# Kinematics of a Delta Configuration Fused Deposition Modelling Device


Varun Uday Nayak
ID: 2014A4PS086G


A project presented for partial fulfilment of the requirements of the course
ME F313: **Production Techniques II**


Department of Mechanical Engineering
**BITS Pilani**, K K Birla Goa Campus

# Contents

# Kinematics of a Delta Configuration Fused Deposition Modelling Device

Varun Uday Nayak

April 30, 2017

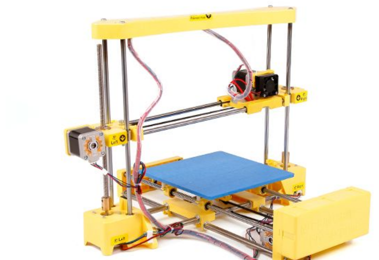
## 1 Introduction

*Rapid Prototyping* has enabled industries to revolutionise the design and development phase of their products. From reduction in time required for prototyping to the improvement in product testing and demonstration, the impact of rapid prototyping technology and its development is rising exponentially. Such devices have also proved to be a useful resource for students for projects and research activities and an indispensable component in *Fab-Labs* (Maker's Spaces).

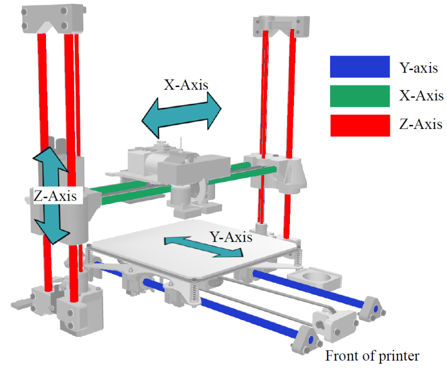
Fused Deposition Modelling, popularly known as *3-D Printing* is the most common type of rapid prototyping device. It uses artificial polymer (such as ABS or PLA) filaments as raw material and extrudes it in its fused form onto a heated bed thus creating the profile of the model to be fabricated. The extruder is translated across space using stepper motors (or any other accurate form of actuation) along with appropriate mechanical linkages. Material is deposited layer by layer and it leads to the formation of a complete model derived from a CAD file. Axis speeds, print patterns, extruder and heated bed temperatures, feed rates, etc. are some of the parameters concerning this process. The process variables are computer controlled and can be set by the user depending on the application.

### 1.1 Cartesian Configuration

The most popular type of 3-D Printer works in Cartesian coordinates. This implies the movement of three mutually perpendicular axes *X, Y* and *Z* independently to position the extruder. This design is simple and the kinematics are straightforward. However, this requires the movement of the heavy bed which leads to higher power requirement for the motors as well as a slow response.



(a) A typical Cartesian printer



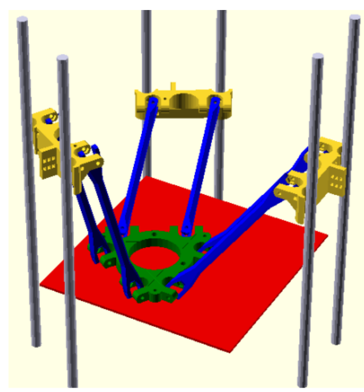

(b) The three axes of a Cartesian printer

Figure 1: Cartesian Configuration

### 1.2 Delta Configuration

The delta configuration devices uses a unique type of mechanism which consists of three independently moving carriages riding over three vertical guide-way columns placed at the vertices of a triangle. The heated bed is stationary and the extruder moves in 3-D space using three parallel arms connecting its platform to the independent carriage units. Hence, the position of the extruder is controlled by varying the position of the carriages along their respective columns. The kinematics are relatively complicated but the computation speed can be made up for by increasing processing power which is quite cheap these days. Thus, the delta configuration printer gives a faster print with a larger maximum print height.

(a) A typical Delta printer



(b) The three arms of a Delta printer

Figure 2: Delta Configuration

## 2 Literature

Since the kinematics of a delta printer are not as simple as a cartesian configuration printer, considerable research has been done on its mechanical design and control. Baroui [1] has summarised the structure, mechanism and electronic control of a self-assembled delta printer. The inverse and forward kinematics have been developed in a comprehensive manner by Graves in his paper [2] on Rostock's design of the Delta Printer.

The geometry and two GUI simulations of the delta printer kinematics have been presented in RepRap's database availble at [3] and [4]. RepRap is an open source community project on developing rapid prototyping machines.

## 3 Methodology

There will be three components in this project.

A mathematical model of the kinematics of the delta printer was developed in this project. The inverse kinematics of the extruder position with respect to the vertical position of the carriages was derived for the positions $X, Y$ and $Z$.

A SolidWorks model of a standard-sized delta configuration printer was modelled and assembled keeping all design parameters as known constants (these values can be altered by using the parametric design technique in SolidWorks). The assembly file was created following which the kinematics was studied using the *Motion Study* toolbox by applying necessary assembly relations and constraints.

Once the mathematical model was ready, the inverse kinematics of the model was simulated on MATLAB depicting the above analytical results graphically. The configuration space (space of operation) of the extruder was determined and plotted.

# 4 Analysis

## 4.1 Inverse Kinematics

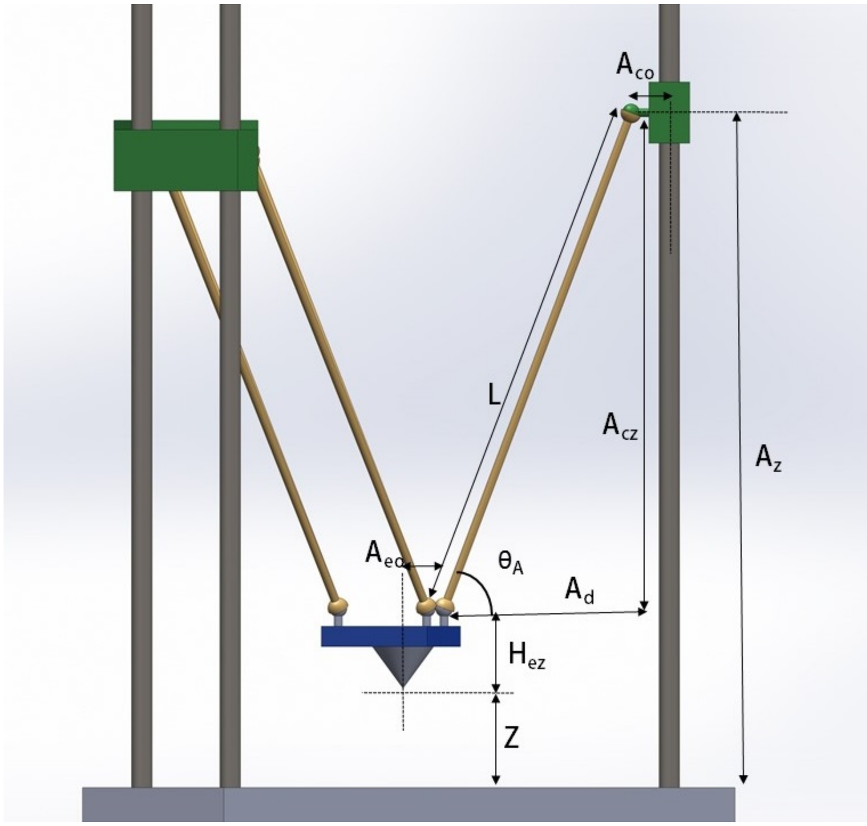



Figure 3: Side-view of the arm-pairs

The three vertical columns will be referred to as arm $A, B$ and $C$. The vertical height of the carriage from the base is given by $A_z$ and that from the base of the extruder platform is given by $A_{cz}$. The arm length is $L$, the horizontal distance is $A_d$ and the arm angle is $\theta_A$. The extruder an the carriage offsets are given by $A_{eo}$ and $A_{co}$ respectively. Similar notations are used for arm-pairs $B$ and $C$. Note that we are using the line of action for each arm-pair and not any one arm itself. The line of action is defined by the line joining the centres of the line joining the two spherical joint-pairs at the carriage and at the extruder platform. This construction simplifies our analysis. Therefore, using pythagoras theorem

$$L^2 = A_d^2 + A_{cz}^2 \tag{1}$$
$$L^2 = B_d^2 + B_{cz}^2 \tag{2}$$
$$L^2 = C_d^2 + C_{cz}^2 \tag{3}$$

Also, for the Z-position, we can write

$$A_z = A_{cz} + H_{ez} + Z \tag{4}$$
$$B_z = B_{cz} + H_{ez} + Z \tag{5}$$
$$C_z = C_{cz} + H_{ez} + Z \tag{6}$$

Now, we shall proceed with our analysis of the $X$ and $Y$ positions of the centre of the extruder.

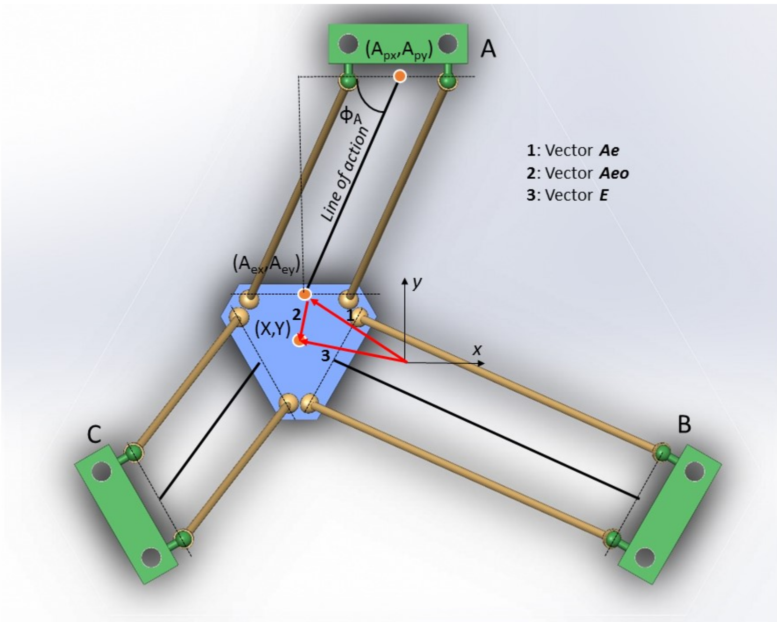



Figure 4: Top-view of the arm-pairs

$(A_{px}, A_{py})$ represents the coordinates of the position where the line of action intersects the line joining the spherical joints at the carriage. $(A_{ex}, A_{ey})$ represents the intersection of the line of action and the line joining the spherical joints on the extruder platform. $(X, Y)$ represents the coordinates of the extruder centre. The origin of coordinates is selected as the centre of the circle passing through $(A_{px}, A_{py})$,$(B_{px}, B_{py})$ and $(C_{px}, C_{py})$.
From a simple vector sum, we get the following relation

$$\vec{E} = \vec{A_e} + \vec{A_{eo}} \tag{7}$$

where $\vec{E}$ is the position vector of the extruder center $\vec{A_e}$ is the position vector of the centre of the joints on the platform. There is constant vector $\vec{A_{eo}}$ which is the extruder offset vector.

From equation (7), we get

$$X = A_{eox} + Aex \tag{8}$$
$$Y = A_{eoy} + Aey \tag{9}$$

Similar equations can be written for carriages $B$ and $C$.
Using distance formula and pythogoras theorem

$$(A_{px} - A_{ex})^2 + (A_{py} - A_{ey})^2 = A_d^2 \tag{10}$$

Substituting 8 and 9 in 10, we get

$$(A_{px} - X + A_{eox})^2 + (A_{py} - Y + A_{eoy})^2 = A_d^2 \tag{11}$$
$$\Rightarrow (X - (A_{eox} + A_{px}))^2 + (Y - (A_{eoy} + A_{py}))^2 = A_d^2 \tag{12}$$
$$\Rightarrow (X - A_{vx})^2 + (Y - A_{vy})^2 = A_d^2 \tag{13}$$

where $A_{v_{x/y}} = A_{eo_{x/y}} + A_{p_{x/y}}$ is the arm offset vector. Note that this vector is a constant.
Replacing $A_d^2$ from equation (1), we get

$$\Rightarrow (X - A_{vx})^2 + (Y - A_{vy})^2 = L^2 - A_{cz}^2 \tag{14}$$
$$\Rightarrow A_{cz} = \sqrt{L^2 - (X - A_{vx})^2 - (Y - A_{vy})^2} \tag{15}$$

and from equation (4)

$$\Rightarrow A_z - H_{ez} - Z = \sqrt{L^2 - (X - A_{vx})^2 - (Y - A_{vy})^2} \tag{16}$$
$$\Rightarrow A_z = \sqrt{L^2 - (X - A_{vx})^2 - (Y - A_{vy})^2} + H_{ez} + Z \tag{17}$$

Using similarity, we can write

$$B_z = \sqrt{L^2 - (X - B_{vx})^2 - (Y - B_{vy})^2} + H_{ez} + Z \tag{18}$$
$$C_z = \sqrt{L^2 - (X - C_{vx})^2 - (Y - C_{vy})^2} + H_{ez} + Z \tag{19}$$

Therefore, equations (17), (18) ad (19) represent the *inverse kinematics* of the Delta Configuration Printer.

## 4.2 Geometrical Constraints

The equations above have been developed assuming no constraint on the movement of the arms or carriages. Therefore, in this section, we shall impose practical limits on the kinematic variables. Each carriage is constrained to move between two limits (upper and lower) so as to prevent any collisions with the frame of the delta printer.

$$A_{z(min)} < A_z < A_{z(max)} \tag{20}$$
$$B_{z(min)} < B_z < B_{z(max)} \tag{21}$$
$$C_{z(min)} < C_z < C_{z(max)} \tag{22}$$

The arm angles $\theta$ and $\phi$ are also limited by the design of the spherical joints. Therefore,

$$\theta_{A(min)} < \theta_A < \theta_{A(max)} \tag{23}$$
$$\theta_{B(min)} < \theta_B < \theta_{B(max)} \tag{24}$$
$$\theta_{C(min)} < \theta_C < \theta_{C(max)} \tag{25}$$

From Figure 3, this can be written as

$$\theta_{A(min)} < \tan^{-1}(\frac{A_{cz}}{A_d}) < \theta_{A(max)} \tag{26}$$

$$\theta_{B(min)} < \tan^{-1}(\frac{B_{cz}}{B_d}) < \theta_{B(max)} \tag{27}$$

$$\theta_{C(min)} < \tan^{-1}(\frac{C_{cz}}{C_d}) < \theta_{C(max)} \tag{28}$$

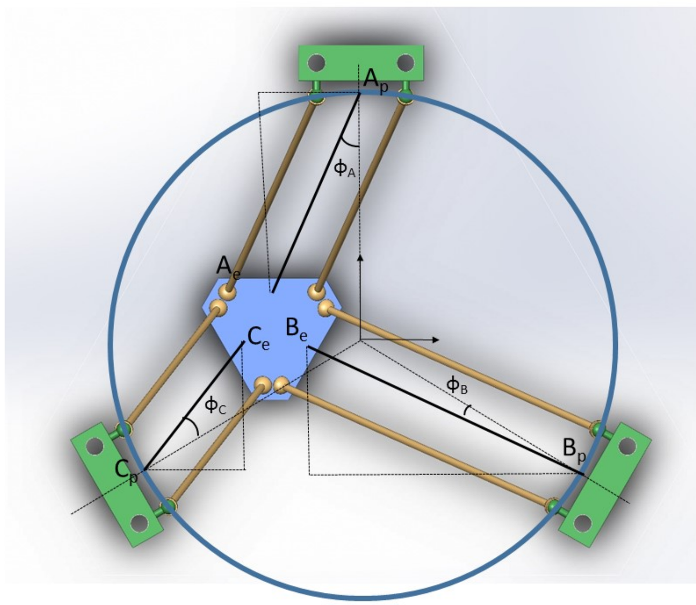



Figure 5: Top-view of the arm-pairs showing arm angles about $Z$ direction

For the arm angles about the $Z$ axis, we have

$$\phi_A = -\tan^{-1}\left(\frac{A_{ex} - A_{px}}{A_{ey} - A_{py}}\right) \tag{29}$$

$$\phi_B = \frac{\pi}{6} + \tan^{-1}\left(\frac{B_{ey} - B_{py}}{B_{ex} - B_{px}}\right) \tag{30}$$

$$\phi_C = \tan^{-1}\left(\frac{C_{ey} - C_{py}}{C_{ex} - C_{px}}\right) - \frac{\pi}{6} \tag{31}$$

Adding constraints,

$$\mid \phi_A \mid < \phi_{A_{limit}} \tag{32}$$

$$\mid \phi_B \mid < \phi_{B_{limit}} \tag{33}$$

$$\mid \phi_C \mid < \phi_{C_{limit}} \tag{34}$$

## 5 SolidWorks Assembly

A typical delta printer has been modelled on SolidWorks in order to visually understand its kinematics. The required mechanical constraints were applied to the joints, shafts, etc. and the motion of the carriages by varying the $(X, Y, Z)$ position of the extruder was observed.

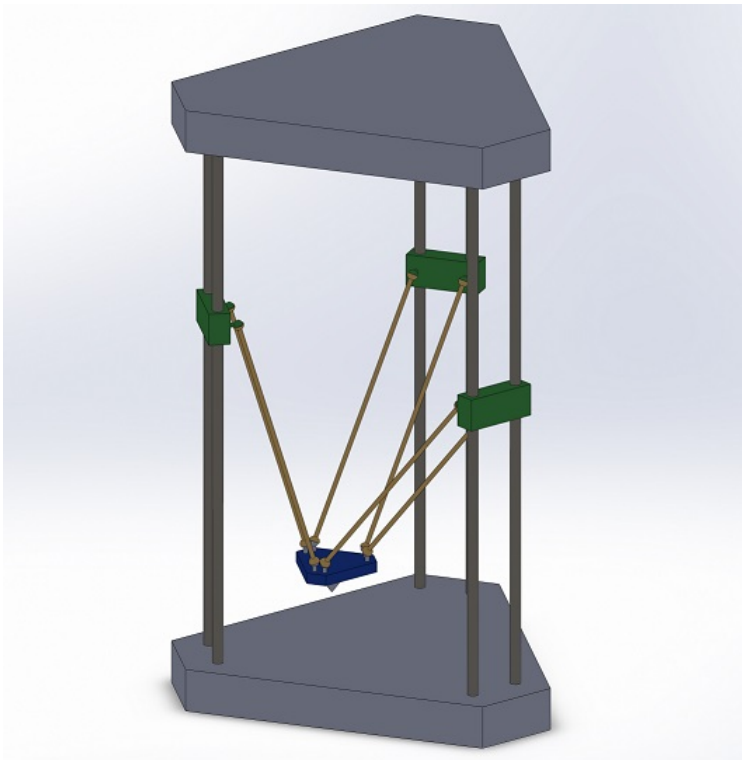


Figure 6: Assembly of the Delta Printer on SolidWorks

The parts include the following:

1. Delta Printer Frame (Structure)
2. Vertical Shaft (6 nos, shown in dark grey)
3. Carriage (3 nos, shown in green)
4. Arm (6 nos, shown in orange)
5. Extruder (end effector, shown in blue)

# 6 Simulation

## 6.1 Inverse Kinematics on MATLAB

The inverse kinematics of the delta printer were used to devise a MATLAB code which would output a set of values of carriage heights $A_z, B_z$ and $C_z$ for input values of extruder coordinates $(X, Y, Z)$. The geometrical constraints mentioned above are imposed upon the inverse kinematics in order to restrict movement of the arm spherical joints and vertical carriage movements.

```
function [Az,Bz,Cz]=deltaprinter(X,Y,Z)
%% Delta Printer Inverse Kinematics
% Function takes in desired X Y Z as input and gives required positions Az
% Bz and Cz of vertically moving carriages
% Author: Varun Uday Nayak %
%% Delta Printer Constants
L=250; %arm length
Hez=39.87; %extruder vertical offset
Aeox=0; %extruder horizontal offsets
Aeoy=-20.33;
Beox=-20.33*cos(30*pi/180);
Beoy=20*sin(30*pi/180);
Ceox=20*cos(30*pi/180);
Ceoy=20*sin(30*pi/180);
Apx=0; %pivotoffsets
Apy=139.09;
Bpx=139.09*cos(30*pi/180);
Bpy=-139.09*sin(30*pi/180);
Cpx=-139.09*cos(30*pi/180);
Cpy=-139.09*sin(30*pi/180);
Avx=Aeox+Apx; %total arm offset
Avy=Aeoy+Apy;
Bvx=Beox+Bpx;
Bvy=Beoy+Bpy;
Cvx=Ceox+Cpx;
Cvy=Ceoy+Cpy;
%% The restrictions on Z axis angle movement
Aex=X-Aeox;
Aey=Y-Aeoy;
Bex=X-Beox;
Bey=Y-Beoy;
Cex=X-Ceox;
Cey=Y-Ceoy;
if not((abs(atan((Aex-Apx)/(Aey-Apy)))<pi/3) &...
        (abs(pi/6+atan((Bey-Bpy)/(Bex-Bpx)))<pi/3) ...
        & (abs(atan((Cey-Cpy)/(Cex-Cpx))-pi/6)<pi/3))
    %disp('Z arm error: Beyond Configuration Space. Try another X,Y,Z next time');
    Az=0;
    Bz=0;
    Cz=0;
    return
end
%% Calculating Az Bz and Cz
Az=Z+Hez+sqrt(L^2-(X-Avx)^2-(Y-Avy)^2);
Bz=Z+Hez+sqrt(L^2-(X-Bvx)^2-(Y-Bvy)^2);
Cz=Z+Hez+sqrt(L^2-(X-Cvx)^2-(Y-Cvy)^2);
%% Checking whether desired X,Y,Z lies in configuration space
if not(Az<470 & Az>260 & Bz<470 & Bz>260 & Cz<470 & Cz>260)
    %disp('Carriage error: Beyond Configuration Space. Try another X,Y,Z next time');
    Az=0;
    Bz=0;
    Cz=0;
    return
end
Ad=sqrt((X-Avx)^2+(Y-Avy)^2);
Bd=sqrt((X-Bvx)^2+(Y-Bvy)^2);
Cd=sqrt((X-Cvx)^2+(Y-Cvy)^2);
Acz=Az-Hez-Z;
Bcz=Bz-Hez-Z;
Ccz=Cz-Hez-Z;
if not(atan(Acz/Ad)<80*pi/180 & atan(Acz/Ad)>35*pi/180 &...
        atan(Bcz/Bd)<80*pi/180 & atan(Bcz/Bd)>35*pi/180 ...
        & atan(Ccz/Cd)<80*pi/180 & atan(Ccz/Cd)>35*pi/180)
    %disp('XY Arm Error: Beyond Configuration Space. Try another X,Y,Z next time');
    Az=0;
    Bz=0;
    Cz=0;
    return
end
end
```

### 6.2 Configuration Space

The configuration space of the extruder (end-effector) is the set of all coordinates $(X, Y, Z)$ that it can locate itself to given the kinematic constraints of the delta printer. Another MATLAB function was used to call the inverse kinematics function and plot those values of $X, Y$ and $Z$ that satisfy the motion constraints. The configuration space gives us a good idea of the print volume of the printer.

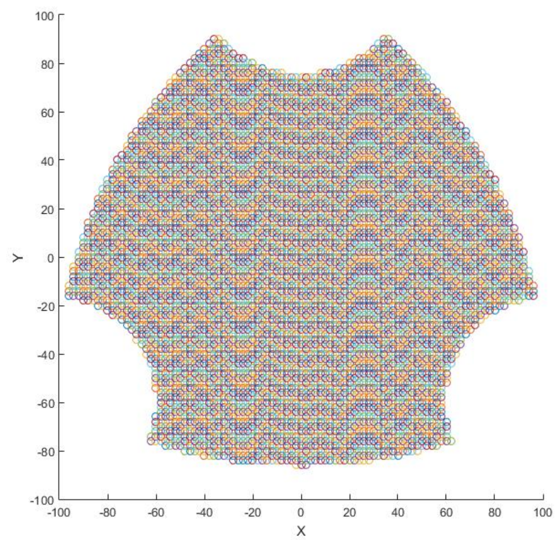



Figure 7: X-Y Configuration Space of the Delta Printer Extruder

Note that other practical considerations such as clashing of the extruder platform with the vertical columns need to be considered as well. Also, the Z-direction movement of the extruder is restricted between 0 mm (bed) and 200 mm. Thus the maximum print height of this printer is 20 cm.

### 6.3 SolidWorks Motion Study

The SolidWorks Motion Study toolbox was used to create an animation of the movement of the delta printer mechanism.

## 7 Conclusion

Hence, we have observed how the delta printer mechanism enables precise control over extruder position in 3-D space. The inverse kinematics as would be implemented on a microcontroller was implemented and simulated on MATLAB in this project.The print volume, which was observed qualitatively from the configuration space, is more than adequate and the print height is larger than conventional cartesian printers of similar size. Although the kinematics are not as straighforward as that of a cartesian printer, it has been possible to completely eliminate the need for the movement of the heated bed. This saves electrical power and improves the overall accuracy of deposition of the filament material as inertial resistance reduces.

## 8 References


1. Baroui N., *Delta 3D Printer*, Journal of Industrial Design and Engineering Graphics, Volume 11, Issue 1, July 2016

2. Graves S., *Rostock Style Delta Robot Kinematics*, Centre for Bits and Atoms, MIT Media Lab, Massachusetts Insititute of Technology

3. reprap.org/wiki/Delta_geometry

4. reprap.org/wiki/Linear_Delta_Printer_Kinematics