The Law of Large Numbers

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Law of Large Numbers

Law of Large Numbers

In repeated **independent tests** with the same **actual probability** p of a particular outcome in each test,

the chance that the **fraction of times** that outcome occurs differs from p converges to zero as the number of trials goes to infinity.

Gambler's Fallacy

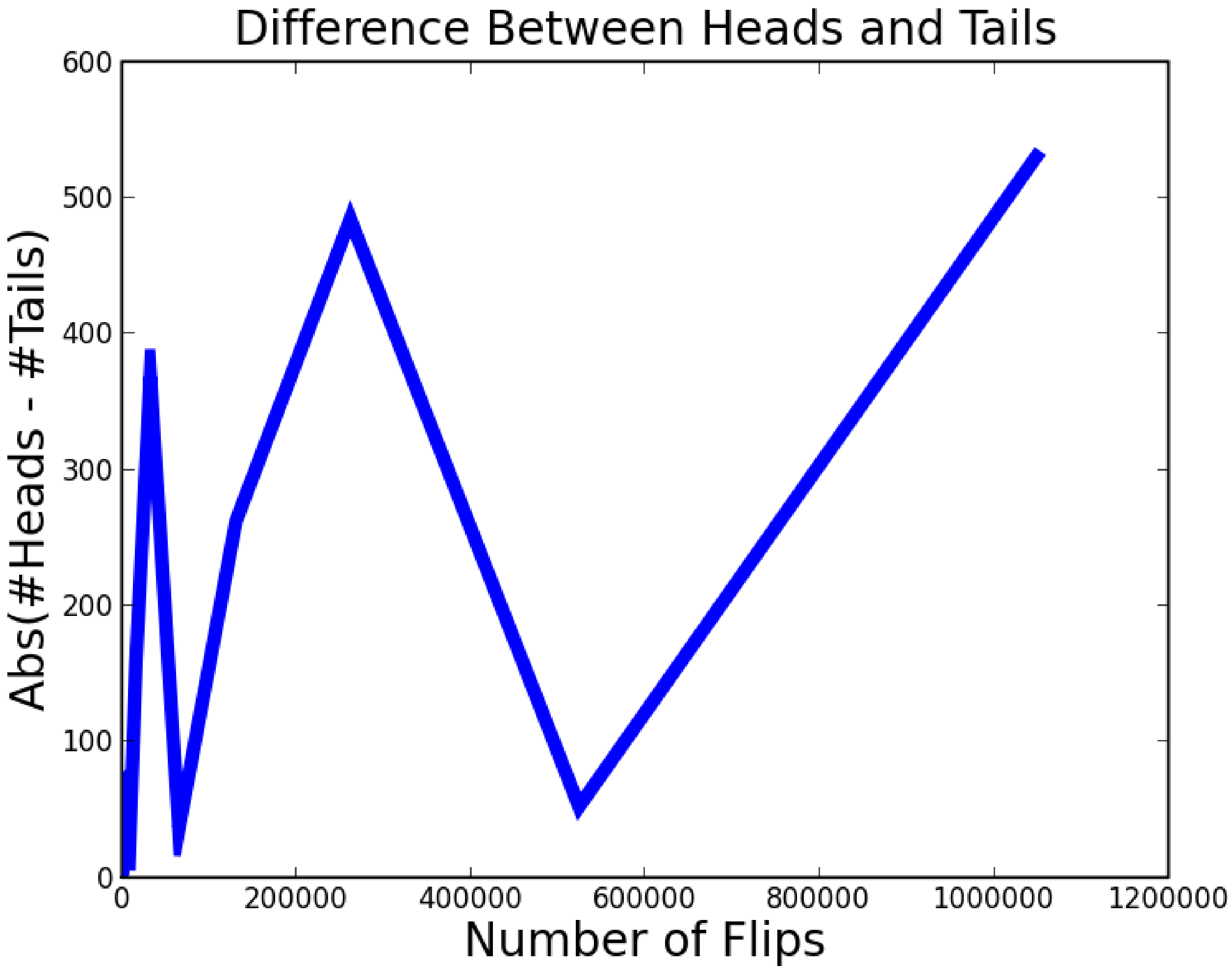
If deviations from expected behavior occur, these deviations are likely to be evened out by opposite deviations in the future.

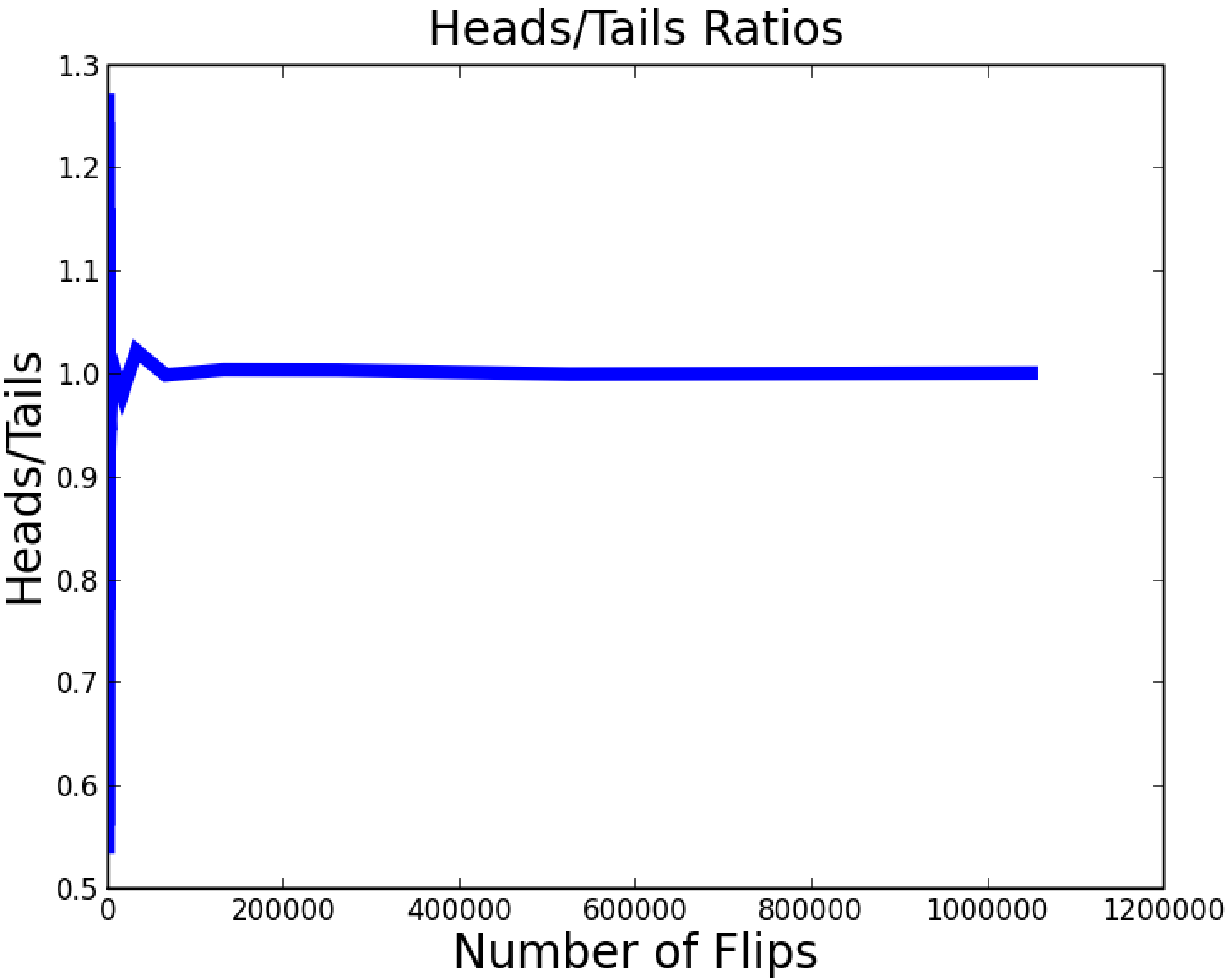
```
def flipPlot(minExp, maxExp):
    """Assumes minExp and maxExp positive
          integers; minExp < maxExp
       Plots results of 2**minExp to
         2**maxExp coin flips"""
    ratios = []
    diffs = []
    xAxis = []
    for exp in range(minExp, maxExp + 1):
        xAxis.append(2**exp)
. . .
```

```
for numFlips in xAxis:
        numHeads = 0
        for n in range(numFlips):
            if random.random() < 0.5:
                numHeads += 1
        numTails = numFlips - numHeads
        ratios.append(numHeads/float(numTails))
        diffs.append(abs(numHeads - numTails))

. . .
```

```
pylab.title('Difference Between Heads and Tails')
    pylab.xlabel('Number of Flips')
    pylab.ylabel('Abs(#Heads - #Tails)')
    pylab.plot(xAxis, diffs)
    pylab.figure()
    pylab.title('Heads/Tails Ratios')
    pylab.xlabel('Number of Flips')
    pylab.ylabel('Heads/Tails')
    pylab.plot(xAxis, ratios)
```


Difference Between Heads and Tails
600
500
400
300
200
100
0
0
200000
400000
600000
800000
1000000
1200000
Abs(#Heads - #Tails)
Number of Flips


Heads/Tails Ratios
Heads/Tails
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0
200000
400000
600000
800000
1000000
1200000
Number of Flips