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Polynomial Multiplication

Back to Week 4

3/3 points earned (100%)

Quiz passed!

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1 / 1
points

1.
For $n = 1024$, compute how many operations will the faster divide and conquer algorithm from the lectures perform, using the formula $3^{\log_2 n}$ for the number of operations.

- 1024
- 1048576
- 59049 (selected)

Correct Response
$\log_2 n = \log_2 1024 = 10$, so $3^{\log_2 n} = 3^{10} = 59049$.

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1 / 1
points

2.
What is the key formula used in the faster divide and conquer algorithm to decrease the number of multiplications needed from 4 to 3?

- $a_1 b_0 + a_0 b_1 = (a_0 + a_1)(b_0 + b_1) - a_0 b_0 - a_1 b_1$ (selected)

Correct Response
Correct! This means that we only need to do 3 multiplications $a_0 b_0$, $a_1 b_1$ and $(a_0 + a_1)(b_0 + b_1)$ instead of 4 multiplications $a_0 b_0$, $a_1 b_1$, $a_0 b_1$ and $a_1 b_0$.

- $a_1 (b_0 + b_1) = a_1 b_0 + a_1 b_1$

- ○ $a_0 + b_0 = a_1 + b_1$
- ○ $(a_0 + a_1)(b_0 + b_1) = a_0 b_0 + a_0 b_1 + a_1 b_0 + a_1 b_1$

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1 / 1
points

3.
(*This is an advanced question.*)

How to apply fast polynomial multiplication algorithm to multiply very big integer numbers (containing hundreds of thousands of digits) faster?

- ● For a number $A = \overline{a_1 a_2 \ldots a_n}$ with n digits create a corresponding polynomial $a(x) = a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n$. Then $a(10) = A$. Do the same with number $B = \overline{b_1 b_2 \ldots b_n}$ and create polynomial $b(x)$. Multiply polynomials $a(x)$ and $b(x)$, get polynomial $c(x) = \overline{c_1 c_2 \ldots c_n}$. If we create a number $C = \overline{c_1 c_2 \ldots c_n}$, it is almost the same as product of A and B, but some of its "digits" may be 10 or bigger. If the last "digit" is 52, for example, make the last digit just 2 and add 5 to the previous digit. Go on until all the digits are from 0 to 9.

  Suppose we need to multiply numbers 13 and 24. The correct result is 312. To get this result, we first create polynomials $a(x) = x + 3$ and $b(x) = 2x + 4$ corresponding to numbers 13 and 24 respectively. We then use Karatsuba's algorithm to multiply those polynomials and get polynomial $c(x) = 2x^2 + 10x + 12$. To get the answer, we need to compute $c(10) = 2 \times 10^2 + 10 \times 10 + 12$. You see that some of the coefficients of polynomial c are not digits, because they are bigger than 9. To fix that, for each such coefficient from right to left we subtract 10 from it and add 1 to the previous coefficient:
  $c(10) = 2 \times 10^2 + 10 \times 10 + 12 = 2 \times 10^2 + 11 \times 10 + 2 = 3 \times 10^2 + 1 \times 10 + 2 = 312$
  .

Correct Response
First we need to convert number with n digits to polynomial with n coefficients in $O(n)$ time. Then we need to multiply two polynomials of degree n in $O(3^{log_2 n})$ time. After that, we need to convert the polynomial back to number and "fix" it in $O(n)$. The total time for multiplication of the numbers will be $O(n) + O(3^{\log_2 n}) + O(n) = O(3^{\log_2 n})$ as opposed to $O(n^2)$ time for the grade school number multiplication algorithm.

- ○ For number A, create a polynomial $a(x) = A$, for number B create a polynomial $b(x) = B$, multiply those polynomials and get the answer.

Suppose we need to multiply numbers 13 and 24. The correct result is 312. To get this result, we first create polynomials $a(x) = 13$ and $b(x) = 24$ corresponding to numbers 13 and 24 respectively. We then use Karatsuba's algorithm to multiply those polynomials and get polynomial $c(x) = 312$. Now we know that $13 \times 24 = 312$.