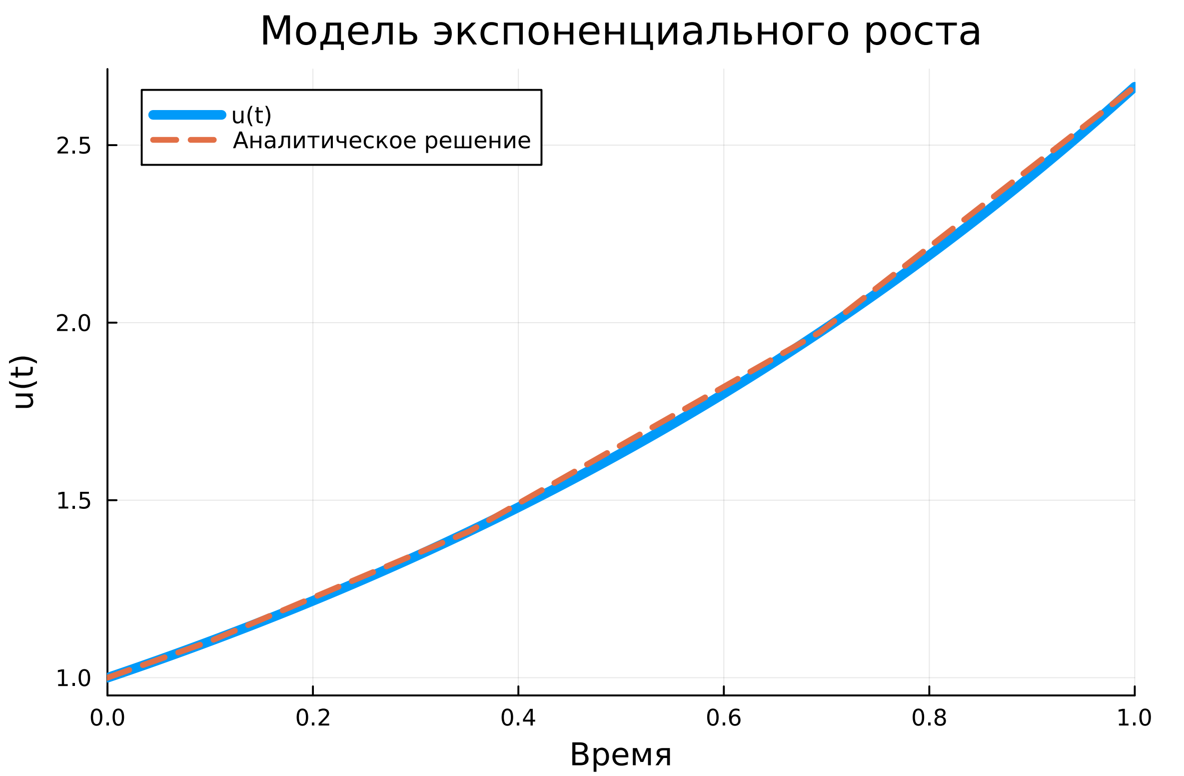
Повторение примеров

Решение обыкновенных дифференциальных уравнений

Модель экспоненциального роста

In [1]:
```
using DifferentialEquations
# задаём описание модели с начальными условиями:
a = 0.98
f(u,p,t) = a*u
u0 = 1.0
# задаём интервал времени:
tspan = (0.0,1.0)
# решение:
prob = ODEProblem(f,u0,tspan)
sol = solve(prob)
# подключаем необходимые пакеты:
using Plots
# строим графики:
plot(sol, linewidth=5,title="Модель экспоненциального роста", xaxis="Время",yaxis="
plot!(sol.t, t->1.0*exp(a*t),lw=3,ls=:dash,label="Аналитическое решение")
```

Out[1]:



In [2]:
```
# задаём точность решения:
sol = solve(prob,abstol=1e-8,reltol=1e-8)
```

```julia
println(sol)
# строим график:
plot(sol, lw=2, color="black", title="Модель экспоненциального роста", xaxis="Время
plot!(sol.t, t->1.0*exp(a*t),lw=3,ls=:dash,color="red",label="Аналитическое решение
```

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```

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```

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ter!, step_limiter! = trivial_limiter!, thread = static(false), lazy = true,), Rodas
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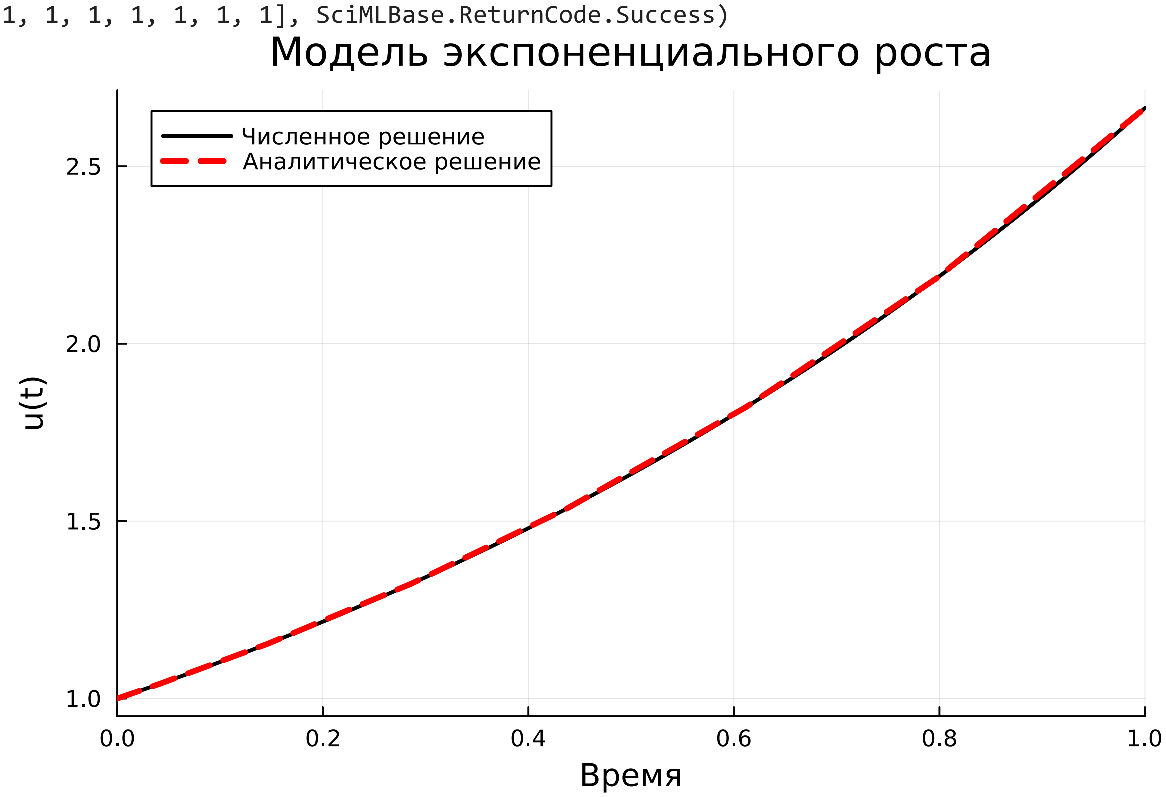
```

Out[2]:



Система Лоренца

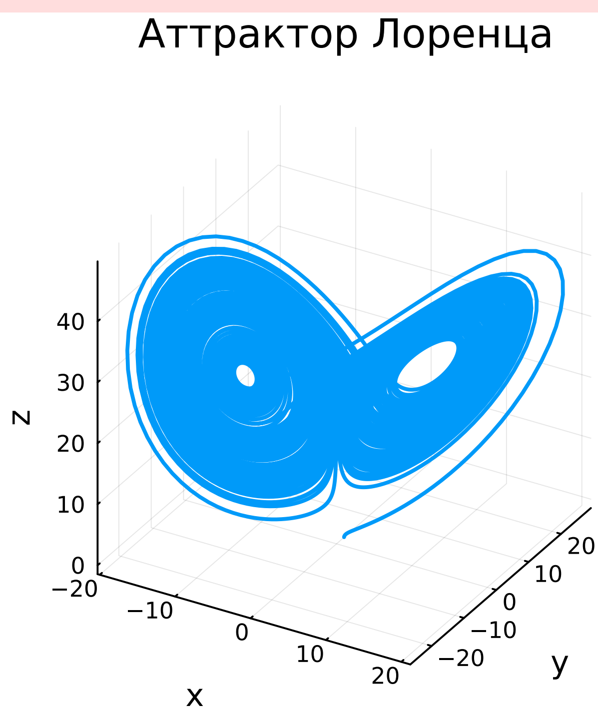
In [3]:

```
# задаём описание модели:
function lorenz!(du,u,p,t)
```

```
σ,ρ,β = p
    du[1] = σ*(u[2]-u[1])
    du[2] = u[1]*(ρ-u[3]) - u[2]
du[3] = u[1]*u[2] - β*u[3]
end
# задаём начальное условие:
u0 = [1.0,0.0,0.0]
# задаём знанчения параметров:
p = (10,28,8/3)
# задаём интервал времени:
tspan = (0.0,100.0)
# решение:
prob = ODEProblem(lorenz!,u0,tspan,p)
sol = solve(prob)
# строим график:
plot(sol, vars=(1,2,3), lw=2, title="Аттрактор Лоренца", xaxis="x",yaxis="y", zaxis
```

```
┌ Warning: To maintain consistency with solution indexing, keyword argument vars wil
l be removed in a future version. Please use keyword argument idxs instead.
│   caller = ip:0x0
└ @ Core :-1
```

Out[3]:

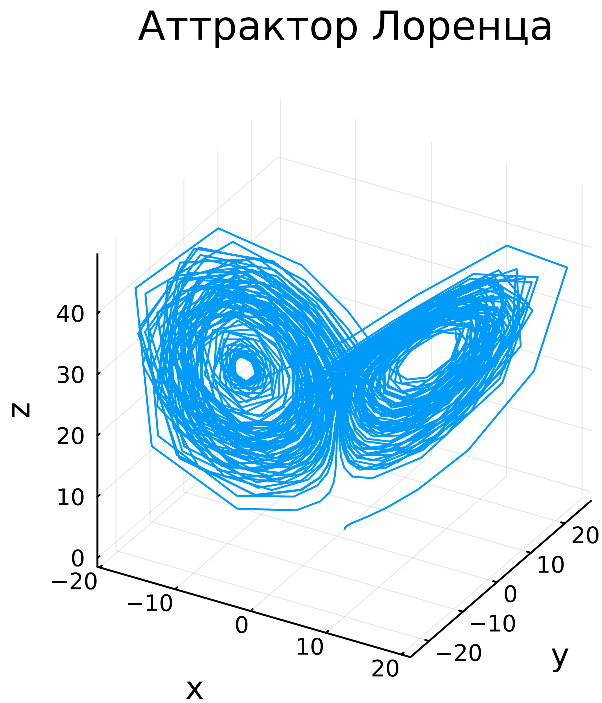


Без интерполяции

In [4]:

```
# отключаем интерполяцию:
plot(sol,vars=(1,2,3),denseplot=false, lw=1, title="Аттрактор Лоренца", xaxis="x",y
```

Out[4]:

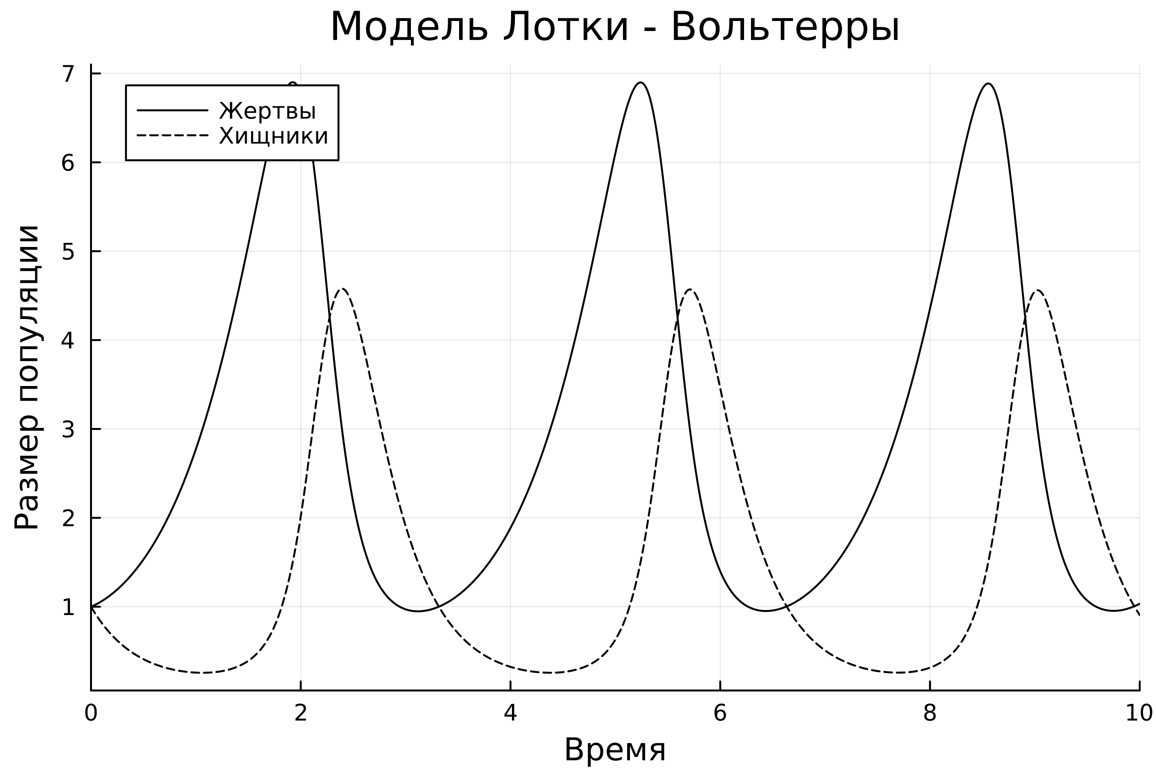


Модель Лотки–Вольтерры

In [5]:

```
using ParameterizedFunctions, DifferentialEquations, Plots;
# задаём описание модели:
lv! = @ode_def LotkaVolterra begin
dx = a*x - b*x*y
dy = -c*y + d*x*y
end a b c d
# задаём начальное условие:
u0 = [1.0,1.0]
# задаём значения параметров:
p = (1.5,1.0,3.0,1.0)
# задаём интервал времени:
tspan = (0.0,10.0)
# решение:
prob = ODEProblem(lv!,u0,tspan,p)
sol = solve(prob)
plot(sol, label = ["Жертвы" "Хищники"], color="black", ls=[:solid :dash], title="Mo
```

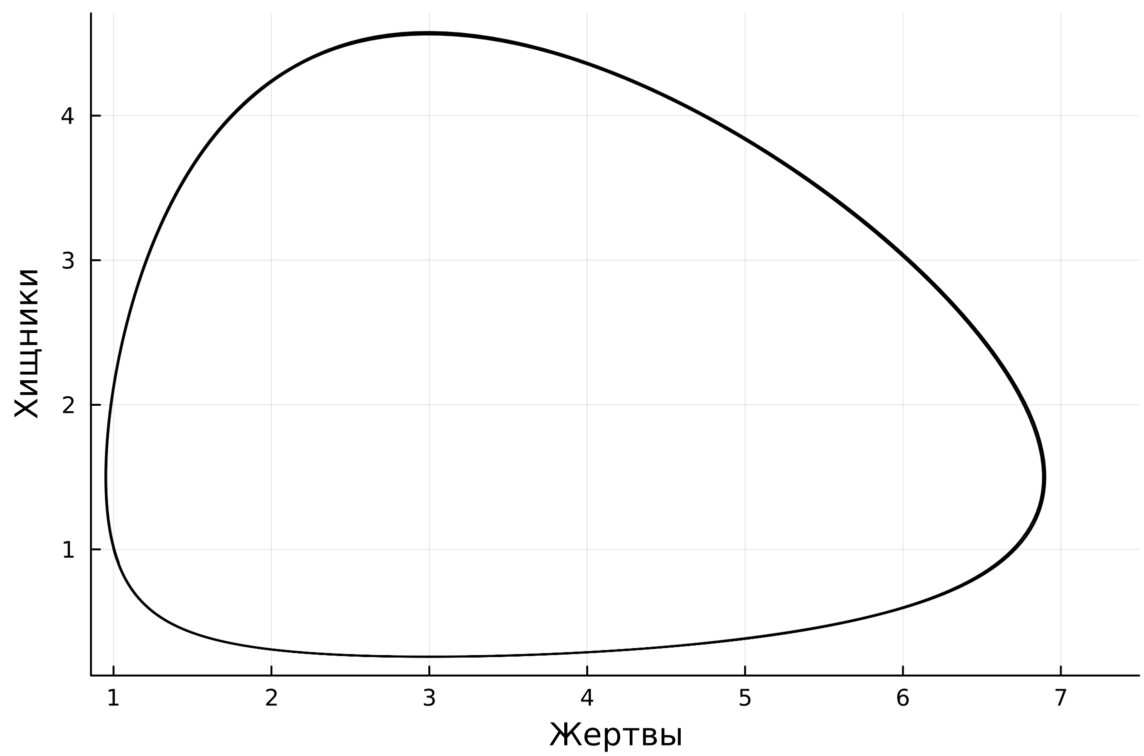
Out[5]:



In [6]:

```
# фазовый портрет:
plot(sol,vars=(1,2), color="black", xaxis="Жертвы",yaxis="Хищники", legend=false)
```

Out[6]:



Самостоятельная работа

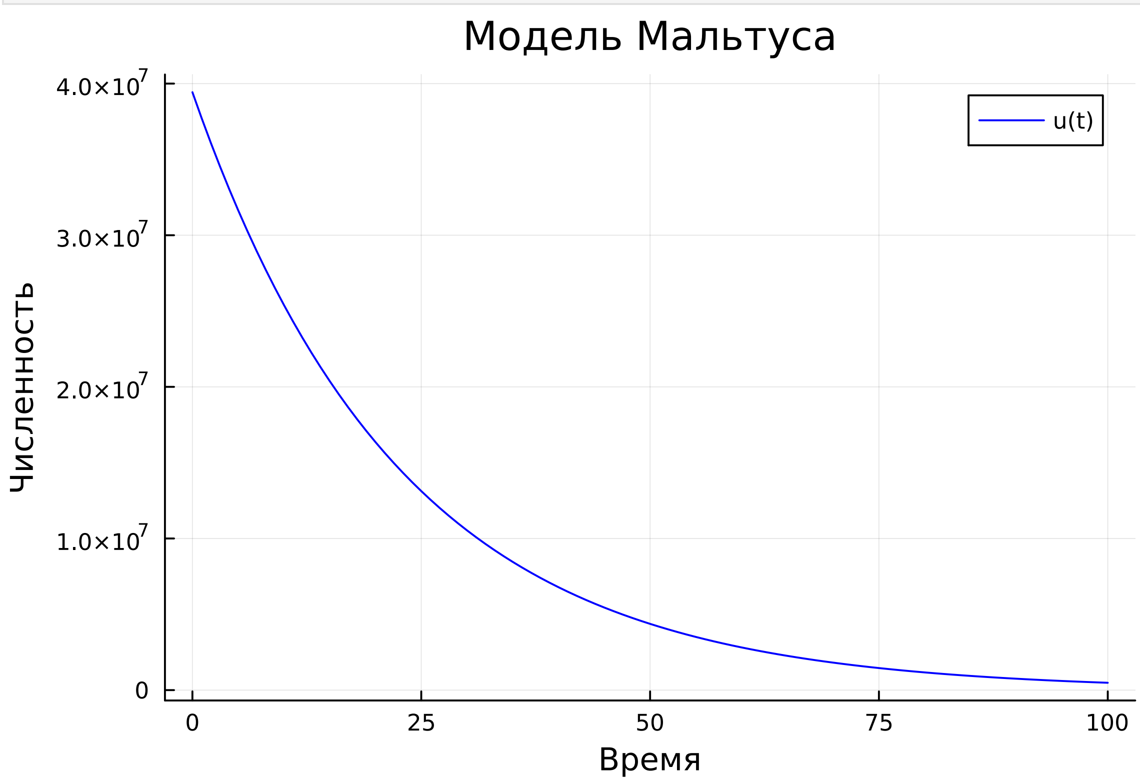
Модель Мальтуса

Модель Мальтуса --- модель роста численности изолированной популяции, где изменение роста популяции контролируется численностью уже существующей популяции, домноженной на коэффициент a, который является разницей между рождаемостью и смертностью ($b - c$). Коэффициенты b и c было предложено выбрать самостоятельно, и я выставлю для системы значения $b = 1.09$ и $c = 1.134$ (что является соответственно коэффициентами рождаемости и смертности за январь-август в 2022 году в Центральном федеральном округе РФ). Изначальная численность населения (39433556 человек) также взята из статистики Росстата за 2022 год (с учётом переписи населения).

Модель Мальтуса подразумевает, что коэффициенты рождаемости и смертности не изменяются, так что если b превышает c, численность популяции будет расти (и наоборот).

In [7]:
```
function Maltus!(du,u,p,t)
    du[1] = (p[1]-p[2])*u[1]
end
u0 = [39433556.0]
tspan = (0.0,100.0)
p = Float64[1.09, 1.134]
prob = ODEProblem(Maltus!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=1.0)
R1 = [tu[1] for tu in sol.u]
plot(sol.t, R1, title="Модель Мальтуса", xaxis="Время",yaxis="Численность",label="u
```

Out[7]:



In [8]:
```
anim = @animate for i in 1:length(sol.t)
    plot(sol.t[1:i], R1[1:i], title="Модель Мальтуса", xaxis="Время",yaxis="Численн
end
gif(anim, "presentation//image//1.gif")
```

```
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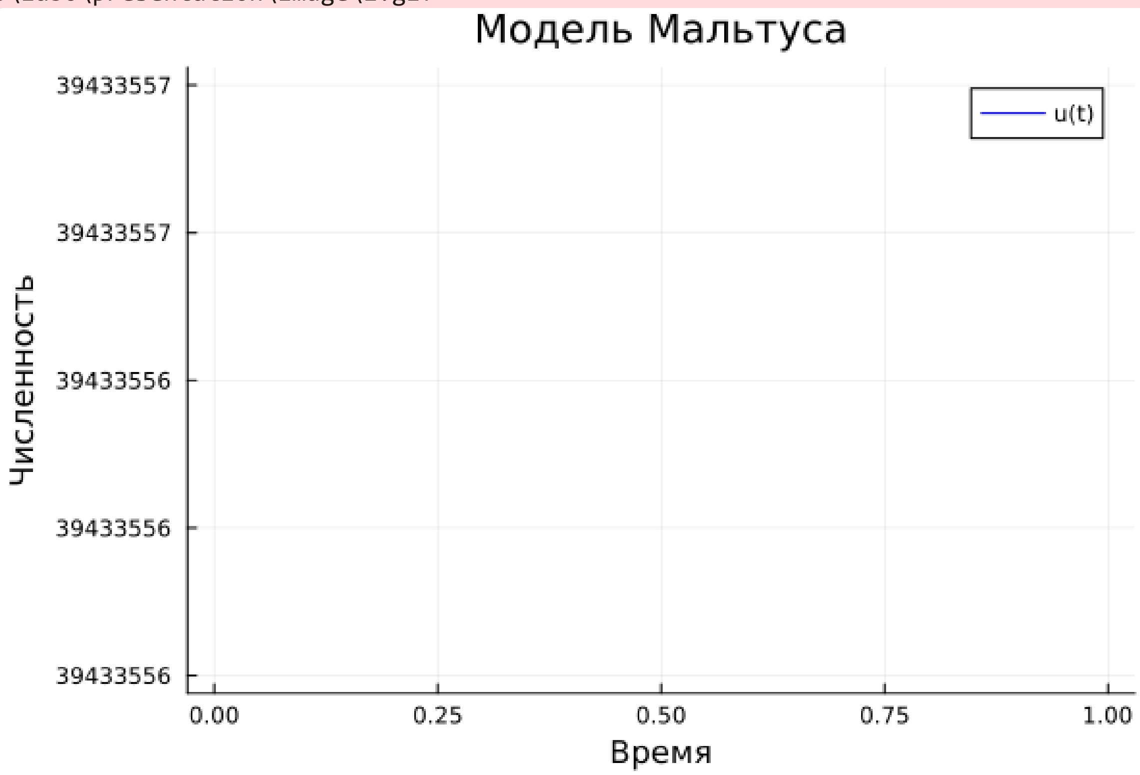
o\lab6\presentation\image\1.gif
```

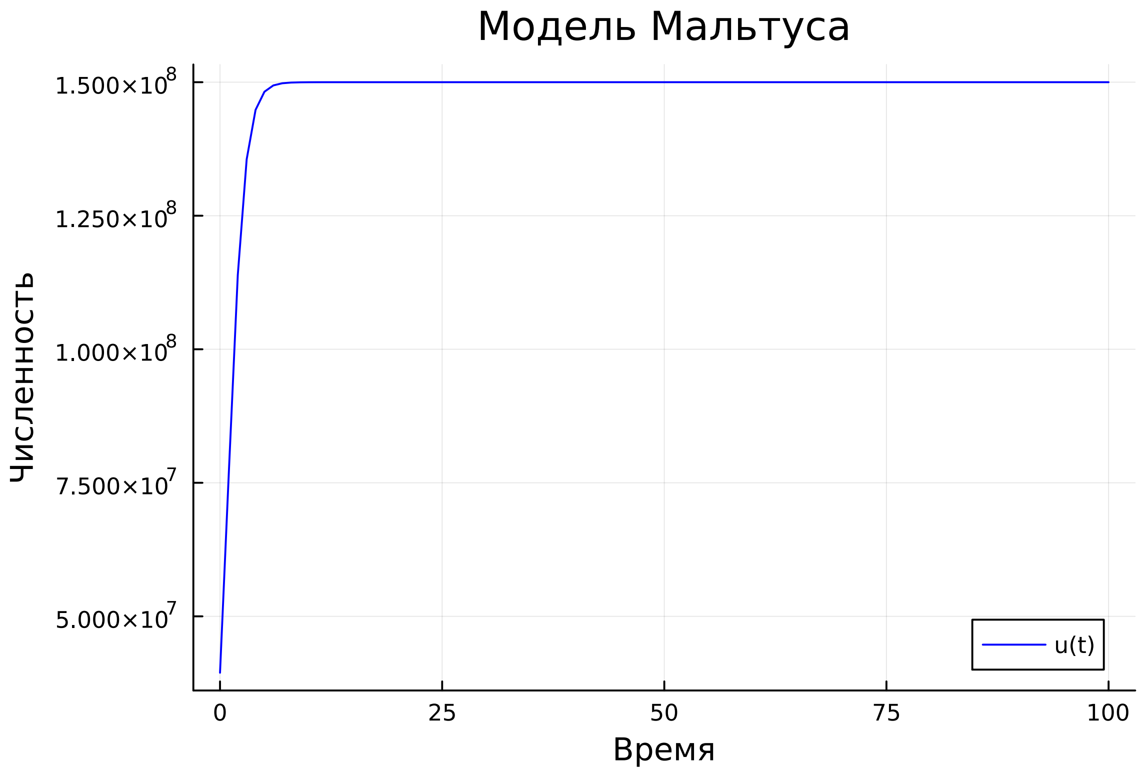
Out[8]:



Логистическая модель роста популяции

In [9]:
```
function LogModPop!(du,u,p,t)
    du[1] = p[1]*u[1]*(1-u[1]/p[2])
end
u0 = [39433556.0]
tspan = (0.0,100.0)
p = Float64[1.09, 15e7]
prob = ODEProblem(LogModPop!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=1.0)
R1 = [tu[1] for tu in sol.u]
plot(sol.t, R1, title="Логистическая модель роста популяции", xaxis="Время",yaxis="
```

Out[9]:



In [10]:

```
anim = @animate for i in 1:length(sol.t)
    plot(sol.t[1:i], R1[1:i], title="Логистическая модель роста популяции", xaxis="
end
gif(anim, "presentation//image//2.gif")
```

```
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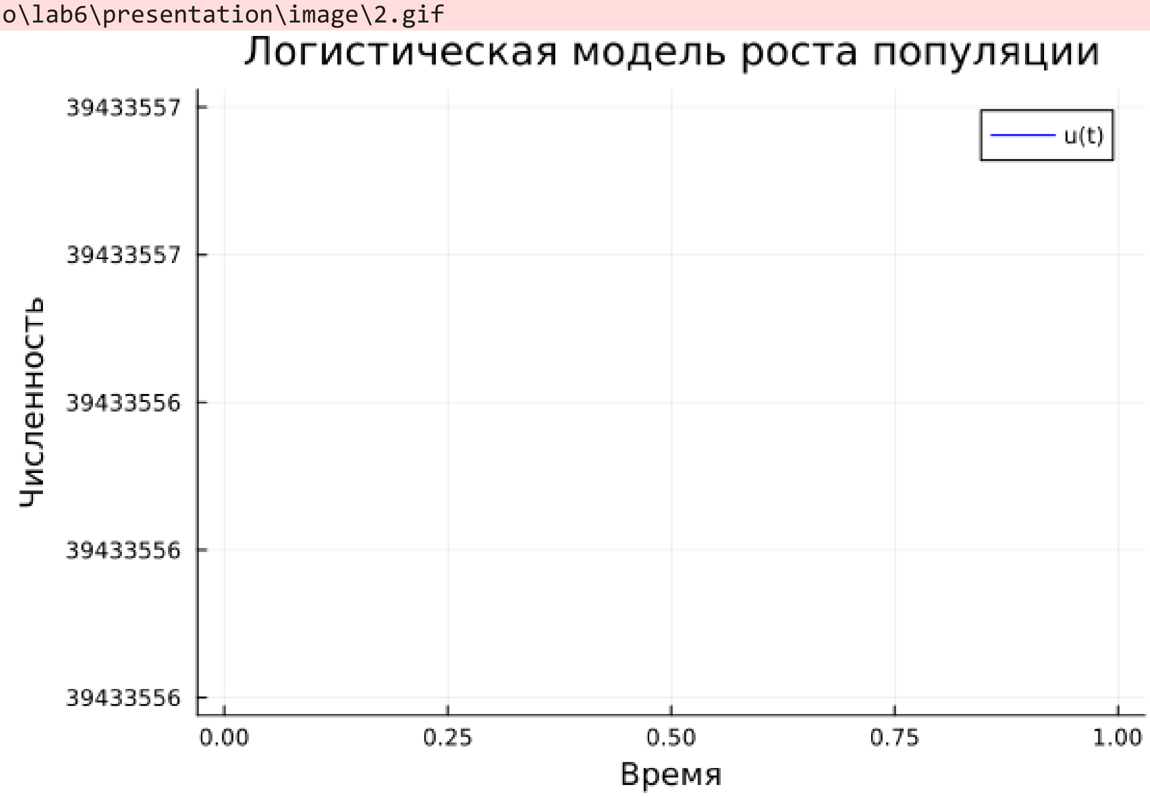
o\lab6\presentation\image\2.gif
```

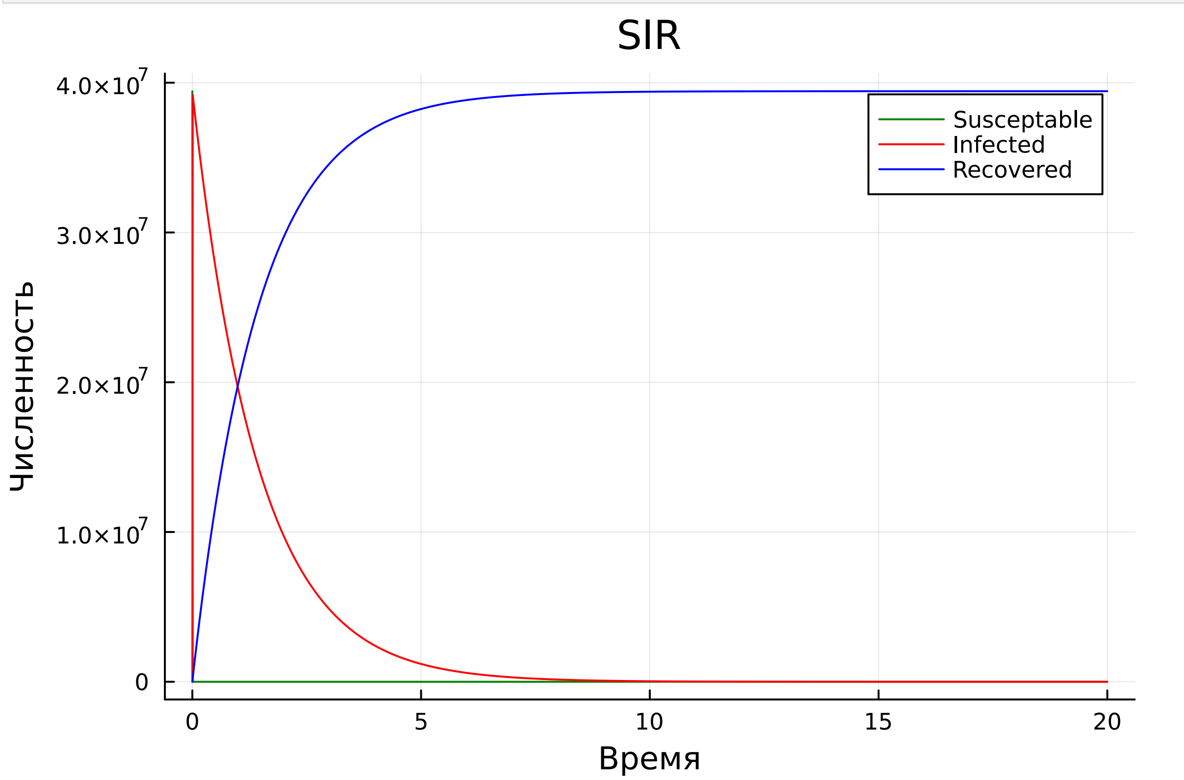
Out[10]:



SIR

In [11]:
```
function SIR!(du,u,p,t)
    du[1] = -p[1]*u[1]*u[2] # S
    du[2] = p[1]*u[2]*u[1]-p[2]*u[2] # I
    du[3] = p[2]*u[2] # R
end
u0 = [39433553.0, 3.0, 0.0]
tspan = (0.0,20.0)
p = Float64[0.3,0.7]
prob = ODEProblem(SIR!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=0.01)
R1 = [tu[1] for tu in sol.u]
R2 = [tu[2] for tu in sol.u]
R3 = [tu[3] for tu in sol.u]
plot(sol.t, R1, title="SIR", xaxis="Время",yaxis="Численность",label="Susceptable",
plot!(sol.t, R2, title="SIR", label="Infected", c=:red, leg=:topright)
plot!(sol.t, R3, label="Recovered", c=:blue, leg=:topright)
```

Out[11]:



In [12]:
```
anim = @animate for i in 1:length(sol.t)
    plot(sol.t[1:i], R1[1:i], title="SIR", xaxis="Время",yaxis="Численность",label=
    plot!(sol.t[1:i], R2[1:i], title="SIR", label="Infected", c=:red, leg=:topright
    plot!(sol.t[1:i], R3[1:i], label="Recovered", c=:blue, leg=:topright)
end
gif(anim, "presentation//image//3.gif")
```

```
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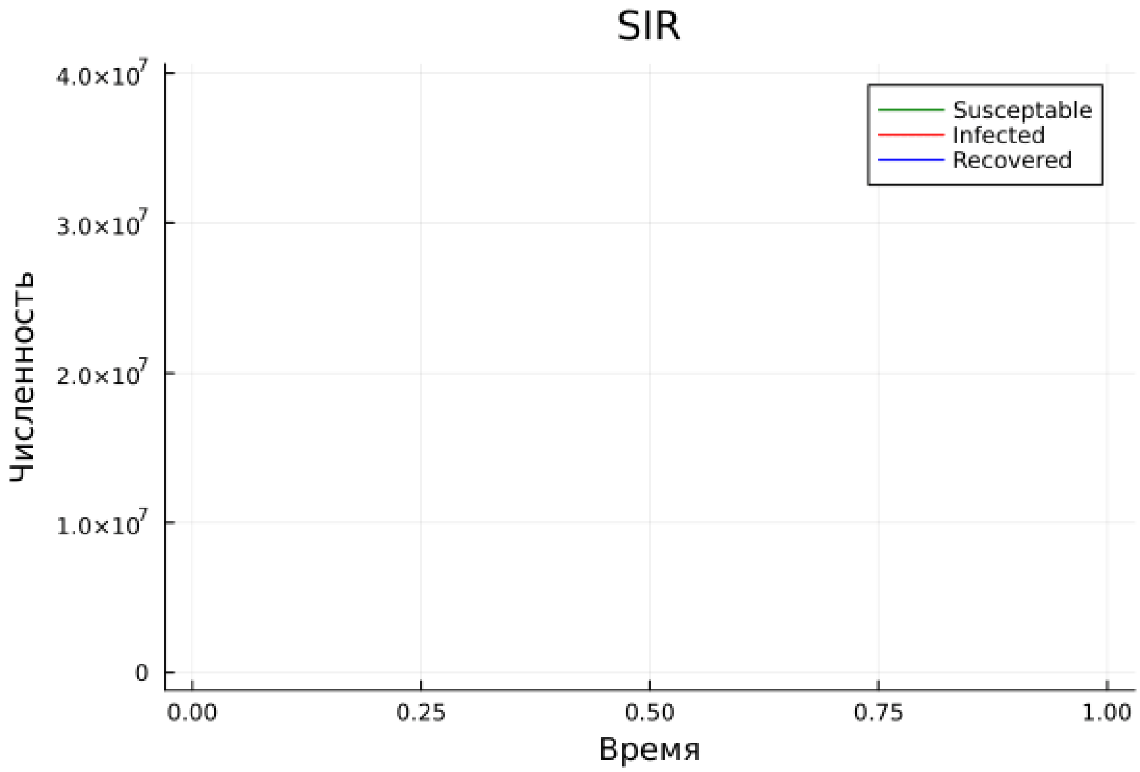
o\lab6\presentation\image\3.gif
```

Out[12]:

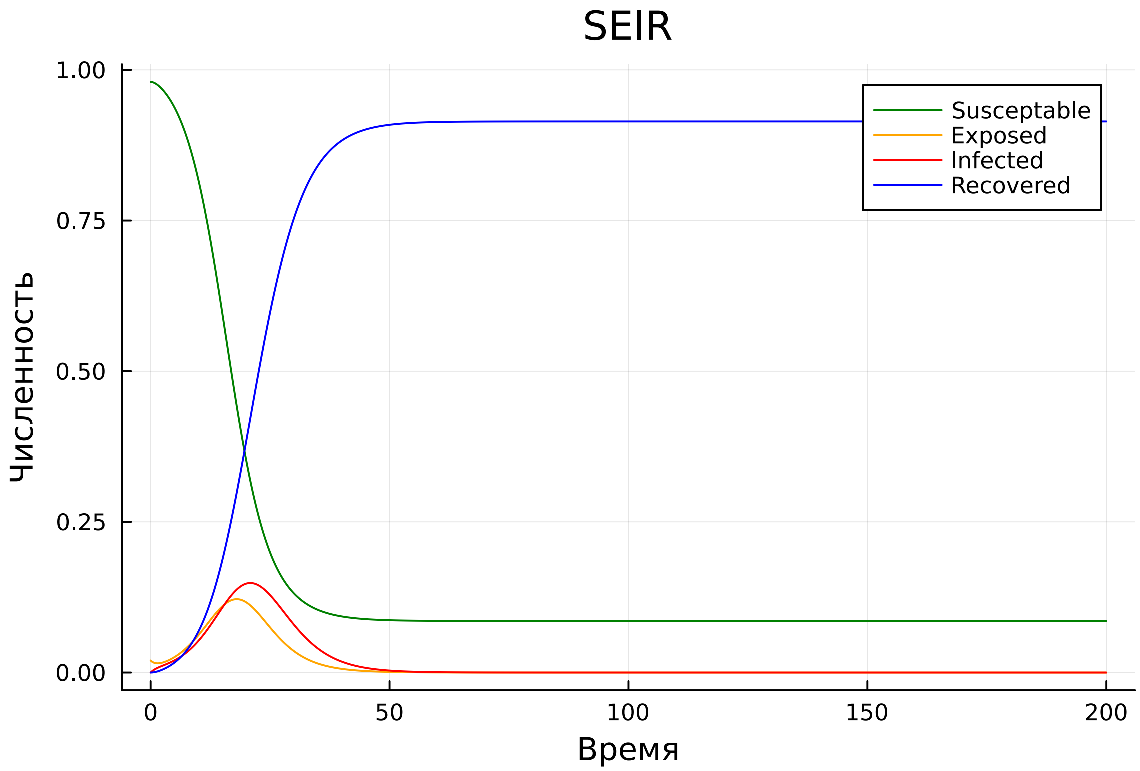


SEIR

In [13]:

```
function SEIR!(du,u,p,t)
    betta, delta, gamma, N = p
    s, e, i, r = u
    du[1] = -betta / N * s * i
    du[2] = betta / N * s * i - delta * e
    du[3] = delta * e - gamma * i
    du[4] = gamma * i
end
u0 = [0.98, 0.02, 0.0, 0.0]
tspan = (0.0,200.0)
p = Float64[0.8,0.4,0.3,1.0]
prob = ODEProblem(SEIR!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=0.1)
R1 = [tu[1] for tu in sol.u]
R2 = [tu[2] for tu in sol.u]
R3 = [tu[3] for tu in sol.u]
R4 = [tu[4] for tu in sol.u]
plot(sol.t, R1, title="SEIR", xaxis="Время",yaxis="Численность",label="Susceptable"
plot!(sol.t, R2, label="Exposed", c=:orange, leg=:topright)
plot!(sol.t, R3, label="Infected", c=:red, leg=:topright)
plot!(sol.t, R4, label="Recovered", c=:blue, leg=:topright)
```

Out[13]:



In [14]:

```
anim = @animate for i in 1:length(sol.t)
    plot(sol.t[1:i], R1[1:i], title="SEIR", xaxis="Время",yaxis="Численность",label
    plot!(sol.t[1:i], R2[1:i], label="Exposed", c=:orange, leg=:topright)
    plot!(sol.t[1:i], R3[1:i], label="Infected", c=:red, leg=:topright)
    plot!(sol.t[1:i], R4[1:i], label="Recovered", c=:blue, leg=:topright)
end
gif(anim, "presentation//image//4.gif")
```

```
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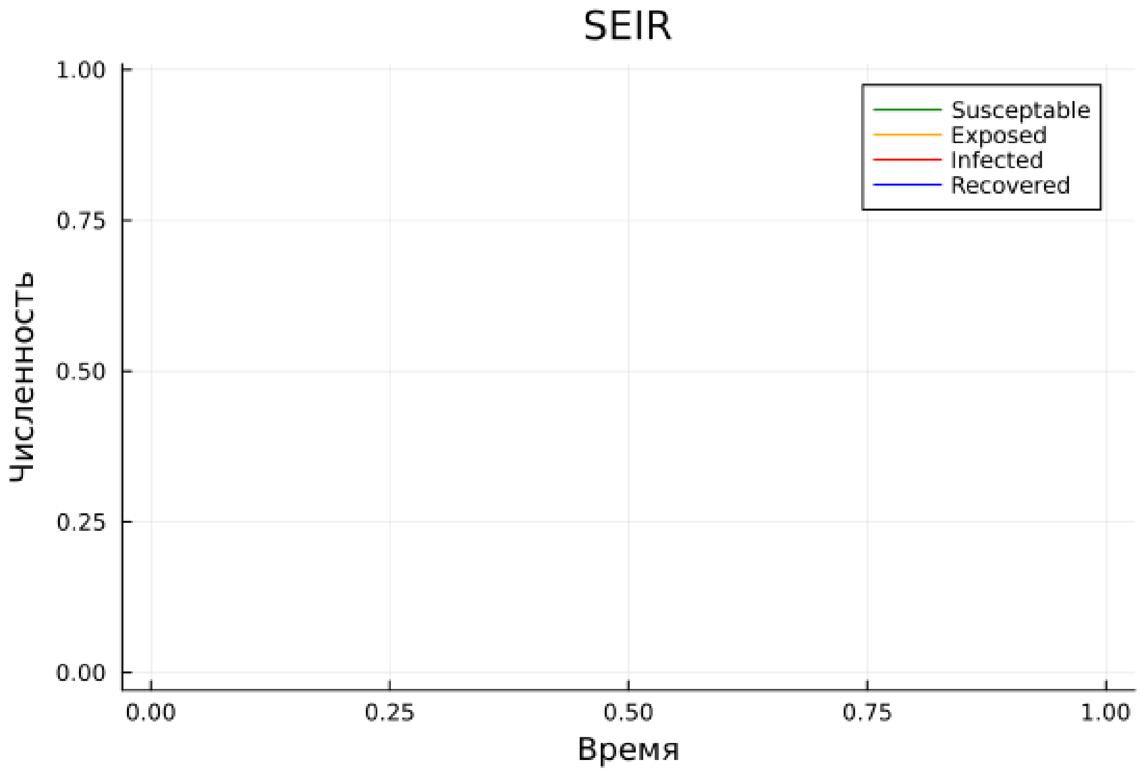
o\lab6\presentation\image\4.gif
```

Out[14]:



Лотки-Вольтерры

In [15]:

```
using NLsolve
# Аналитическое решение
function find_equilibrium(a, c, d)
    function system!(du, u)
        du[1] = a*u[1]*(1-u[1]) - u[1]*u[2]
        du[2] = -c*u[2] + d*u[1]*u[2]
    end

    initial_guess = [0.5, 0.5]
    result = nlsolve(system!, initial_guess)

    equilibrium_point = result.zero
    return equilibrium_point
end
# Численное решение
function LotkiVolterry(a, c, d, x1_0, x2_0, dt, num_steps)
    x1 = x1_0
    x2 = x2_0
    results = [(x1, x2)]

    for _ in 1:num_steps
        x1_new = x1 + dt * (a * x1 * (1 - x1) - x1 * x2)
        x2_new = x2 + dt * (-c * x2 + d * x1 * x2)
        x1, x2 = x1_new, x2_new
        push!(results, (x1, x2))
    end

    return results
end
```

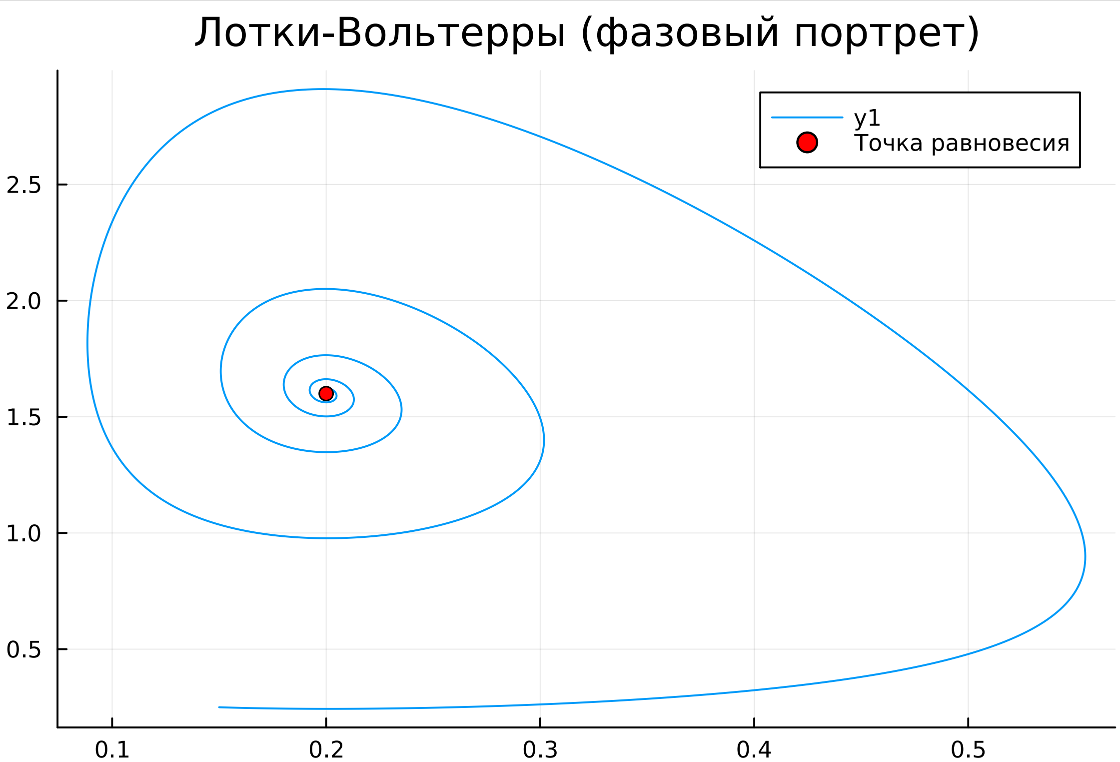
```julia
a = 2.0
c = 1.0
d = 5.0
x1_0 = 0.15
x2_0 = 0.25
dt = 0.01
num_steps = 10000

results = LotkiVolterry(a, c, d, x1_0, x2_0, dt, num_steps)
R1 = [x[1] for x in results]
R2 = [x[2] for x in results]

equilibrium = find_equilibrium(2,1,5)

plot(R1, R2, title="Лотки-Вольтерры (фазовый портрет)", leg=:topright)
scatter!([equilibrium[1]], [equilibrium[2]], color="red", label="Точка равновесия")
```

Out[15]:



In [16]:

```julia
anim = @animate for i in 1:length(R1)
    plot(R1[1:i], R2[1:i], title="Лотки-Вольтерры (фазовый портрет)", leg=:topright
    scatter!([equilibrium[1]], [equilibrium[2]], color="red", label="Точка равновес
end
gif(anim, "presentation//image//5.gif")
```

```
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o\lab6\presentation\image\5.gif
```
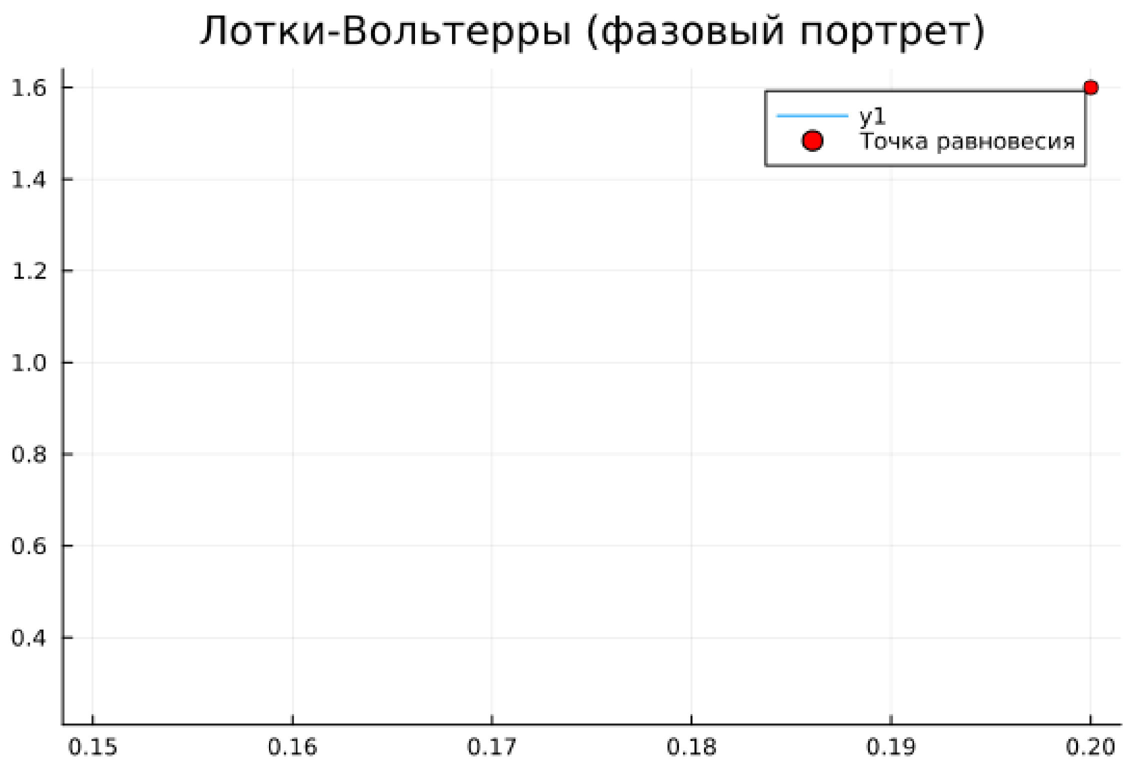

Out[16]:

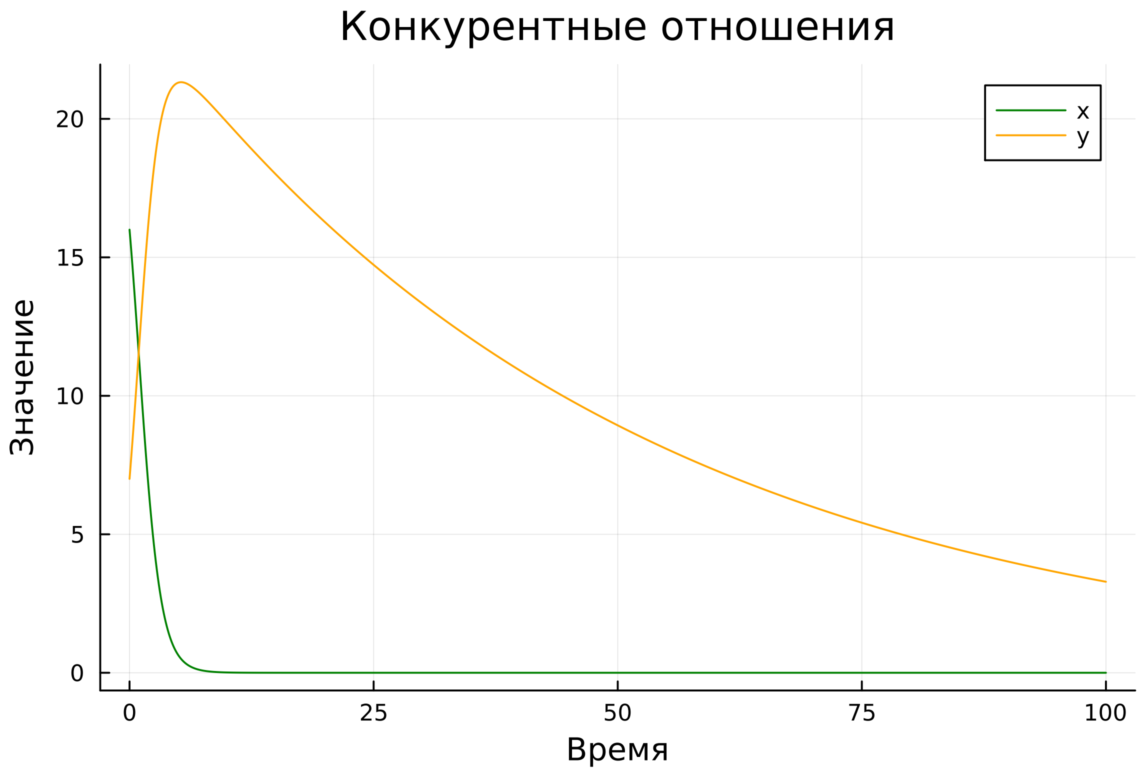


Конкурентные отношения

In [2]:

```
function KonkOtn!(du,u,p,t)
    du[1] = p[1] * u[1] - p[2] * u[1] * u[2]
    du[2] = -p[1] * u[2] + p[2] * u[1] * u[2]
end
u0 = [16.0,7.0]
tspan = (0.0,100.0)
p = Float64[0.02, 0.04]
prob = ODEProblem(KonkOtn!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=0.1)
R1 = [tu[1] for tu in sol.u]
R2 = [tu[2] for tu in sol.u]
plot(sol.t, R1, title="Конкурентные отношения", xaxis="Время",yaxis="Значение",labe
plot!(sol.t, R2, label="y", c=:orange, leg=:topright)
```

Out[2]:



In [3]:

```
anim = @animate for i in 1:length(R1)
    plot(sol.t[1:i], R1[1:i], title="Конкурентные отношения", xaxis="Время",yaxis="
    plot!(sol.t[1:i], R2[1:i], label="y", c=:orange, leg=:topright)
end
gif(anim, "presentation//image//6.gif")
```

```
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o\lab6\presentation\image\6.gif
```
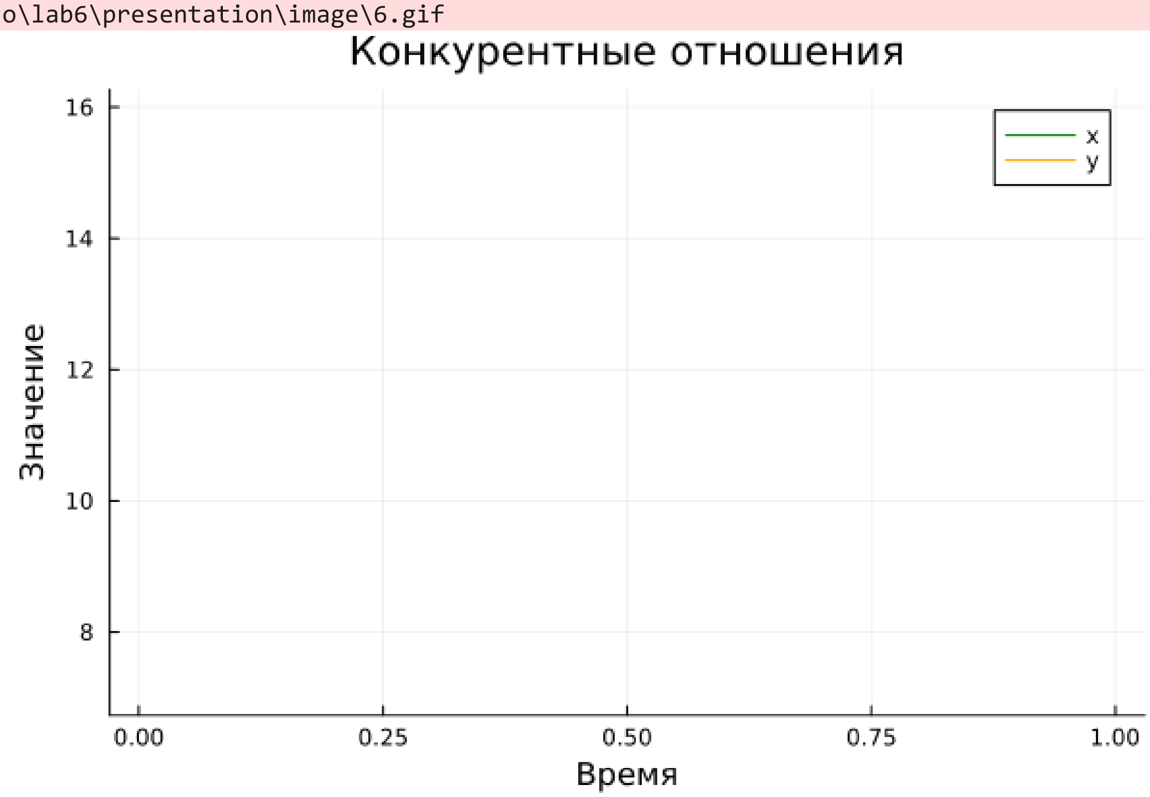
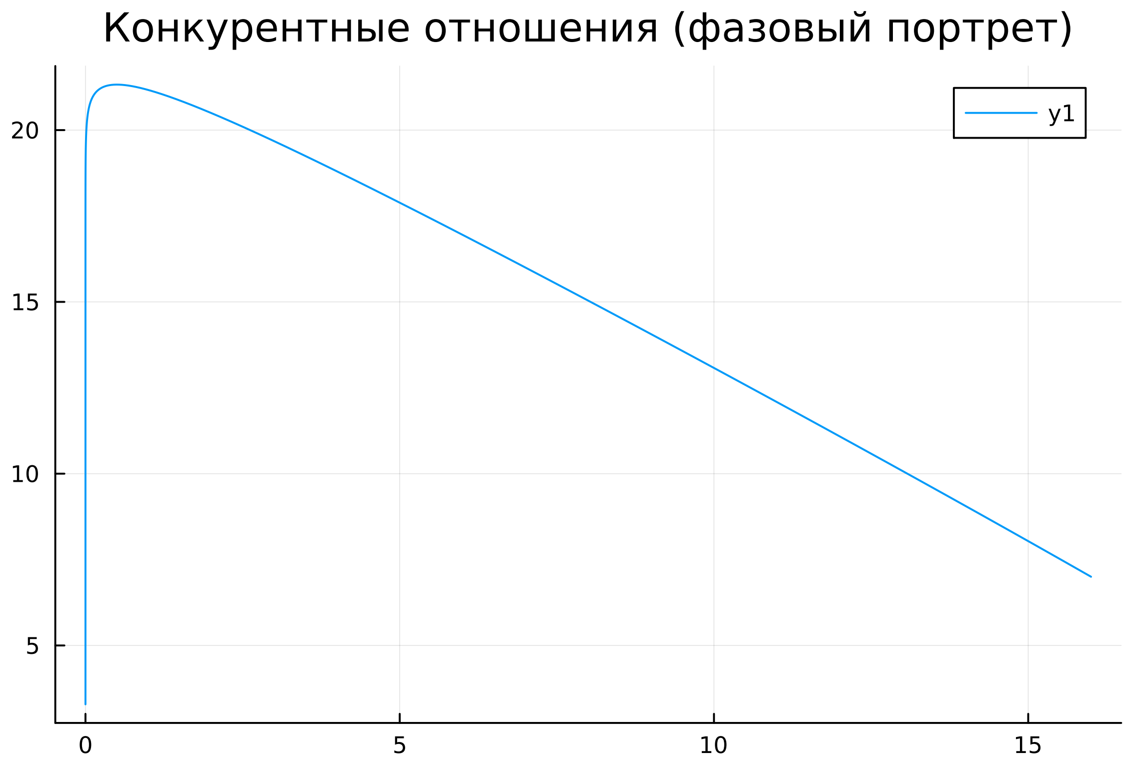

Out[3]:



In [4]:
```
plot(R1, R2, title="Конкурентные отношения (фазовый портрет)", leg=:topright)
```

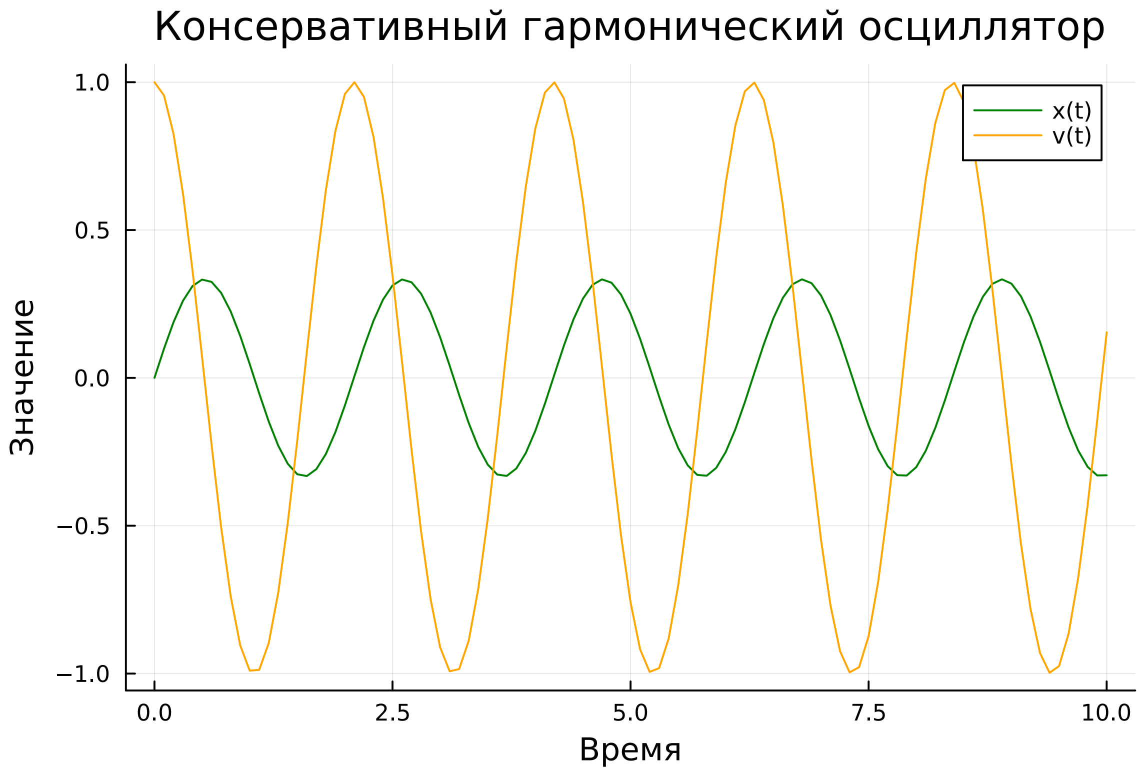
Out[4]:



Консервативный гармонический осциллятор

In [22]:
```
function KGO!(du,u,p,t)
    du[1] = u[2]
    du[2] = -p[1]^2 * u[1]
end
u0 = [0.0, 1.0]
tspan = (0.0,10.0)
p = Float64[3.0]
prob = ODEProblem(KGO!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=0.1)
R1 = [tu[1] for tu in sol.u]
R2 = [tu[2] for tu in sol.u]
plot(sol.t, R1, title="Консервативный гармонический осциллятор", xaxis="Время",yaxi
plot!(sol.t, R2, label="v(t)", c=:orange, leg=:topright)
```

Out[22]:



In [23]:

```
anim = @animate for i in 1:length(R1)
    plot(sol.t[1:i], R1[1:i], title="Консервативный гармонический осциллятор", xaxi
    plot!(sol.t[1:i], R2[1:i], label="v(t)", c=:orange, leg=:topright)
end
gif(anim, "presentation//image//7.gif")
```

```
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o\lab6\presentation\image\7.gif
```
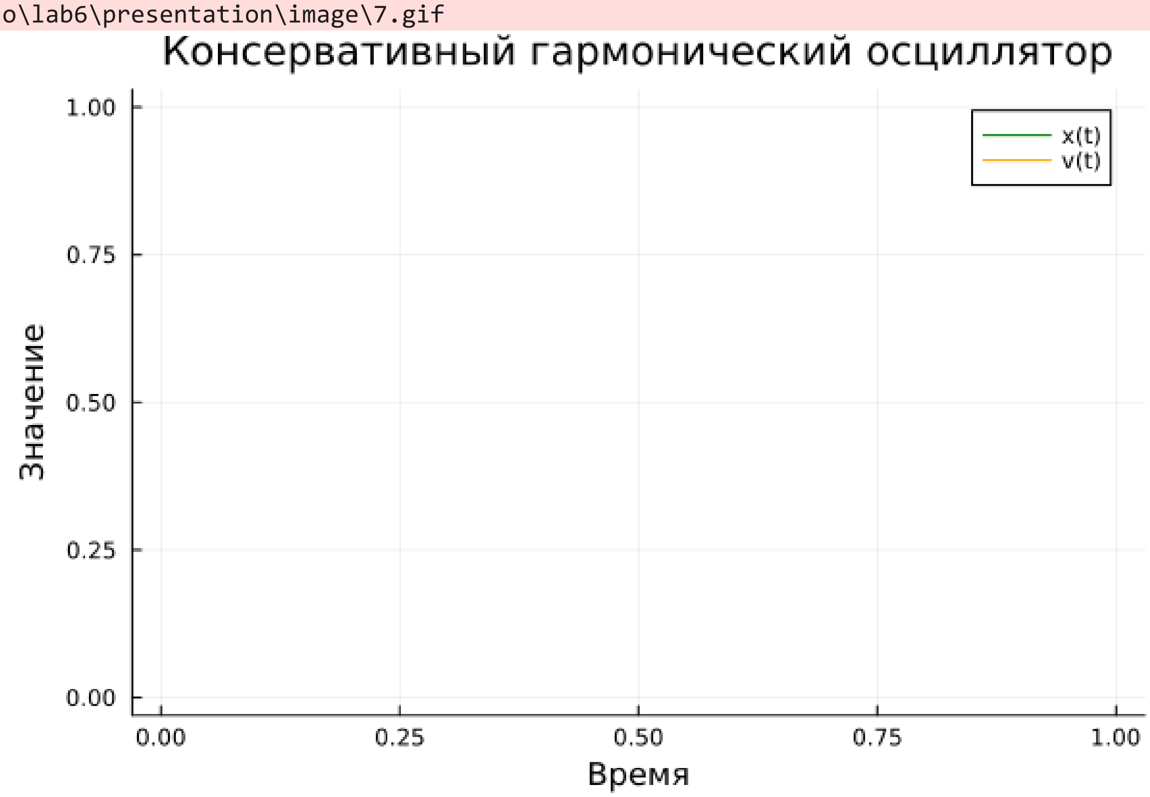
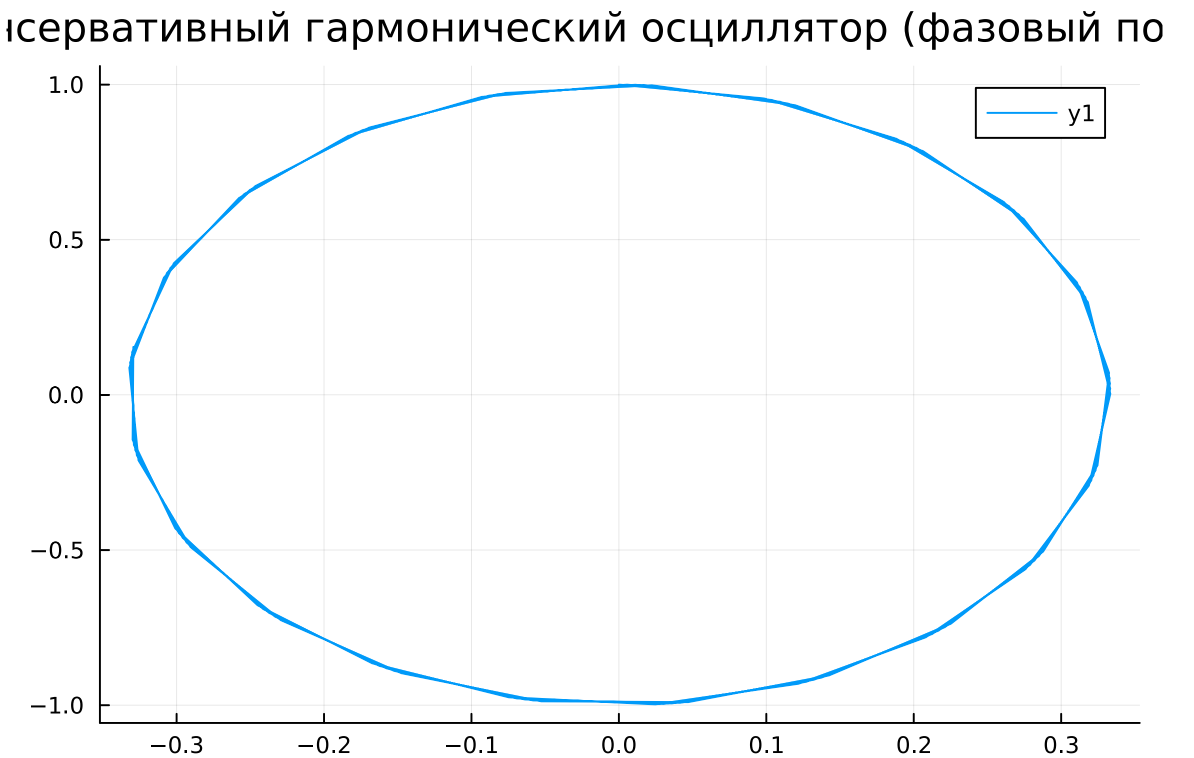

Out[23]:



In [24]:
```
plot(R1, R2, title="Консервативный гармонический осциллятор (фазовый портрет)", leg
```

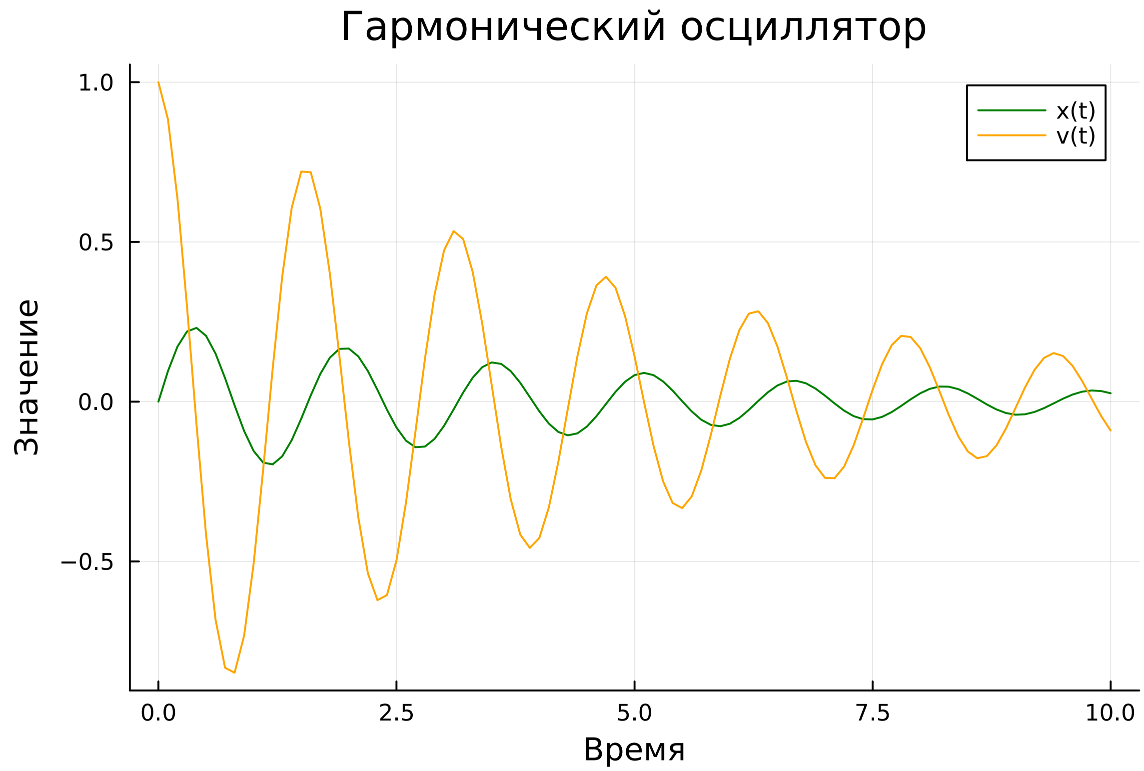
Out[24]:



Гармонический осциллятор

In [30]:
```
function GO!(du,u,p,t)
    du[1] = u[2]
    du[2] = -2.0*p[2]*u[2] - p[1]^2*u[1]
end
u0 = [0.0, 1.0]
tspan = (0.0,10.0)
p = Float64[4.0, 0.2]
prob = ODEProblem(GO!,u0,tspan,p)
sol = solve(prob,abstol=1e-6,reltol=1e-6, saveat=0.1)
R1 = [tu[1] for tu in sol.u]
R2 = [tu[2] for tu in sol.u]
plot(sol.t, R1, title="Гармонический осциллятор", xaxis="Время",yaxis="Значение",la
plot!(sol.t, R2, label="v(t)", c=:orange, leg=:topright)
```

Out[30]:



In [33]:

```
anim = @animate for i in 1:length(R1)
    plot(sol.t[1:i], R1[1:i], title="Гармонический осциллятор", xaxis="Время",yaxis
    plot!(sol.t[1:i], R2[1:i], label="v(t)", c=:orange, leg=:topright)
end
gif(anim, "presentation//image//8.gif")
```

```
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o\lab6\presentation\image\8.gif
```
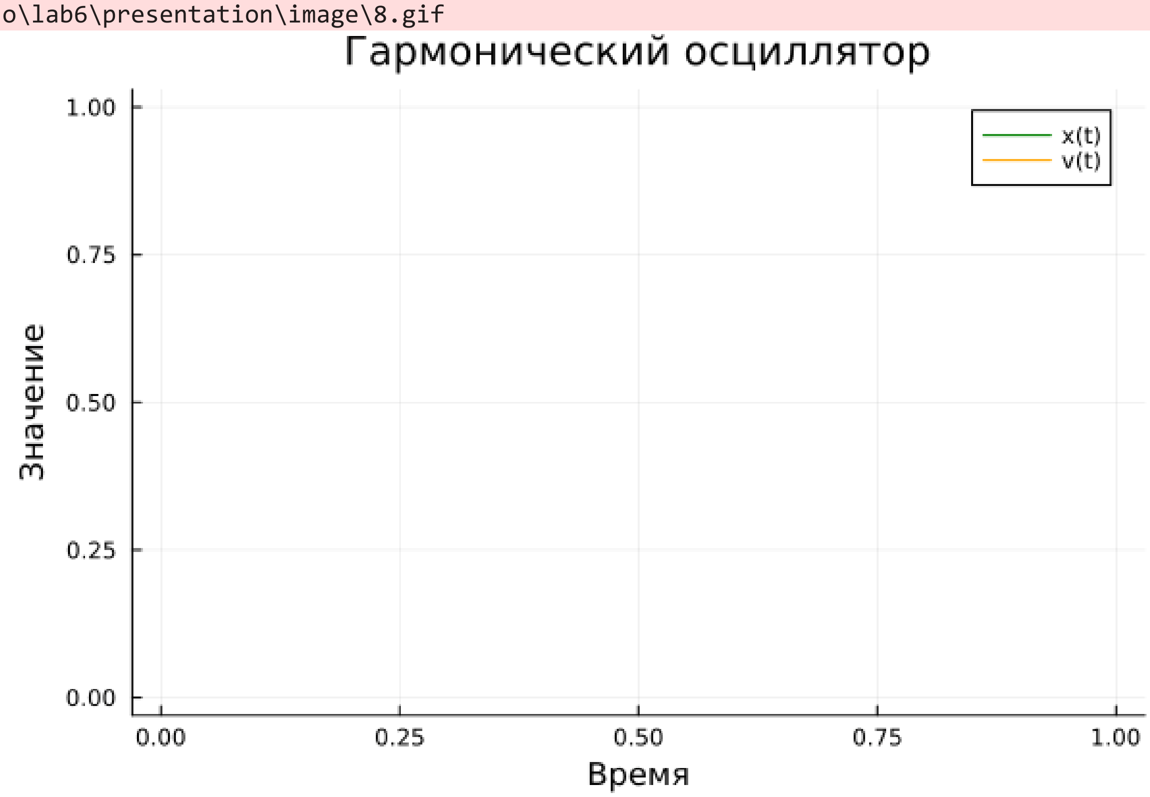

Out[33]:



In [32]:

```
plot(R1, R2, title="Гармонический осциллятор (фазовый портрет)", leg=:topright)
```

Out[32]: