Del operator → $(\vec{\nabla})$

$$\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}.$$

1) **Gradient** (grad ϕ) of ~~Sector~~ scalar field →

Let $\phi = \phi(x, y, z)$ be the scalar field, which is function of (x, y, z).

Now change in ϕ is $d\phi$ →

$$d\phi = \left(\frac{\partial \phi}{\partial x}\right)dx + \left(\frac{\partial \phi}{\partial y}\right)dy + \left(\frac{\partial \phi}{\partial z}\right)dz$$

$$= \left(\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$d\phi = (\vec{\nabla}\phi)\cdot\vec{dl}$$

here $\vec{dl} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

here the quantity

$$\boxed{\vec{\nabla}\phi = \text{grad}\,\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}} \quad -(1)$$

represents the Gradient of ϕ, it is a vector quantity.

Physical Significance → since $d\phi = (\vec{\nabla}\cdot\phi)\cdot\vec{dl}$

$\therefore\ d\phi = |\vec{\nabla}\phi||\vec{dl}|\cos\theta$

$\Rightarrow$

$$\frac{d\phi}{dl} = \nabla\phi\cos\theta.$$

for maximum change in ϕ, $\theta \to 0$.

$\therefore \cos\theta = 1$

$$\therefore\ \boxed{\left(\frac{d\phi}{dl}\right)_{max} = \vec{\nabla}\phi = \text{grad}\,\phi}$$

The gradient $\nabla\phi$, in its direction along the normal to the surface. ~~more over~~,

thus $\nabla\phi$, represents the maximum change in ϕ.

Divergence $(\vec{\nabla}\cdot\vec{A})$

let us consider a vector field $\vec{A}$, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

then

$$\text{div}\,\vec{A} = (\vec{\nabla}\cdot\vec{A}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).$$

$$\Rightarrow\ \boxed{\text{div}\,\vec{A} = \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right)}$$

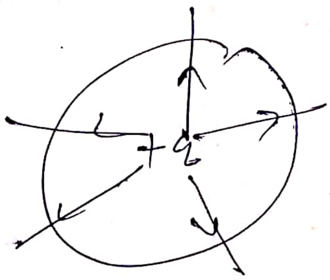
physical meaning → if a volume dv is placed in vector field $\vec{A}(x,y,z)$; the outgoing flux from surface S is given by – $\oint_S \vec{A}\cdot\vec{da}$

$$\therefore\ \boxed{\text{Div}\,\vec{A} = \vec{\nabla}\cdot\vec{A} = \lim_{dv\to 0}\frac{\int_S \vec{A}\cdot\vec{da}}{dv}}$$

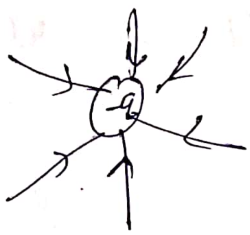
Thus, the net outgoing flux, per unit volume from the closed surface of small volume placed in vector field, is called Divergence of that vector field.

Divergence of vector $\vec{A}$ is a measure of how much $\vec{A}$, spreads out (Diverges), from the point of observation.

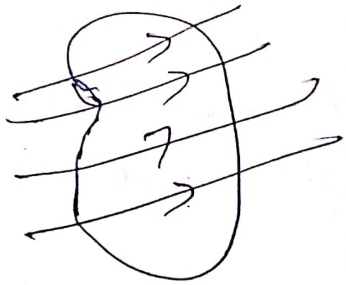
It may be +ive, -ive or zero.



$\mathrm{Div}\,\vec{E} = +\text{ive}$



$\mathrm{Div}\,\vec{E} = -\text{ive}$



$\mathrm{div}\,\vec{E} = 0$

Curl

$$\text{curl } \vec{A} = \vec{\nabla} \times \vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$$

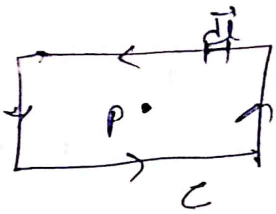
or

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

Physical meaning of curl $\vec{A}$ →

Let us consider a point P, in vector field $\vec{A}$, and path C, enclosing the point.

Now, we divide the path into small displacement $\vec{dl}$.



If the surface area enclosed by path C, is small, then $\vec{A}$ remain same on each point of the path.

then $\oint_C \vec{A}\cdot\vec{dl}$ is line integral of $\vec{A}$, over closed path C. So →

Curl of any vector is defined as the line integral per unit small surface area enclosed by the path, so

$$\boxed{\text{curl } \vec{A} = \lim_{da \to 0} \left(\frac{\oint_C \vec{A}\cdot\vec{dl}}{da} \right) \cdot \hat{n}}$$

Curl means rotation of in vector $\vec{A}$.

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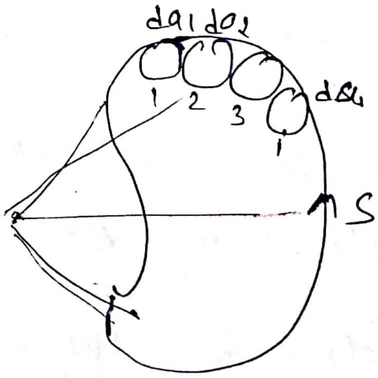
Gauss's Divergence Theorem →

let we consider a vector field $\vec{A}$. The vol^m enclosed by it V.

Id this volume is enclosed by an imaginary surface S.

Volume can be considered as made of many rectangular hexagons.

outgoing flux from each small element -



$$\phi_i = \oint_S \vec{A_i} \cdot \vec{da_i} \quad -(1)$$

∴ total flux from this surface S →

$$\oint_S \vec{A} \cdot \vec{da} = \sum_i \oint_S \vec{A_i} \cdot \vec{da_i} \quad -(2)$$

divergence of $\vec{A}$ on small area element da_i

$$\therefore \quad \operatorname{div} \vec{A_i} = \frac{1}{dv_i} \sum_i \oint_S \vec{A_i} \cdot \vec{da_i} \quad dv_i \to 0$$

$$\therefore \oint_S \vec{A_i} \cdot \vec{da_i} = \lim_{dv_i \to 0} \text{div}\, \vec{A_i} \cdot dv_i \quad -③$$

from ② and ③

$$\therefore \oint_S \vec{A} \cdot \vec{da} = \sum_{i} \text{div}\vec{A_i} \cdot dv_i \quad dv_i \to 0$$

if $dv_i \to 0$, the summation sign can be replaced by integration sign -

$$\boxed{\oint_S \vec{A} \cdot \vec{da} = \int_V (\text{div}\vec{A})\, dv}$$

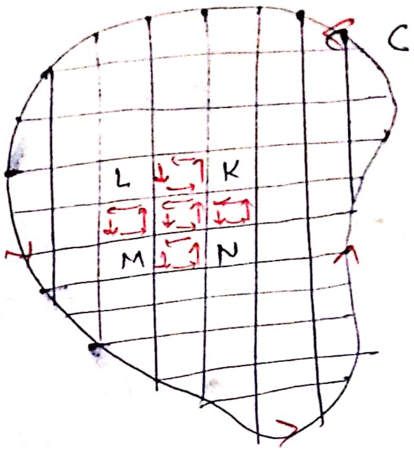
Thus, the surface integration of any vector field over a closed surface, is equal to the volume integration of it's divergence.

This is called Gauss's divergence theorem.

Stoke's Curl Theorem → Let us consider an area enclosed by closed path c in a vector field $\vec{A}$.

To determine the line integral along the path c, we divide whole surface in small rectangles, and find line integral over each closed path of rectangle, then sum them all.

Let us consider path KLMN,



For each boundary of path KLMN, the path integration is equal and opposite to that of neighbouring path like

Since $\int_K^L \vec{A}\cdot\vec{dl} = -\int_L^K \vec{A}\cdot\vec{dl}$

$\therefore \int_K^L \vec{A}\cdot\vec{dl} + \int_L^K \vec{A}\cdot\vec{dl} = 0$

The sum of all the integrals over common path becomes zero.

Thus, sum only comes from the integration over boundary C, ..

$$\therefore \oint_C \vec{A}\cdot\vec{dl} = \sum_i \oint_{dc} \vec{A}\cdot\vec{dl_i} \quad \text{--- (1)}$$

As we know the curl over the small path (i th cell)

$$\therefore \text{curl}\,\vec{A} = \lim_{da_i \to 0}\left[\frac{1}{da_i}\sum_i \oint_{dc} \vec{A}\cdot\vec{dl_i}\right]\hat{n}$$

$$\therefore \oint_{dc} \sum \vec{A}\cdot\vec{dl_i} = \lim_{da_i \to 0} \text{curl}\,\vec{A}\cdot\hat{n}\,da_i \quad \text{--- (2)}$$

$\therefore$ from (1) and (2)

$$\oint_C \vec{A}\cdot\vec{dl} = \lim_{da_i \to 0} \sum_i (\vec{\nabla}\times\vec{A})\cdot\hat{n}\,da_i$$

for small surface element → $\Sigma \rightarrow \int$

$$\therefore \boxed{\oint_C \vec{A} \cdot \vec{dl} = \oint_S (\vec{\nabla} \times \vec{A}) \cdot \vec{da}} \quad \left\{ \begin{array}{l} \hat{n}\, da = \vec{da} \end{array} \right.$$

⇒ Thus the line integration of any vector along the closed path is equal to the surface integration of curl of that surface vector, over the surface enclosed by the path.

This is called stoke's curl theorem.

Gauss Law of electrostatic →

According to this law → the net electric flux through a closed surface, is equal to ratio of the charge enclosed by that surface, and ϵ_0.

$$\boxed{\phi_E = \frac{q}{\epsilon_0}} \quad -(1) \qquad \text{or} \qquad \phi_E = \frac{1}{\epsilon_0} \int_V \rho \cdot dv \quad -(2)$$

where ρ → is the volume charge density.

eq^n (2) is gauss law in integral form.

Now, from the defination of flux

$$\phi_E = \oint_S \vec{E} \cdot \vec{da} \quad -(3)$$

if the surface S is divided into small area element $\vec{da}$.

then $d\phi = \vec{E} \cdot \vec{da}$

from Gauss divergence theorem

$$\phi_E = \oint_S \vec{E} \cdot \vec{da} = \int_V (\vec{\nabla} \cdot \vec{E}) \, dv \quad -③$$

from eqⁿ ② and ③

$$\frac{1}{\epsilon_0} \oint_V \rho \cdot \vec{dv} = \oint_V (\vec{\nabla} \cdot \vec{E}) \, dv$$

$$\Rightarrow \frac{\rho}{\epsilon_0} = \vec{\nabla} \cdot \vec{E} \Rightarrow \boxed{\nabla \cdot \vec{D} = \rho_v} \quad -④$$

Eqⁿ ④ represents the Gauss law in differential form.

{ as $\vec{D} = \epsilon \vec{E}$ electric displacement vector

Now as we know $E = -\nabla V$

$\therefore$ from ④ $D = -\epsilon \nabla V$

$$\vec{\nabla} \cdot (-\epsilon \nabla V) = \rho_v$$

$$\Rightarrow -\nabla^2 V = \frac{\rho_v}{\epsilon} \Rightarrow \boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \quad -⑤$$

Eqⁿ ⑤ is known as Poisson's Equation.

Now for charge free region $\rho_v = 0$

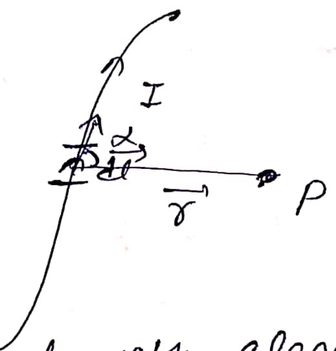
$$\therefore \boxed{\nabla^2 V = 0} \quad -⑥$$

This eqⁿ is known as Laplace Equation.

Bio - Savart law → This law is used to determine the resultant magnetic field $\vec{B}$, at position r in 3-D space generated by a steady current. [Steady - current means a continuous flow of charge, which does not change with time].

let us consider a wire carrying a current I,

we want to find out magnetic field at point P, which is distant $\vec{r}$, from wire.



we divide wire into small length elements.
let we take a small length element $\vec{dl}$,
the direction of $\vec{dl}$ is in direction of I.

Now the magnetic field at $\vec{P}$, due to current flow in element $\vec{dl}$, is dB

$$dB \propto I$$

$$dB \propto \frac{1}{r^2}$$

$$dB \propto dl$$

$$dB \propto \sin\alpha$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\alpha}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, \vec{dl} \times \hat{r}}{r^2}$$

where $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ is unit vector.

$$\therefore \boxed{B = \int dB = \frac{\mu_0}{4\pi} \int_l \frac{I\, \vec{dl} \times \vec{r}}{r^3}}$$

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~~The sources of light from which phase difference between the waves from them reaching at any point in space does not change with time rather remains constant, are known as coherent source.~~

~~This property of the waves is known as coherence. it is an important property of waves which refers to the connection b/w the phase of light waves.~~

~~coherence effect are mainly divided into two categories.~~

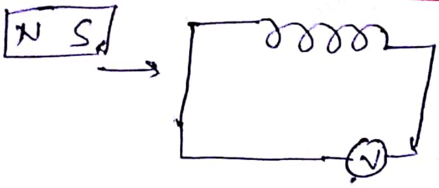
~~Tempo~~

Faraday's law of Electromagnetic Induction →

According to this law – when magnetic flux linked with any circuit changes, an e.m.f., is induced in the circuit, and this induced e.m.f., is proportional to the rate of change of magnetic flux linked with the circuit.

and direction of this e.m.f. is always opposite to the changes responsible for its production

$$\boxed{e \propto \frac{d\phi}{dt}, \quad e = -\frac{\partial \phi}{\partial t}}$$



Ampere's circuital law → according to this law — Line integral of magnetic field over a closed path is equal to μ times the net current passes through the closed path. —

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu \sum I}$$

Displacement current → In electromagnetism, a displacement current is a quantity, defined in terms of the rate of change of electric displacement field.

If we have a varying magnetic field linked with a conductor, then electric field induced in conductor will also vary.

Thus due to change in electric field, ~~a current~~ ~~is~~ caused a current, known as displacement current

$$I_D = \frac{\partial \vec{D}}{\partial t} = \frac{\epsilon \, \partial \vec{E}}{\partial t}$$

This current is not due to moving charges, its due to time varying electric field.

Maxwell's Equation

1. first eq^n is based on Gauss's law for electric field.

according to it
$$\phi_E = \oint_S \vec{E}\cdot\vec{da} = \frac{Q}{\epsilon_0} \quad -①$$

if charge inside the closed path is continuously distributed the ρ - is volume charge density defined as -
$$Q = \int_V \rho \cdot dv \quad -②$$

from eq^n ① and eq^n ② -
$$\boxed{\oint_S \vec{E}\cdot\vec{da} = \frac{1}{\epsilon_0}\int_V \rho\cdot dv} \quad -③$$

eq^n ③ is integral form of Maxwell's eq^n.

Now, according to Gauss divergence theorem -
$$\oint_S \vec{E}\cdot\vec{da} = \int_V (\vec{\nabla}\cdot\vec{E})\, dv$$

$$\therefore \int_V (\vec{\nabla}\cdot\vec{E})\, dv = \frac{1}{\epsilon_0}\int_V \rho\, dv$$

$$\Rightarrow \quad \nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0} \quad \Rightarrow \quad \mathrm{div}\,\vec{E} = \frac{\rho}{\epsilon_0}$$

This is the differential form of Maxwell's eq^n.

from displacement vector $\vec{D} = \epsilon\vec{E}$

$$\therefore \quad \nabla\cdot(\epsilon\vec{E}) = \rho \quad \Rightarrow \quad \boxed{\nabla\cdot\vec{D} = \rho}$$

for free space $\quad \rho = J = 0$

$$\therefore \quad \nabla\cdot\vec{E} = 0$$

$$\boxed{\nabla\cdot\vec{D} = 0}$$

Maxwell's Second eq^n → This eq^n is derived from Gauss law for magnetic field.

According to this law – net outgoing flux from any closed surface is always zero. (b'coz the magnetic field lines always form closed loops).

since $\phi_B = 0 \quad$ or $\quad \oint_S \vec{B}\cdot\vec{da} = 0$

This is the integral form of Maxwell's IInd eq^n.

from Gauss Divergence theorem –

$$\oint_S \vec{B}\cdot\vec{da} = \int_V (\nabla\cdot\vec{B})\,dV = 0 \quad \Rightarrow \quad \boxed{\nabla\cdot\vec{B} = 0}$$

This is differential form of Maxwell IInd eq^n.

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This equation prove that monopole does not exist in nature.

Maxwell's Third Equation

This Equation is derived from Faraday's law of electromagnetic induction.

According to this law, — whenever magnetic flux linked with a circuit, changes, an e.m.f. is induced in the circuit and this induced e.m.f. is directly proportional to the rate of change of magnetic flux linked with the circuit.

$$\therefore \quad e \propto \frac{d\phi_B}{dt}$$

$$\text{or} \quad e = \frac{-d\phi_B}{dt} \quad \text{—} \; (1)$$

this e.m.f. is defined as the work done in carrying a unit positive charge around a open loop. Thus

$$e = \oint_C \vec{E}\cdot\vec{dl} \quad \text{—} \; (2)$$

As we know the magnetic field flux $\phi_B = \oint_S \vec{B} \cdot \vec{da}$ —③

$\therefore \oint_C \vec{E} \cdot \vec{dr} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot \vec{da}$

$$\Rightarrow \boxed{\oint_C \vec{E} \cdot \vec{dl} = \oint_S \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot \vec{da}} \quad —④$$

This is the integral form of Maxwell's third eqn.

from Stoke's curl theorem -

$$\oint_C \vec{E} \cdot \vec{dr} = \oint_S (\nabla \times \vec{E}) \cdot \vec{da}$$

$$\therefore \oint_S (\nabla \times \vec{E}) \cdot \vec{da} = \oint_S \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot \vec{da}$$

$$\Rightarrow \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{or} \quad \boxed{\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

This is differential form of Maxwell's third eqn.

for static field $\frac{\partial \vec{B}}{\partial t} = 0 = \frac{\partial \vec{E}}{\partial t}$

$\therefore \nabla \times \vec{E} = 0$

or $\oint_S \vec{E} \cdot \vec{dl} = 0$

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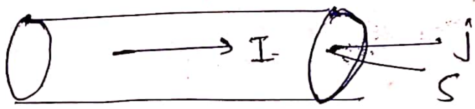
Equation of continuity →

As we know $I = \int_S \vec{J} \cdot \vec{da}$ —(1)

Let we consider a current carrying conductor, and ρ be the volume charge density, then the total charge in the volume V is

$$q = \oint_V \rho_v \cdot dv$$



So $I = -\frac{dq}{dt} = -\frac{d}{dt} \int_V \rho \cdot dv$ —(2)

from eq^n (1) and eq^n (2)

$$\oint_S \vec{J} \cdot \vec{da} = -\frac{d}{dt} \int_V \rho \cdot dv \quad —(3)$$

from Gauss Divergence theorem –

$$\oint_S \vec{J} \cdot \vec{da} = \int_V (\vec{\nabla} \cdot \vec{J}) \cdot dv \quad —(4)$$

∴ from (3) and (4)

$$\Rightarrow \int_V (\vec{\nabla} \cdot \vec{J})\, dv = -\frac{d}{dt} \int_V \rho \cdot dv$$

$$\Rightarrow \boxed{\operatorname{div} \vec{J} = \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} + \frac{d\rho}{dt} = 0}$$

This Eqⁿ is known as equation of continuity and represents the physical fact of conservation for charge.

Maxwell's fourth Eqⁿ

This Eqⁿ is based upon Ampere's circuital law. acc. to this law - line integral of magnetic field over a closed path is equal to μ times the net current passes through the closed path.

$$\oint_C \vec{B}\cdot\vec{dl} = \mu \Sigma I \quad -(1)$$

as we know $\Sigma I = \oint_S \vec{J}\cdot\vec{da}$ -(2)

$$\therefore \oint_C \vec{B}\cdot\vec{dl} = \mu \oint_S \vec{J}\cdot\vec{da}$$

from stoke's curl theorem - $\oint_C \vec{B}\cdot\vec{dl} = \oint_S (\vec{\nabla}\times\vec{B})\cdot\vec{da}$ -(3)

∴ from (2) and (3)

$$\Rightarrow \quad \nabla\times\vec{B} = \mu\vec{J} \quad -(4)$$

Eqⁿ (4) is valid only for steady close current. For changing electric field it does not holds true b'coz if we take divergence of eqⁿ (4) -

$$\nabla\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot\vec{J}$$

since $\nabla \cdot (\vec{\nabla} \times \vec{A}) = 0$ $\quad div(curl \vec{A}) = 0$

$\therefore \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$

$$\therefore \vec{\nabla} \cdot \vec{J} = 0 \quad \text{--- (5)}$$

But from eq^n of continuity $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

So eq^n (5) is confilicts with the Equation of continuity $div \vec{J} = -\frac{\partial \rho}{\partial t}$

Thus here it is clear that Ampere's law relates magnetic field to the steady current only.
while for the current, produces due to change in electric field w. rt. to time this law is not valid.

Maxwell removed this inconsistancy by giving a suggestion that total current density is made of two types of currents -

1) free current Density $\vec{J_f}$ due to motion of free charges.
2) Displacement current $\vec{J_D}$ due to change in electric field.

$$\vec{J} = \vec{J_f} + \vec{J_d}$$

Eq^n (5) becomes

$$\therefore \vec{\nabla} \times \vec{B} = \mu(\vec{J_f} + \vec{J_d})$$

On taking divergence $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu(\nabla \cdot \vec{J_f} + \nabla \cdot \vec{J_d})$

$$\therefore \nabla \cdot \vec{J_f} = -\nabla \cdot \vec{J_d}$$

but from eq^n of continuity $\nabla \cdot \vec{J_f} + \frac{\partial \rho}{\partial t} = 0$

$$\therefore \nabla \cdot \vec{J_f} = \frac{-\partial \rho}{\partial t}$$

$$\Rightarrow \nabla \cdot \vec{J_d} = \frac{\partial \rho}{\partial t}$$

$\left\{ \begin{array}{l} \nabla \cdot \vec{D} = \rho \\ \text{Maxwell's } I^{st} \text{ eq}^n \end{array} \right.$

$$\text{or } \nabla \cdot \vec{J_d} = \frac{\partial}{\partial t}(\nabla \cdot \vec{D}).$$

$$\Rightarrow \vec{J_d} = \frac{\partial \vec{D}}{\partial t} \quad \text{--- (6)}$$

Eq^n (6) represents the displacement current, which is due to change in electric field (w.r.t. to time)

Now Maxwell' forth eq^n becomes.

$$\vec{\nabla} \times \vec{B} = \mu(\vec{J_f} + \vec{J_d}).$$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu \vec{J_f} + \mu \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu \vec{J_f} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}}$$

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subtracting Eqⁿ ② from Eqⁿ ③ →

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{H}\cdot(\vec{\nabla}\times\vec{E}) = E\cdot\frac{\partial}{\partial t}(\epsilon E) + H\cdot\frac{\partial(\mu H)}{\partial t}$$

$$\therefore\ \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \frac{-1}{2}\frac{\partial}{\partial t}(\epsilon E^2 + \mu H^2) \quad -④$$

$$\left\{\begin{aligned}&\vec{A}\cdot(\vec{B}\times\vec{C})\\ &= \vec{B}(\vec{C}\times\vec{A}) - \vec{C}(\vec{A}\times\vec{B})\end{aligned}\right.$$

integrating Eqⁿ ④ over a volume V, then

$$\oint_V [\vec{\nabla}\cdot(\vec{E}\times\vec{H})]\,dv = -\frac{\partial}{\partial t}\int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv$$

from Gauss divergence theorem

$$\boxed{\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds} = \frac{-\partial}{\partial t}\int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right) dv} \quad -⑤$$

The R.H.S. of Eqⁿ ⑤, represents the rate of change of total EM Energy, Therefore the L.H.S. should also represents the flux of total energy passing per unit time from a closed surface bounding the field and it is denoted by $\vec{P}$ (poynting vector).

$$\Rightarrow \quad \vec{P} = \underset{= \frac{V}{m}\cdot\frac{amp}{m} =}{\vec{E}\times\vec{H}} \ watt/m^2 = \frac{Energy}{time \times area}$$

The $\vec{P}$, has the dimensions of $\frac{Energy}{area \times time}$ and hence is interpreted as the rate at which the energy flows through unit area of surface. This statement is termed as poyting theorem and represents the law of conservation of Energy.

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Energy flow Due to plane EM wave - Poynting vector

As EM waves propogates through space from one point to another they transport energy with them.

Let the EM wave propogates through free space -

$\sigma = \rho = J = 0$

$\left\{ \begin{array}{l} \vec{D} = \epsilon \vec{E} \\ \vec{B} = \mu_0 \vec{H} \end{array} \right.$

$$\therefore \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad -(1)$$

On taking ~~Divergence~~ Dot product with $\vec{H}$

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) = -H \cdot \frac{\partial (\mu_0 \vec{H})}{\partial t} \quad -(2)$$

similarly from maxwell fourth Eq^n

$$\vec{\nabla} \times \vec{B} = \mu J + \mu \frac{\partial \vec{D}}{\partial t} \qquad \left\{ \begin{array}{l} J = 0 \\ B = \mu H \end{array} \right.$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

dot product with $\vec{E}$

$$\therefore \quad \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \frac{\partial (\epsilon \vec{E})}{\partial t} \quad -(3)$$

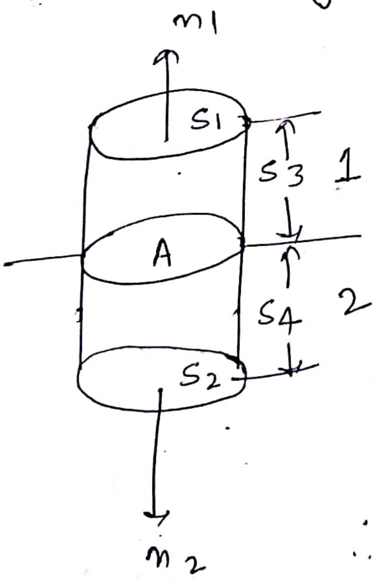
Boundary conditions for Electro Magnetic field vector ($\vec{E}, \vec{D}, \vec{B}$ and $\vec{H}$) →

we shall consider the behaviour of EM waves at the boundaries between different media.

A boundary means – the end of one region and the beginning of the other.

In study boundary conditions – we study the behaviour of tangential and normal component of EM field vector at the boundary surfaces.

(i) Boundary condition for $\vec{B}$ →



Let we consider two media 1 and 2, in a box having four surfaces S_1, S_2, S_3 and S_4.

from Maxwell's II^{nd} Eq^n -

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\therefore \oint_V \vec{\nabla} \cdot \vec{B} \, dV = 0$$

Gauss divergence theorem -

$$\Rightarrow \oint_S \vec{B} \cdot \vec{n} \, ds = 0 \quad \text{--- (1)}$$

applying Eq^n (1) for all surfaces of the box -

$$\oint_{S_1} \vec{B_1}\cdot\vec{n_1}\,ds_1 + \int_{S_2} \vec{B_2}\cdot\vec{n_2}\,ds_2 + \int_{S_3} \vec{B_P}\cdot\vec{n_P}\,ds_3 + \int_{S_4} \vec{B_2}\cdot\vec{n_2}\,ds_4 = 0$$

At the boundary ; the height of the box approaches zero , and S_1 and S_2 approaches same area A, and contribution from S_3 and $S_4 \to 0$

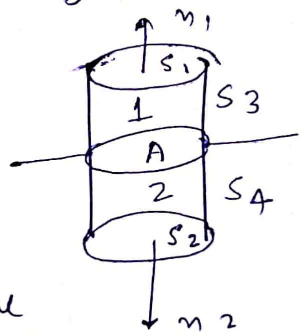
$$\Rightarrow \int_A \left(\vec{B_1}\cdot\vec{n_1} + \vec{B_2}\cdot\vec{n_2}\right) dA = 0 \qquad \text{as } \vec{n_1} = -\vec{n_2}$$

$$\therefore \quad B_1\cdot n_1 = B_2\cdot n_1$$

$$\Rightarrow \boxed{B_{1n} = B_{2n}}$$

i.e., the ~~no~~ normal component of magnetic induction is continuous across the boundary.

(2) Boundary condition of Displacement vector $\vec{D}$ →



from Maxwell's Eq^n → $\vec{\nabla}\cdot\vec{D} = \rho$

$$\therefore \int_V (\vec{\nabla}\cdot\vec{D})\,dv = \int_V \rho\cdot dv$$

from Gauss Divergence theorem -

$$\oint_S (\vec{D}\cdot\hat{n})\,ds = \int_V \rho\cdot dv \quad \text{for all surfaces of the box -}$$

$$\int_{S_1} \vec{D_1}\cdot\vec{n_1}\,ds + \int_{S_2} \vec{D_2}\cdot\vec{n_2}\,ds + \int_{S_3} \vec{D_1}\cdot\vec{n_1}\,ds + \int_{S_4} \vec{D_2}\cdot\vec{n_2}\,ds = \int_V \rho_v\,dv$$

at boundary $S_1 = S_2 = A$ and contribution from S_3 and $S_4 \to 0$

$$\text{Now} \quad \int_V \rho_v\,dv = \int_S \sigma\cdot dA = \sigma\cdot A$$

$$\therefore \int_{S_1} \vec{D_1}\cdot\vec{n_1}\,ds + \int_{S_2} \vec{D_2}\cdot\vec{n_2}\,ds = \sigma A$$

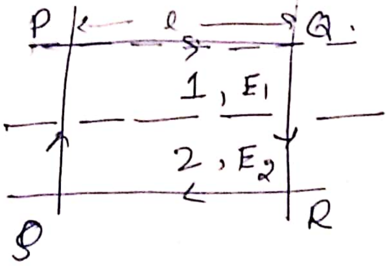
$$\Rightarrow \cancel{A}\left(D_1\cdot n_1 - D_2\cdot n_2\right) = \sigma\cancel{A}$$

$$\Rightarrow \boxed{D_1 n_1 - D_2 n_2 = \sigma}$$

Thus the normal component of Electric displacement vector is not continuous across the interface and changes by an amount Equal to surface current density at the interface.

(3) Boundary condition for $\vec{E}$ → Let us consider a rectangular loop PQRS bounding the surface S.

from maxwell eq^n –



$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

Now $$\int_S (\nabla \times \vec{E}) \cdot \hat{n}\, ds = -\int_S \frac{\partial}{\partial t} \vec{B} \cdot \hat{n}\, ds.$$

using stoke's theorem

Now
$$\int_{PQRS} \vec{E} \cdot \vec{dl} = -\int_S \frac{\partial B}{\partial t} \cdot \hat{n}\, ds$$

$$\therefore \int_{PQ} \vec{E_1} \cdot \vec{dl} + \int_{RS} \vec{E_2} \cdot \vec{dl} + \text{contribution from sides QR and SP}$$

At boundary the height of the loop is zero. So contribution from sides QR and PS will be zero. and $\int \vec{B} \cdot \hat{n}\, ds \to 0$ b'coz no flux can be enclosed when $\delta h \to 0$

$$\therefore \int_{PQ} \vec{E_1} \cdot dl + \int_{RS} \vec{E_2} \cdot \vec{dl} = 0$$

$$\vec{E_1}\, \vec{PQ} - E_2\, \vec{RS} = 0 \Rightarrow \boxed{E_{1t} = E_{2t}}$$

here $\vec{E}_{1t}$ and $\vec{E}_{2t}$ are since $(\vec{PQ} = -\vec{RS})$

the tangential component of the electric field in two media.

Thus, the tangential component of $\vec{E}$, are continuous across the interface.