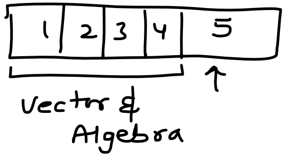
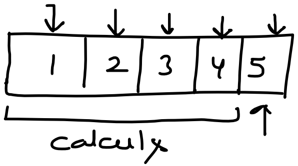


Revision



1. Line / hyperplane — General Eq. of the line $\underbrace{w_1 x_1 + w_2 x_2}_{\text{input features}} + w_0 = 0$ [$w_1, w_2 \dots w_n \dots$ weight and w_0 is the bias]

2. vector $\vec{V}$ $\bar{V}$ $\vec{w}$ $\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ $\vec{x}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

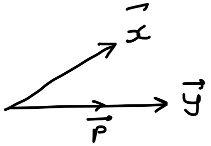
3. Norm / magnitude / Length $\|\vec{w}\|$ $\sqrt{w_1^2 + w_2^2}$

4. Dot product $\vec{x}$ and $\vec{y}$ $\vec{x}^T \cdot \vec{y}$

5. $\cos(\theta) = \dfrac{\vec{x}^T \cdot \vec{y}}{\|\vec{x}\| \, \|\vec{y}\|}$

6. unit vector $\hat{x} = \dfrac{\vec{x}}{\|\vec{x}\|}$

7. Projection of $\vec{x}$ on $\vec{y}$



$$\boxed{\|\vec{p}\| = \vec{x}^T \cdot \hat{y}}$$

$$\hat{p} = \frac{\vec{p}}{\|\vec{p}\|} \qquad \vec{p} = \|\vec{p}\| \cdot \hat{p}$$

$$\boxed{\vec{p} = \|\vec{p}\| \cdot \hat{y}}$$

8. Distance of the line from origin

$$\|\vec{d}\| = \frac{w_0}{\|\vec{w}\|}$$

9. weight vector is always perpendicular to the hyperplane

10. Distance of a point (x_1, x_2) from the hyperplane (w_0, w_1, w_2)

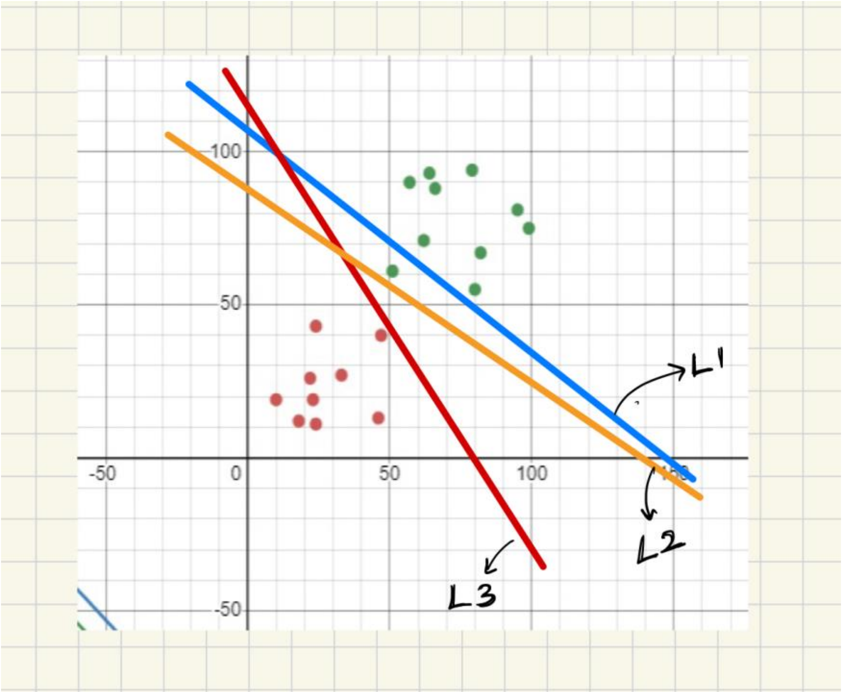
$$\|\vec{d}\| = \frac{w_0 + w_1 x_1 + w_2 x_2}{\|w\|}$$

11. Half space [the direction of the weight vector +ve half space]

$\text{sign}(\vec{d})$ = +ve data point is above the line

$$\vec{x}^T \cdot \vec{y} = \|x\| \, \|y\| \cos(\theta)$$

Revision

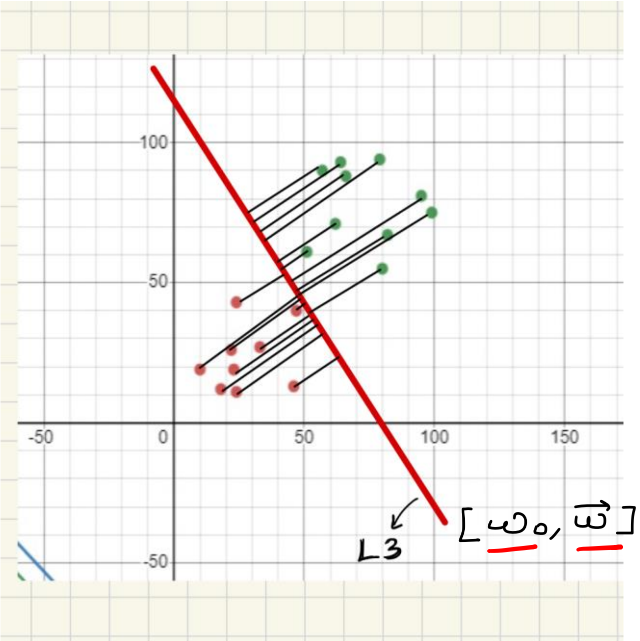


① minimize misclassification

② Total distance need to be maximized

Revision

$$G(\vec{x}, \underline{\vec{w}}, \underline{w_0}) = \sum_{i=1}^{n} \left(\frac{\vec{w}^T \cdot x_i + w_0}{\|\vec{w}\|} \right) \cdot y_i$$



$[\underline{w_0}, \underline{\vec{w}}]$

total distance

gain function

Rotation to optimize. (If there is a mis classification)

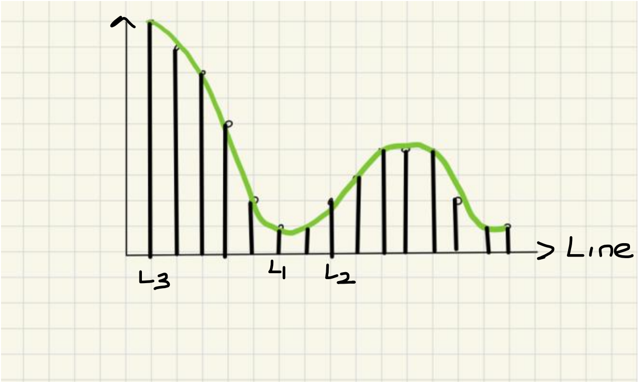
update rule

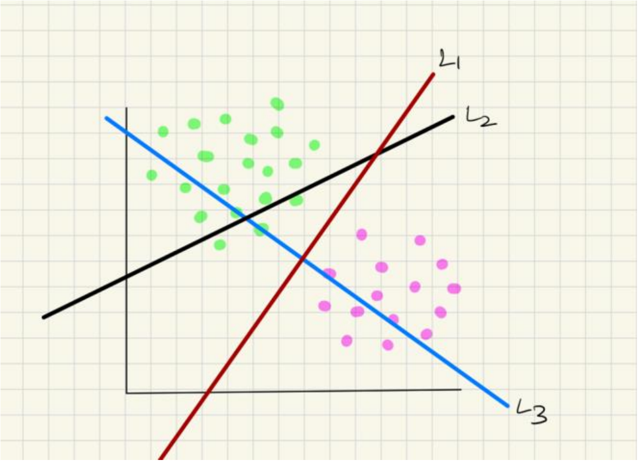
$$\begin{cases} \vec{w} = \vec{w} + (\vec{x} \cdot y_i) \\ w_0 = w_0 + y_i \end{cases}$$

Intuition Behind Classification Algorithm Using Functions

Line $\longrightarrow$ parameter $(\vec{w}, w_0)$

error rate





−10 ⟶ 10

Optimisation Using $\boxed{\text{Linear Search}}$ and It’s Drawback

(−10 — 10)

100 100

W0 (−10 ...... −1.2 −1.1 −1, 0, 1, 1.1, 1.2, 1.3 ...... +10) ⟶ 201

x_1 W1 ( ) ⟶ 201

x_2 W2 ( ) ⟶ 201

Number of possible combinations to test?

w_0 w_1

$201 \times 201 \times 201$

w_1 w_2 w_0

(0, 0, 0) ⟶ to process each combination (10^{-6} secs)

(1, 1, 1)

(1, 0, 1)

⋮

$(201)^3$

8 sec ⟶ best $\vec{w}$, w_0

finding the best $\vec{w}$, w_0 where error is min.

What’s the solution?

Gradient Descent   optimisation Algo

1. Limits, continuity
2. Derivaties, slopes & tangets
3. calculas with single var
4. calculas with multiple var
5. maxima, minima

(Loss function)

Defining Classification Problem Mathematically

Laballed data

$$D = \{ (\vec{x}_i, y_i) : \vec{x}_i \in \mathbb{R}^d, y_i \in \{-1, 1\} \}_{i=1}^{n}$$

Goal classification

$$[f(\vec{x}_i) = y_i]$$

y_i Actual Label

$\hat{y}_i \rightarrow$ predicted Label

minimise the misclassifications

$$G(D, \underbrace{\vec{w}, w_0}) =$$

the error is minimum

D features [target Label]

	x1	x2	x3	y
1 x_i				
2				
3				
⋮				
n				

Defining Classification Problem Mathematically

$$g(\vec{x}_i, y_i, \vec{w}, w_0) = \left[\frac{\vec{w}^T \cdot x_i + w_0}{\|\vec{w}\|}\right] \cdot y_i$$

$$\boxed{\text{optimal } \bar{w}, w_0} = \underset{\bar{w}, w_0}{\operatorname{argmax}} \overbrace{\sum_{i=1}^{n} \left[\frac{\bar{w}^T \cdot x_i + w_0}{\|\bar{w}\|}\right] \cdot y_i}^{G(D, \bar{w}, w_0)}$$

I want that $\bar{w}, w_0$ where the gain is maximum

$$a = \text{np.array}([1, 2, 9, 0]) \qquad \begin{array}{cccc} 0 & 1 & 2 & 3 \\ 1 & 2 & 9 & 0 \end{array}$$

$$\text{argmax}(a) = 2$$

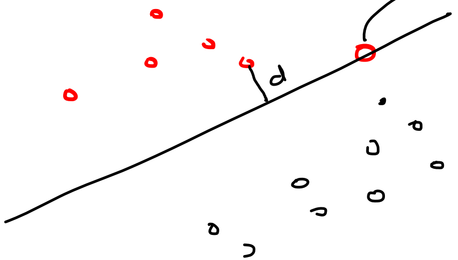
$$\begin{array}{ccccccc} \bar{w}', w_0' & \bar{w}'', w_0'' & [\bar{w}''', w_0'''] & & & & \\ D_1 & D_2 & D_3 & \_ & \_ & \_ & \_ \end{array}$$

Gain Function

Σ Sum.

$\prod$ multiplication

$$= \left\{ \prod_{i=1}^{n} \underbrace{\left(\frac{\bar{w}^T \cdot x_i + w_0}{\|\bar{w}\|} \right)}_{d = 0} \cdot y_i \right\} \longrightarrow 0$$



Functions Basics

① one to one relation

x f(x)

fx { (2,-1), (1,0), (3,4) } function

gx { (2,2), (3,0), (2,7), (4,1)} not a function

i/p → mathematical Exp. → o/p

The function is defined with var. f and it takes input x

$f(x) = x^2$ → o/p

$[y = x]$ function

x	y
1	1
2	2
3	3
⋮	⋮

$[y = x^2]$ function

x	y
1	1
2	4
3	9
⋮	⋮

x 2 → f(x) → x^2 4

x	f(x)
2	-1
1	0
3	4

x	g(x)
2	2
3	0
2	7
4	1

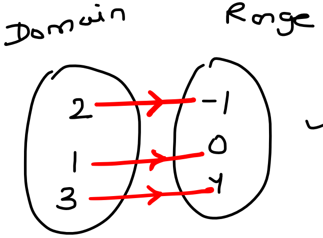
$f(a) = a^2$

$g(a) = a^2$

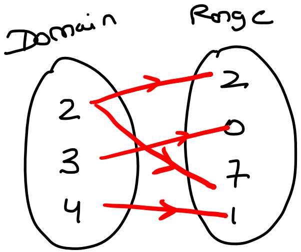
g $h(a) = a$

Functions Basics

Domain & Range of function
(input) (output)

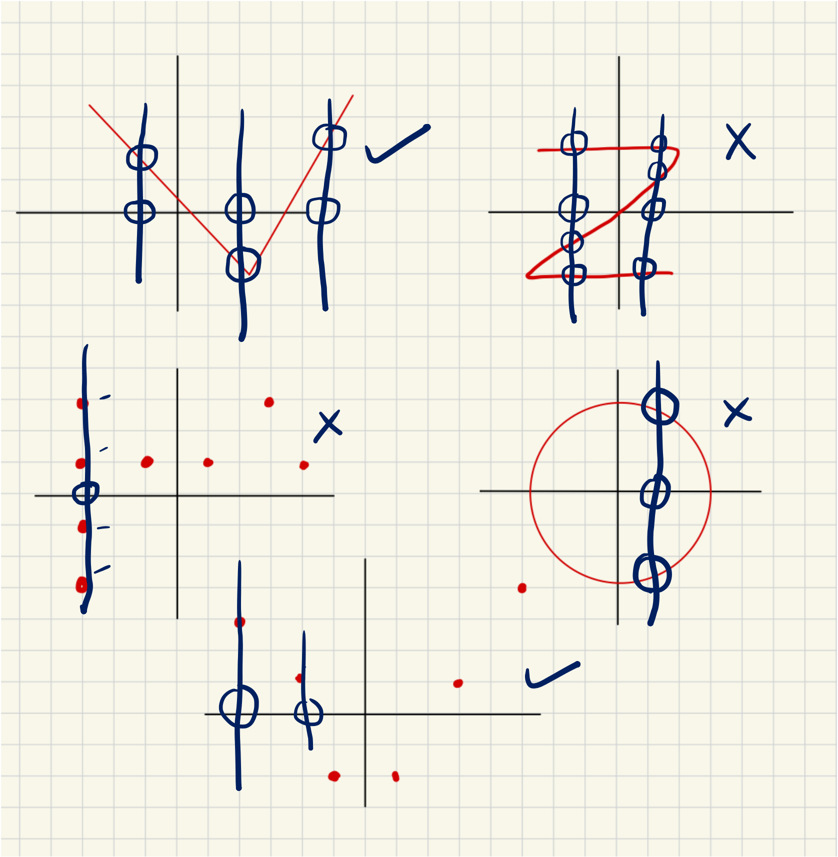


✓ function



Not a function
It doesn't follow one to one relation

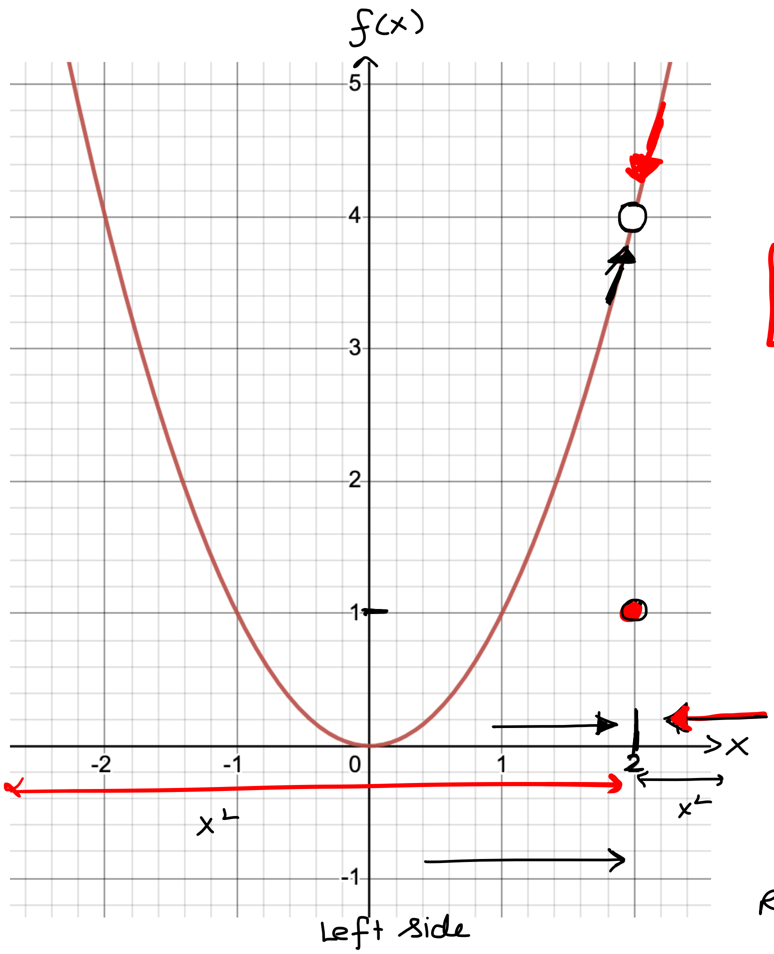
Functions Basics



Parabola

Limits Basics

$\boxed{f(x) = x^2}$



$$f(x) \begin{cases} x^2 & x \neq 2 \\ 1 & x = 2 \end{cases}$$

$f(x=2) = 1$

$[f(x=2) = 1]$

$$\left[\lim_{x \to 2^-} f(x) = x^2\right]$$

$x = 1.9$	$f(x) = 3.6$
$x = 1.999$	$f(x) = 3.96$
$x = 1.9999$	$f(x) = 3.996$
approaching 2	approaching 4

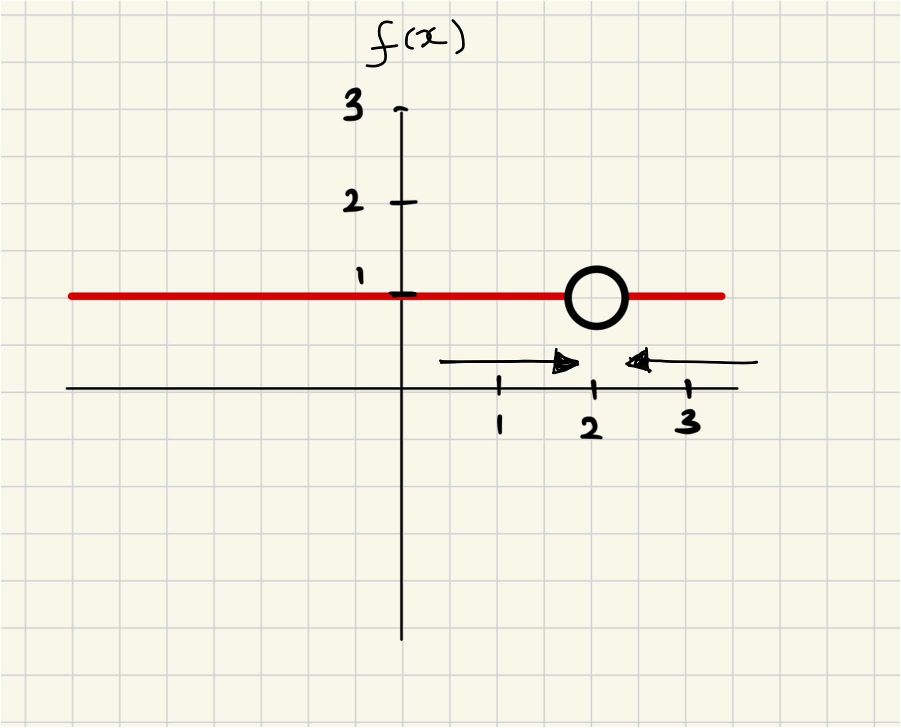
$x = 2.01$	$f(x) = 4.01$
$x = 2.001$	$f(x) = 4.001$
$x = 2.0001$	$f(x) = 4.0001$
approaching 2	approaching 4

Right side

$$\left[\lim_{x \to 2^+} f(x) = x^2\right]$$

Limits Basics

$$f(x) = \frac{x-2}{x-2}$$



$$f(x) = \begin{cases} \frac{x-2}{x-2} & x \neq 2 \\ \text{undefined} & x = 2 \end{cases}$$

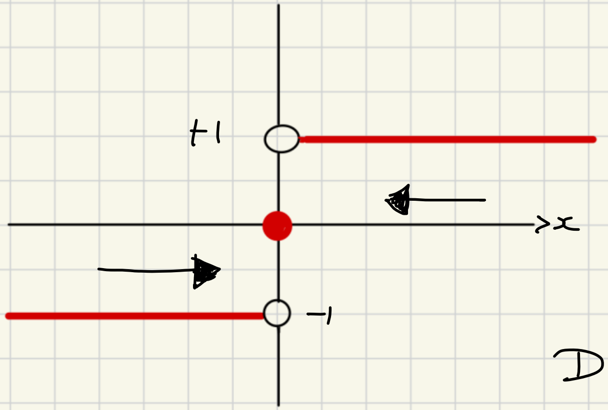
$$\lim_{x \to 2^-} f(x) = 1$$

$$\lim_{x \to 2^+} f(x) = 1$$

$$f(x=2) = \frac{2-2}{2-2} = \text{undefined}$$

Signum function

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$



Discontinuous function

$$\lim_{x \to 0^-} f(x) = -1$$

$$f(x=0) = 0$$

$$\lim_{x \to 0^+} f(x) = +1$$

Limits Basics

$$f(x) = 1/x$$

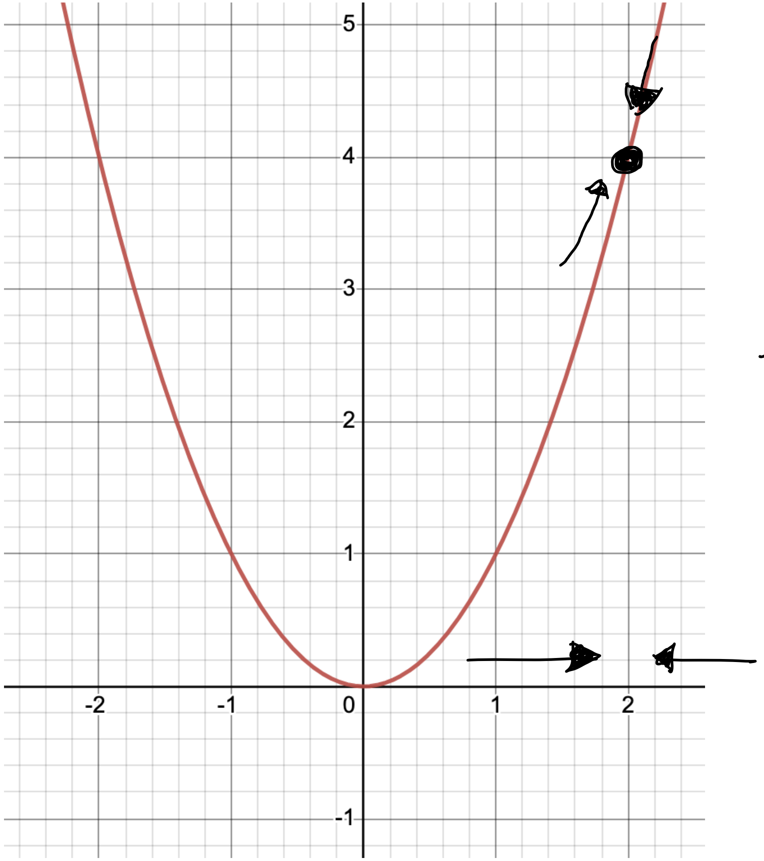
$$\lim_{x \to 0^-} f(x) = -\infty$$

$$\lim_{x \to 0^+} f(x) = +\infty$$

$$f(x=0) = \text{undefined}$$

The function that you can draw without lifting pen

Continuous Function



$$\lim_{x \to a^-} f(a) = \lim_{x \to a^+} f(a) = f(x=a)$$

$$\lim_{x \to 2^-} f(x) = 4$$

$$\lim_{x \to 2^+} f(x) = 4$$

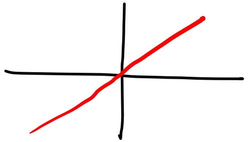
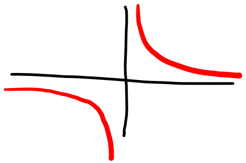
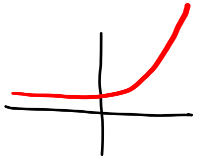
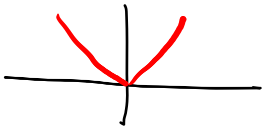
$$f(x=2) = 4$$

Continuous function

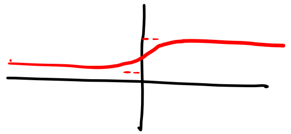
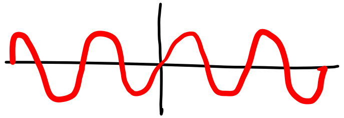
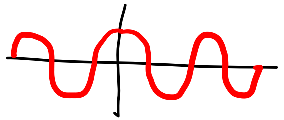
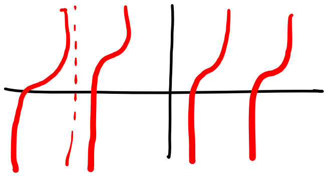
$$LHL = RHL = f(a)$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Some Important Function

Function	**Domain**	**Range**	**Is Continuous**	**Plot**
1 $f(x) = x$	$(-\infty, +\infty)$	$(-\infty, +\infty)$	Yes	
2 $y = \frac{1}{x}$	$(-\infty, +\infty) - \{0\}$	$(-\infty, +\infty) - \{0\}$	NO	
3 $y = e^{x}$	$(-\infty, +\infty)$	$(0, \infty) - \{0\}$	Yes	
4 $y = \lvert x \rvert$	$(-\infty, +\infty)$	$(0, \infty)$	Yes	

Some Important Function

Function	Domain	Range	Is Continuous	Plot
(6) $f(x) = \frac{1}{1+e^{-x}}$ Sigmoid function	$(-\infty, +\infty)$	$(0, 1)$	Yes	
(7) $y = \sin(\theta)$	$(-\infty, +\infty)$	$(-1, +1)$	Yes	
(8) $y = \cos(\theta)$	$(-\infty, +\infty)$	$(-1, +1)$	Yes	
(9) $y = \tan(\theta)$	$(-\infty, +\infty)$	$(-\infty, +\infty)$	No	

$d \propto w_0$

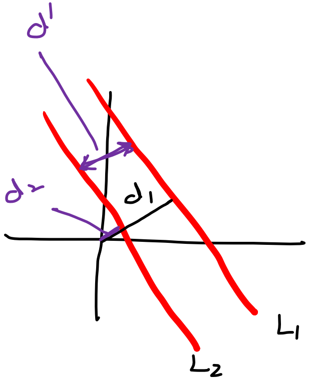
Doubt

$w_1, \; w_2$

$$4x_1 + 3x_2 - 5 = 0 \quad \text{(given)}$$

New line = $4x_1 + 3x_2 + w_0' = 0$

$$4x_1 + 3x_2 - 20 = 0$$



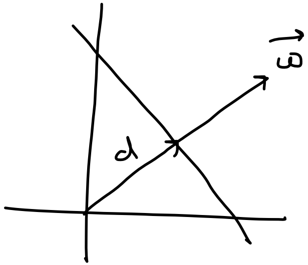
$$d_1 = \frac{w_0}{||\bar{w}||}$$

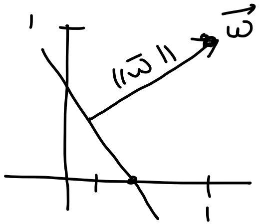
$$d_2 = \frac{w_0'}{||\bar{w}||}$$

$$d' = d_1 - d_2$$

$$= \frac{w_0}{||\bar{w}||} - \frac{w_0'}{||w||}$$

$$3 = \frac{w_0 - w_0'}{||w||}$$





$w_1 = +1$

$w_2 = +1$

$$\sqrt{w_1^2 + w_2^2}$$

$$w_0 - 3 \times ||w|| = w_0'$$

$$-5 - (3 \times 5)$$

$$w_0' = -20$$