# Mode Shape Determination of a Dynamic Model of a Human Body in Seated Position

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A project presented for partial fulfilment of the requirements of the course Mechanical Vibrations

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# Contents

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## 1 Introduction

The human body is subject to forces and displacements in everyday life: while one exercises, while one sits inside a moving car, while one performs manual labour and numerous other scenarios. Studying the dynamics of a human body has become an interesting topic of research in domains such as ergonomics, biomechanics, prosthetics, rehabilitation robotics and human-machine interaction. Since the human body is a complicated system, it had always been difficult to model it using simple analytical techniques. With improvements in computational techniques and rapid development in the field of biomechanics, it has become possible to accurately model such systems. Experimental testing has also evolved significantly due to the use of bio-instrumentation and carefully controlled dynamic environments.

## 2 Literature Used

A lot of research has been carried out in the domain of seat vibration models of a human body among other configurations. Behari and Noga [1] have developed five different biodynamic models of a seated human body and have analysed the transmissibility of seat vibrations onto the head. They have provided comprehensive anatomical data and have plotted results that give us an idea of the effect of frequency on the head transmissibility. Amar Kishore *et al* [3] have also done a transmissibility analysis of a seated human, specifically in the low frequency region. This makes prefect sense for regular car and bus commute on non-ideal roads. The model in this paper has most of its roots in the model presented in their paper due to its simplicity and effectiveness. Ksiazek and Lacny [4] have studied a 2-DOF active human body model with two distinct seat positions and analysed response due to impulse inputs. Liang and Chiang [2] have presented a rather complicated model of a human body considering all possible joints and have tabulated the results for seat-to-head transmissibility like [1] and [3].

# 3 Human Body Model

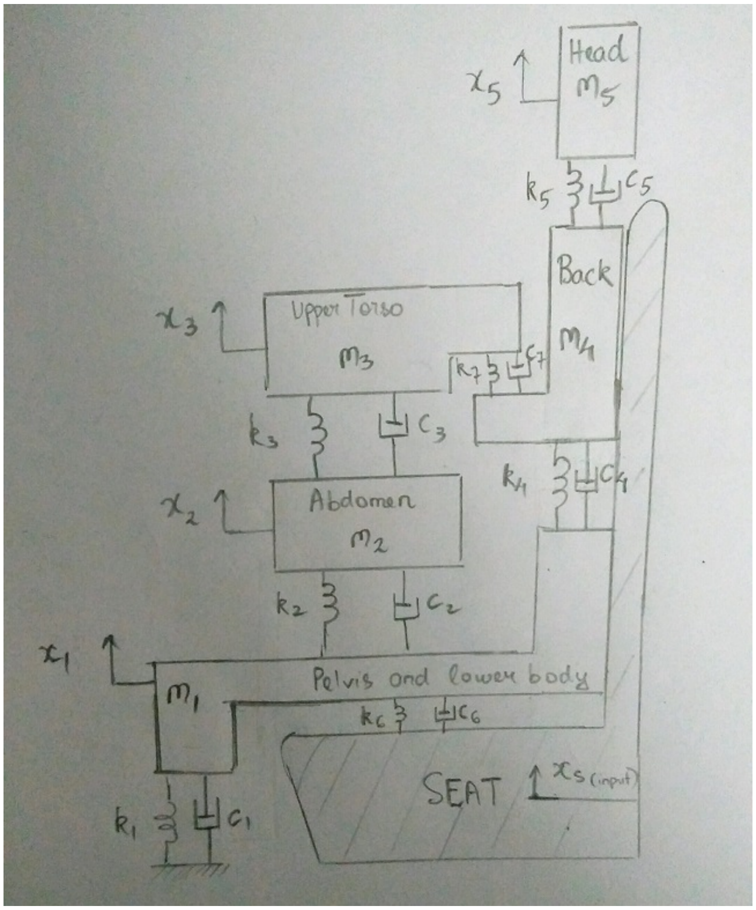


Figure 1: 5-DOF Seated Human Body Biomechanical Model

The human body can be modelled as a 5-DOF system as shown below. Note that as the objective of this paper is to develop a biomechanical model of a human body on a vibrating seat in vertical direction only, the masses have been lumped in a manner which results out of the consideration of only that direction. The abdomen has been assumed to be not connected to the back directly. The lower body and the pelvis have been merged into one to avoid the complexities involving joints in the leg and feet. Also, the feet have been assumed to be placed (fixed) flat on the ground. This model is a a great model for seat vibration analysis of a human body.

## 4 Equation of Motion

Applying Force Balance on (1)Pelvis and Lower Body (2)Abdomen (3)Upper Torso (4)Back and (5)Head, we get

$$m_1\ddot{x_1} + \dot{x_1}(c_1+c_2+c_4+c_6) + \dot{x_2}(-c_2) + \dot{x_4}(-c_4) + x_1(k_1+k_2+k_4+k_6) + x_2(-k_2) + x_4(-k_4) = 0 \quad (1)$$

$$m_2\ddot{x_2} + \dot{x_2}(c_2+c_3) + \dot{x_1}(-c_2) + \dot{x_3}(-c_3) + x_2(k_2+k_3) + x_1(-k_2) + x_3(-k_3) = 0 \quad (2)$$

$$m_3\ddot{x_3} + \dot{x_3}(c_3+c_7) + \dot{x_2}(-c_3) + \dot{x_4}(-c_7) + x_3(k_3+k_7) + x_2(-k_3) + x_4(-k_7) = 0 \quad (3)$$

$$m_4\ddot{x_4} + \dot{x_4}(c_4+c_5+c_7) + \dot{x_3}(-c_7) + \dot{x_1}(-c_4) + \dot{x_5}(-c_5) + x_4(k_4+k_5+k_7) + x_3(-k_7) + x_1(-k_4) + x_5(-k_5) = 0 \quad (4)$$

$$m_5\ddot{x_5} + \dot{x_5}(c_5) + \dot{x_4}(-c_5) + x_5(k_5) + x_4(-k_5) = 0 \quad (5)$$

In matrix form

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix} \ddot{\mathbf{X}} + \begin{bmatrix} c_1+c_2+c_4+c_6 & -c_2 & 0 & -c_4 & 0 \\ -c_2 & c_2+c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3+c_7 & -c_7 & 0 \\ -c_4 & 0 & -c_7 & c_4+c_5+c_7 & -c_5 \\ 0 & 0 & 0 & -c_5 & c_5 \end{bmatrix} \dot{\mathbf{X}}$$
$$+ \begin{bmatrix} k_1+k_2+k_4+k_6 & -k_2 & 0 & -k_4 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_7 & -k_7 & 0 \\ -k_4 & 0 & -k_7 & k_4+k_5+k_7 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \mathbf{X} = 0 \quad (6)$$

Assuming the solution $x_i = A_i e^{st}$ and substituting it into (6), we get

$$\begin{bmatrix} m_1s^2+(c_1+c_2+c_4+c_6)s+(k_1+k_2+k_4+k_6) & -c_2s-k_2 & 0 & -c_4s-k_4 & 0 \\ -c_2s-k_2 & m_2s^2+(c_2+c_3)s+k_2+k_3 & -c_3s-k_3 & 0 & 0 \\ 0 & -c_3s-k_3 & m_3s^2+(c_3+c_7)s+k_3+k_7 & -c_7s-k_7 & 0 \\ -c_4s-k_4 & 0 & -c_7s-k_7 & m_4s^2+(c_4+c_5+c_7)s+k_4+k_5+k_7 & -c_5s-k_5 \\ 0 & 0 & 0 & -c_5s-k_5 & m_5s^2+c_5s+k_5 \end{bmatrix} = \mathbf{0} \quad (7)$$

## 5 Mode Shapes

The amplitude of the pelvis and lower body lumped mass is taken to be of unit magnitude while plotting the mode shapes. Using MATLAB, the mode shapes thus obtained are shown in the figure below.

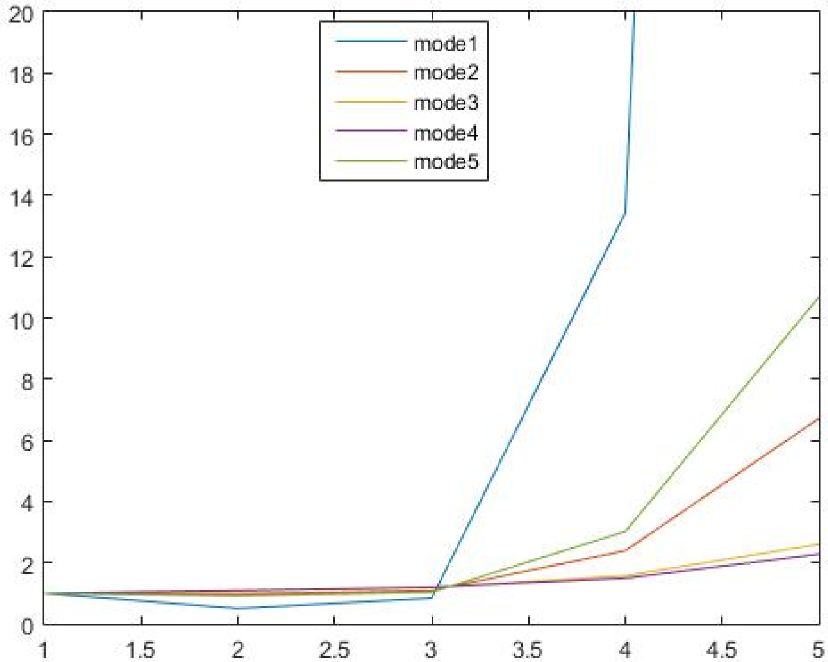


Figure 2: Mode Shapes for the Five Distinct Modes
Note: Point No. 5 for mode1 with an amplitude of about 160, is not shown in graph

# 6 MATLAB Code

The MATLAB code contains two different functions. (1)To calculate the five complex frequencies and plot the mode shape using the values of amplitudes after solving the linear equations and (2)To call the plotting function five times for the five modes.

```
  function Soln=thehuman(i)
%%HUMAN BODY MODE SHAPE PLOTTER%%
%%AUTHOR VARUN UDAY NAYAK%%
m1=28;m2=6;m3=28.5;m4=7;m5=5.4;
k1=0;k2=187000;k3=208000;k4=250000;k5=310000;k6=103000;k7=183000;
c1=0;c2=5190;c3=5400;c4=3585;c5=1400;c6=2370;c7=2850;
syms s;
A=[m1*s^2+(c1+c2+c4+c6)*s+(k1+k2+k4+k6),-c2*s-k2,0,-c4*s-k4,0;-c2*s-k2,m2*s^2+(c2+c3)*s+k2+k3,-c3*s-k3,0,0;0,-c3*s-k3,m3*s^2+(c3+c7)*s+k3+k7,-c7*s-k7,0;-c4*s-k4,0,-c7*s-k7,m4*s^2+(c4+c5+c7)*s+k4+k5+k7,-c5*s-k5;0,0,0,-c5*s-k5,m5*s^2+c5*s+k5];
R=det(A);
S=simplify(R);
d=vpa(solve(S,'s'));
svalues=[sqrt(d(1)*d(2)),sqrt(d(3)*d(4)),sqrt(d(5)*d(6)),sqrt(d(7)*d(8)),sqrt(d(9)*d(10))];
syms a2 a3 a4 a5;
B=[1;a2;a3;a4;a5];
syms C;
C=A*B;
C=subs(C,s,svalues(i));
D=[0;0;0;0;0];
eqn1=C(1)==0;
eqn2=C(2)==0;
eqn3=C(3)==0;
eqn4=C(4)==0;
sol = solve([eqn1,eqn2,eqn3,eqn4],[a2,a3,a4,a5]);
c=zeros(1,5);
c(1)=1;
c(2)=double(sol.a2);
c(3)=double(sol.a3);
c(4)=double(sol.a4);
c(5)=double(sol.a5);
t=1:1:5;
plot(t,c);
ylim([0,20]);
xlim([1 5]);
end
```

## 7 Conclusion

Therefore, we see that the mode shapes give vital information regarding the response of a seated human body. The effect on the head ($X_5$) is seen evidently as there is a significant difference in its amplitude for different frequency ranges. The pelvis ($X_1$), abdomen ($X_2$) and upper torso ($X_3$) seem to vibrate closer in phase with each other as seen from the mode shapes. All in all, this model gives a fairly accurate idea of the dynamic behaviour of a human body under seat-vibration conditions.

## 8 References


1. Behari N., Noga M., *Vibration Transmissibility Behaviour of Simple Biodynamic Models used in Vehicle Seat Design*, Technical Transactions Mechanika, 5-M/2016

2. Liang C., Chiang C., *Modelling of a Seated Human Body Exposed to Vertical Vibrations in Various Automotive Postures*, Industrial Health 2008 **46**, 125-127, 2008

3. NV Amar Kishore *et al*, *Transmissibility and DPMI analysis of the seated posture of Human under Low frequency vibration*, Mechanical and Industrial Engineering Department, Indian Institute of Technology, Roorkee

4. Kisazek M, Lacny L, *A Comparison of the Human Body Seat Model Responses to Several Types of Impulse Excitations*, Journal of Theoretical and Applied Mechanics, **52**, 3, pp. 839-845, Warsaw 2014