1 Problem 1

1.1 a

$(\gamma v)^T w = \gamma^T v^T w$, but since γ is a constant, the transpose of a constant is itself. Therefore, this is equal to $\gamma v^T w = \gamma(v^T w)$, since scalars are able to be distributed.

1.2 b

$(u + v)^T w = (u^T + v^T)w = u^T w + v^T w$

2 Problem 2

2.1 a

We know the inner product, or the dot product, of two vectors is equal to the transpose of the first vector times the second. Therefore, $\sum_{i=1}^{n} v_i^2 = \sum_{i=1}^{n} v_i v_i$, so this is equal to the inner product of v_i^T and v_i, or $v_i^T v_i$

2.2 b

I don't know

2.3 c

w can be written as the combination of two vectors, the first being $[v_1, 0, v_2, 0, v_3, 0...v_n, 0]^T$, with dimension $(2nx1)$, and the second being the 1s vector, [1], with dimension $(1x1)$. This produces vector w of dimension $(2nx1)$.

3 Problem 3

Since S contains 3 integer points, x_1, x_2, x_3, we know that in order for the conditions of S to be true, $x_1 = x_3$ and $x_2 = -2x_3$. Let us say that there exist 3 arbitrary integers, a, b, c. This would mean that $ax_1 + 2bx_2 + 3cx_3 = 0$ and $3ax_1 + 2bx_2 + cx_3 = 0$. We know that this will be a subset of R^3 because when we substitute $x_1 = x_3$ and $x_2 = -2x_3$ in, all values cancel out, leaving us with 0=0. Therefore, the set is closed under addition, multiplication, and (0,0,0) is in the subset, therefore it is a subspace.

4 Problem 4

4.1 a

The gradient is equal to a vector of $[\frac{\partial f}{\partial x_1}(x_1, x_2...x_n), \frac{\partial f}{\partial x_2}(x_1, x_2...x_n)... \frac{\partial f}{\partial x_n}(x_1, x_2...x_n)]$.

4.2 b

The hessian is equal to a 2-dimensional matrix of $[\frac{\partial^2 f_1}{\partial^2 x_1}(x_1, x_2...x_n),\ \frac{\partial^2 f_1}{\partial^2 x_2}(x_1, x_2...x_n)...\ \frac{\partial^2 f_1}{\partial^2 x_n}(x_1, x_2...x_n)],\ [\frac{\partial^2 f_2}{\partial^2 x_1}(x_1, x_2...x_n),\ \frac{\partial^2 f_2}{\partial^2 x_2}(x_1, x_2...x_n)...\ \ \frac{\partial^2 f_2}{\partial^2 x_n}(x_1, x_2...x_n)],\ ...\ [\frac{\partial^2 f_m}{\partial^2 x_1}(x_1, x_2...x_n),\ \frac{\partial^2 f_m}{\partial^2 x_2}(x_1, x_2...x_n)...\ \frac{\partial^2 f_m}{\partial^2 x_n}(x_1, x_2...x_n)],$

5 Problem 5

$$A = \begin{matrix} 1 & 3 & 0 & 7 \\ 2 & 6 & 5 & 9 \\ 3 & 9 & 5 & 16 \end{matrix}$$

$$v = \begin{matrix} -1 \\ -2 \\ 1 \\ 1 \end{matrix}$$

$$w = \begin{matrix} 1 \\ 2 \\ 4 \end{matrix}$$

5.1 a

For v to be in the null space of A, Av must equal 0, and $Av = 0$, so it is in the null space

5.2 b

Row reducing A gives us the matrix

$$\begin{matrix} 1 & 3 & 0 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

This means the null space has a basis of $[-3, 1, 0, 0]^T, [-7, 0, 1, 1]^T$.

5.3 c

If w is in the range of A, then there must be some vector x such that $Ax = w$, so we can row reduce $Ax = w$ for vector $x = [x_1, x_2, x_3, x_4]^T$, giving us the row-reduced matrix

$$\begin{matrix} 1 & 3 & 0 & 7 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Since the final row has a row of 0s $= 1$, this means that w is not in the range of A.

5.4 d

Row reducing A gives us $\begin{matrix} 1 & 3 & 0 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{matrix}$

This means a basis for the range can be found with $[1, 0, 0] and [0, 1, 0]$.

6 Problem 6

6.1 a

Let there be two vectors $u = [u_1, u_2, u_3, u_4]$ and $v = [v_1, v_2, v_3, v_4]$.

For the given map T, $T(u) + T(v) = [u_4, u_3, u_2, u_1] + [v_4, v_3, v_2, v_1] = [u_4 + v_4, u_3 + v_3, u_2 + v_2, u_1 + v_1]$.

$T(u + v) = T([u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4) = [u_4 + v_4, u_3 + v_3, u_2 + v_2, u_1 + v_1 = T(u) + T(v)$.

For the given map T and some constant c, $T(cu) = T(c[u_1, u_2, u_3, u_4]) = T([cu_1, cu_2, cu_3, cu_4]) = [cu_4, cu_3, cu_2, cu_1]$.

$cT(u) = c[u_4, u_3, u_2, u_1] = [cu_4, cu_3, cu_2, cu_1] = T(cu)$.

Therefore, T is a linear transformation.

6.2 b

For the map T and some vector x, there exists the following matrix A such that $T(x) = Ax$:

$$A = \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$$

7 Problem 7

Let $x = [x_1, x_2...x_n]$, which makes $x^T = [x_1, x_2...x_n]^T$, and let $A = [A_{11}, A_{12}...A_{1n}], [A_{21}, A_{22}...A_{2n}], ...[A_{n1}, A_{n2}$

This means this is equal to $(x_1A_{11}+x_2A_{21}...+x_nA_{n1})x_1+(x_1A_{12}+x_2A_{22}...+x_nA_{n1})x_2 + ...(x_1A_{n1} + x_2A_{2n}... + x_nA_{nn})x_n$

If A is a diagonal matrix, then all value of A where i and j are not equal to each other are 0, so this is then equal to $(x_1^2A_{11} + x_2^2A_{22} + ...x_n^2A_{nn})$, which is much simpler.

8 Problem 8

For the following matrix, we know the eigenvalue is 1. To find the eigenvector, we plug 1 into the diagonals and then get the matrix

$$\begin{matrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{matrix}$$

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We then solve this matrix, and we get the eigenvector of $[1, 1, 1]$.