Question 1

I will be exploring Medical Resident Matching (NRMP or programs in other countries) as my basis for question 1.

(a) Theorem 1.19 ([261]). Let I be a solvable instance of SRI. The same set of agents are unassigned in every stable matching in I.

Manlove, David. Algorithmics Of Matching Under Preferences, World Scientific Publishing Company, 2013. ProQuest Ebook Central, http://ebookcentral.proquest.com/lib/washington/detail.action?docID=1168176. Created from washington on 2021-01-22 12:36:28.

In terms of horses and riders, an instance with incomplete lists is when all riders (proposers in this case) rank m horses, where m is less than the total number of horses. Let us say that we begin with n horses, n riders, and each agent ranks m members of the opposing set. If this is the case and there are multiple possible stable matchings, in every possible stable matching instance, the same horses will always not be matched to a rider, and the same riders will not be matched to horses.

In terms of Medical Resident Matching, the theorem changes slightly because in this scenario, there are many more medical resident applications than there are medical resident institutions (Or more students than hospitals) and each hospital can be paired with multiple students. If this is the case, we can assume that hospitals have the advantage of proposer-optimality because they will have a more important say than those applying and all hospitals will be paired to the same amount of students. If there are n hospitals, p students, each hospital ranks s students where $s < p$, and each hospital is paired with m students where $nm < p$, then we know there will always be at least 1 student left over since there are more students applying than students taken in. Moreover, in accordance to the theorem, the students left over will always be the same students every time.

(b) In this instance, the three groups of people who are affected by this theorem are the students who are accepted, the students who are rejected, and the hospitals. The hospitals have the biggest advantage because regardless of what happens, they have the advantage of proposer-optimality so they are guaranteed to get a high number of their top choices. The students who are accepted have the next biggest advantage because there is no scenario where they are not accepted into any one program; While they may not get into their top choice, they will still be in a residency program. The students who are rejected have the least advantage because no matter what the pattern of choice is, they will never be placed into a program and therefore lose the most.

Question 2

(a) One possible scenario where this stable matching could be used is the NFL draft. Right now, the NFL draft is entirely based on teams selecting in order of the number of games they won and players going to whichever team drafts them irrespective of choice. With a stable matching, the problem of teams intentionally losing games could be slightly alleviated because the players they are most interested won't be selected purely based off of wins and losses. In this case, the two groups of agents are the NFL teams and the players who are being selected. The preference list would be for every NFL team to rank all players eligible for the NFL draft and all players ranking each of the NFL team. In the end, all players will be drafted so multiple players will be drafted to each team; One way to make this easier would be to mandate that the number of draft-eligible players is a multiple of the number of teams so it is even, but if this is not possible then there will be m-pn undrafted (unmatched) players where m is the total number of players, p is the number of players selected by each team, and n is the total number of teams.

(b) I will be reusing the same theorem from part 1.1.

In this example of the NFL draft, the teams are always the proposers since they have higher priority than players. Therefore, they will benefit from proposer-optimality. The three groups of people in this scenario are the players who get drafted, the players who do not get drafted, and the NFL teams. The teams have the biggest advantage because regardless of what happens, they have the advantage of proposer-optimality so they are guaranteed to get a high number of their choices. The players who are drafted have the next biggest advantage because there is no scenario where they are not drafted onto any team; While they may not be on their favorite team, they will still be on any one team. The players who go undrafted have the least advantage because no matter what the pattern of drafting is, they will never be drafted to any team and therefore lose the most. I think the best choice is option 3. Announce that you will flip a coin after receiving the preference lists. If its heads one side proposes, if it is tails the other side proposes. This is because, while it would be nice to be truthful to both groups of agents, truthfulness holds no value if the results are unfair. If you can assure that the results are fair, then you can be assured that neither group would intentionally lie about their preference order because it is in their best interest to assume they are proposers. If they are choosers, lying still does not benefit them as the blocking pairs may end up being worse than if they were truthful still, and if they are proposers, they get their best case scenario of choices.

(c) I do not think Gale-Shapley should be used in my scenario of the NFL draft because it is not fair to rule out one group of players from potentially being drafted. With Gale-Shapley and the theorem I described, it results in the same players being drafted and undrafted every time. Since we cannot have a scenario where players are proposers due to the nature of NFL teams having draft picks, then this results in unfair drafting. It doesn't make sense to even attempt to go into the NFL if because of the nature of the algorithm, you not only are not going to be drafted, but you are also not told beforehand that you will not be drafted. Therefore, stable matchings should not be used in the NFL draft.