Лабораторная работа №8

Дисциплина: Компьютерный практикум по статистическому анализу данных

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Цели и задачи работы

Цель лабораторной работы

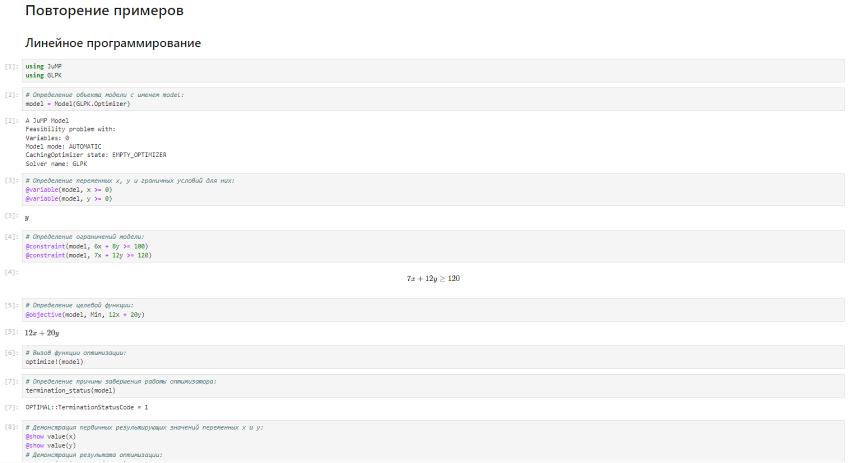
Освоить пакеты Julia для решения задач оптимизации.

Задачи

1. Повторить примеры из раздела 8.2
2. Выполнить задания для самостоятельной работы из раздела 8.4

Выполнение лабораторной работы

Повторение примеров



Повторение примеров

Линейное программирование

```
[1]: using JuMP
     using GLPK

[2]: # Определение объекта модели с именем model:
     model = Model(GLPK.Optimizer)

[2]: A JuMP Model
     Feasibility problem with:
     Variables: 0
     Model mode: AUTOMATIC
     CachingOptimizer state: EMPTY_OPTIMIZER
     Solver name: GLPK

[3]: # Определение переменных x, y и граничных условий для них:
     @variable(model, x >= 0)
     @variable(model, y >= 0)
```

[3]: y

```
[4]: # Определение ограничений модели:
     @constraint(model, 6x + 8y >= 100)
     @constraint(model, 7x + 12y >= 120)
```

[4]: $$7x + 12y \geq 120$$

```
[5]: # Определение целевой функции:
     @objective(model, Min, 12x + 20y)
```

[5]: $12x + 20y$

```
[6]: # Вызов функции оптимизации:
     optimize!(model)

[7]: # Определение причины завершения работы оптимизатора:
     termination_status(model)

[7]: OPTIMAL::TerminationStatusCode = 1

[8]: # Демонстрация первичных результирующих значений переменных x и y:
     @show value(x)
     @show value(y)
     # Демонстрация результата оптимизации:
```

Рис. 1: Повторение примеров (1)

```
value(x) = 14.999999999999993
value(y) = 1.2500000000000047
objective_value(model) = 205.0
```

[8]:
```
205.0
```

Векторизованные ограничения

[9]:
```
# Определение объекта модели с именем vector_model:
vector_model = Model(GLPK.Optimizer)
```

[9]:
```
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

[10]:
```
# Определение начальных данных:
A= [ 1 1 9 5; 3 5 0 8; 2 0 6 13]
b = [7; 3; 5]
c = [1; 3; 5; 2]
```

[10]:
```
4-element Vector{Int64}:
 1
 3
 5
 2
```

[11]:
```
# Определение вектора переменных:
@variable(vector_model, x[1:4] >= 0)
```

[11]:
```
4-element Vector{VariableRef}:
 x[1]
 x[2]
 x[3]
 x[4]
```

[12]:
```
# Определение ограничений модели:
@constraint(vector_model, A * x .== b)
```

[12]:
```
3-element Vector{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64}, MathOptInterface.EqualTo{Float64}}, ScalarShape}}:
 x[1] + x[2] + 9 x[3] + 5 x[4] == 7
 3 x[1] + 5 x[2] + 8 x[4] == 3
 2 x[1] + 6 x[3] + 13 x[4] == 5
```

[13]:
```
# Определение целевой функции:
@objective(vector_model, Min, c' * x)
```

[13]: $x_1 + 3x_2 + 5x_3 + 2x_4$

[14]:
```
# Вызов функции оптимизации:
optimize!(vector_model)
```

Рис. 2: Повторение примеров (2)

Повторение примеров (3)

```
[15]: # Определение причины завершения работы оптимизатора:
termination_status(vector_model)
```

```
[15]: OPTIMAL::TerminationStatusCode = 1
```

```
[16]: # Демонстрация результата оптимизации:
@show objective_value(vector_model)
```

```
objective_value(vector_model) = 4.9230769230769225
[16]: 4.9230769230769225
```

Оптимизация рациона

```
[17]: category_data = JuMP.Containers.DenseAxisArray(
    [1800 2200;
    91 Inf;
    0 65;
    0 1779],
    ["calories", "protein", "fat", "sodium"],
    ["min", "max"])
```

```
[17]: 2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
    Dimension 2, ["min", "max"]
And data, a 4×2 Matrix{Float64}:
 1800.0  2200.0
   91.0    Inf
    0.0    65.0
    0.0  1779.0
```

```
[18]: # массив данных с наименованиями продуктов:
foods = ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza","salad", "milk", "ice cream"]
```

```
[18]: 9-element Vector{String}:
 "hamburger"
 "chicken"
 "hot dog"
 "fries"
 "macaroni"
 "pizza"
 "salad"
 "milk"
 "ice cream"
```

```
[19]: # Массив стоимости продуктов:
cost = JuMP.Containers.DenseAxisArray(
[2.49, 2.89, 1.50, 1.89, 2.09, 1.99, 2.49, 0.89, 1.59],
foods)
```

```
[19]: 1-dimensional DenseAxisArray{Float64,1,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
And data, a 9-element Vector{Float64}:
 2.49
```

Рис. 3: Повторение примеров (3)

Повторение примеров (4)

```
[19]: 1-dimensional DenseAxisArray{Float64,1,...} with index sets:
          Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
      And data, a 9-element Vector{Float64}:
       2.49
       2.89
       1.5
       1.89
       2.09
       1.99
       2.49
       0.89
       1.59
```

```
[20]: food_data = JuMP.Containers.DenseAxisArray(
          [410 24 26 730;
          420 32 10 1190;
          560 20 32 1800;
          380 4 19 270;
          320 12 10 930;
          320 15 12 820;
          320 31 12 1230;
          100 8 2.5 125;
          330 8 10 180],
          foods,
          ["calories", "protein", "fat", "sodium"])
```

```
[20]: 2-dimensional DenseAxisArray{Float64,2,...} with index sets:
          Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
          Dimension 2, ["calories", "protein", "fat", "sodium"]
      And data, a 9×4 Matrix{Float64}:
       410.0  24.0  26.0   730.0
       420.0  32.0  10.0  1190.0
       560.0  20.0  32.0  1800.0
       380.0   4.0  19.0   270.0
       320.0  12.0  10.0   930.0
       320.0  15.0  12.0   820.0
       320.0  31.0  12.0  1230.0
       100.0   8.0   2.5   125.0
       330.0   8.0  10.0   180.0
```

```
[21]: # Определение объекта модели с именем model:
      model_calories = Model(GLPK.Optimizer)
```

```
[21]: A JuMP Model
      Feasibility problem with:
      Variables: 0
      Model mode: AUTOMATIC
      CachingOptimizer state: EMPTY_OPTIMIZER
      Solver name: GLPK
```

```
[22]: # Определим массив:
      categories = ["calories", "protein", "fat", "sodium"]
```

Рис. 4: Повторение примеров (4)

Повторение примеров (5)

```
[23]: # Определение переменных:
@variables(model_calories, begin
    category_data[c, "min"] <= nutrition[c = categories] <= category_data[c, "max"]
    # Сколько покупать продуктов:
    buy[foods] >= 0
    end)
```

```
[23]: (1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
And data, a 4-element Vector{VariableRef}:
 nutrition[calories]
 nutrition[protein]
 nutrition[fat]
 nutrition[sodium], 1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
And data, a 9-element Vector{VariableRef}:
 buy[hamburger]
 buy[chicken]
 buy[hot dog]
 buy[fries]
 buy[macaroni]
 buy[pizza]
 buy[salad]
 buy[milk]
 buy[ice cream])
```

```
[24]: # Определение целевой функции:
@objective(model_calories, Min, sum(cost[f] * buy[f] for f in foods))
```

[24]: $2.49buy_{hamburger} + 2.89buy_{chicken} + 1.5buy_{hotdog} + 1.89buy_{fries} + 2.09buy_{macaroni} + 1.99buy_{pizza} + 2.49buy_{salad} + 0.89buy_{milk} + 1.59buy_{icecream}$

```
[25]: # Определение ограничений модели:
@constraint(model_calories, [c in categories],
sum(food_data[f, c] * buy[f] for f in foods) == nutrition[c])
```

```
[25]: 1-dimensional DenseAxisArray{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64}, MathOptInterface.EqualTo{Float64}}, ScalarShape},1,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
And data, a 4-element Vector{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64}, MathOptInterface.EqualTo{Float64}}, ScalarShape}}:
 -nutrition[calories] + 410 buy[hamburger] + 420 buy[chicken] + 560 buy[hot dog] + 380 buy[fries] + 320 buy[macaroni] + 320 buy[pizza] + 320 buy[salad] + 100 buy[milk] + 330 buy[ice cream] == 0
 -nutrition[protein] + 24 buy[hamburger] + 32 buy[chicken] + 20 buy[hot dog] + 4 buy[fries] + 12 buy[macaroni] + 15 buy[pizza] + 31 buy[salad] + 8 buy[milk] + 8 buy[ice cream] == 0
 -nutrition[fat] + 26 buy[hamburger] + 10 buy[chicken] + 32 buy[hot dog] + 19 buy[fries] + 10 buy[macaroni] + 12 buy[pizza] + 12 buy[salad] + 2.5 buy[milk] + 10 buy[ice cream] == 0
 -nutrition[sodium] + 730 buy[hamburger] + 1190 buy[chicken] + 1800 buy[hot dog] + 270 buy[fries] + 930 buy[macaroni] + 820 buy[pizza] + 1230 buy[salad] + 125 buy[milk] + 180 buy[ice cream] == 0
```

```
[26]: # Вызов функции оптимизации:
JuMP.optimize!(model_calories)
term_status = JuMP.termination_status(model_calories)
```

```
[26]: OPTIMAL::TerminationStatusCode = 1
```

```
[27]: hcat(buy.data,JuMP.value.(buy.data))
```

```
[27]: 9×2 Matrix{AffExpr}:
 buy[hamburger]  0.6045138888888888
 buy[chicken]    0
```

Рис. 5: Повторение примеров (5)

Путешествие по миру

```
[2]: using DelimitedFiles
     using CSV
```

```
[29]: passportdata = readdlm("data/passport-index-matrix.csv",',')
```

```
[29]: 200×200 Matrix{Any}:
 "Passport"                "Albania"  …   "Afghanistan"
 "Afghanistan"             "e-visa"     -1
 "Albania"               -1               "visa required"
 "Algeria"                 "e-visa"       "visa required"
 "Andorra"               90               "visa required"
 "Angola"                  "e-visa"   …   "visa required"
 "Antigua and Barbuda"   90               "visa required"
 "Argentina"             90               "visa required"
 "Armenia"               90               "visa required"
 "Australia"             90               "visa required"
 "Austria"               90           …   "visa required"
 "Azerbaijan"            90               "visa required"
 "Bahamas"               90               "visa required"
 ⋮                                    ⋱
 "United Arab Emirates"  90               "visa required"
 "United Kingdom"        90               "visa required"
 "United States"        360           …   "visa required"
 "Uruguay"               90               "visa required"
 "Uzbekistan"              "e-visa"       "visa required"
 "Vanuatu"                 "e-visa"       "visa required"
 "Vatican"               90               "visa required"
 "Venezuela"             90           …   "visa required"
 "Vietnam"                 "e-visa"       "visa required"
 "Yemen"                   "e-visa"       "visa required"
 "Zambia"                  "e-visa"       "visa required"
 "Zimbabwe"                "e-visa"       "visa required"
```

```
[30]: # Задаём переменные:
      cntr = passportdata[2:end,1]
      vf = (x -> typeof(x)==Int64 || x == "VF" || x == "VOA" ? 1 : 0).(passportdata[2:end,2:end])
```

```
[30]: 199×199 Matrix{Int64}:
 0  0  0  0  0  0  0  0  0  0  0  0  0  …  0  0  0  0  0  0  0  0  0  0  0  1
 1  0  1  0  1  0  1  0  1  0  0  0  0     0  0  0  0  0  1  0  0  0  0  0  0
 0  1  0  1  0  0  0  0  0  0  0  0  0     0  0  0  0  0  0  0  0  0  0  0  0
 1  0  1  0  1  1  1  0  1  0  1  0  0     1  0  1  1  1  1  1  0  0  0  0  0
 0  0  0  1  0  0  0  0  0  0  0  0  0     0  0  0  0  0  0  0  0  0  1  1  0
 1  0  1  1  1  0  0  0  1  0  1  0  0  …  1  0  0  1  1  1  1  0  0  1  1  0
 1  0  1  1  1  1  0  0  1  0  1  0  0     1  0  1  1  1  1  1  0  0  0  0  0
 1  0  0  0  1  1  1  0  0  0  1  0  0     0  0  1  0  0  0  0  0  0  0  0  0
 1  0  1  1  1  1  1  1  1  0  1  0  0     1  0  1  1  1  1  1  0  0  1  0  0
 1  0  0  1  1  1  1  0  1  0  1  0  0     1  0  1  1  1  0  1  0  0  1  0  0
 1  0  0  0  1  0  0  0  0  1  1  0  0  …  0  0  0  0  1  0  0  0  0  0  0  0
 1  0  1  1  1  0  0  0  1  0  1  0  0     1  0  1  1  1  1  0  0  0  1  1  0
 1  0  0  0  0  0  0  0  0  0  1  1  0     0  0  0  1  1  0  0  0  0  1  0  0
```

Рис. 6: Повторение примеров (6)

```
[31]: # Определение объекта модели с именем model:
      model_passports = Model(GLPK.Optimizer)

[31]: A JuMP Model
      Feasibility problem with:
      Variables: 0
      Model mode: AUTOMATIC
      CachingOptimizer state: EMPTY_OPTIMIZER
      Solver name: GLPK

[32]: # Переменные, ограничения и целевая функция:
      @variable(model_passports, pass[1:length(cntr)], Bin)
      @constraint(model_passports, [j=1:length(cntr)], sum( vf[i,j]*pass[i] for i in 1:length(cntr)) >= 1)
      @objective(model_passports, Min, sum(pass))
```

[32]: $pass_1 + pass_2 + pass_3 + pass_4 + pass_5 + pass_6 + pass_7 + pass_8 + pass_9 + pass_{10} + pass_{11} + pass_{12} + pass_{13} + pass_{14} + pass_{15} + pass_{16} + pass_{17} + pass_{18} + pass_{19} + pass_{20} + pass_{21} + pass_{22} + pass_{23} + pass_{24} + pass_{25} + pass_{26} + pass_{27} + p$

```
[33]: # Вызов функции оптимизации:
      JuMP.optimize!(model_passports)
      termination_status(model_passports)

[33]: OPTIMAL::TerminationStatusCode = 1

[34]: print(JuMP.objective_value(model_passports)," passports: ",join(cntr[findall(JuMP.value.(pass) .== 1)],", "))

      34.0 passports: Afghanistan, Australia, Bahrain, Cameroon, Canada, Comoros, Congo, Denmark, Djibouti, Eritrea, Guinea-Bissau, Hong Kong, Iran, Kenya, Kuwait, Liberia, Libya, Madagascar, Maldives, Mauritania, Morocco, Nauru, Ne
      pal, New Zealand, North Korea, Palestine, Papua New Guinea, Qatar, Saudi Arabia, Singapore, Somalia, Sri Lanka, Syria, Turkmenistan
```

Портфельные инвестиции

```
[3]: using DataFrames
     using XLSX
     using Plots
     pyplot()
     using Convex
     using SCS
     using Statistics

[36]: # Считываем данные и размещаем их во фрейм:
      T = DataFrame(XLSX.readtable("data/stock_prices.xlsx","Sheet2"))
```

[36]: 13×3 DataFrame

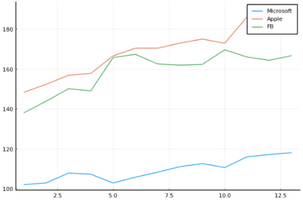
Row	MSFT	FB	AAPL
	Any	Any	Any
1	101.93	137.95	148.26
2	102.8	143.8	152.29

Рис. 7: Повторение примеров (7)

Повторение примеров (8)

```
[37]: # Построение графика:
plot(T[!,:MSFT],label="Microsoft")
plot!(T[!,:AAPL],label="Apple")
plot!(T[!,:FB],label="FB")
```

[37]:




```
[38]: # Данные о ценах на акции размещаем в матрице:
prices_matrix = Matrix(T)
```

```
[38]: 13×3 Matrix{Any}:
 101.93  137.95  148.26
 102.8   143.8   152.29
 107.71  150.04  156.82
 107.17  149.01  157.76
 102.78  165.71  166.52
 105.67  167.33  170.41
 108.22  162.5   170.42
 110.97  161.89  172.97
 112.53  162.28  174.97
 110.51  169.6   172.91
 115.91  165.98  186.12
 117.05  164.34  191.05
 117.94  166.69  189.95
```

```
[39]: # Вычисление матрицы доходности за период времени:
M1 = prices_matrix[1:end-1,:]
M2 = prices_matrix[2:end,:]
# Матрица доходности:
R = (M2.-M1)./M1
```

```
[39]: 12×3 Matrix{Float64}:
  0.00853527   0.0424067    0.027182
```

Рис. 8: Повторение примеров (8)

Повторение примеров (9)

```
[40]: # Матрица рисков:
      risk_matrix = cov(R)
      # Проверка положительной определённости матрицы рисков:
      isposdef(risk_matrix)

[40]: true

[41]: # Доход от каждой из компаний:
      r = mean(R,dims=1)[:]

[41]: 3-element Vector{Float64}:
       0.012532748705136572
       0.016563036855293173
       0.02114580465503291

[42]: # Вектор инвестиций:
      x = Variable(length(r))

[42]: Variable
      size: (3, 1)
      sign: real
      vexity: affine
      id: 122…222

[43]: # Объект модели:
      problem = minimize(Convex.quadform(x,risk_matrix),[sum(x)==1;r'*x>=0.02;x.>=0])

[43]: minimize
      └─ * (convex; positive)
         ├─ 1
         └─ qol_elem (convex; positive)
            ├─ norm2 (convex; positive)
            │  └─ …
            └─ [1.0;;]
      subject to
      ├─ == constraint (affine)
      │  ├─ sum (affine; real)
      │  │  └─ 3-element real variable (id: 122…222)
      │  └─ 1
      ├─ >= constraint (affine)
      │  ├─ * (affine; real)
      │  │  ├─ [0.0125327 0.016563 0.0211458]
      │  │  └─ 3-element real variable (id: 122…222)
      │  └─ 0.02
      ├─ >= constraint (affine)
      │  ├─ index (affine; real)
      │  │  └─ 3-element real variable (id: 122…222)
      │  └─ 0
      ├─ >= constraint (affine)
      │  ├─ index (affine; real)
      │  │  └─ 3-element real variable (id: 122…222)
      │  └─ 0
      └─ >= constraint (affine)
         ├─ index (affine; real)
```

Рис. 9: Повторение примеров (9)

Повторение примеров (10)

```
[44]: # Находим решение:
      solve!(problem, SCS.Optimizer)

------------------------------------------------------------------
               SCS v3.2.4 - Splitting Conic Solver
        (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 6, constraints m: 14
cones:    z: primal zero / dual free vars: 2
          l: linear vars: 5
          q: soc vars: 7, qsize: 2
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
          alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
          max_iters: 100000, normalize: 1, rho_x: 1.00e-006
          acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
          nnz(A): 24, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.71e+001 1.00e+000 1.62e+001 -8.03e+000 1.00e-001 1.78e-004
    75|8.16e-005 1.46e-004 5.60e-005 5.56e-004 1.00e-001 2.48e-004
------------------------------------------------------------------
status:  solved
timings: total: 2.50e-004s = setup: 1.25e-004s + solve: 1.25e-004s
         lin-sys: 3.27e-005s, cones: 2.31e-005s, accel: 4.50e-006s
------------------------------------------------------------------
objective = 0.000556
------------------------------------------------------------------

[45]: x

[45]: Variable
      size: (3, 1)
      sign: real
      vexity: affine
      id: 122…222
      value: [0.06922834751660403, 0.11730158220227511, 0.813469514654251]

[46]: sum(x.value)

[46]: 0.9999994443731302

[47]: r'*x.value

[47]: 1×1 adjoint(::Vector{Float64}) with eltype Float64:
       0.020011959361601172

[48]: x.value .* 1000

[48]: 3×1 Matrix{Float64}:
        69.22834751660403
       117.30158220227511
       813.469514654251
```

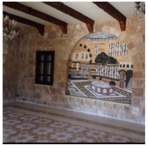
Рис. 10: Повторение примеров (10)

Восстановление изображения

[4]:
```
using Images
```

[50]:
```
# Считывание исходного изображения:
Kref = load("data/khiam-small.jpg")
```

[50]:



[51]:
```
K = copy(Kref)
p = prod(size(K))
missingids = rand(1:p,400)
```

[51]:
```
400-element Vector{Int64}:
 13926
 75313
 71284
 16985
 40139
 54158
 54979
 14764
 38463
 33118
 68673
 16737
 66617
     ⋮
 36699
 36677
  6733
 50628
  4083
 10212
 62751
 50227
 37820
```

Рис. 11: Повторение примеров (11)

Повторение примеров (12)

```
[52]: K[missingids] .= RGBX{N0f8}(0.0,0.0,0.0)
      K
      Gray.(K)
```

[52]:

```
[53]: # Матрица цветов:
      Y = Float64.(Gray.(K))
```

```
[53]: 283×283 Matrix{Float64}:
 0.101961   0.0627451  0.0784314  0.0941176  …  0.509804  0.552941  0.666667
 0.0666667  0.0980392  0.0745098  0.054902      0.505882  0.584314  0.501961
 0.0784314  0.0862745  0.0784314  0.0901961     0.6       0.701961  0.615686
 0.0862745  0.0666667  0.0745098  0.0941176     0.658824  0.705882  0.145098
 0.0784314  0.101961   0.0901961  0.0745098     0.713725  0.682353  0.231373
 0.0745098  0.0745098  0.0784314  0.0862745  …  0.729412  0.701961  0.168627
 0.12549    0.0980392  0.0862745  0.0862745     0.0       0.466667  0.192157
 0.439216   0.447059   0.305882   0.137255      0.231373  0.184314  0.137255
 0.458824   0.454902   0.45098    0.454902      0.196078  0.101961  0.117647
 0.45098    0.466667   0.458824   0.45098       0.584314  0.121569  0.137255
 0.458824   0.458824   0.458824   0.454902   …  0.521569  0.513725  0.12549
 0.466667   0.45098    0.458824   0.47451       0.576471  0.741176  0.117647
 0.45098    0.45098    0.462745   0.458824      0.560784  0.67451   0.117647
 ⋮                                           ⋱  ⋮
 0.494118   0.47451    0.47451    0.462745      0.427451  0.435294  0.443137
 0.47451    0.482353   0.470588   0.470588      0.439216  0.431373  0.431373
 0.494118   0.501961   0.470588   0.45098       0.447059  0.447059  0.45098
 0.470588   0.494118   0.490196   0.482353      0.431373  0.419608  0.419608
 0.458824   0.482353   0.47451    0.466667   …  0.454902  0.435294  0.423529
 0.443137   0.458824   0.45098    0.45098       0.309804  0.32549   0.34902
 0.47451    0.478431   0.462745   0.462745      0.341176  0.345098  0.360784
 0.482353   0.478431   0.458824   0.458824      0.423529  0.372549  0.321569
 0.552941   0.552941   0.541176   0.533333      0.447059  0.411765  0.372549
 0.552941   0.545098   0.576471   0.552941   …  0.435294  0.423529  0.407843
 0.564706   0.552941   0.54902    0.505882      0.439216  0.431373  0.419608
 0.568627   0.552941   0.517647   0.462745      0.439216  0.431373  0.427451
```

Рис. 12: Повторение примеров (12)

[54]:
```
correctids = findall(Y[:].!=0)
X = Convex.Variable(size(Y))
problem = minimize(nuclearnorm(X))
problem.constraints += X[correctids]==Y[correctids]
```

[54]:
```
1-element Vector{Constraint}:
 == constraint (affine)
├─ index (affine; real)
│  └─ 283×283 real variable (id: 141…996)
└─ 79690-element Vector{Float64}
```

[55]:
```
# Находим решение:
solve!(problem, SCS.Optimizer)
```

```
------------------------------------------------------------------
               SCS v3.2.4 - Splitting Conic Solver
        (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 240268, constraints m: 400047
cones:    z: primal zero / dual free vars: 239586
          s: psd vars: 160461, ssize: 1
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
          alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
          max_iters: 100000, normalize: 1, rho_x: 1.00e-006
          acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
          nnz(A): 400330, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.50e+001 9.96e-001 8.34e+003 1.76e+002 1.00e-001 1.01e+000
   250|3.11e-004 2.58e-005 6.72e-006 4.46e+002 3.40e-001 9.48e+001
------------------------------------------------------------------
status:  solved
timings: total: 9.48e+001s = setup: 5.96e-001s + solve: 9.42e+001s
         lin-sys: 4.95e+000s, cones: 8.73e+001s, accel: 3.16e-001s
------------------------------------------------------------------
objective = 445.530879
------------------------------------------------------------------
```

[56]:
```
@show norm(float.(Gray.(Kref))-X.value)
@show norm(-X.value)
colorview(Gray, X.value)
```

```
norm(float.(Gray.(Kref)) - X.value) = 1.2447077854577675
norm(-(X.value)) = 124.33581441728488
```

[56]:

Рис. 13: Повторение примеров (13)

Самостоятельная работа

Самостоятельная работа (1)

Самостоятельная работа

Линейное программирование

[41]:
```
model = Model(GLPK.Optimizer)
```

[41]:
```
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

[42]:
```
@variable(model, x[1:3] >= 0)
```

[42]:
```
3-element Vector{VariableRef}:
 x[1]
 x[2]
 x[3]
```

[43]:
```
@constraint(model, -x[1] + x[2] + 3x[3] <= -5)
@constraint(model, x[1] + 3x[2] - 7*x[3] <= 10)
@constraint(model, 0 <= x[1] <= 10)
```

[43]:

$$x_1 \in [0, 10]$$

[44]:
```
@objective(model, Max, x[1] + 2x[2] + 5x[3])
```

[44]: $x_1 + 2x_2 + 5x_3$

[45]:
```
optimize!(model)
```

[46]:
```
println("Оптимальное значение целевой функции: ", objective_value(model))
println("Оптимальное значение переменных: ", value.(x))
```

```
Оптимальное значение целевой функции: 19.0625
Оптимальное значение переменных: [10.0, 2.1875, 0.9375]
```

Рис. 14: Самостоятельная работа (1)

Линейное программирование. Использование массивов

```
[160]: c = [1, 2, 5]
       A = [-1 1 3; 1 3 -7]
       b = [-5, 10]
       display(c); display(A); b
```

```
3-element Vector{Int64}:
 1
 2
 5
2×3 Matrix{Int64}:
 -1  1   3
  1  3  -7
```

```
[160]: 2-element Vector{Int64}:
 -5
 10
```

```
[161]: model = Model(GLPK.Optimizer)
       @variable(model, x[1:3] >= 0)
```

```
[161]: 3-element Vector{VariableRef}:
 x[1]
 x[2]
 x[3]
```

```
[162]: @constraint(model, 0 <= x[1] <= 10)
```

[162]:

$$x_1 \in [0, 10]$$

```
[163]: @objective(model, Max, transpose(c)*x)
```

[163]: $x_1 + 2x_2 + 5x_3$

```
[164]: @constraint(model, A * x .<= b)
```

```
[164]: 2-element Vector{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64}, MathOptInterface.LessThan{Float64}}, ScalarShape}}:
 -x[1] + x[2] + 3 x[3] <= -5
 x[1] + 3 x[2] - 7 x[3] <= 10
```

```
[165]: optimize!(model)
```

```
[166]: println("Оптимальное значение целевой функции: ", objective_value(model))
       println("Оптимальное значение переменных: ", value.(x))
```

```
Оптимальное значение целевой функции: 19.0625
Оптимальное значение переменных: [10.0, 2.1875, 0.9375]
```

Рис. 15: Самостоятельная работа (2)

Выпуклое программирование

```
[167]: n = rand(3:5)
       m = n-rand(0:2)
       display(n); m

5

[167]: 5
```

```
[168]: A = rand(m, n)
       b = rand(m)
       x = Variable(n)
       display(A); display(b); x

5×5 Matrix{Float64}:
 0.770232   0.240449  0.77553   0.0444783   0.258416
 0.47234    0.872164  0.357746  0.272792    0.035957
 0.0725477  0.237383  0.608813  0.607776    0.291872
 0.679407   0.25419   0.631587  0.00426607  0.182371
 0.514284   0.563756  0.191832  0.261296    0.180975
5-element Vector{Float64}:
 0.9624458021448501
 0.2624239322987302
 0.8558835816793745
 0.3059378263841269
 0.5229702845366548

[168]: Variable
size: (5, 1)
sign: real
vexity: affine
id: 289…482
```

```
[169]: objective = minimize(square(norm(A * x - b, 2)), x >= 0)
       solve!(objective, SCS.Optimizer)

------------------------------------------------------------------
               SCS v3.2.4 - Splitting Conic Solver
        (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 8, constraints m: 16
cones:    z: primal zero / dual free vars: 1
          l: linear vars: 6
          q: soc vars: 9, qsize: 2
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
          alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
          max_iters: 100000, normalize: 1, rho_x: 1.00e-006
          acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
          nnz(A): 36, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.71e+001 1.00e+000 1.62e+001 -8.02e+000 1.00e-001 1.26e-004
```

Рис. 16: Самостоятельная работа (3)

```
----------------------------------------------------------------
 iter | pri res | dua res |   gap    |   obj    |  scale   | time (s)
----------------------------------------------------------------
     0|1.71e+001 1.00e+000 1.62e+001 -8.02e+000 1.00e-001 1.26e-004
   125|1.81e-005 2.57e-006 1.57e-005 1.03e-001 1.00e-001 8.90e-003
----------------------------------------------------------------
status:  solved
timings: total: 8.91e-003s = setup: 1.03e-004s + solve: 8.80e-003s
         lin-sys: 5.91e-005s, cones: 3.04e-005s, accel: 8.63e-003s
----------------------------------------------------------------
objective = 0.102876
----------------------------------------------------------------
```

[170]:

```
println("Оптимальное значение: ", objective.optval)
println("Оптимальное решение: ", Convex.evaluate(x))
```

```
Оптимальное значение: 0.1028679143351298
Оптимальное решение: [2.3833981663828694e-7, 0.08100161792417866, 0.12067386282564724, 0.09562982046568322, 2.482421306212329З]
```

Рис. 17: Самостоятельная работа (4)

Самостоятельная работа (5)

Оптимальная рассадка по залам

[85]:
```
using Random
```

[139]:
```
zals_str = collect(1:5)
zals_data = JuMP.Containers.DenseAxisArray(
    [180 250;
    180 250;
    220 220;
    180 250;
    180 250],
    zals_str,
    ["min", "max"])
```

[139]:
```
2-dimensional DenseAxisArray{Int64,2,...} with index sets:
    Dimension 1, [1, 2, 3, 4, 5]
    Dimension 2, ["min", "max"]
And data, a 5×2 Matrix{Int64}:
 180  250
 180  250
 220  220
 180  250
 180  250
```

[140]:
```
# Переделаны обозначения, потому что не нашла способа для оптимизации
N = 1000
peopl = collect(1:N)
people_pref = copy(hcat([shuffle([1, 2, 3, 10000, 10000]) for i in peopl]...))
```

[140]:
```
5×1000 Matrix{Int64}:
 10000  10000  10000  10000  10000  …  10000      3  10000      3  10000
     2      1      3      3      3     10000  10000      3      2      3
 10000      3      1      1      2         1      2  10000  10000      2
     3  10000  10000      2      1         3  10000      2  10000      1
     1      2      2  10000  10000         2      1      1      1  10000
```

[141]:
```
model_zal = Model(GLPK.Optimizer)
```

[141]:
```
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

Рис. 18: Самостоятельная работа (5)

Самостоятельная работа (6)

```
[142]: @variable(model_zal, answ[peopl, zals_str], Bin)
```

```
[142]: 2-dimensional DenseAxisArray{VariableRef,2,...} with index sets:
           Dimension 1, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10  …  991, 992, 993, 994, 995, 996, 997, 998, 999, 1000]
           Dimension 2, [1, 2, 3, 4, 5]
       And data, a 1000×5 Matrix{VariableRef}:
        answ[1,1]     answ[1,2]     answ[1,3]     answ[1,4]     answ[1,5]
        answ[2,1]     answ[2,2]     answ[2,3]     answ[2,4]     answ[2,5]
        answ[3,1]     answ[3,2]     answ[3,3]     answ[3,4]     answ[3,5]
        answ[4,1]     answ[4,2]     answ[4,3]     answ[4,4]     answ[4,5]
        answ[5,1]     answ[5,2]     answ[5,3]     answ[5,4]     answ[5,5]
        answ[6,1]     answ[6,2]     answ[6,3]     answ[6,4]     answ[6,5]
        answ[7,1]     answ[7,2]     answ[7,3]     answ[7,4]     answ[7,5]
        answ[8,1]     answ[8,2]     answ[8,3]     answ[8,4]     answ[8,5]
        answ[9,1]     answ[9,2]     answ[9,3]     answ[9,4]     answ[9,5]
        answ[10,1]    answ[10,2]    answ[10,3]    answ[10,4]    answ[10,5]
        answ[11,1]    answ[11,2]    answ[11,3]    answ[11,4]    answ[11,5]
        answ[12,1]    answ[12,2]    answ[12,3]    answ[12,4]    answ[12,5]
        answ[13,1]    answ[13,2]    answ[13,3]    answ[13,4]    answ[13,5]
        ⋮
        answ[989,1]   answ[989,2]   answ[989,3]   answ[989,4]   answ[989,5]
        answ[990,1]   answ[990,2]   answ[990,3]   answ[990,4]   answ[990,5]
        answ[991,1]   answ[991,2]   answ[991,3]   answ[991,4]   answ[991,5]
        answ[992,1]   answ[992,2]   answ[992,3]   answ[992,4]   answ[992,5]
        answ[993,1]   answ[993,2]   answ[993,3]   answ[993,4]   answ[993,5]
        answ[994,1]   answ[994,2]   answ[994,3]   answ[994,4]   answ[994,5]
        answ[995,1]   answ[995,2]   answ[995,3]   answ[995,4]   answ[995,5]
        answ[996,1]   answ[996,2]   answ[996,3]   answ[996,4]   answ[996,5]
        answ[997,1]   answ[997,2]   answ[997,3]   answ[997,4]   answ[997,5]
        answ[998,1]   answ[998,2]   answ[998,3]   answ[998,4]   answ[998,5]
        answ[999,1]   answ[999,2]   answ[999,3]   answ[999,4]   answ[999,5]
        answ[1000,1]  answ[1000,2]  answ[1000,3]  answ[1000,4]  answ[1000,5]
```

```
[143]: for i in peopl
           @constraint(model_zal, sum(answ[i, :]) == 1)
       end
       for i in zals_str
           @constraint(model_zal, zals_data[i, "min"] <= sum(answ[:, i]) <= zals_data[i, "max"])
       end
```

```
[146]: @objective(model_zal, Min, sum([sum([answ[t, c]*people_pref[c, t] for c in zals_str]) for t in peopl]))
```

[146]: $10000 answw_{1,1} + 2 answ_{1,2} + 10000 answ_{1,3} + 3 answ_{1,4} + answ_{1,5} + 10000 answ_{2,1} + answ_{2,2} + 3 answ_{2,3} + 10000 answ_{2,4} + 2 answ_{2,5} + 10000 answ_{3,1} + 3 answ_{3,2} + answ_{3,3} + 10000 answ_{3,4} + 2 answ_{3,5} + 10000 answ_{4,1} + 3 answ_{4,2} + answ_{4,3} + 2an$

```
[148]: optimize!(model_zal)
```

```
[149]: termination_status(model_zal)
```

```
[149]: OPTIMAL::TerminationStatusCode = 1
```

Рис. 19: Самостоятельная работа (6)

Самостоятельная работа (7)

```
[153]: ress = value.(answ)

[153]: 2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10  …  991, 992, 993, 994, 995, 996, 997, 998, 999, 1000]
    Dimension 2, [1, 2, 3, 4, 5]
And data, a 1000×5 Matrix{Float64}:
 0.0  0.0  0.0  0.0  1.0
 0.0  1.0  0.0  0.0  0.0
 0.0  0.0  1.0  0.0  0.0
 0.0  0.0  1.0  0.0  0.0
 0.0  0.0  0.0  1.0  0.0
 0.0  1.0  0.0  0.0  0.0
 1.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  1.0
 0.0  1.0  0.0  0.0  0.0
 1.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  1.0
 0.0  0.0  1.0  0.0  0.0
 1.0  0.0  0.0  0.0  0.0
 ⋮
 0.0  0.0  0.0  1.0  0.0
 0.0  1.0  0.0  0.0  0.0
 0.0  0.0  0.0  1.0  0.0
 1.0  0.0  0.0  0.0  0.0
 0.0  0.0  1.0  0.0  0.0
 0.0  0.0  1.0  0.0  0.0
 0.0  0.0  0.0  0.0  1.0
 0.0  0.0  1.0  0.0  0.0
 0.0  0.0  0.0  0.0  1.0
 0.0  0.0  0.0  0.0  1.0
 0.0  0.0  0.0  0.0  1.0
 0.0  0.0  0.0  1.0  0.0
```

```
[157]: zals_filling = zeros(5)
recomendationss = zeros(N)
for i in peopl
    for j in zals_str
        zals_filling[j] += ress[i, j]
        if ress[i, j] == 1
            recomendationss[i] = j
        end
    end
end
```

Рис. 20: Самостоятельная работа (7)

Самостоятельная работа (8)

```
[158]: zals_filling

[158]: 5-element Vector{Float64}:
        200.0
        180.0
        220.0
        198.0
        202.0

[159]: recomendationss

[159]: 1000-element Vector{Float64}:
        5.0
        2.0
        3.0
        3.0
        4.0
        2.0
        1.0
        5.0
        2.0
        1.0
        5.0
        3.0
        1.0
        ⋮
        4.0
        2.0
        4.0
        1.0
        3.0
        3.0
        5.0
        3.0
        5.0
        5.0
        5.0
        4.0
```

Рис. 21: Самостоятельная работа (8)

План приготовления кофе

```
[12]: model = Model(GLPK.Optimizer)
      @variable(model, raf >= 0)
      @variable(model, cappuccino >= 0)
```

[12]: $cappuccino$

```
[13]: const grain_limit = 500
      @constraint(model, raf * 40 + cappuccino * 30 <= grain_limit)
```

[13]:

$$40raf + 30cappuccino \leq 500$$

```
[14]: const milk_limit = 2000
      @constraint(model, raf * 140 + cappuccino * 120 <= milk_limit)
```

[14]:

$$140raf + 120cappuccino \leq 2000$$

```
[15]: const sugar_limit = 40
      @constraint(model, raf * 5 == sugar_limit)
```

[15]:

$$5raf = 40$$

```
[16]: objective = 400 * raf + 300 * cappuccino
      @objective(model, Max, objective)
```

[16]: $400raf + 300cappuccino$

```
[17]: optimize!(model)
```

```
[18]: println("Раф кофе: ", round(value(raf)))
      println("Капучино: ", round(value(cappuccino)))
      println("Прибыль: ", value(objective))
```

```
Раф кофе: 8.0
Капучино: 6.0
Прибыль: 5000.0
```

Рис. 22: Самостоятельная работа (9)

Выводы по проделанной работе

Вывод

В результате выполнения работы мы освоили пакеты Julia для решения задач оптимизации.

Были записаны скринкасты выполнения и защиты лабораторной работы.