The 0/1 Knapsack problem – finding an optimal solution

- Each item is represented by a pair, <value, weight>.
- The knapsack can accommodate items with a total weight of no more than w.
- A vector, I, of length n, represents the set of available items. Each element of the vector is an item.
- A vector, V, of length n, is used to indicate whether or not each item is taken by the burglar. If V[i] = 1, item I[i] is to be taken. If V[i] = 0, item I[i] is not taken.
- Find a V that maximizes the sum of V[i]*I[i].value over all values of i, subject to the constraint that the sum of V[i]*I[i].weight over all values of i is no more than w.

An approach to solving this problem

- Enumerate all possible combinations of items, this is called the **power set**,
- Remove all of the combinations whose weight exceeds the allowed weight,
- From the remaining combinations choose any one whose value is at least as large as the value of the other combinations.

But this is going to be slow?

- How big is a power set?
  - Suppose we have two elements {a, b}
  - Then the power set is {}, {a}, {b}, {a, b}
  - Now suppose we have three elements {a, b, c}
  - Then the power set is {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}

  - And if we have four elements?

Representing the power set

- We can represent any combination of items by a vector of 0’s and 1’s.
- The combination containing no items would be represented by a vector of all 0’s.
- The combination containing all of the items would be represented by a vector of all 1’s.
- The combination containing only the first and last elements would be represented by 100…001.
- And so on
- If we look at four items, the possible choices are:

a	b	c	d	combos
0	0	0	0	{}
0	0	0	1	{d}
0	0	1	0	{c}
0	0	1	1	{c,d}
0	1	0	0	{b}
0	1	0	1	{b,d}
0	1	1	0	{b,c}
0	1	1	1	{b,c,e}
1	0	0	0	{a}
1	0	0	1	{a,d}
1	0	1	0	{a,c}
1	0	1	1	{a,c,d}
1	1	0	0	{a,b}
1	1	0	1	{a,b,d}
1	1	1	0	{a,b,c}
1	1	1	1	{a,b,c,d}

Capturing the power set

- Just looking at the right hand column, this may seem confusing
- But looking at the left side columns, there is a clear method to generating the power set
  - We are just enumerating all possible four digit binary numbers