

## ABSTRACT

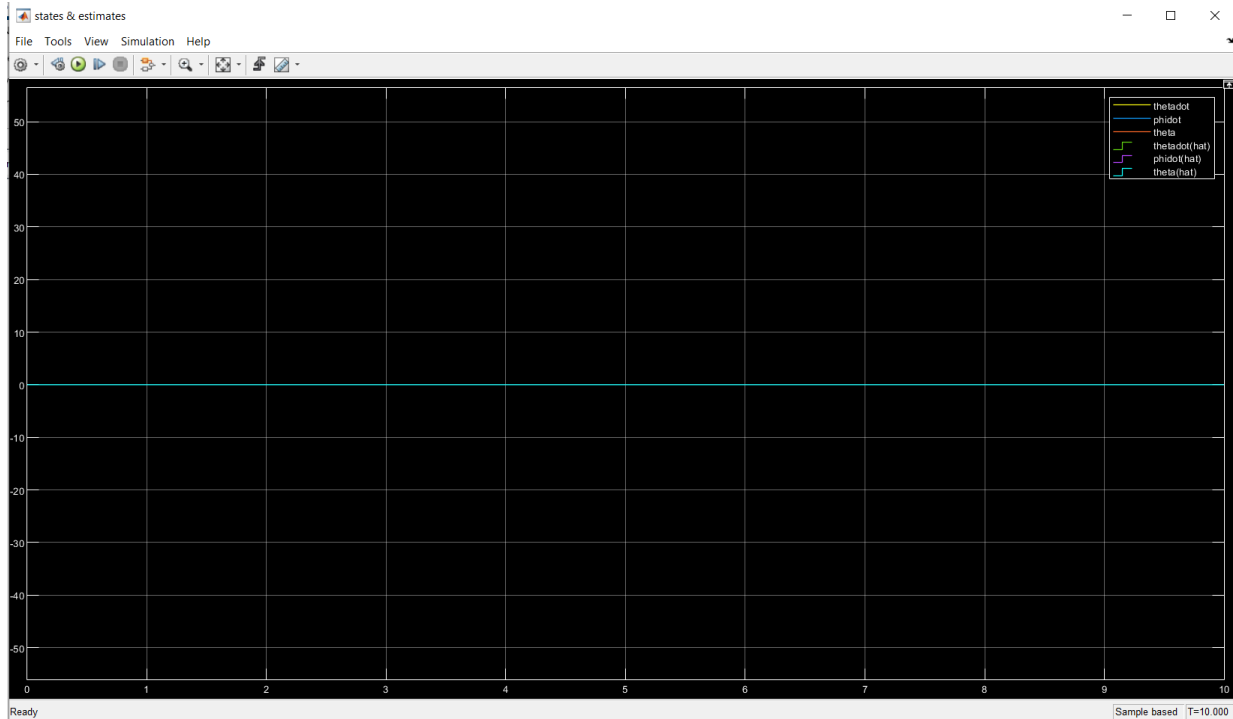
This report deals with the control of MIP robot. MIP is a mobile inverted pendulum system which is balanced by two wheels. MIP can navigate on the horizontal plane while balancing the pendulum body. The task is to design a discrete linear state-estimate feedback controller for MiP that performs better than the one developed by Zhu Zhuo.

## PROBLEM STATEMENT

The given system has 4 states  $\theta$ ,  $\phi$ ,  $\theta_{\text{dot}}$ ,  $\phi_{\text{dot}}$ . Our task is to design a controller for this system such that we get linear quadratic gaussian controller consisting of a linear feedback controller coupled with a state observer. The problem requires us to experiment with the eigen values of the feedback and the observer system, and to arrive at the optimal controller parameters. We are trying to control the reduced-order system.

## TESTING ZHUO'S MODEL

1) **Zero Initial Condition of  $\theta$**  i.e.  $\theta_{ic} = 0$  and Noise power = 0



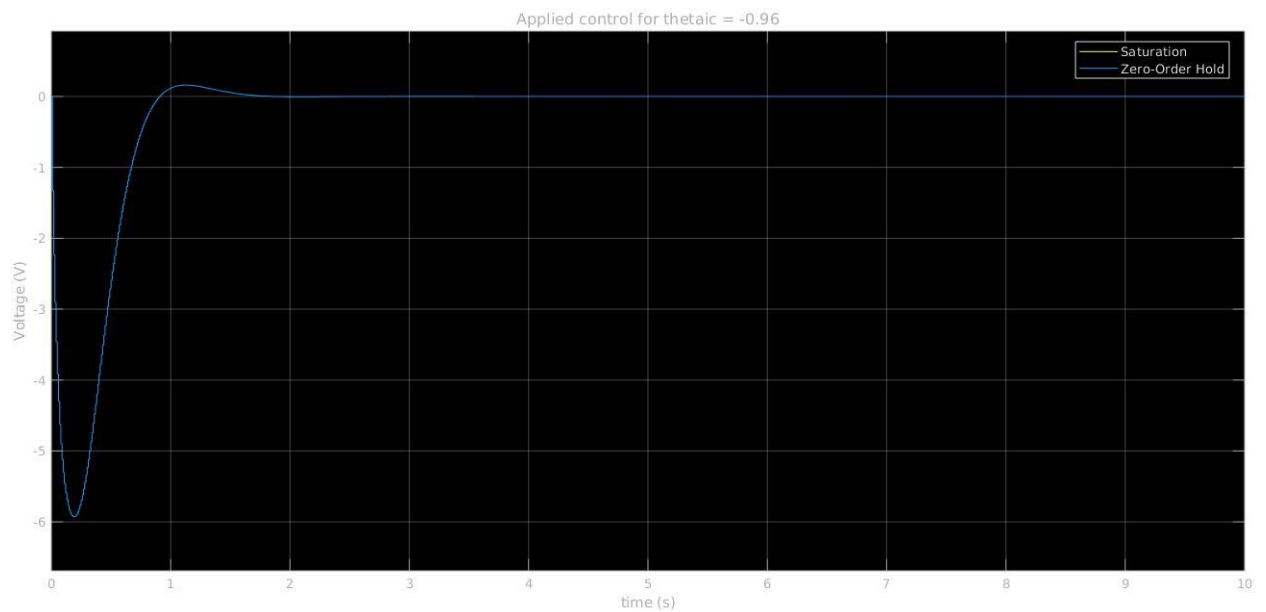
We can see that all states and its estimates are zero, and the system is in equilibrium.

## Zhu Zhuo's controller

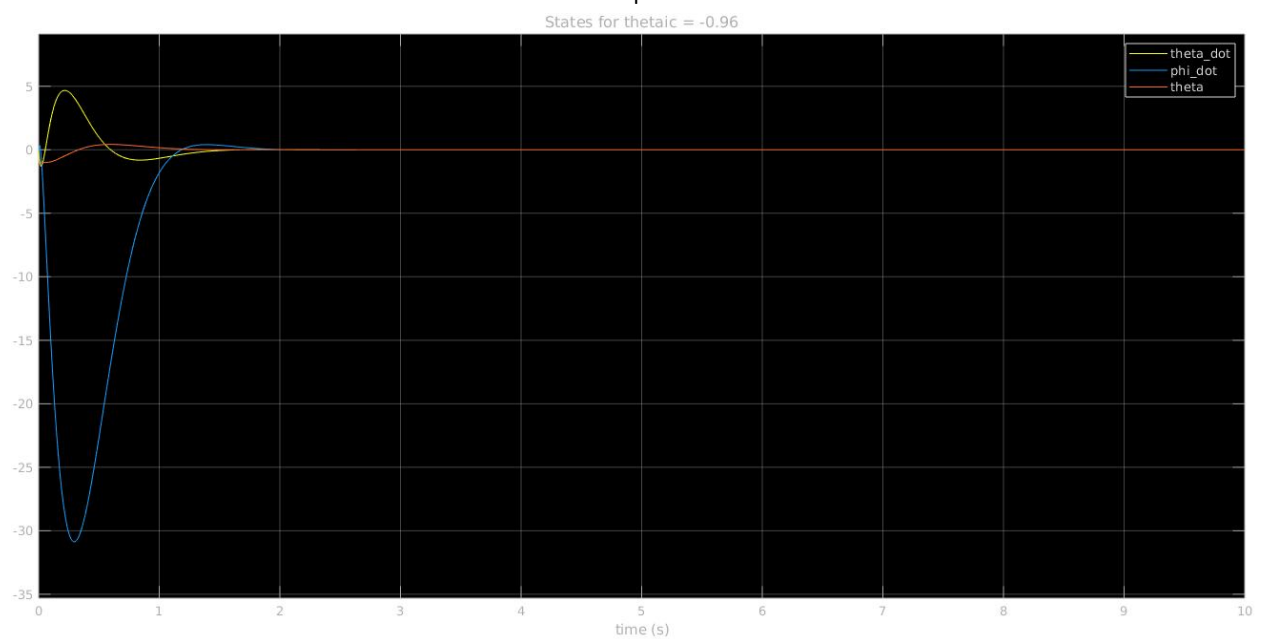
Checking with various  $\theta_{ic}$  conditions for the controller based on Zhuo's thesis.

For  $\theta_{ic} = -0.96$

### 1. Applied control



### 2. States plot



Next, in order to check for upper bound, we simply have to change theta to +ve.

Hence we can infer that the applied control is between  $\pm 6V$  for  $-0.96 < \theta_{ic} < 0.97$ .  
Hence the system is stable between these values.

The **region of attraction** is hence,  $-0.96 < \theta_{ic} < 0.97$  for Zhu Zhuo's Controller  
Also, the **response time** seen from the plot is about **1.8** seconds.

## DESIGNING CONTROLLER

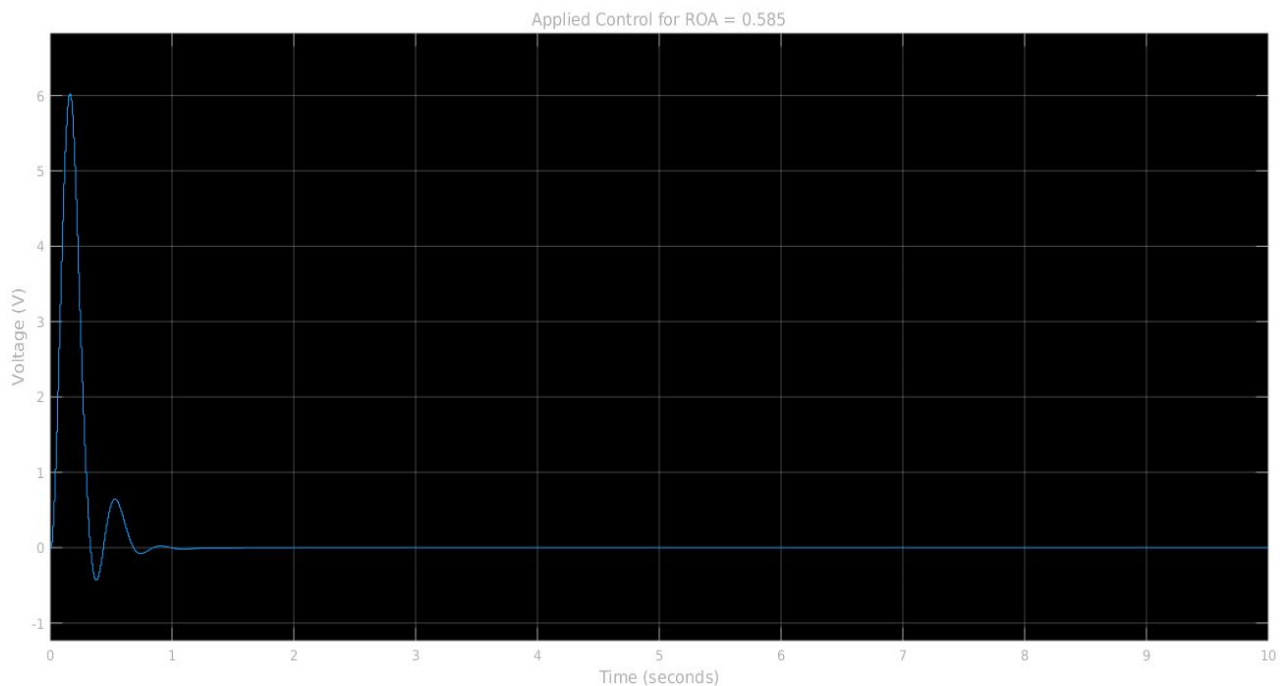
We choose various eigen values for LSVF (Linear State Variable Feedback) gain (K) and Observer Gain (L). By repeated experimentation, I found some values which gave me some decent controller performance.

### CASE 1 :

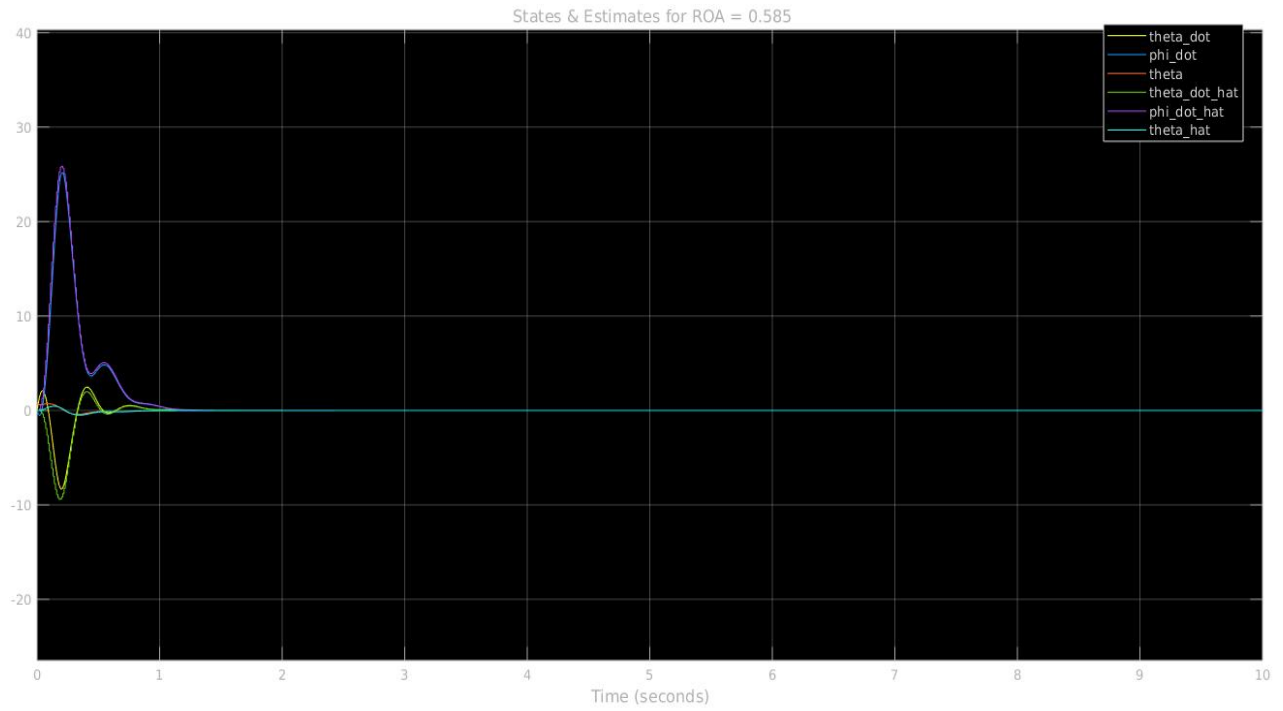
$$\text{Eig}(K) = [-12 + 2.2*i, -12 - 2.2*i, -12]$$

$$\text{Eig}(L) = [-12.5 + 3.2*i, -12.5 - 3.2*i, -11]$$

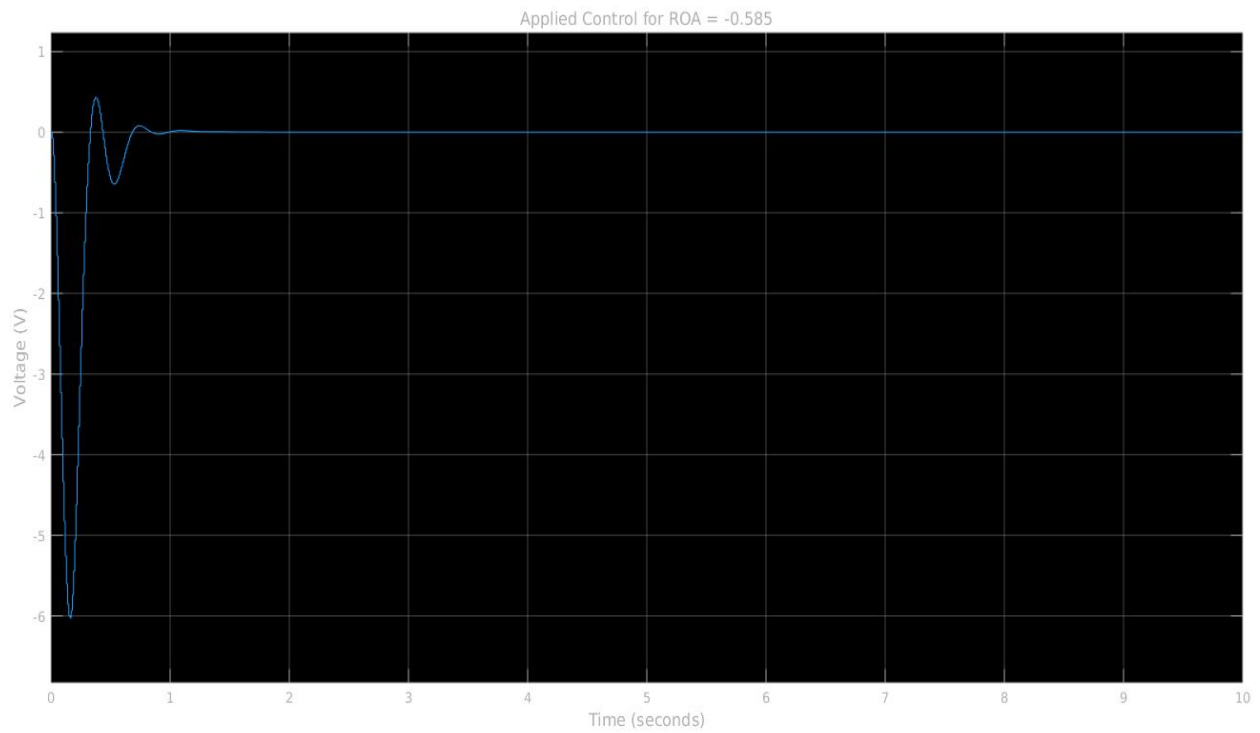
1. Applied control for  $\theta_{ic} = 0.585$  radians



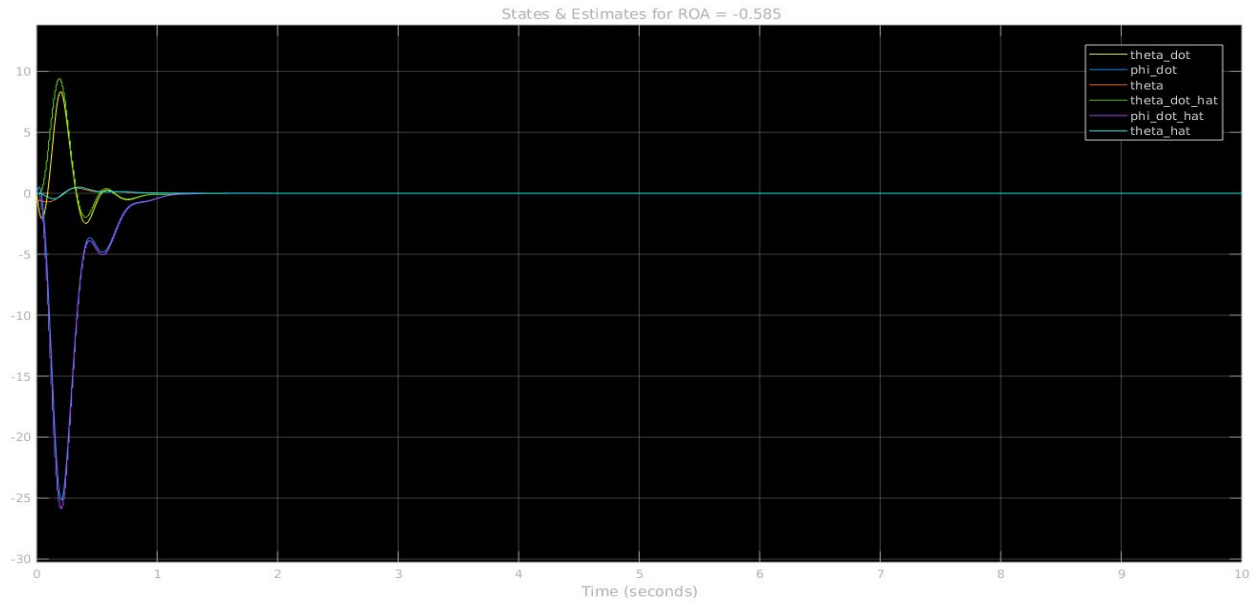
## 2. State plots and its estimates for $\theta_{ic} = 0.585$



## 1. Applied control for $\theta_{ic} = -0.585$ radians



## 2. State plots and its estimates for $\theta_{ic} = 0.585$ radians



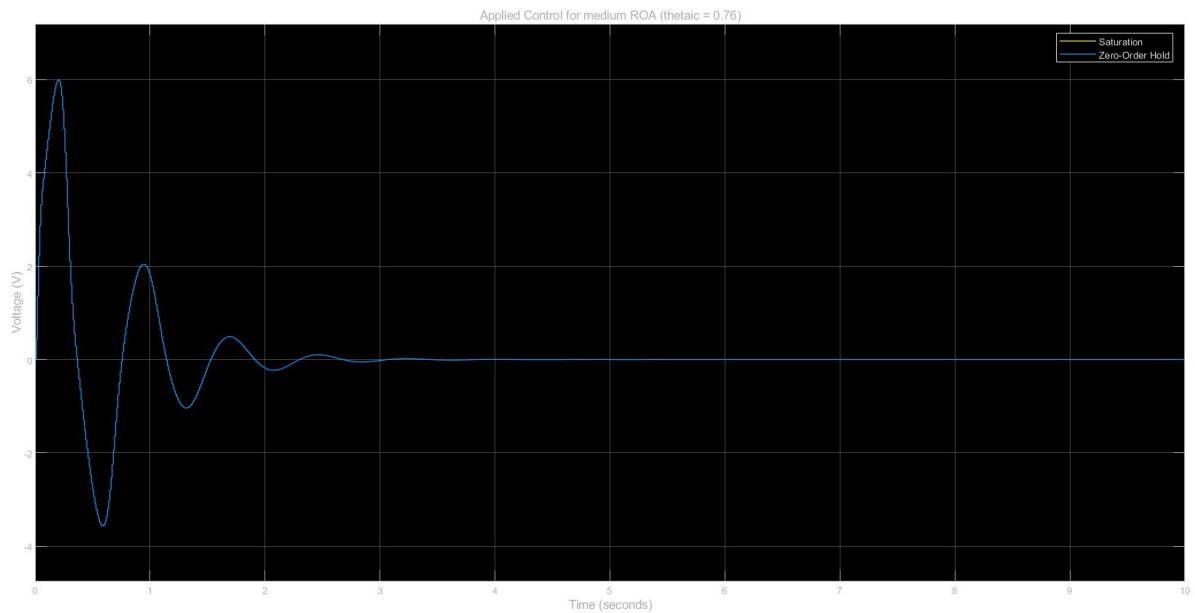
So we can see that the region of attraction in this case is  $-0.585 < \theta_{ic} < 0.585$ . The response time is very less.

### CASE 2 :

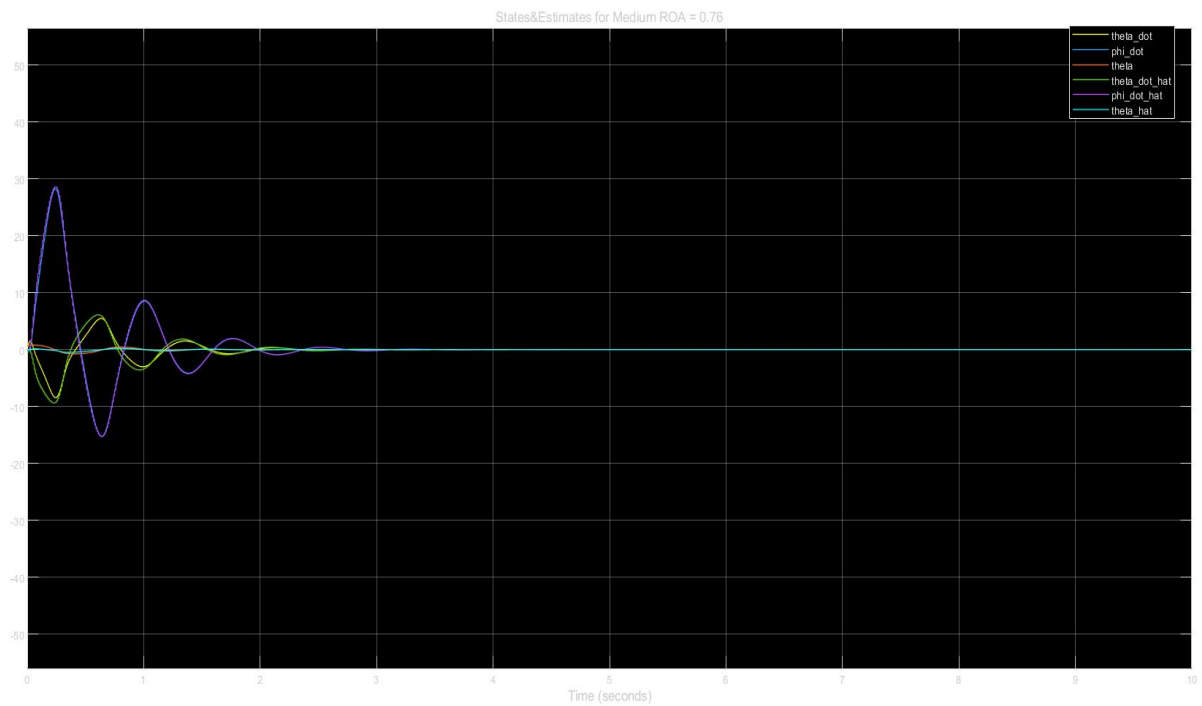
$$\text{Eig}(K) = [-30 + 2.2*j, -30 - 2.2*j, -6.4]$$

$$\text{Eig}(L) = [-31 + 5.2*j, -31 - 5.2*j, -12]$$

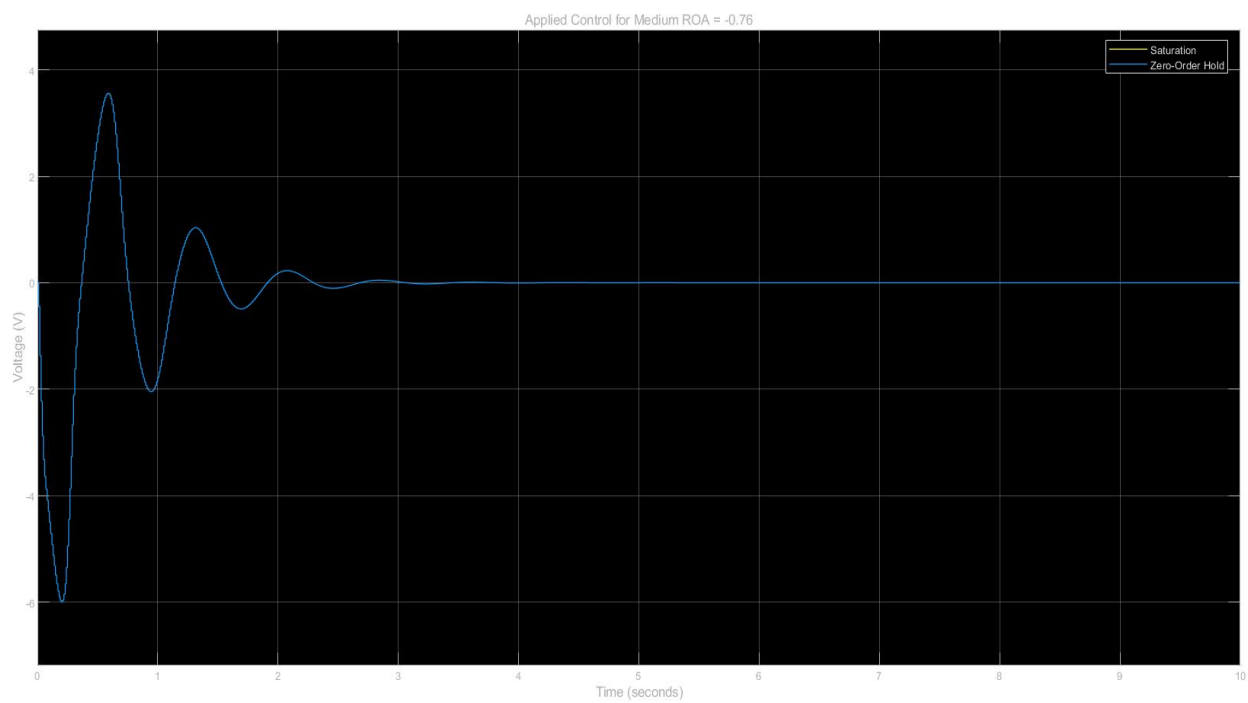
## 1. Applied control for $\theta_{ic} = 0.76$ radians



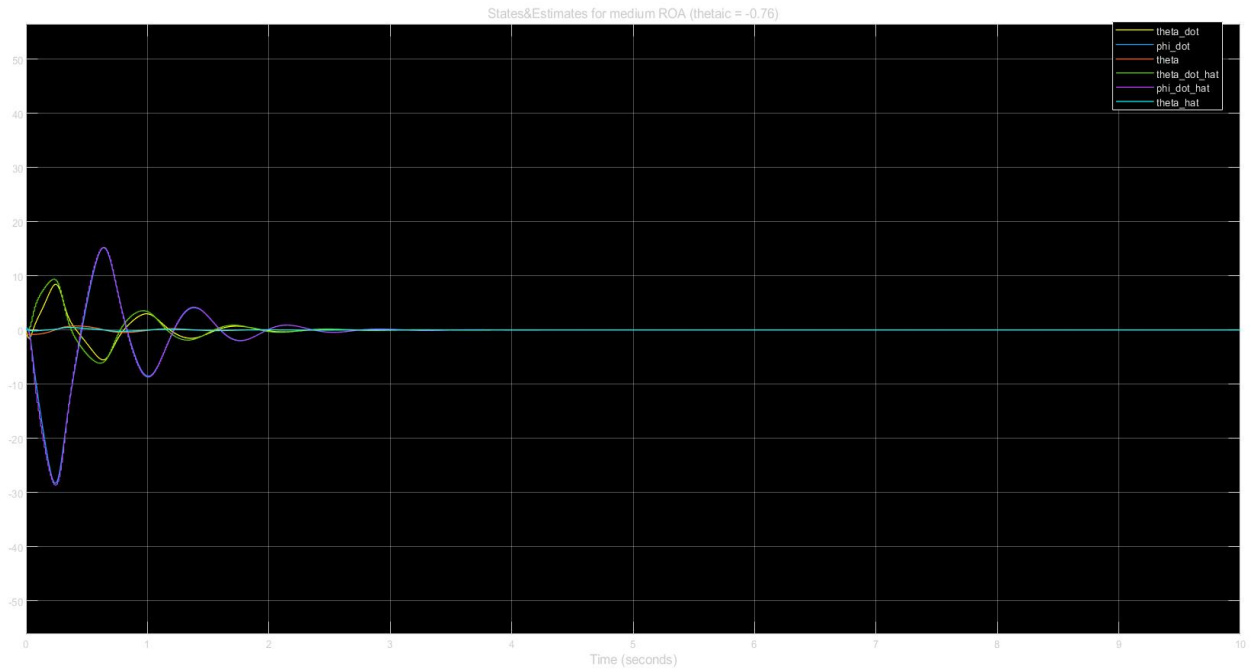
## 2. State plots and its estimates for $\theta_{ic} = 0.76$ radians



## 1. Applied control for $\theta_{ic} = -0.76$ radians



## 2. State plots and estimates for $\theta_{ic} = -0.76$ radians



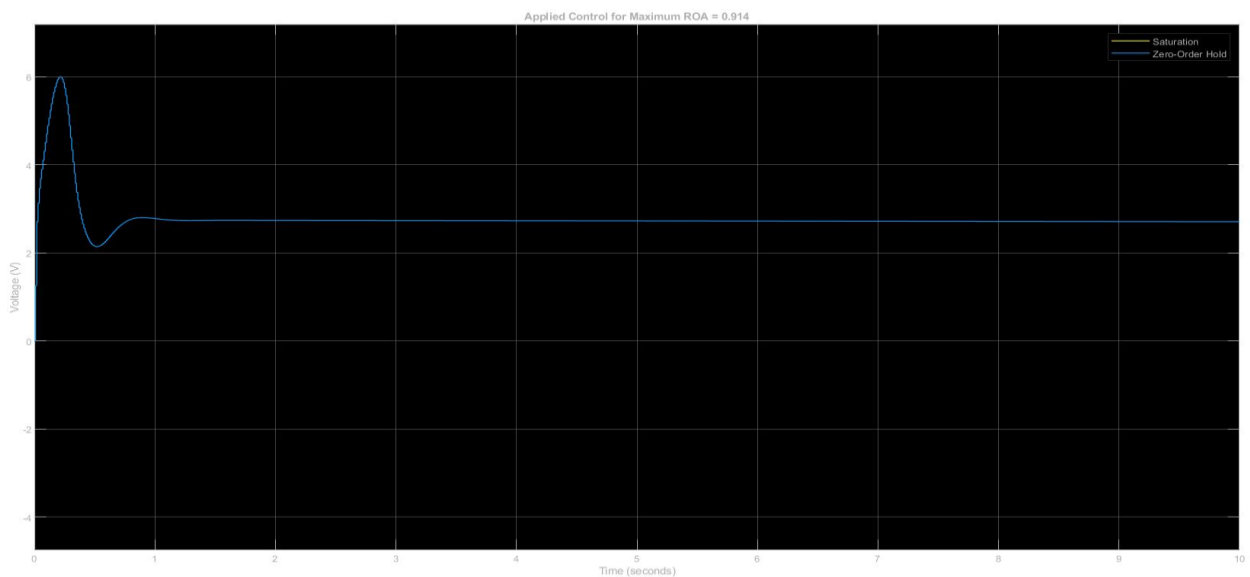
Response time is slightly higher, around 3 seconds and the Region of attraction in this case is  $-0.76 < \theta_{ic} < 0.76$ .

### CASE 3 :

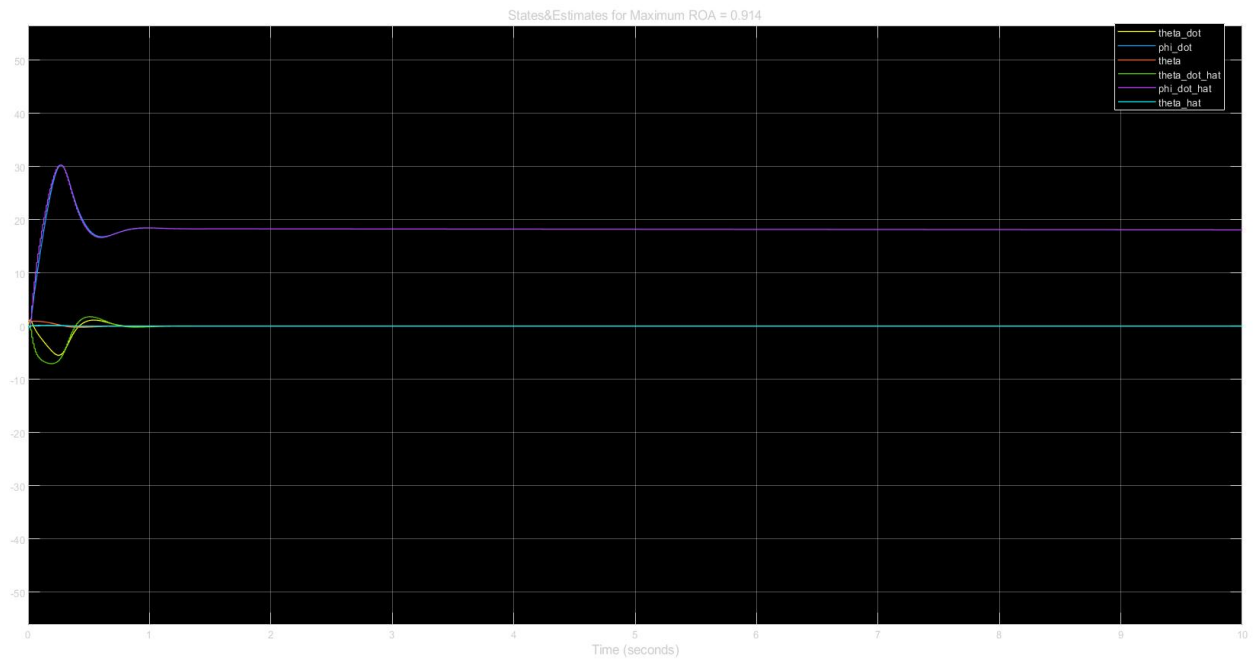
$$\text{Eig}(K) = [-98 + 8*j, -98 - 8*j, -0.01]$$

$$\text{Eig}(L) = [-31 + 5.2*j, -31 - 5.2*j, -12]$$

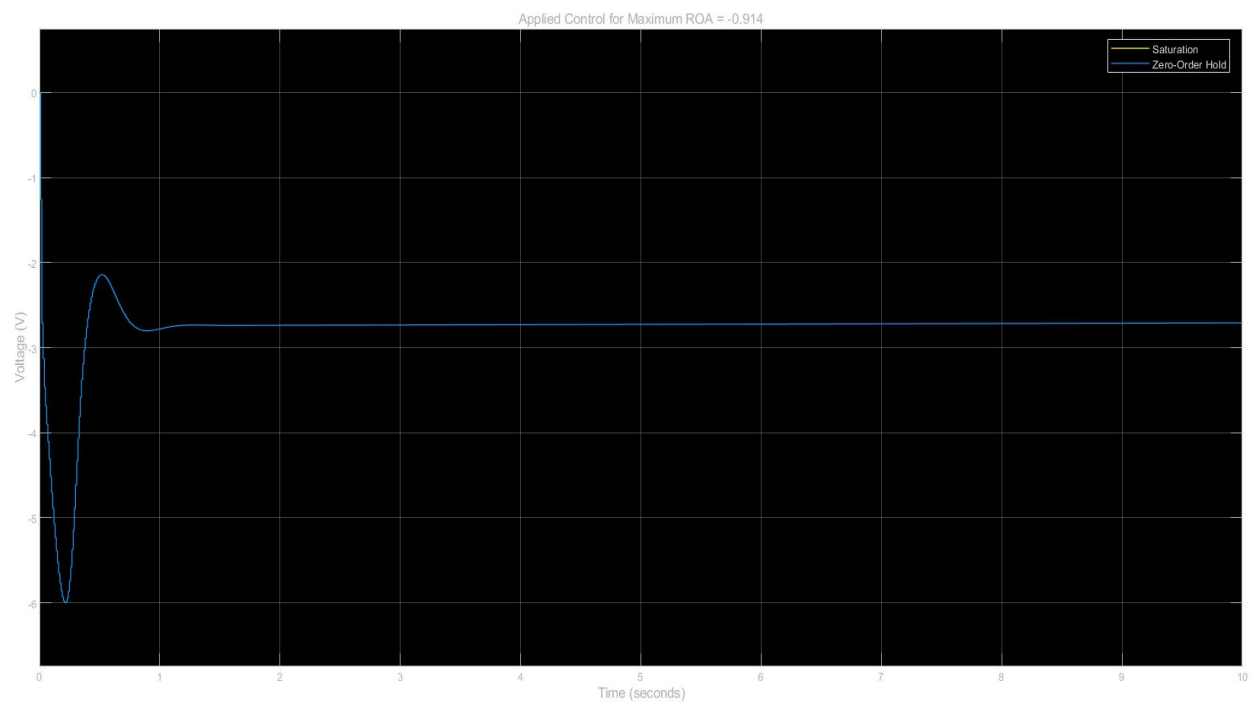
## 1. Applied control for $\theta_{ic} = 0.914$ radians



## 2. State plots and its estimates for $\theta_{ic} = 0.914$ radians

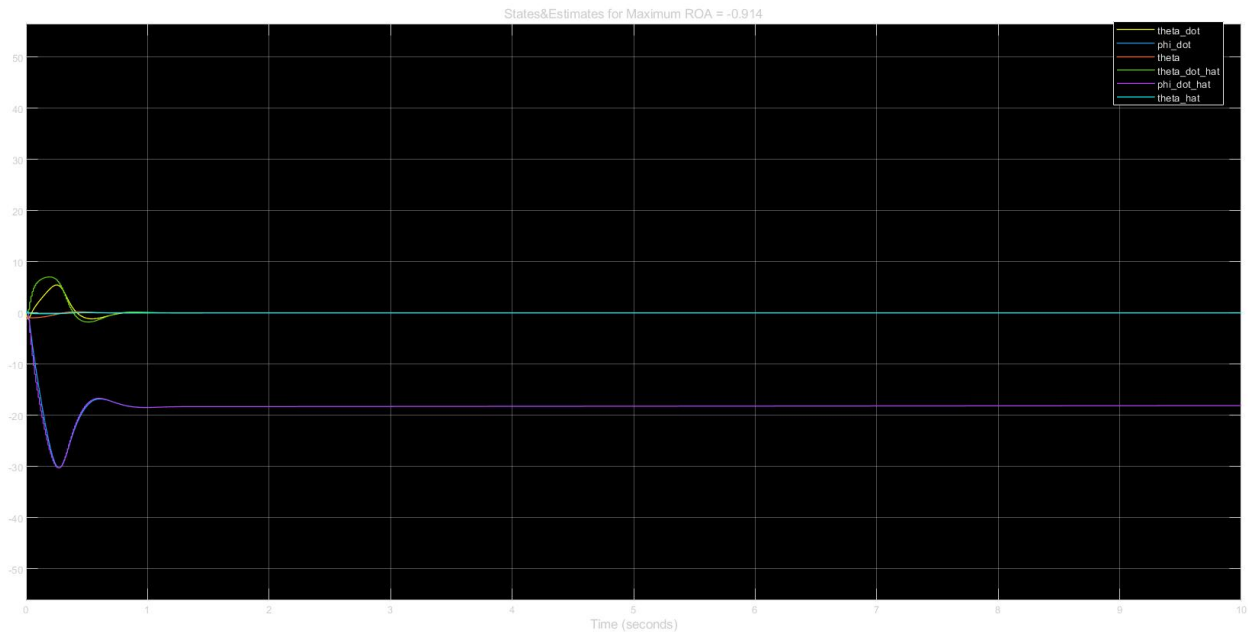


## 1. Applied control for $\theta_{ic} = -0.914$ radians





## 2. State plots and its estimates for $\theta_{ic} = -0.914$ radians

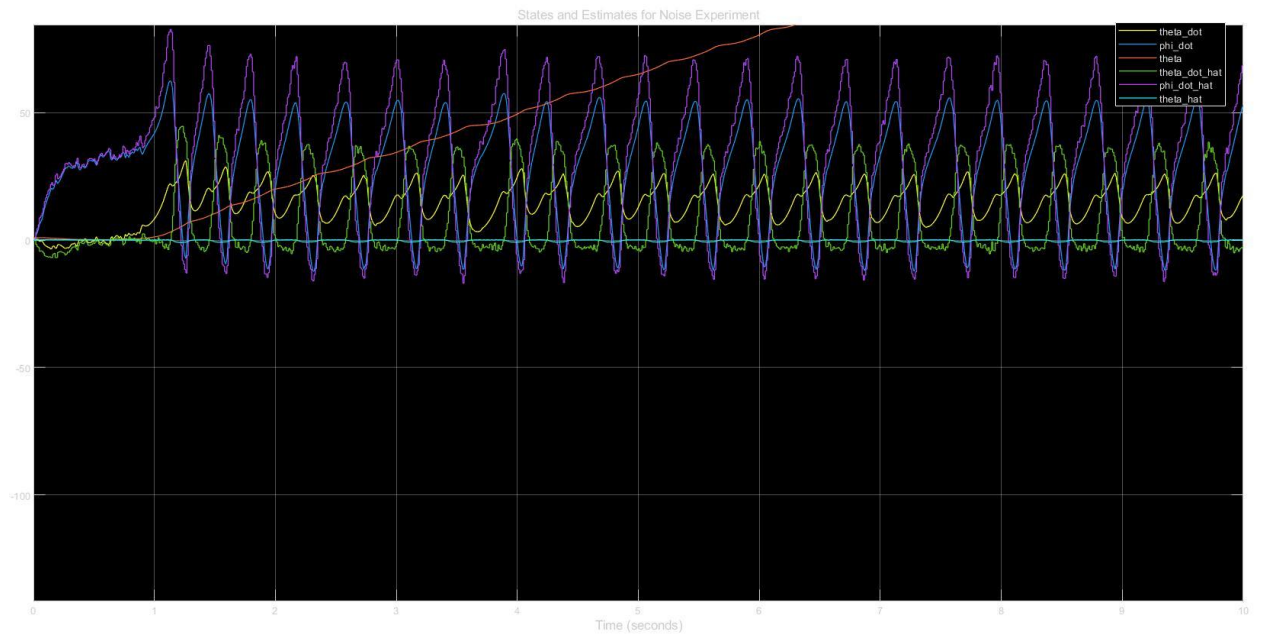


These controller settings give me the most beautiful performance, with a region of attraction of  $-0.914 < \theta_{ic} < 0.914$ . It fares well on all accounts, and with all such things, this also has a caveat. The  $\phi_{dot}$  saturates at a constant value greater than 0. This means that it would be moving at a constant velocity. Even though other controllers give poorer performance compared to this, this fails in terms of energy consumption. The bot would have to be in a constant state of motion to be stable.

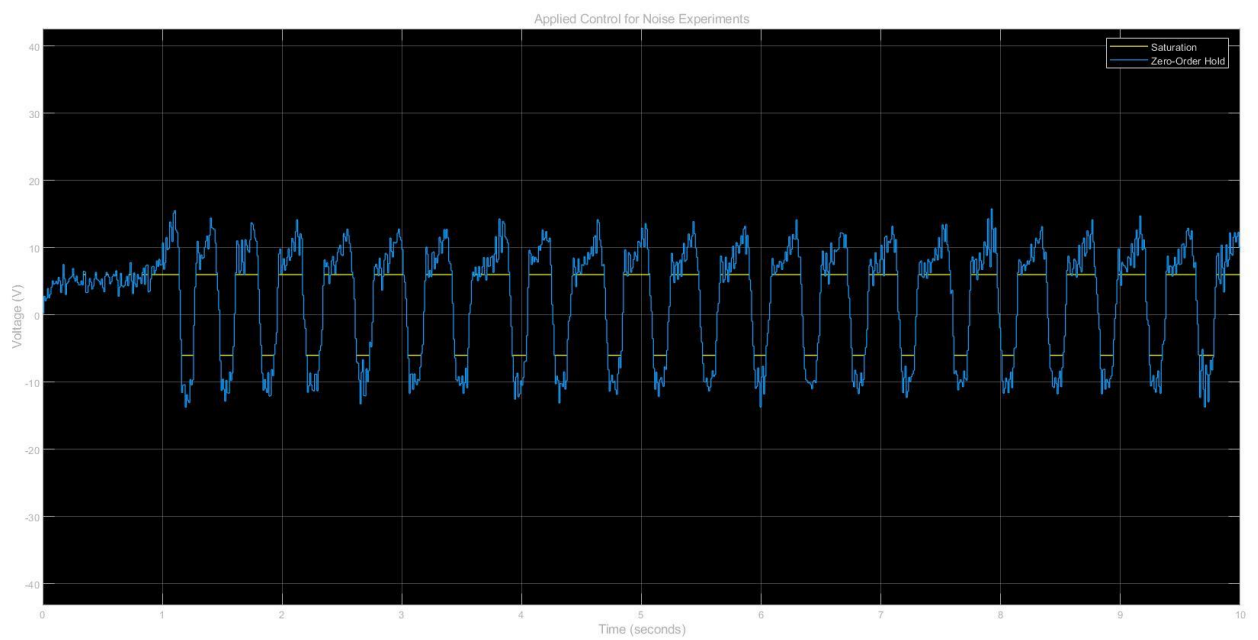
## NOISE REJECTION PERFORMANCE

The best controller(worst in terms of energy) was also tested with noise added to the theta dot and phi dot measurements. Upon testing, it is clear that the system attains stability only for noise power values equal to or below 0.0001 for the thetadot. Noise of 0.001 leads to a huge disturbance and consequently diminished stability.

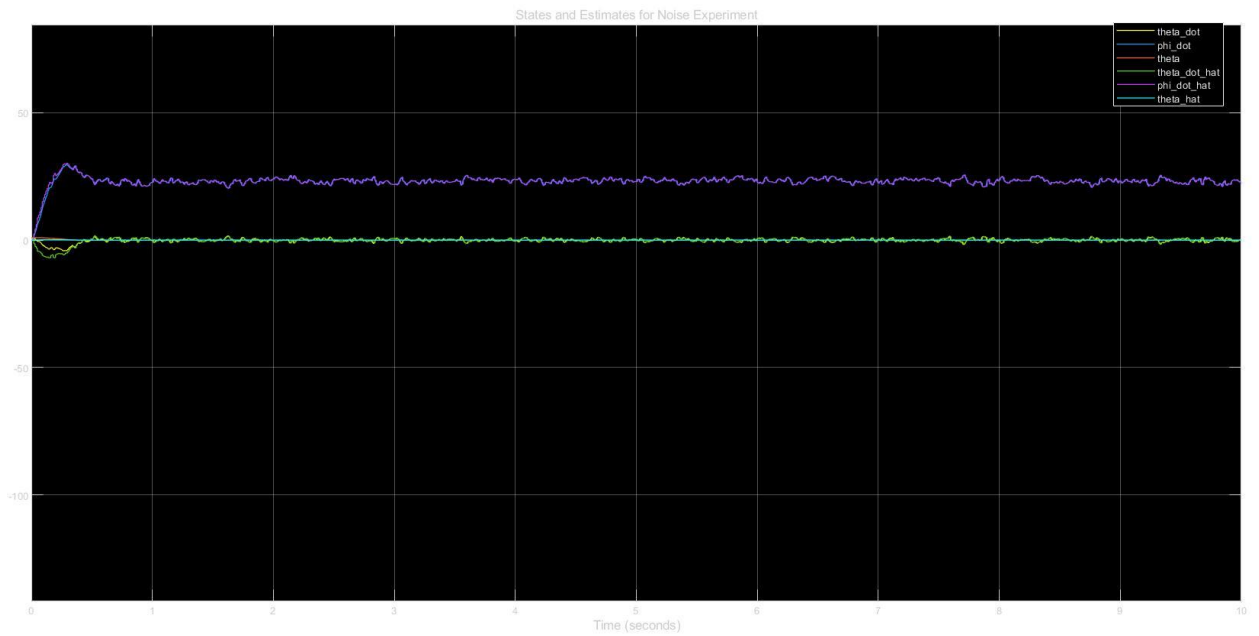
Please turn over.



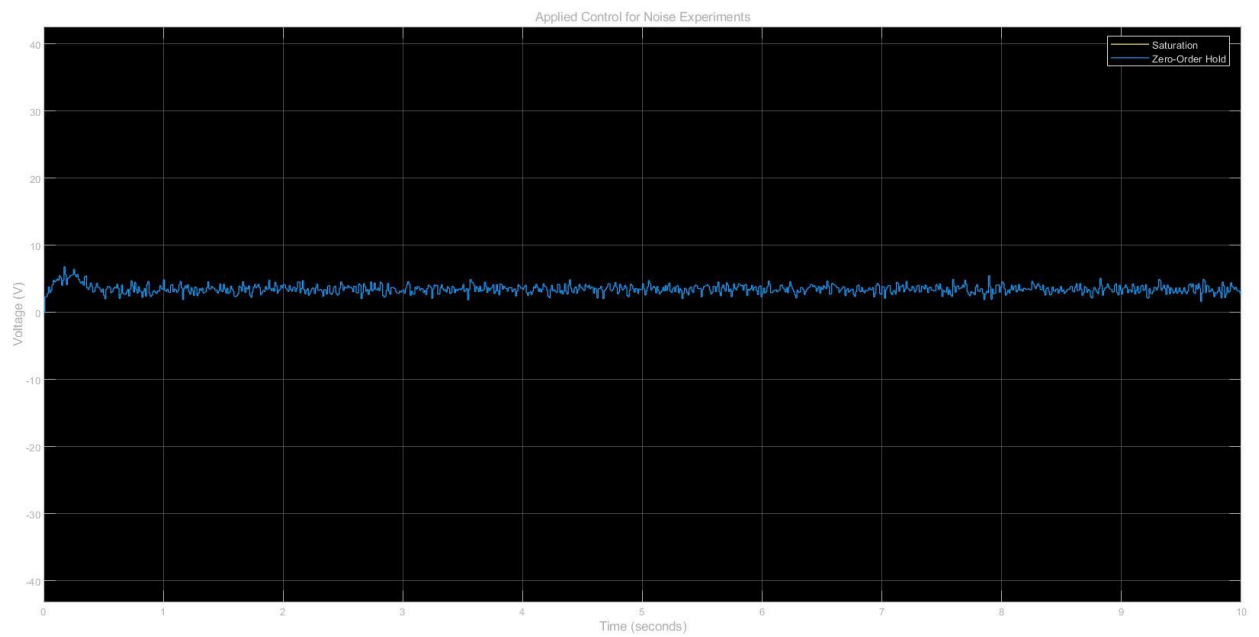
States and estimates noise at noise power of 0.001 and above



Applied control for noise at noise power 0.001 and above



States and Estimates for Noise at 0.0001



Applied control for noise at 0.0001

## Conclusion

Hence, after completing my assignment, my biggest learning has been the fact that designing a controller is an extremely arduous task. A controller has many performance metrics such as response time, region of attraction, stability and noise rejection, and one cannot satisfy them all. One needs to prioritise a few performance metrics, and target them, like I have done in this assignment.

We arrive at three controller configurations, each facing a different performance metric. Noise rejection turns out to be an extremely arduous task for all 3 controllers, emphasising the difficulty of the non linear control problem that we are facing.