THE DIRECTION-BASED FLATTENING ENERGY

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For triangle $ijk \in T$ define the angle at vertex i

$$\beta^i_{jk} = |\alpha_{ij} - \alpha_{ki}|$$

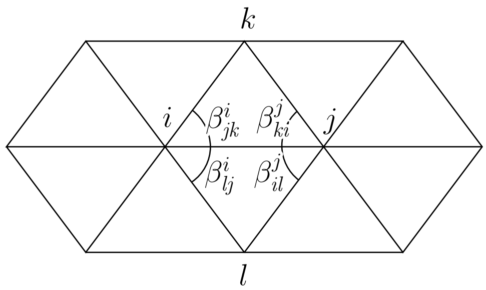

FIGURE 1. Energy labels at edge ij

The direction-based flattening functional at edge $ij \in E$ is defined as

$$\begin{aligned} S_{ij}(\alpha) &= \alpha_{ij} \sum_{\substack{imn \ni i \\ ij \notin imn}} (\log\sin\beta^m_{ni} - \log\sin\beta^n_{im}) + \alpha_{ij} \sum_{\substack{jmn \ni j \\ ij \notin jmn}} \left(\log\sin\beta^m_{nj} - \log\sin\beta^n_{jm}\right) \\ &\quad + Л(\beta^j_{ki}) + Л(\beta^j_{il}) + Л(\beta^i_{lj}) + Л(\beta^i_{jk}) \end{aligned}$$

For boundary vertices drop the corresponding sums and Л terms.
The gradient of S is given by

$$\frac{\partial S}{\partial \alpha_{ij}} = \sum_{ijk \ni i} \left(\log\sin\beta^j_{ki} - \log\sin\beta^k_{ij}\right) + \sum_{jlm \ni j} \left(\log\sin\beta^l_{mj} - \log\sin\beta^m_{jl}\right)$$

For boundary vertices drop the corresponding sum.

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