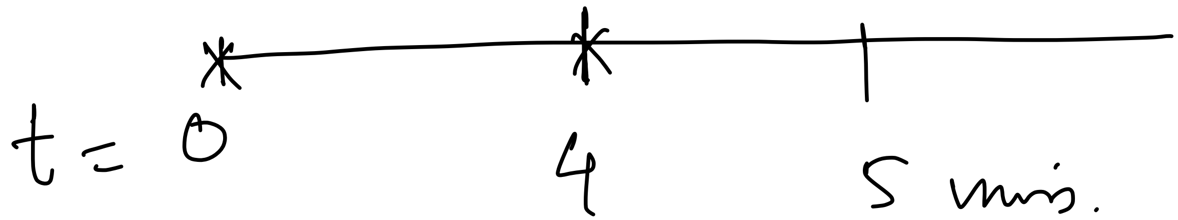
5 min $\rightarrow$ Error.

1 min $\rightarrow$ $\left(\frac{1}{5}\right)$ Error

$$\boxed{\lambda_1 = \left(\frac{1}{5}\right)}$$

- $P[T \le x] = 1 - e^{\lambda_1 * x}$
- expon.cdf(x, scale).

Q1) Find the probability of debugging in 4 to 5 minutes



$$P[4 < T < 5] = P[X \le 5] - P[X \le 4]$$

$$= \left[1 - e^{-\left(\frac{1}{5}\right)\cdot 5}\right] - \left[1 - e^{-\left(\frac{1}{5}\right)\cdot 4}\right]$$

$$= \left(e^{-\frac{4}{5}} - e^{-1}\right) = 0.081 \checkmark$$

$$\approx 8\%$$

$$P[4 < T < 5] = \text{expon.cdf(scale} = 5, x = 5)$$

$$- \text{expon.cdf(scale} = 5, x = 4)$$

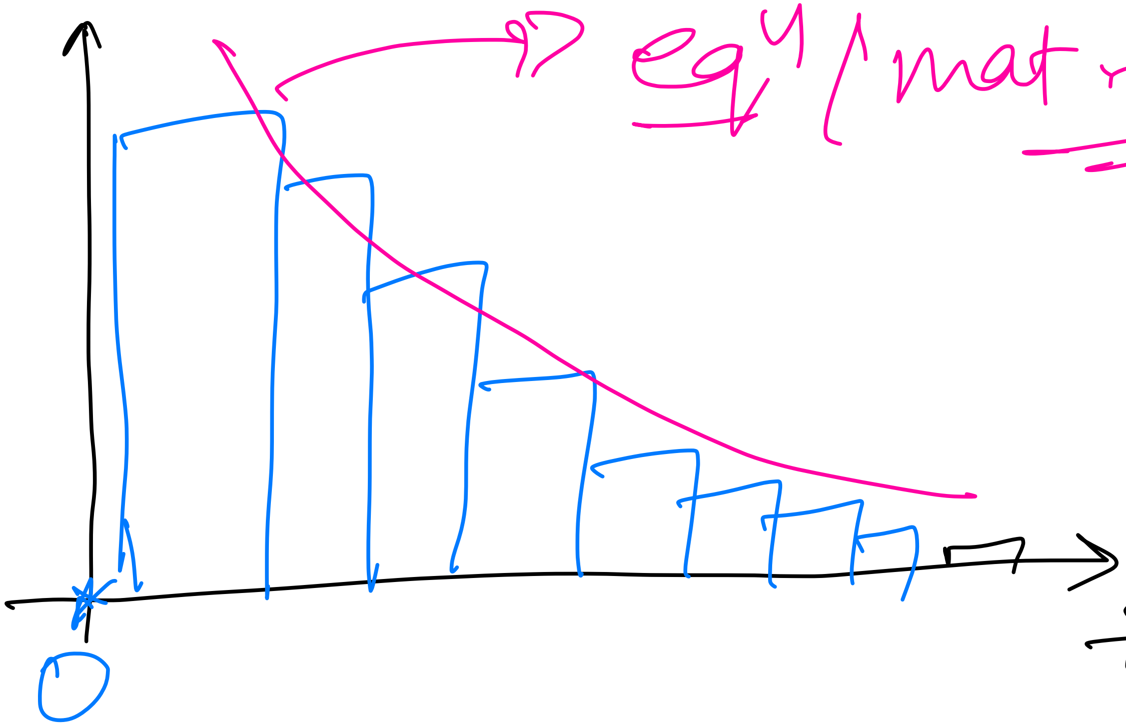
$$= 0.081 \checkmark$$

Expo. time taken to find first Error.

. 2, 1, 0.5, 2, 3, 4, 5.5, ---- 7, ---

R.V

eq^n / mat fn



t.

Q2) Find the probability of needing more than 6 minutes to debug

- $P[T \le t] = 1 - e^{-\lambda_1 t}$.
- expon.cdf(scale, x)

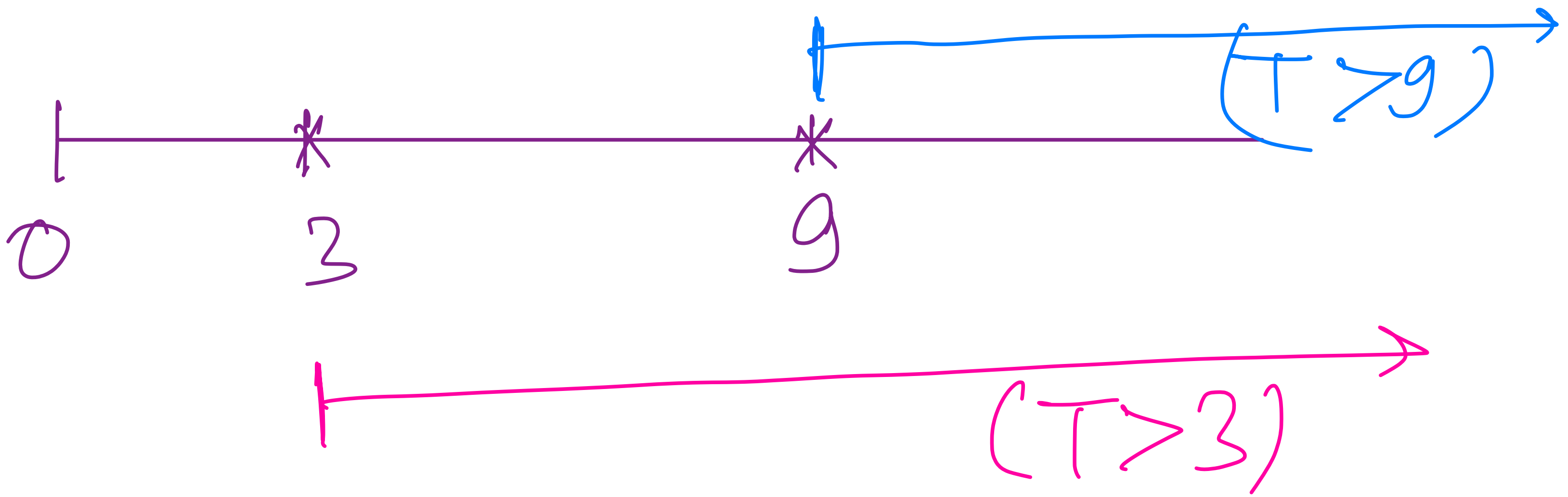
- $P(T>6) = 1 - P[T \le 6]$

$= 1 - \text{expon.cdf}(\text{scale}=5, x=6)$

$P(T>6) = 0.3011$

Can be any general interval

Given that you have already spent 3 minutes, what is the probability of needing more than 9 minutes



$$P[T>9 \mid T>3] = ? \qquad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P[(T>9) \cap (T>3)]}{P(T>3)}$$

$$= \frac{P[T>9]}{P[T>3]} = \frac{1 - P[T \leq 9]}{1 - P[T \leq 3]}$$

$$= \frac{1 - [1 - e^{-\frac{1}{5} \cdot 9}]}{1 - [1 - e^{-1/5 \cdot 3}]} = \frac{e^{-9/5}}{e^{-3/5}}$$

$$= e^{\frac{3}{5} - \frac{9}{5}} = e^{-6/5}$$

$$P(T>6) = 1 - \left[1 - e^{-\lambda_1 \cdot 6}\right]$$

$$= \cancel{1} - \cancel{1} + e^{-\frac{1}{5} \cdot 6}\Big] = e^{-6/5}$$

$$\boxed{P(T>9 \mid T>3) = P(T>6)} \; *$$

Memoryless:

The fact that you took three minutes so far does not affect how much more you might take to debug

* Poisson: (avg / time) Rate → 100 trees / acre

(time or space bound activity) ** ↑

Rate → typos → 100 pages

acre → space

Errors ← 0, 1, 0, 5, 1, 2, 0, 3, 0, 5, 7, 0, 0, 1, 2, 3, 0, ...

Rate of Errors/typos $\Rightarrow$ Rate = 3/page

?

mu

$P(X=0)$ = poisson.pmf(mu=3, k=0)