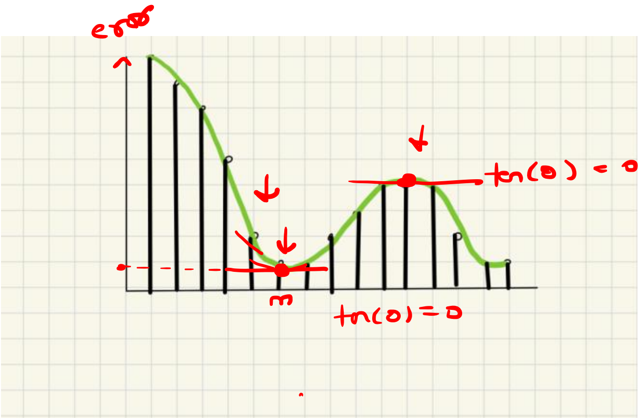
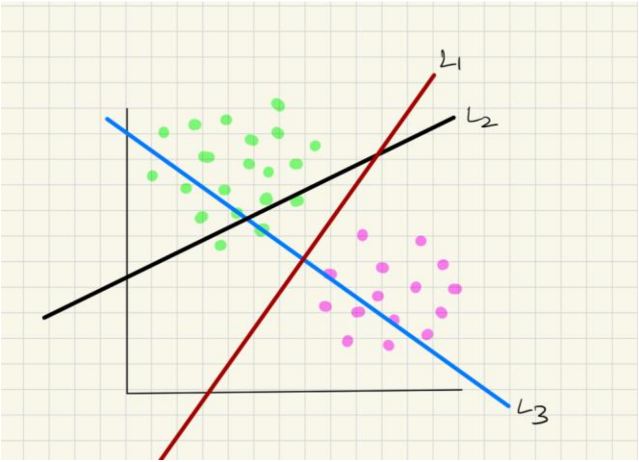
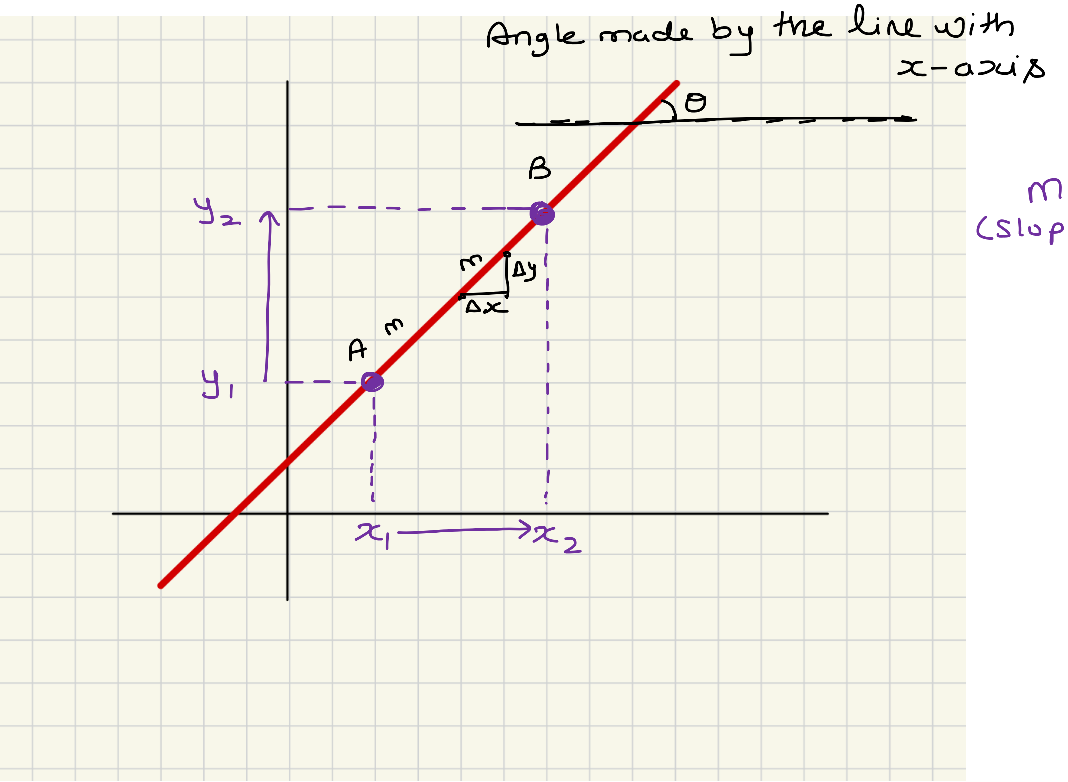
Intuition Behind Classification Algorithm Using Functions





Slope Of A Line



Slope

The rate of change in y direction w.r.t to the x direction

$$\underset{\text{(slope)}}{m} = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \tan(\theta)$$

$m_4 = \frac{4\ 1}{2 \cdot 1} = 2(+ve)$

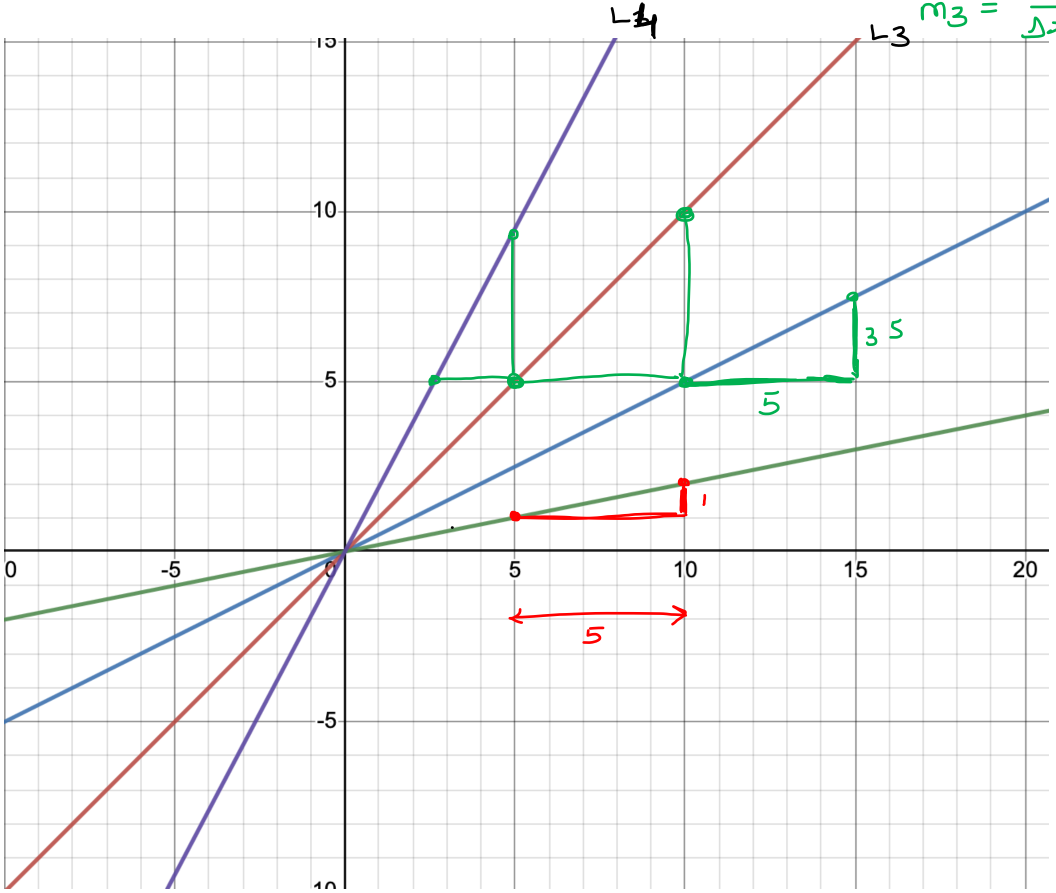
Slope Of A Line

$m_3 = \frac{\Delta y}{\Delta x} = \frac{5}{5} = 1\ (+ve)$

$\tan(45^\circ) = 1$

$L_1\ L_2\ L_3\ L_4$

+ve slope



$L_2\ m_2 = \frac{\Delta y}{\Delta x} = \frac{3.5}{5}\ (+ve)$

$L_1\ m_1 = \frac{\Delta y}{\Delta x} = \frac{1}{5} = (+ve)$

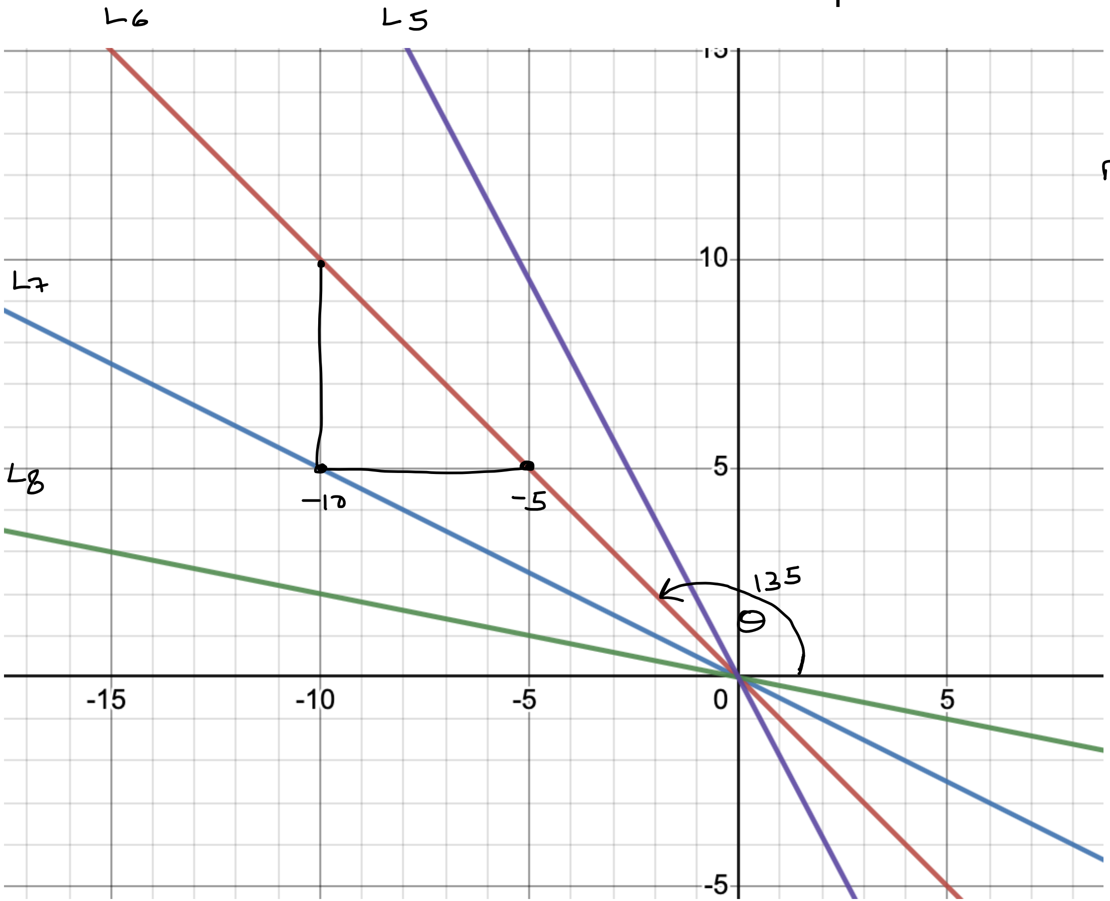
θ is increasing my slope is increasing

$m_1 < m_2 < m_3 < m_4$

$m = -1$

Slope Of A Line

$\tan(135°)$

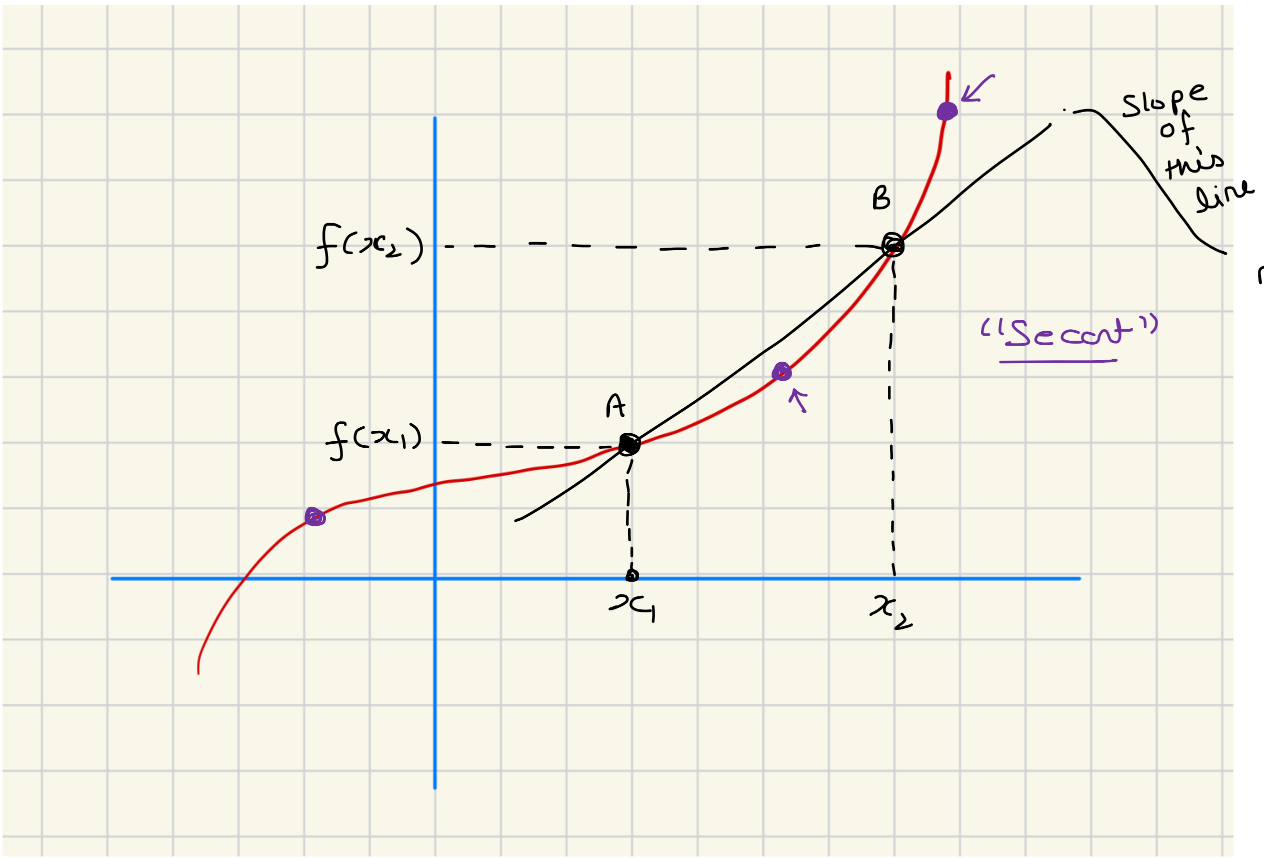


$$m = \frac{\Delta y}{\Delta x} = \frac{(10-5)}{-10-(-5)} = \frac{5}{-5} = -1$$

$L_5 \ L_6 \ L_7 \ L_8 = (-ve)$ slope

$f(x)$

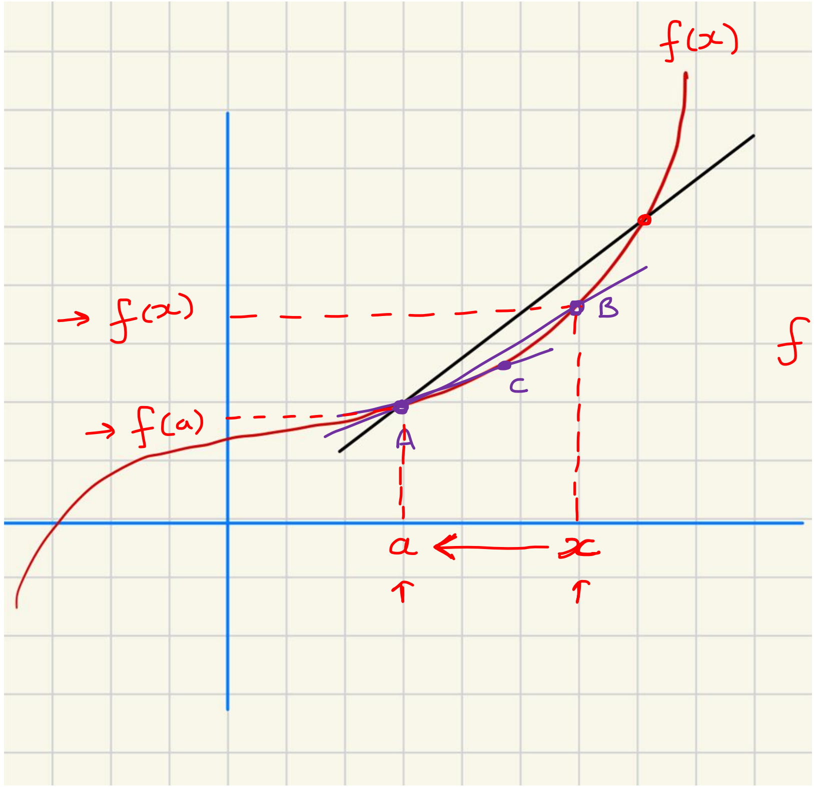
Slope Of A Curve



$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Slope Of A Curve



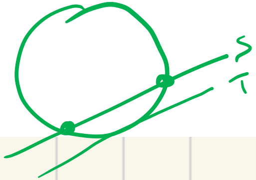
$$m = \frac{\Delta y}{\Delta x}$$

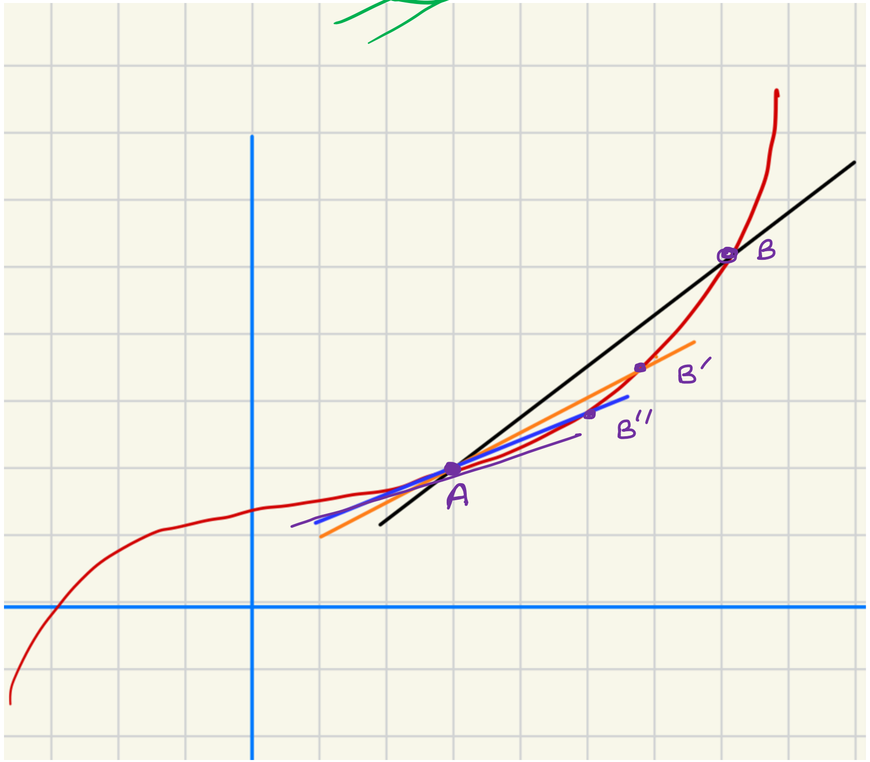
$$= \frac{f(x) - f(a)}{x - a}$$

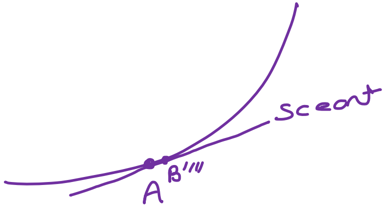
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ } slope of the curve at point a

The Derivate of a given function is the slope of the function at a point

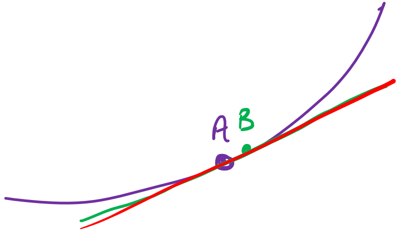
Slope Of A Curve







tangent is a line that touches the curve at a given point "only one point"



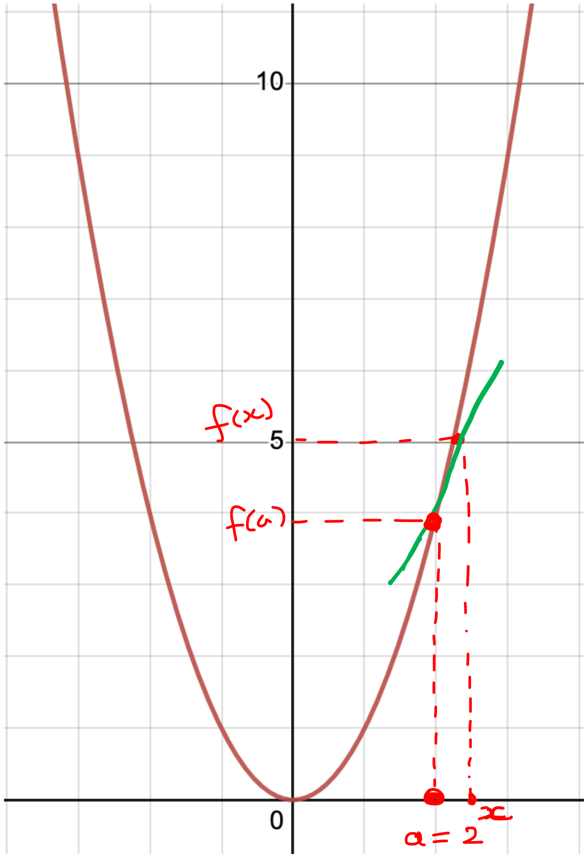
{ Secant is a line that crosses the curve at two point "only two points)

If the point B is very very close to A then don't you think that secant would overlap the tangent

$f(x) = \underline{\underline{x^2}}$ $\quad 2x$ $\quad 4$

Slope Of A Curve

Q: find the slope of the curve at $x=2$



$$m = \frac{\Delta y}{\Delta x} = \frac{f(x)-f(a)}{x-a}$$

$$\boxed{\lim_{x \to a} \frac{f(x)-f(a)}{x-a}}$$

$$\lim_{x \to a} \frac{x^2 - a^2}{x-a}$$

$$\lim_{x \to a} \frac{(x+a)\cancel{(x-a)}}{\cancel{(x-a)}}$$

$$\boxed{\lim_{x \to a} (x+a)}$$

x

$m = 2x$

$$\lim_{x \to 2} (x+2)$$

$x = 2.0001$

$m(x=2) = 2.0001 + 2 = 4.0001$

$\cong 4$

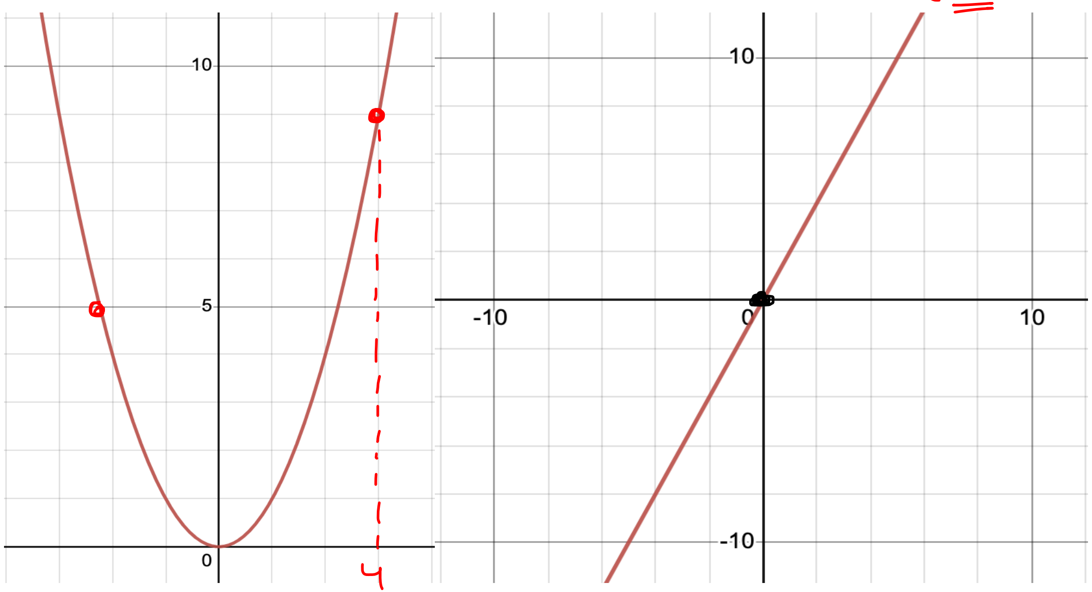
Slope Of A Curve

$y = x^2 \qquad = \underline{\underline{2x}}$

$f''(x) \longrightarrow \underline{\underline{(2x)}}$

$f'(x) =$ derivate of the function $f(x)$

The Rate of change of slope is constant w.r.t x



$$f(x) = x^2$$

$$f'(x) = 2x$$

$$f''(x) = 2$$

The rate of change for the slope of the function $f(x)=x^2$ is 2

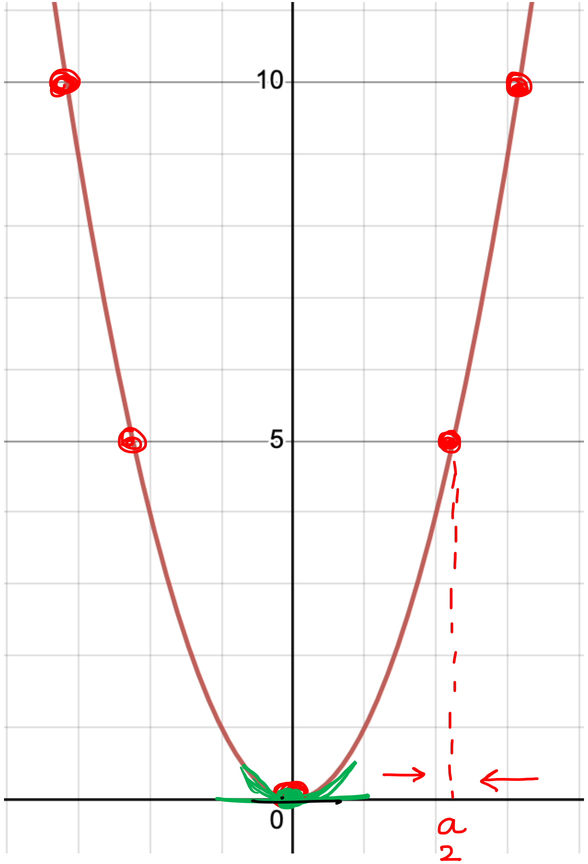
$$\left\{ \lim_{x \to 4} \frac{f(x) - f(4)}{x-4} \right\} \qquad \lim_{x \to 4} (x+4) \simeq 8$$

$$x \simeq 4$$

$f'(x) = \frac{d}{dx} x^2 = 2x$

Are there any function that are not differential at a given point

Can We Differentiate Every Given Function At Any Given Point?



$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

RHL

$\lim_{x \to a^+} (x+a)$

x	$f'(a)$
2.01	4.01
2.001	4.001
2.0001	4.0001

4

LHL

$\lim_{x \to a^-} (x+a)$

x	$f'(a)$
1.99	3.99
1.999	3.999
1.999	3.999

4

$f'(x=a)$

$2x$

2×2

$= 4$

4

$\underline{LHL = RHL}$

$$\begin{bmatrix} LHL = RHL = \cancel{f'(a)} \\ f'(a) \quad f'(a) \end{bmatrix}$$

(Continuous & Differentiable)

Can We Differentiate Every Given Function At Any Given Point? Example

$\rightarrow \; 3 \; \leftarrow$

Continuous

$$f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$$

$$\lim_{x \to 3^-} x^2 \qquad \lim_{x \to 3^+} 6x - 9$$

(8.999) (9.0001)

$$f(x=3) = 6x - 9 = 6 \times 3 - 9 = 9$$

$$[\,LHL = RHL = f(x=a)\,]$$

Check if this function is continuous & differential at 3 or not?

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

$$\lim_{x \to 3^-} \frac{x^2 - 3^2}{x - 3}$$

$$\lim_{x \to 3^-} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$\lim_{x \to 3^-} (x+3)$$

5.9999

LHL

$$\lim_{x \to 3^+} \frac{6x - 9 - (18 - 9)}{x - 3}$$

$$\lim_{x \to 3^+} \frac{6x - 9 - 9}{x - 3}$$

$$\lim_{x \to 3^+} \frac{6x - 18}{x - 3}$$

$$\lim_{x \to 3^+} 6 \frac{\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$\lim_{x \to 3^+} 6$$

RHL

$f'(x=3)$

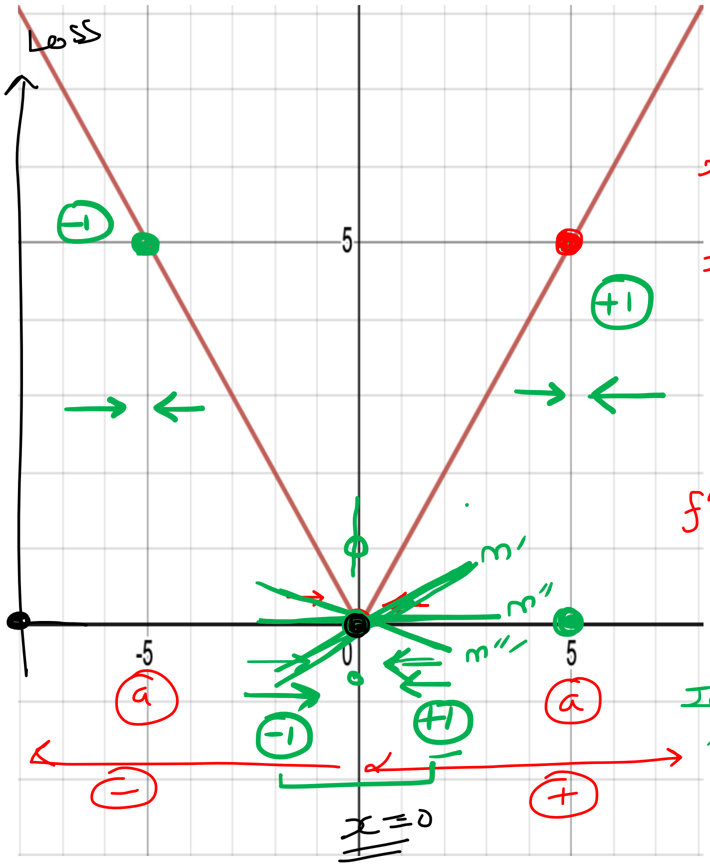
$6x - 9$

6

$f'(x=3)$

continuous but not differentiable at $x=0$

Can We Differentiate Every Given Function At Any Given Point? Example



$$f(x)\begin{cases} x & ; x>0 \\ -x & ; x<0 \\ \boxed{0 \quad ; x=0} \end{cases}$$

$f(x=-5) = 5$

$f(x=a) = a$

$f'(x)$

$$\lim_{x\to 5} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to 5} \frac{x-5}{x-5}$$

+1

$$\lim_{x\to -5} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to -5} \frac{-x-5}{x-(-5)}$$

$$\lim_{x\to -5} \frac{-(x+5)}{(x+5)}$$

−1

$$f'(x=0) \lim_{x\to 0^-} = -1$$

$$f'(x=0) \lim_{x\to 0^+} = +1$$

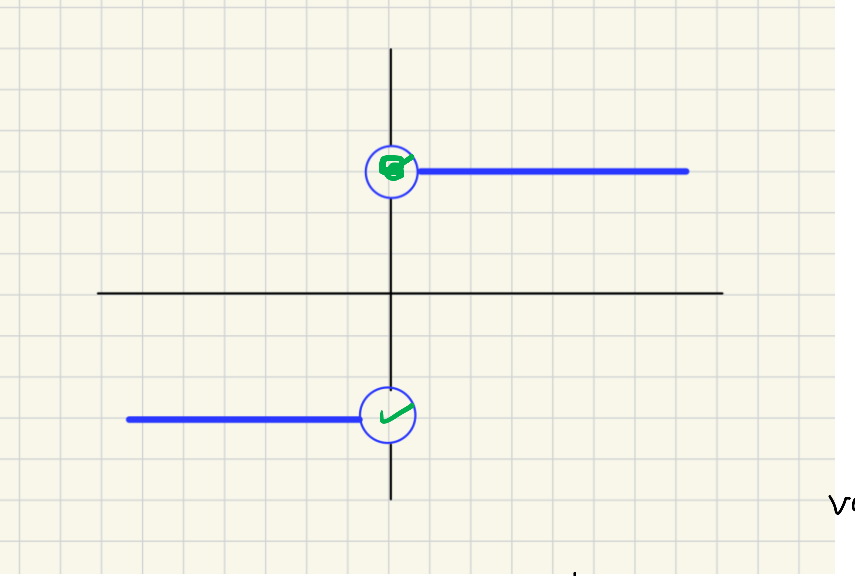
Indefinite value of m at $x=0$

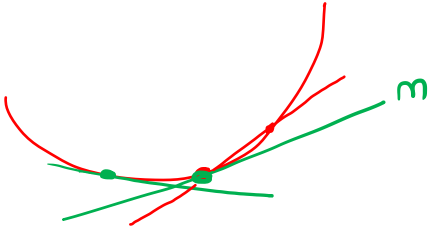
at $x=0$

LHL $\neq$ RHL

this function is not differentiable at $x=0$

Can We Differentiate Every Given Function At Any Given Point? Example

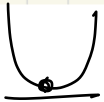




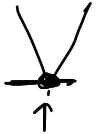
$(x - \bar{x})$

variance $\downarrow$

$$\sum \frac{(y-\bar{y})^2}{n} \qquad \sum \frac{|y-\bar{y}|}{n}$$



f''



Some Common Derivatives

ⓐ $\frac{d}{dx} x^n = n x^{n-1}$

ⓑ $\frac{d}{dx} \log(x) = \frac{1}{x}$

ⓒ $\frac{d}{dx} e^x = e^x$

ⓓ $\frac{d}{dx} c = 0$

ⓔ $\frac{d}{dx} \sin(x) = \cos(x)$

ⓕ $\frac{d}{dx} \cos(x) = -\sin(x)$

ⓖ $\frac{d}{dx} \tan(x) = \sec^2(x)$

$x^3 = x^n = nx^{n-1}$

$3x^{3-1} = 3x^2$

Rules Of Differentiation

① Linearity Rule

$f(x) = x^3 + \log(x)$

$f'(x) = 3x^2 + \frac{1}{x}$

$f(x) = 3\sin(x)$

$f'(x) = 3\cos(x)$

$h(x) = g(x) + f(x)$

$h'(x) = g'(x) + f'(x)$

$h(x) = c \cdot f(x)$

$h'(x) = c\, f'(x)$

Rules Of Differentiation

② Product rule

$$h(x) = f(x) \cdot g(x)$$

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$h(x) = x \sin x$$

$$\sin(x) \cdot \frac{d}{dx} x + x \cdot \frac{d}{dx} \sin(x)$$

$$\sin(x) + x \cdot \cos(x)$$

Rules Of Differentiation

③ Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$

$$f(x) = \frac{\sin(x)}{\cos(x)} = \tan(x)$$

$$h'(x) = \frac{g(x) \cdot f'(x) - g'(x) \cdot f(x)}{[g(x)]^2}$$

$$f'(x) = \frac{\cos(x)\frac{d}{dx}\sin(x) - \sin(x) \cdot \frac{d}{dx}\cos(x)}{[\cos(x)]^2}$$

$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$f'(x) = \sec^2(x)$$

Rules Of Differentiation

(4) <u>chain Rule</u>

$$h(x) = f(g(x))$$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h(x) = \log(x^2)$$

$$h'(x) = \frac{d}{dx}\log(x^2) \cdot \frac{d}{dx}x^2$$

$$= \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x}$$

$$h'(x) = \frac{2}{x}$$

$$h(x) = e^{-x}$$

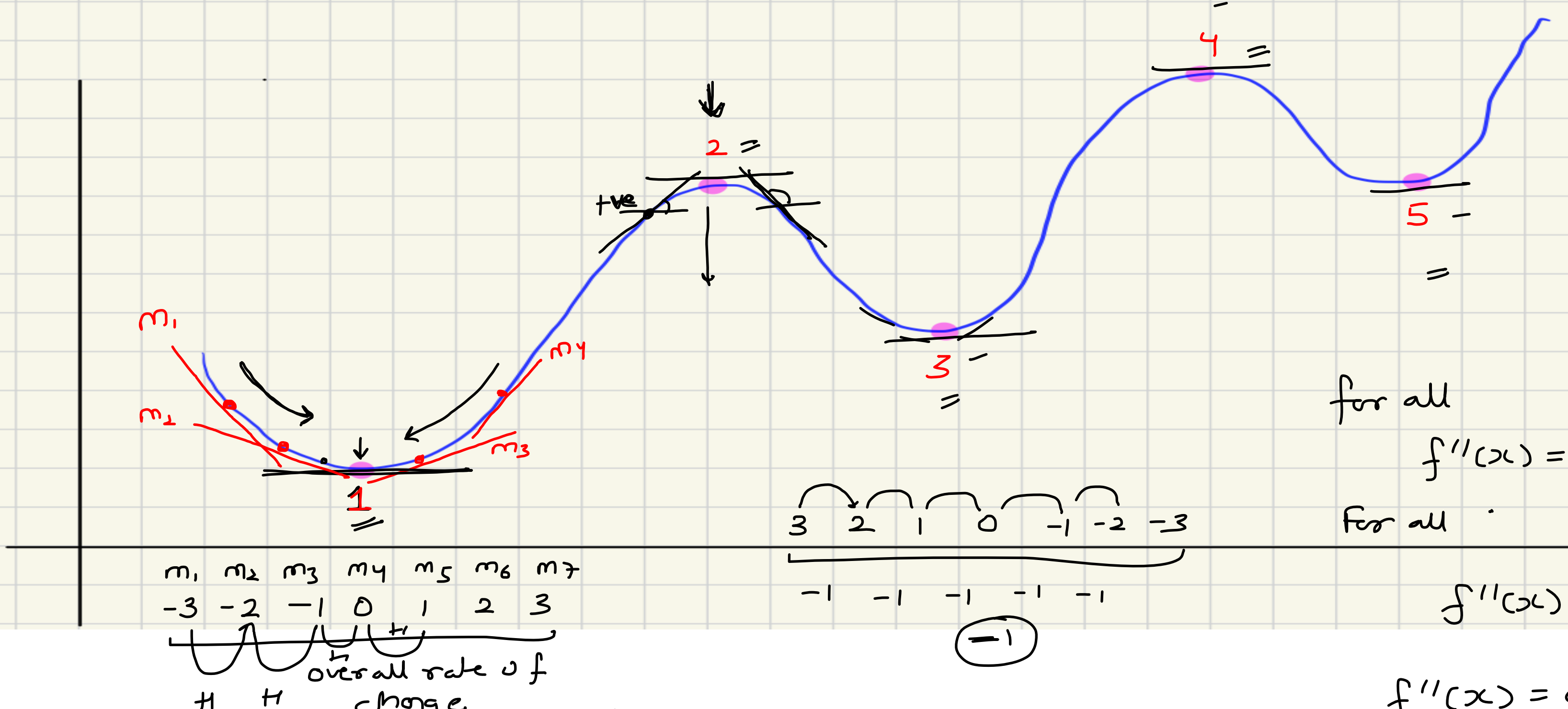
$$h'(x) = \frac{d}{dx}e^{-x} \cdot \frac{d}{dx}(-x)$$

$$= e^{-x}(-1)$$

$$h'(x) = -e^{-x}$$

Maxima and Minima

1, 3, 5 → minima
2, 4 → maxima



+1

$f''(x=1) = +ve$

for all minima

$f''(x) = +ve$

for all maxima

$f''(x) = -ve$

$f''(x) = 0$ Saddle point

Maxima and Minima

How to find candidate points for testing Minima and Maxima?

$$f'(x) = 0$$

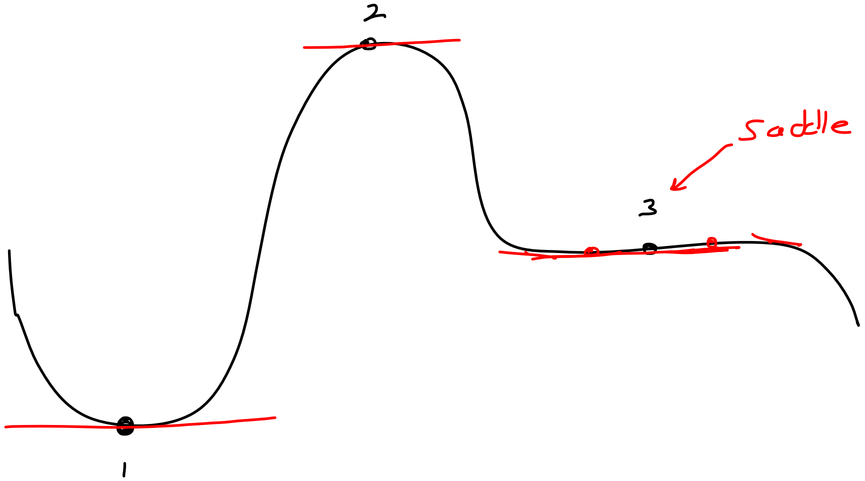
Once we identify candidate points, how to classify them in maxima point and minima point?

$$f''(x) > 0 \longrightarrow \text{minima}$$

$$f''(x) < 0 \longrightarrow \text{maxima}$$

$$f''(x) = 0 \longrightarrow \text{Saddle point}$$

Maxima and Minima



Saddle

$f''(x_3) = 0$

m is not changing

$f'(x_1) = 0$

$f'(x_2) = 0$

$f'(x_3) = 0$

$f''(x_1) = +ve$

$f''(x_2) = -ve$