

Prime numbers

10^1	+1	+3	+7	+9	+13	+19	+21	+27	−3	−5	−7	−8
10^2	+1	+3	+7	+9	+13	+27	+31	+37	−3	−11	−17	−21
10^3	+9	+13	+19	+21	+31	+33	+39	+49	−3	−9	−17	−23
10^4	+7	+9	+37	+39	+61	+67	+69	+79	−27	−33	−51	−59
10^5	+3	+19	+43	+49	+57	+69	+103	+109	−9	−11	−29	−39
10^6	+3	+33	+37	+39	+81	+99	+117	+121	−17	−21	−39	−41
10^7	+19	+79	+103	+121	+139	+141	+169	+189	−9	−27	−29	−57
10^8	+7	+37	+39	+49	+73	+81	+123	+127	−11	−29	−41	−59
10^9	+7	+9	+21	+33	+87	+93	+97	+103	−63	−71	−107	−117
10^{10}	+19	+33	+61	+69	+97	+103	+121	+141	−33	−57	−71	−119
10^{11}	+3	+19	+57	+63	+69	+73	+91	+103	−23	−53	−57	−93
10^{12}	+39	+61	+63	+91	+121	+163	+169	+177	−11	−39	−41	−63
10^{13}	+37	+51	+99	+129	+183	+259	+267	+273	−29	−137	−201	−237
10^{14}	+31	+67	+97	+99	+133	+139	+169	+183	−27	−29	−41	−69
10^{15}	+37	+91	+159	+187	+223	+241	+249	+259	−11	−53	−117	−123
10^{16}	+61	+69	+79	+99	+453	+481	+597	+613	−63	−83	−113	−149
10^{17}	+3	+13	+19	+21	+49	+81	+99	+141	−3	−23	−39	−57
10^{18}	+3	+9	+31	+79	+177	+183	+201	+283	−11	−33	−123	−137

Primitive Roots

<i>mod</i>	$12 \cdot 2^{10} + 1$	$13 \cdot 2^{10} + 1$	$15 \cdot 2^{10} + 1$	$57 \cdot 2^{10} + 1$	$58 \cdot 2^{10} + 1$	$60 \cdot 2^{10} + 1$	$148 \cdot 2^{10} + 1$
<i>root</i>	49	7	84	29	9	21	38
<i>mod</i>	$6 \cdot 2^{11} + 1$	$9 \cdot 2^{11} + 1$	$20 \cdot 2^{11} + 1$	$56 \cdot 2^{11} + 1$	$65 \cdot 2^{11} + 1$	$140 \cdot 2^{11} + 1$	$150 \cdot 2^{11} + 1$
<i>root</i>	7	19	32	16	39	106	91
<i>mod</i>	$3 \cdot 2^{12} + 1$	$10 \cdot 2^{12} + 1$	$15 \cdot 2^{12} + 1$	$66 \cdot 2^{12} + 1$	$70 \cdot 2^{12} + 1$	$75 \cdot 2^{12} + 1$	$127 \cdot 2^{12} + 1$
<i>root</i>	41	28	19	114	19	41	71
<i>mod</i>	$136 \cdot 2^{12} + 1$	$141 \cdot 2^{12} + 1$	$5 \cdot 2^{13} + 1$	$8 \cdot 2^{13} + 1$	$14 \cdot 2^{13} + 1$	$51 \cdot 2^{13} + 1$	$78 \cdot 2^{13} + 1$
<i>root</i>	66	114	12	13	2	67	87
<i>mod</i>	$90 \cdot 2^{13} + 1$	$113 \cdot 2^{13} + 1$	$4 \cdot 2^{14} + 1$	$7 \cdot 2^{14} + 1$	$9 \cdot 2^{14} + 1$	$63 \cdot 2^{14} + 1$	$69 \cdot 2^{14} + 1$
<i>root</i>	96	63	15	15	22	94	86
<i>mod</i>	$73 \cdot 2^{14} + 1$	$139 \cdot 2^{14} + 1$	$2 \cdot 2^{15} + 1$	$5 \cdot 2^{15} + 1$	$17 \cdot 2^{15} + 1$	$81 \cdot 2^{15} + 1$	$110 \cdot 2^{15} + 1$
<i>root</i>	31	20	9	7	19	89	117
<i>mod</i>	$114 \cdot 2^{15} + 1$	$135 \cdot 2^{15} + 1$	$1 \cdot 2^{16} + 1$	$12 \cdot 2^{16} + 1$	$18 \cdot 2^{16} + 1$	$55 \cdot 2^{16} + 1$	$88 \cdot 2^{16} + 1$
<i>root</i>	27	126	3	3	14	30	10
<i>mod</i>	$102 \cdot 2^{16} + 1$	$112 \cdot 2^{16} + 1$	$117 \cdot 2^{16} + 1$	$6 \cdot 2^{17} + 1$	$9 \cdot 2^{17} + 1$	$21 \cdot 2^{17} + 1$	$51 \cdot 2^{17} + 1$
<i>root</i>	51	83	15	8	74	83	43
<i>mod</i>	$53 \cdot 2^{17} + 1$	$63 \cdot 2^{17} + 1$	$104 \cdot 2^{17} + 1$	$108 \cdot 2^{17} + 1$	$123 \cdot 2^{17} + 1$	$3 \cdot 2^{18} + 1$	$22 \cdot 2^{18} + 1$
<i>root</i>	47	10	13	54	26	5	74
<i>mod</i>	$28 \cdot 2^{18} + 1$	$52 \cdot 2^{18} + 1$	$54 \cdot 2^{18} + 1$	$63 \cdot 2^{18} + 1$	$108 \cdot 2^{18} + 1$	$127 \cdot 2^{18} + 1$	$147 \cdot 2^{18} + 1$
<i>root</i>	79	4	25	70	108	99	34
<i>mod</i>	$11 \cdot 2^{19} + 1$	$14 \cdot 2^{19} + 1$	$26 \cdot 2^{19} + 1$	$54 \cdot 2^{19} + 1$	$57 \cdot 2^{19} + 1$	$71 \cdot 2^{19} + 1$	$134 \cdot 2^{19} + 1$
<i>root</i>	12	25	2	106	20	86	49
<i>mod</i>	$7 \cdot 2^{20} + 1$	$13 \cdot 2^{20} + 1$	$22 \cdot 2^{20} + 1$	$66 \cdot 2^{20} + 1$	$67 \cdot 2^{20} + 1$	$106 \cdot 2^{20} + 1$	$115 \cdot 2^{20} + 1$
<i>root</i>	5	3	50	54	7	85	138
<i>mod</i>	$148 \cdot 2^{20} + 1$	$11 \cdot 2^{21} + 1$	$33 \cdot 2^{21} + 1$	$39 \cdot 2^{21} + 1$	$53 \cdot 2^{21} + 1$	$54 \cdot 2^{21} + 1$	$63 \cdot 2^{21} + 1$
<i>root</i>	81	38	45	94	54	134	46
<i>mod</i>	$110 \cdot 2^{21} + 1$	$119 \cdot 2^{21} + 1$	$123 \cdot 2^{21} + 1$	$25 \cdot 2^{22} + 1$	$27 \cdot 2^{22} + 1$	$33 \cdot 2^{22} + 1$	$55 \cdot 2^{22} + 1$
<i>root</i>	68	135	95	21	66	30	63
<i>mod</i>	$90 \cdot 2^{22} + 1$	$99 \cdot 2^{22} + 1$	$20 \cdot 2^{23} + 1$	$56 \cdot 2^{23} + 1$	$77 \cdot 2^{23} + 1$	$107 \cdot 2^{23} + 1$	$119 \cdot 2^{23} + 1$
<i>root</i>	139	65	4	53	19	45	31
<i>mod</i>	$132 \cdot 2^{23} + 1$	$10 \cdot 2^{24} + 1$	$28 \cdot 2^{24} + 1$	$66 \cdot 2^{24} + 1$	$73 \cdot 2^{24} + 1$	$108 \cdot 2^{24} + 1$	$120 \cdot 2^{24} + 1$
<i>root</i>	64	2	40	8	149	126	21
<i>mod</i>	$148 \cdot 2^{24} + 1$						
<i>root</i>	25						

Misc

Gomory-Hu tree (Gusfield's algorithm): label nodes from 0 to $(|V| - 1)$ and set $p_i = 0 \forall i > 0$. $\forall i > 0$: find min-cut (S, T) between i and p_i , where $i \in S, p_i \in T$; for each node j , s.t. $i < j, j \in S, p_j = p_i$ set $p_j = i$

$$d_i = v_i - \sum_{j < i} \frac{\langle v_i, d_j \rangle}{\langle d_j, d_j \rangle} d_j \quad \sum_{k=1}^n \mu(k) \lfloor \frac{n}{k} \rfloor = 1 \quad g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

$$\mu_A^*(a, b) = \begin{cases} 1, & a = b \\ - \sum_{a \preceq z \prec b} \mu_A^*(a, z), & a \prec b \\ 0, & b \prec a \end{cases} \quad \sum_{d|n} \varphi(d) = n \quad \varphi(n) = \sum_{d|n} d \cdot \mu\left(\frac{n}{d}\right)$$

$$F(N) = \sum_{n=1}^N \varphi(n) \Rightarrow F(N) = \frac{N(N+1)}{2} - \sum_{k=2}^N F\left(\lfloor \frac{N}{k} \rfloor\right)$$

$$Gx = \{y \in X \mid \exists a \in G: a \star x = y\} \quad G_x = \{a \in G: a \star x = x\} \quad |G| = |Gx| \cdot |G_x| \quad X^a = \{x \in X: a \star x = x\}$$

$$|X/G| = \frac{1}{|G|} \sum_{a \in G} |X^a| \quad \prod_{k=1}^{\infty} (1 - x^k) = \sum_{q=-\infty}^{+\infty} (-1)^q x^{\frac{3q^2+q}{2}}$$

$$\prod_{k=1}^{\infty} \frac{1}{1-x^k} = \sum_{n=0}^{+\infty} p(n) x^n \Rightarrow p(0) = 1, p(n) = \sum_{q=1}^{+\infty} (-1)^{q+1} \left[p\left(n - \frac{3q^2-q}{2}\right) + p\left(n - \frac{3q^2+q}{2}\right) \right]$$

$$M_1 = (S, I_1) \cap M_2 = (S, I_2). J.y \rightarrow z(J - y + z \in I_1). y \leftarrow z(J - y + z \in I_2)$$

$$X_2 = \{z \in S/J: J + z \in I_1\}. X_2 = \{z \in S/J: J + z \in I_2\}. \text{ Находим кратчайший путь из } X_1 \text{ в } X_2. \text{ Ксорим.}$$

$$x = \frac{B_1 C_2 - B_2 C_1}{A_1 B_2 - A_2 B_1}, y = \frac{A_2 C_1 - A_1 C_2}{A_1 B_2 - A_2 B_1}$$

$$\text{Теорема Пика. } S = I + \frac{B}{2} - 1, \text{ где } S - \text{площадь, } B - \# \text{ на сторонах, } I - \# \text{ строго внутри}$$

$$s(0, 0) = c(0, 0) = 1. s(n, k) = s(n-1, k-1) - (n-1)s(n-1, k). c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k).$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1)(x+2) \cdot \dots \cdot (x+n-1). \sum_{k=0}^n s(n, k) x^k = x(x-1)(x-2) \cdot \dots \cdot (x-n+1)$$

$$c(n, k) = (-1)^{n-k} s(n, k) \quad \sum_{n=0}^{\infty} \sum_{k=0}^n \frac{c(n, k)}{n!} x^n y^k = \exp(-y \log(1-x)) \quad \sum_{n=0}^{\infty} \sum_{k=0}^n \frac{s(n, k)}{n!} x^n y^k = \exp(y \log(1+x))$$

Suffix Tree

```
// Ukkonen's algorithm O(n)
const int A = 27; // Alphabet size
struct SuffixTree {
    struct Node { // [l, r) !!!
        int l, r, link, par;
        int nxt[A];
        Node() : l(-1), r(-1), link(-1), par(-1) {
            fill(nxt, nxt + A, -1);
        }
        Node(int _l, int _r, int _link, int _par)
            : l(_l), r(_r), link(_link), par(_par) {
            fill(nxt, nxt + A, -1);
        }
        int& next(int c) { return nxt[c]; }
        int get_len() const { return r - l; }
    };
    struct State {
        int v, len;
    };
    vec<Node> t;
    State cur_state;
    vec<int> s;
    SuffixTree() : cur_state({0, 0}) {
        t.push_back(Node());
    }
    // v -> v + s[l, r) !!!
```

```
State go(State st, int l, int r) {
    while (l < r) {
        if (st.len == t[st.v].get_len()) {
            State nx = State({t[st.v].next(s[l]),
→ 0});
            if (nx.v == -1) return nx;
            st = nx;
            continue;
        }
        if (s[t[st.v].l + st.len] != s[l])
            return State({-1, -1});
        if (r - l < t[st.v].get_len() - st.len)
            return State({st.v, st.len + r - l});
        l += t[st.v].get_len() - st.len;
        st.len = t[st.v].get_len();
    }
    return st;
}

int get_vertex(State st) {
    if (t[st.v].get_len() == st.len) return st.v;
    if (st.len == 0) return t[st.v].par;
    Node& v = t[st.v];
    Node& pv = t[v.par];
    Node add(v.l, v.l + st.len, -1, v.par);
    // nxt
    pv.next(s[v.l]) = (int)t.size();
    add.next(s[v.l + st.len]) = st.v;
```

```

// par
v.par = (int)t.size();
// [l, r)
v.l += st.len;
t.push_back(add); // !!!
return (int)t.size() - 1;
}

int get_link(int v) {
    if (t[v].link != -1) return t[v].link;
    if (t[v].par == -1) return 0;
    int to = get_link(t[v].par);
    to = get_vertex(
        go(State{to, t[to].get_len()}),
        t[v].l + (t[v].par == 0), t[v].r));
    return t[v].link = to;
}

void add_symbol(int c) {
    assert(0 <= c && c < A);
    s.push_back(c);
    while (1) {
        State hlp = go(cur_state, (int)s.size() -
→ 1,
                    (int)s.size());
        if (hlp.v != -1) {
            cur_state = hlp;
            break;
        }
        int v = get_vertex(cur_state);
        Node add((int)s.size() - 1, +inf, -1, v);
        t.push_back(add);
        t[v].next(c) = (int)t.size() - 1;
        cur_state.v = get_link(v);
        cur_state.len = t[cur_state.v].get_len();
        if (!v) break;
    }
}
};

```

Suffix Array

```

const int LOG = 21;
struct SuffixArray {
    string s;
    int n;
    vec<int> p;
    vec<int> c[LOG];
    SuffixArray() : n(0) {}
    SuffixArray(string ss) : s(ss) {
        s.push_back(0);
        n = (int)s.size();
        vec<int> pn, cn;
        vec<int> cnt;
        p.resize(n);
        for (int i = 0; i < LOG; i++)
            c[i].resize(n);
        pn.resize(n);
        cn.resize(n);
        cnt.assign(300, 0);
        for (int i = 0; i < n; i++)
            cnt[s[i]]++;
        for (int i = 1; i < (int)cnt.size(); i++)
            cnt[i] += cnt[i - 1];
    }
};

```

```

for (int i = n - 1; i >= 0; i--)
    p[--cnt[s[i]]] = i;
for (int i = 1; i < n; i++) {
    c[0][p[i]] = c[0][p[i - 1]];
    if (s[p[i]] != s[p[i - 1]]) c[0][p[i]]++;
}
for (int lg = 0, k = 1; k < n;
    k <= 1, lg++) {
    for (int i = 0; i < n; i++) {
        if ((pn[i] = p[i] - k) < 0) pn[i] += n;
    }
    cnt.assign(n, 0);
    for (int i = 0; i < n; i++)
        cnt[c[lg][pn[i]]]++;
    for (int i = 1; i < (int)cnt.size(); i++)
        cnt[i] += cnt[i - 1];
    for (int i = n - 1; i >= 0; i--)
        p[--cnt[c[lg][pn[i]]]] = pn[i];
    for (int l1, r1, l2, r2, i = 1; i < n;
        i++) {
        cn[p[i]] = cn[p[i - 1]];
        l1 = p[i - 1];
        l2 = p[i];
        if ((r1 = l1 + k) >= n) r1 -= n;
        if ((r2 = l2 + k) >= n) r2 -= n;
        if (c[lg][l1] != c[lg][l2] ||
            c[lg][r1] != c[lg][r2])
            cn[p[i]]++;
    }
    c[lg + 1] = cn;
}
p.erase(p.begin(), p.begin() + 1);
n--;
}

int get_lcp(int i, int j) {
    int res = 0;
    for (int lg = LOG - 1; lg >= 0; lg--) {
        if (i + (1 << lg) > n || j + (1 << lg) > n)
            continue;
        if (c[lg][i] == c[lg][j]) {
            i += (1 << lg);
            j += (1 << lg);
            res += (1 << lg);
        }
    }
    return res;
}
};

```

Suffix Automaton

```

const int ALPHSIZE = 26; // alphabet size
struct SuffixAutomaton {
    struct Node {
        int link, len;
        int next[ALPHSIZE];
        Node() {
            link = -1;
            len = 0;
            for (int i(0); i < ALPHSIZE; i++)
                next[i] = -1;
        }
    };
};

```

```

    }
};
string s;
vector<Node> sa;
int last;
SuffixAutomaton() {
    sa.emplace_back();
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    for (int i(0); i < ALPHASIZE; i++)
        sa[0].next[i] = -1;
}
void add(const int& c) {
    s.push_back(c + 'a');
    int cur = (int)sa.size();
    sa.emplace_back();
    sa[cur].len = sa[last].len + 1;
    int p;
    for (p = last; p != -1 && sa[p].next[c] ==
→ -1;
        p = sa[p].link) {
        sa[p].next[c] = cur;
    }
    if (p == -1) {
        sa[cur].link = 0;
    } else {
        int q = sa[p].next[c];
        if (sa[p].len + 1 == sa[q].len) {
            sa[cur].link = q;
        } else {
            int clone = (int)sa.size();
            sa.emplace_back();
            sa[clone].len = sa[p].len + 1;
            sa[clone].link = sa[q].link;
            for (int i(0); i < ALPHASIZE; i++)
                sa[clone].next[i] = sa[q].next[i];
            sa[cur].link = sa[q].link = clone;
            for (; p != -1 && sa[p].next[c] == q;
                p = sa[p].link) {
                sa[p].next[c] = clone;
            }
        }
    }
    last = cur;
}
};

```

LCP

```

vector<int> get_lcp(const string& s,
                   const vector<int>& suf) {
    int n = (int)suf.size();
    vector<int> back(n);
    for (int i = 0; i < n; i++)
        back[suf[i]] = i;
    vector<int> lcp(n - 1);
    for (int i = 0, k = 0; i < n; i++) {
        int x = back[i];
        k = max(0, k - 1);
        if (x == n - 1) {
            k = 0;

```

```

        continue;
    }
    while (s[suf[x] + k] == s[suf[x + 1] + k])
        k++;
    lcp[x] = k;
    }
    return lcp;
}

```

Manacker

```

pair<vector<int>, vector<int>>
manacker(const string& s) {
    // -> {d0, d1}. RUN test!
    int n = (int)s.size();
    vector<int> d0(n), d1(n);
    for (int l = 0, r = -1, i = 0; i < n; i++) {
        d1[i] =
            i <= r ? min(r - i, d1[l + r - i]) : 0;
        while (i >= d1[i] && i + d1[i] < n &&
            s[i - d1[i]] == s[i + d1[i]])
            d1[i]++;
        d1[i]--;
        if (i + d1[i] > r)
            l = i - d1[i], r = i + d1[i];
    }
    for (int l = 0, r = -1, i = 0; i < n; i++) {
        d0[i] =
            i < r ? min(r - i, d0[l + r - i - 1]) : 0;
        while (i >= d0[i] && i + d0[i] + 1 < n &&
            s[i - d0[i]] == s[i + d0[i] + 1])
            d0[i]++;
        if (d0[i] > 0 && i + d0[i] > r)
            l = i - d0[i] + 1, r = i + d0[i];
    }
    return {d0, d1};
}

```

Prefix Function

```

vector<int> get_pi(const string& s) {
    int n = (int)s.length();
    vector<int> pr(n);
    for (int i = 1; i < n; i++) {
        int k = pr[i - 1];
        while (k && s[k] != s[i])
            k = pr[k - 1];
        if (s[k] == s[i]) k++;
        pr[i] = k;
    }
    return pr;
}

```

Z-Function

```

vector<int> get_z(const string& s) {
    int n = (int)s.length();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {

```

```

    if (i < r) z[i] = min(r - i, z[i - 1]);
    while (i + z[i] < n && s[z[i]] == s[i +
→ z[i]])
        z[i]++;
    if (i + z[i] > r) l = i, r = i + z[i];
}
return z;
}

```

Tandem (Lorentz)

```

struct Tandem {
    int l, r, k;
    // [l, l + 2 * k) [l + 1, l + 1 + 2 * k) [l
    // + 2, l + 2 + 2 * k), ..., [r, r + 2 * k)
};
vector<int> z_func(const string& s) {
    int n = (int)s.size();
    vector<int> z(n);
    for (int l = 0, r = -1, i = 1; i < n; i++) {
        int k = i > r ? 0 : min(z[i - 1], r - i + 1);
        while (i + k < n && s[i + k] == s[k])
            k++;
        z[i] = k;
        if (i + k - 1 > r) {
            r = i + k - 1;
            l = i;
        }
    }
    return z;
}
const int SIZE = (1000006) * 30;
const int MAXL = (1000006) * 4;
Tandem tandems[SIZE], hlp[MAXL];
int tsz;
void rec(const string& s, int L, int R) {
    if (R - L + 1 <= 1) { return; }
    int M = (L + R) / 2;
    rec(s, L, M);
    rec(s, M + 1, R);
    int nu = M - L + 1;
    int nv = R - M;
    string vu =
        s.substr(M + 1, nv) + "#" + s.substr(L, nu);
    string urvr = vu;
    reverse(urvr.begin(), urvr.end());
    vector<int> z1 = z_func(urvr);
    vector<int> z2 = z_func(vu);
    for (int x = L; x <= R; x++) {
        if (x <= M) {
            int k = M + 1 - x;
            int k1 = L < x ? z1[nu - x + L] : 0;
            int k2 = z2[nv + 1 + x - L];
            int lsh = max(0, k - k2);
            int rsh = min(k1, k - 1);
            if (lsh <= rsh) {
                tandems[tsz++] = {x - rsh, x - lsh, k};
            }
        } else {
            int k = x - M;
            int k1 = x < R ? z2[x - M] : 0;
            int k2 = z1[nu + nv - x + M + 1];

```

```

        int lsh = max(1, k - k1);
        int rsh = min(k2, k - 1);
        if (lsh <= rsh) {
            tandems[tsz++] = {x - rsh + 1 - k,
                               x - lsh + 1 - k, k};
        }
    }
}
void compress() { // O(n*log(n)*log(n)) can be
                  // replace with count sort
                  // (O(n*log(n))) BE careful
→ with
                  // ML !!!
// O(n*log(n)) --> O(n)
sort(tandems, tandems + tsz,
    [](const Tandem& t1, const Tandem& t2) {
        return t1.k < t2.k ||
            (t1.k == t2.k && t1.l < t2.l);
    });
int hlp_sz = 0;
for (int i = 0; i < tsz; i++) {
    int j = i;
    while (j + 1 < tsz &&
        tandems[i].k == tandems[j + 1].k &&
        tandems[j].r + 1 == tandems[j + 1].l)
→ {
        j++;
    }
    hlp[hlp_sz++] = {tandems[i].l, tandems[j].r,
                    tandems[j].k};
    i = j;
}
memcpy(tandems, hlp, sizeof(Tandem) * hlp_sz);
tsz = hlp_sz;
}
void main_lorentz(const string& s) {
    // n = 10^6 time = 1.8 sec MEM = nlog(n) * 12
    // bytes
    int n = (int)s.size();
    tsz = 0;
    rec(s, 0, n - 1);
    compress();
}

```

Aho-Corasick

```

const int A = 300; // alphabet size
struct Aho {
    struct Node {
        int nxt[A], go[A];
        int par, pch, link;
        int good;
        Node()
            : par(-1), pch(-1), link(-1), good(-1) {
                fill(nxt, nxt + A, -1);
                fill(go, go + A, -1);
            }
    };
    vec<Node> a;
    Aho() { a.push_back(Node()); }
}

```

```

void add_string(const string& s) {
    int v = 0;
    for (char c : s) {
        if (a[v].nxt[c] == -1) {
            a[v].nxt[c] = (int)a.size();
            a.push_back(Node());
            a.back().par = v;
            a.back().pch = c;
        }
        v = a[v].nxt[c];
    }
    a[v].good = 1;
}

int go(int v, int c) {
    if (a[v].go[c] == -1) {
        if (a[v].nxt[c] != -1) {
            a[v].go[c] = a[v].nxt[c];
        } else {
            a[v].go[c] = v ? go(get_link(v), c) : 0;
        }
    }
    return a[v].go[c];
}

int get_link(int v) {
    if (a[v].link == -1) {
        if (!v || !a[v].par)
            a[v].link = 0;
        else
            a[v].link =
                go(get_link(a[v].par), a[v].pch);
    }
    return a[v].link;
}

bool is_good(int v) {
    if (!v) return false;
    if (a[v].good == -1) {
        a[v].good = is_good(get_link(v));
    }
    return a[v].good;
}

bool is_there_substring(const string& s) {
    int v = 0;
    for (char c : s) {
        v = go(v, c);
        if (is_good(v)) { return true; }
    }
    return false;
}
};

```

Eertree

```

const int N = 2e6 + 5;
struct EerTree {
    char s[N];
    int n;
    int sz;
    int link[N];
    int len[N];
    map<char, int> nxt[N];
    int diff[N];
    int dp[N][2];

```

```

    int slink[N];
    int max_suff;
    int ans[N]; // number of partitions into
                // palindromes of even length
    void clr() {
        fill(s, s + N, 0);
        fill(link, link + N, 0);
        fill(len, len + N, 0);
        fill(nxt, nxt + N, map<char, int>());
        fill(diff, diff + N, 0);
        fill((int*)dp, (int*)dp + N * 2, 0);
        fill(slink, slink + N, 0);
        n = 0;
        sz = 0;
        max_suff = 0;
        fill(ans, ans + N, 0);
    }
    EerTree() {
        clr();
        s[0] = '#'; // not in alphabet
        link[0] = 1;
        link[1] = 0;
        len[0] = -1;
        sz = 2;
        ans[0] = 1;
    }
    int get_link(int from) {
        while (s[n] != s[n - len[from] - 1]) {
            from = link[from];
        }
        return from;
    }
    void add_symbol(char c) {
        s[++n] = c;
        max_suff = get_link(max_suff);
        if (!nxt[max_suff].count(c)) {
            int x = get_link(link[max_suff]);
            link[sz] =
                nxt[x].count(c) ? nxt[x][c] : 1;
        }
        len[sz] = len[max_suff] + 2;
        diff[sz] = len[sz] - len[link[sz]];
        slink[sz] = diff[sz] == diff[link[sz]]
            ? slink[link[sz]]
            : link[sz];
        nxt[max_suff][c] = sz++;
    }
    max_suff = nxt[max_suff][c];
    for (int x = max_suff; len[x] > 0;
        x = slink[x]) {
        dp[x][0] = dp[x][1] = 0;
        int j = n - (len[slink[x]] + diff[x]);
        _inc(dp[x][j & 1], ans[j]);
        if (diff[x] == diff[link[x]]) {
            _inc(dp[x][0], dp[link[x]][0]);
            _inc(dp[x][1], dp[link[x]][1]);
        }
        _inc(ans[n], dp[x][n & 1]);
    }
}
};

```

Components of Vertex Duality

```

struct Edge {
    int fr, to, id;
    int get(int v) { return v == fr ? to : fr; }
};

void dfs(const vector<vector<Edge>>& g,
        vector<int>& fup, vector<int>& tin,
        vector<int>& used, int& timer, int v,
        int par = -1) {
    tin[v] = fup[v] = timer++;
    used[v] = 1;
    for (Edge e : g[v]) {
        int to = e.get(v);
        if (to == par) continue;
        if (used[to]) {
            fup[v] = min(fup[v], tin[to]);
        } else {
            dfs(g, fup, tin, used, timer, to, v);
            fup[v] = min(fup[v], fup[to]);
        }
    }
}

void paintEdges(const vector<vector<Edge>>& g,
               vector<int>& fup,
               vector<int>& tin,
               vector<int>& used,
               vector<int>& colors, int v,
               int curColor, int& maxColor,
               int par = -1) {
    used[v] = 1;
    for (Edge e : g[v]) {
        int to = e.get(v);
        if (to == par) continue;
        if (!used[to]) {
            if (tin[v] <= fup[to]) {
                int tmpColor = maxColor++;
                colors[e.id] = tmpColor;
                paintEdges(g, fup, tin, used, colors, to,
                           tmpColor, maxColor, v);
            } else {
                colors[e.id] = curColor;
                paintEdges(g, fup, tin, used, colors, to,
                           curColor, maxColor, v);
            }
        } else if (tin[to] < tin[v]) {
            colors[e.id] = curColor;
        }
    }
}

vector<vector<Edge>>
get2components(const vector<vector<Edge>>& g,
               int m, const vector<Edge>& es) {
    int n = (int)g.size();
    vector<int> fup(n), tin(n), used(n);
    vector<int> colors(m);
    int timer;
    used.assign(n, 0);
    timer = 0;
    for (int v = 0; v < n; v++) {
        if (used[v]) continue;
        dfs(g, fup, tin, used, timer, v);
    }
}

```

```

used.assign(n, 0);
timer = 0;
for (int v = 0; v < n; v++) {
    if (used[v]) continue;
    paintEdges(g, fup, tin, used, colors, v,
               timer, timer, -1);
}
vector<vector<Edge>> res(timer);
for (int i = 0; i < m; i++) {
    res[colors[i]].push_back(es[i]);
}
return res;
}

```

Hungarian Algorithm

```

vector<int>
Hungarian(const vector<vector<int>>&
          a) { // ALARM: INT everywhere
    int n = (int)a.size();
    vector<int> row(n), col(n), pair(n, -1),
               back(n, -1), prev(n, -1);
    auto get = [&](int i, int j) {
        return a[i][j] + row[i] + col[j];
    };
    for (int v = 0; v < n; v++) {
        vector<int> min_v(n, v), A_plus(n),
        → B_plus(n);
        A_plus[v] = 1;
        int jb;
        while (true) {
            int pos_i = -1, pos_j = -1;
            for (int j = 0; j < n; j++) {
                if (!B_plus[j] && (pos_i == -1 ||
                                   get(min_v[j], j) <
                                   get(pos_i, pos_j)))
                    → {
                        pos_i = min_v[j], pos_j = j;
                    }
            }
            int weight = get(pos_i, pos_j);
            for (int i = 0; i < n; i++)
                if (!A_plus[i]) row[i] += weight;
            for (int j = 0; j < n; j++)
                if (!B_plus[j]) col[j] -= weight;
            B_plus[pos_j] = 1, prev[pos_j] = pos_i;
            int x = back[pos_j];
            if (x == -1) {
                jb = pos_j;
                break;
            }
            A_plus[x] = 1;
            for (int j = 0; j < n; j++)
                if (get(x, j) < get(min_v[j], j))
                    min_v[j] = x;
        }
        while (jb != -1) {
            back[jb] = prev[jb];
            swap(pair[prev[jb]], jb);
        }
    }
}

```

```

    return pair;
}

```

General Matching

```

struct GeneralMatching { // O(n^3)
    int n = 0, cc = 10; // [0, n)
    vector<vector<int>> g; // undirected
    vector<int> mt, used, base, p, color;
    queue<int> q;
    GeneralMatching(int nn)
        : n(nn), mt(n, -1), used(n), base(n), p(n),
          color(n), g(n) {}
    void add_edge(int u, int v) {
        g[u].push_back(v), g[v].push_back(u);
    }
    void add(int v) {
        if (!used[v]) used[v] = 1, q.push(v);
    }
    int get_lca(int u, int v) {
        cc++;
        while (1) {
            u = base[u], color[u] = cc;
            if (mt[u] == -1) break;
            u = p[mt[u]];
        }
        while (1) {
            v = base[v];
            if (color[v] == cc) break;
            v = p[mt[v]];
        }
        return v;
    }
    void mark_path(int v, int child, int b) {
        while (base[v] != b) {
            color[base[v]] = color[base[mt[v]]] = cc;
            p[v] = child, child = mt[v], v = p[child];
        }
    }
    int bfs(int root) {
        add(root);
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int to : g[v]) {
                if (base[v] == base[to] || mt[v] == to)
                    continue;
                else if (used[to]) {
                    int w = get_lca(v, to);
                    cc++, mark_path(v, to, w),
                        mark_path(to, v, w);
                    for (int i = 0; i < n; i++)
                        if (color[base[i]] == cc)
                            base[i] = w, add(i);
                } else if (p[to] == -1) {
                    p[to] = v;
                    if (mt[to] == -1) return to;
                    add(mt[to]);
                }
            }
        }
        return -1;
    }
};

```

```

}
void xor_path(int v) {
    while (v != -1) {
        int pv = p[v], ppv = mt[pv];
        mt[v] = pv, mt[pv] = v;
        v = ppv;
    }
}
bool inc(int root) {
    used.assign(n, 0), p.assign(n, -1),
        iota(base.begin(), base.end(), 0);
    while (!q.empty())
        q.pop();
    int v = bfs(root);
    if (v == -1) return false;
    xor_path(v);
    return true;
}
void match() {
    for (int i = 0; i < n; i++)
        if (mt[i] == -1) inc(i);
}
};

```

Hopcroft-Karp

```

struct HopcroftKarp {
    int n, m;
    vec<vec<int>> g;
    vec<int> pl, pr, dist;
    vec<bool> vis;
    HopcroftKarp() : n(0), m(0) {}
    HopcroftKarp(int _n, int _m) : n(_n), m(_m) {
        g.resize(n);
    }
    void add_edge(int u, int v) {
        g[u].push_back(v);
    }
    bool bfs() {
        dist.assign(n + 1, inf);
        queue<int> q;
        for (int u = 0; u < n; u++) {
            if (pl[u] < m) continue;
            dist[u] = 0;
            q.push(u);
        }
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            if (dist[u] >= dist[n]) continue;
            for (int v : g[u]) {
                if (dist[pr[v]] > dist[u] + 1) {
                    dist[pr[v]] = dist[u] + 1;
                    q.push(pr[v]);
                }
            }
        }
        return dist[n] < inf;
    }
    bool dfs(int v) {
        if (v == n) return 1;
    }
};

```



```

vis[v] = true;
for (int to : g[v]) {
    if (dist[pr[to]] != dist[v] + 1) continue;
    if (vis[pr[to]]) continue;
    if (!dfs(pr[to])) continue;
    pl[v] = to;
    pr[to] = v;
    return 1;
}
return 0;
}
int find_max_matching() {
    pl.resize(n, m);
    pr.resize(m, n);
    int result = 0;
    while (bfs()) {
        vis.assign(n + 1, false);
        for (int u = 0; u < n; u++) {
            if (pl[u] < m) continue;
            if (vis[u]) continue;
            result += dfs(u);
        }
    }
    return result;
}
};

```

Dinic

```

struct Dinic {
    struct Edge {
        int fr, to, cp, id, fl;
    };
    int n, S, T;
    vector<Edge> es;
    vector<vector<int>> g;
    vector<int> dist, res, ptr, used;
    Dinic(int n_, int S_, int T_)
        : n(n_), S(S_), T(T_) {
        g.resize(n);
    }
    void add_edge(int fr, int to, int cp, int id) {
        g[fr].push_back((int)es.size());
        es.push_back({fr, to, cp, id, 0});
        g[to].push_back((int)es.size());
        es.push_back({to, fr, 0, -1, 0});
    }
    bool bfs(int K) {
        dist.assign(n, inf);
        dist[S] = 0;
        queue<int> q;
        q.push(S);
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int ps : g[v]) {
                Edge& e = es[ps];
                if (e.fl + K > e.cp) continue;
                if (dist[e.to] > dist[e.fr] + 1) {
                    dist[e.to] = dist[e.fr] + 1;
                    q.push(e.to);
                }
            }
        }
    }
};

```

```

    }
    return dist[T] < inf;
}
int dfs(int v, int _push = INT_MAX) {
    if (v == T || !_push) return _push;
    for (int& iter = ptr[v];
         iter < (int)g[v].size(); iter++) {
        int ps = g[v][ptr[v]];
        Edge& e = es[ps];
        if (dist[e.to] != dist[e.fr] + 1) continue;
        int tmp =
            dfs(e.to, min(_push, e.cp - e.fl));
        if (tmp) {
            e.fl += tmp;
            es[ps ^ 1].fl -= tmp;
            return tmp;
        }
    }
    return 0;
}
ll find_max_flow() {
    ptr.resize(n);
    ll max_flow = 0, add_flow;
    for (int K = 1 << 30; K > 0; K >= 1) {
        while (bfs(K)) {
            ptr.assign(n, 0);
            while ((add_flow = dfs(S))) {
                max_flow += add_flow;
            }
        }
    }
    return max_flow;
}
void assign_result() {
    res.resize(es.size());
    for (Edge e : es)
        if (e.id != -1) res[e.id] = e.fl;
}
int get_flow(int id) { return res[id]; }
bool go(int v, vector<int>& F,
        vector<int>& path) {
    if (v == T) return 1;
    if (used[v]) return 0; used[v] = 1;
    for (int ps : g[v]) {
        if (F[ps] <= 0) continue;
        if (go(es[ps].to, F, path)) {
            path.push_back(ps);
            return 1;
        }
    }
    return 0;
}
vector<pair<int, vector<int>>> decomposition()
→ {
    find_max_flow();
    vector<int> F((int)es.size()), path, add;
    vector<pair<int, vector<int>>> dcmp;
    for (int i = 0; i < (int)es.size(); i++)
        F[i] = es[i].fl;
    used.assign(n, 0);
    while (go(S, F, path)) {
        used.assign(n, 0);
        int mn = INT_MAX;
        for (int ps : path)

```

```

    mn = min(mn, F[ps]);
    for (int ps : path)
        F[ps] -= mn;
    for (int ps : path)
        add.push_back(es[ps].id);
    reverse(add.begin(), add.end());
    dcmp.push_back({mn, add});
    add.clear();
    path.clear();
}
return dcmp;
}
};

```

MCMF

```

struct MCMF {
    struct Edge {
        int fr, to, cp, fl, cs, id;
    };
    int n, S, T;
    vec<Edge> es;
    vec<vec<int>> g;
    vec<ll> dist, phi;
    vec<int> from;
    MCMF(int _n, int _S, int _T)
        : n(_n), S(_S), T(_T) {
        g.resize(n);
    }
    void add_edge(int fr, int to, int cp, int cs,
        int id) {
        g[fr].push_back((int)es.size());
        es.push_back({fr, to, cp, 0, cs, id});
        g[to].push_back((int)es.size());
        es.push_back({to, fr, 0, 0, -cs, -1});
    }
    void init_phi() {
        dist.assign(n, LLONG_MAX);
        dist[S] = 0;
        for (int any, iter = 0; iter < n - 1;
            iter++) { // Ford Bellman
            any = 0;
            for (Edge e : es) {
                if (e.fl == e.cp) continue;
                if (dist[e.to] - dist[e.fr] > e.cs) {
                    dist[e.to] = dist[e.fr] + e.cs;
                    any = 1;
                }
            }
        }
        if (!any) break;
    }
    phi = dist;
}
bool Dijkstra() {
    dist.assign(n, LLONG_MAX);
    from.assign(n, -1);
    dist[S] = 0;
    priority_queue<pair<ll, int>,
        vec<pair<ll, int>>,
        greater<pair<ll, int>>>
        pq;
    pq.push({dist[S], S});
}

```

```

while (!pq.empty()) {
    int v;
    ll di;
    tie(di, v) = pq.top();
    pq.pop();
    if (di != dist[v]) continue;
    for (int ps : g[v]) {
        Edge& e = es[ps];
        if (e.fl == e.cp) continue;
        if (dist[e.to] - dist[e.fr] >
            e.cs + phi[e.fr] - phi[e.to]) {
            dist[e.to] = dist[e.fr] + e.cs +
                phi[e.fr] - phi[e.to];
            from[e.to] = ps;
            pq.push({dist[e.to], e.to});
        }
    }
}
for (int v = 0; v < n; v++) {
    phi[v] += dist[v];
}
return dist[T] < LLONG_MAX;
}
pll find_mcmf() {
    init_phi();
    ll flow = 0, cost = 0;
    while (Dijkstra()) {
        int mn = INT_MAX;
        for (int v = T; v != S;
            v = es[from[v]].fr) {
            mn = min(mn,
                es[from[v]].cp -
→ es[from[v]].fl);
        }
        flow += mn;
        for (int v = T; v != S;
            v = es[from[v]].fr) {
            es[from[v]].fl += mn;
            es[from[v] ^ 1].fl -= mn;
        }
    }
    for (Edge& e : es) {
        if (e.fl >= 0) cost += 1ll * e.fl * e.cs;
    }
    return make_pair(flow, cost);
}
bool go(int v, vec<int>& F, vec<int>& path,
    vec<int>& used) {
    if (used[v]) return 0;
    used[v] = 1;
    if (v == T) return 1;
    for (int ps : g[v]) {
        if (F[ps] <= 0) continue;
        if (go(es[ps].to, F, path, used)) {
            path.push_back(ps);
            return 1;
        }
    }
    return 0;
}
vec<pair<int, vec<int>>>
decomposition(ll& _flow, ll& _cost) {
}

```

```

tie(_flow, _cost) = find_mcmf();
vec<int> F((int)es.size()), path, add,
    used(n);
vec<pair<int, vec<int>>> dcmp;
for (int i = 0; i < (int)es.size(); i++)
    F[i] = es[i].fl;
while (go(S, F, path, used)) {
    used.assign(n, 0);
    int mn = INT_MAX;
    for (int ps : path)
        mn = min(mn, F[ps]);
    for (int ps : path)
        F[ps] -= mn;
    for (int ps : path)
        add.push_back(es[ps].id);
    reverse(ALL(add));
    dcmp.push_back({mn, add});
    add.clear();
    path.clear();
}
return dcmp;
}
};

```

Algorithm of Two Chinese

```

struct Edge {
    int fr, to, w, id;
    bool operator<(const Edge& o) const {
        return w < o.w;
    }
};
// find oriented mst (tree)
// there are no edge --> root (root is 0)
// 0 .. n - 1, weights and vertices will be
// changed, but ids are ok
vector<Edge>
work(const vector<vector<Edge>>& graph) {
    int n = (int)graph.size();
    vector<int> color(n), used(n, -1);
    for (int i = 0; i < n; i++)
        color[i] = i;
    vector<Edge> e(n);
    for (int i = 0; i < n; i++) {
        if (graph[i].empty()) {
            e[i] = {-1, -1, -1, -1};
        } else {
            e[i] = *min_element(graph[i].begin(),
                                graph[i].end());
        }
    }
    vector<vector<int>> cycles;
    used[0] = -2;
    for (int s = 0; s < n; s++) {
        if (used[s] != -1) continue;
        int x = s;
        while (used[x] == -1) {
            used[x] = s;
            x = e[x].fr;
        }
        if (used[x] != s) continue;
        vector<int> cycle = {x};
    }
}

```

```

for (int y = e[x].fr; y != x; y = e[y].fr)
    cycle.push_back(y), color[y] = x;
cycles.push_back(cycle);
}
if (cycles.empty()) return e;
vector<vector<Edge>> next_graph(n);
for (int s = 0; s < n; s++) {
    for (const Edge& edge : graph[s]) {
        if (color[edge.fr] != color[s])
            next_graph[color[s]].push_back(
                {color[edge.fr], color[s],
                 edge.w - e[s].w, edge.id});
    }
}
vector<Edge> tree = work(next_graph);
for (const auto& cycle : cycles) {
    int c1 = color[cycle[0]];
    Edge next_out = tree[c1], out{};
    int from = -1;
    for (int v : cycle) {
        tree[v] = e[v];
        for (const Edge& edge : graph[v])
            if (edge.id == next_out.id)
                from = v, out = edge;
    }
    tree[from] = out;
}
return tree;
}

```

Dominator Tree

```

struct Edge {
    int fr = -1;
    int to = -1;
    int id = -1;
};
struct DSU {
    int n = 0; // [0, n)
    vector<int> p, mn;
    DSU() = default;
    DSU(int nn) {
        n = nn;
        p.resize(n);
        mn.resize(n, inf);
        for (int v = 0; v < n; v++)
            p[v] = v;
    }
    void set_value(int v, int x) { mn[v] = x; }
    int find(int v) {
        if (p[v] == v) return v;
        int pv = find(p[v]);
        mn[v] = min(mn[v], mn[p[v]]);
        p[v] = pv;
        return pv;
    }
    void merge(int P, int S) { p[S] = P; }
};
struct DominatorTree {
    int n = 0; // [0, n)
    vector<Edge> edges;
}

```

```

vector<vector<int>> g, gr;
vector<int> used, tin, sdom, idom, order,
→ depth;
DSU dsu;
vector<vector<int>> cost, parent;
DominatorTree() = default;
DominatorTree(int nn) { n = nn; }
void add_edge(Edge e) { edges.push_back(e); }
void dfs(int v) {
    used[v] = 1;
    tin[v] = (int)order.size();
    order.push_back(v);
    for (int eid : g[v]) {
        const auto& e = edges[eid];
        if (!used[e.to]) {
            depth[e.to] = depth[v] + 1;
            parent[0][e.to] = v;
            dfs(e.to);
        }
    }
}
void init_binary_jumps() {
    int LOG = 0;
    while ((1 << LOG) < n)
        LOG++;
    cost.resize(LOG, vector<int>(n, inf));
    parent.resize(LOG, vector<int>(n, -1));
}
void build_sdom(int s) {
    used.assign(n, 0);
    tin.assign(n, 0);
    depth.assign(n, 0);
    order.clear();
    dfs(s);
    sdom.assign(n, inf);
    idom.assign(n, inf);
    dsu = DSU(n);
    for (int it = (int)order.size() - 1; it >= 0;
        it--) {
        int v = order[it];
        for (int eid : gr[v]) {
            const auto& e = edges[eid];
            if (!used[e.fr]) continue;
            sdom[v] = min(sdom[v], tin[e.fr]);
            if (tin[e.fr] > tin[v]) {
                dsu.find(e.fr);
                sdom[v] = min(sdom[v], dsu.mn[e.fr]);
            }
        }
        dsu.set_value(v, sdom[v]);
        for (int eid : g[v]) {
            const auto& e = edges[eid];
            if (parent[0][e.to] == v) {
                dsu.merge(v, e.to);
            }
        }
    }
}
int get_min_on_path(int P, int S) {
    int res = inf;
    for (int j = (int)cost.size() - 1; j >= 0;
        j--) {
        int pS = parent[j][S];

```

```

        if (pS == -1 || depth[pS] < depth[P])
            continue;
        res = min(res, cost[j][S]);
        S = pS;
    }
    return res;
}
void set_value(int v, int x) {
    cost[0][v] = x;
    for (int j = 1; j < (int)cost.size(); j++) {
        int pv = parent[j - 1][v];
        if (pv == -1) {
            cost[j][v] = cost[j - 1][v];
            parent[j][v] = pv;
        } else {
            cost[j][v] =
                min(cost[j - 1][v], cost[j - 1][pv]);
            parent[j][v] = parent[j - 1][pv];
        }
    }
}
void build_idom(int s) {
    for (int v : order) {
        if (v == s) continue;
        idom[v] = min(
            sdom[v], get_min_on_path(order[sdom[v]],
                                     parent[0][v]));
        set_value(v, idom[v]);
    }
}
void build(int s) {
    init_binary_jumps();
    g.clear();
    g.resize(n);
    gr.clear();
    gr.resize(n);
    for (int i = 0; i < (int)edges.size(); i++) {
        const auto& e = edges[i];
        g[e.fr].push_back(i);
        gr[e.to].push_back(i);
    }
    build_sdom(s);
    build_idom(s);
}
};

```

Factorization

```

__extension__ typedef __int128 int128;
constexpr ll mmul(ll a, ll b, ll mod) {
    return (static_cast<int128>(a) * b) % mod;
}
template <typename T, typename K>
constexpr T mpow(T a, K n, T mod) {
    T res = 1;
    for (T now = a; n;
        n >= 1, now = mmul(now, now, mod)) {
        if (n & 1) res = mmul(res, now, mod);
    }
    return res;
}

```

```

inline bool miller_rabin_det(ll n) {
    static const ll bases[] = {
        2, 3, 5, 7, 11,
        13, 17, 19, 23}; // works for n < 3.8e18
    // static const int bases[] = {2, 3, 5, 7, 11,
    // 13, 17, 19, 23, 29, 31, 37}; // n < 3.1e23
    if (n <= 2) return (n == 2);
    if (n % 2 == 0) return false;
    ll d = n - 1;
    while (!(d & 1))
        d >>= 1;
    for (ll a : bases) {
        if (a == n) return true;
        a = mpow(a, d, n);
        if (a == 1) continue;
        for (ll nd = d; nd != n - 1 && a != n - 1;
            nd <<= 1)
            a = mmul(a, a, n);
        if (a != n - 1) return false;
    }
    return true;
}

inline ll pollard(ll n) {
    static std::mt19937_64 gen;
    static const int LOG = 50;
    if (n <= 1 || miller_rabin_det(n)) return 1;
    if (!(n & 1)) return 2;
    auto f = [n](ll x) {
        return mmul(x, x, n) + 1;
    }; // it is ok if 0 == n
    for (int st = 2, lg = 0;; st = gen() % n) {
        ll cur = 1;
        for (ll x = st, y = f(st); x != y;
            x = f(x), y = f(f(y))) {
            if (ll c = mmul(cur, std::abs(x - y), n);
                ↪ c)
                cur = c;
            if (lg++ == LOG) {
                lg = 0;
                if (ll val = std::__gcd(cur, n); val !=
                    ↪ 1)
                    return val;
            }
        }
    }
    return 1;
}

```

```

ll SUB(ll a, ll b, ll mod) {
    return a >= b ? a - b : a - b + mod;
}

ll BINPOW(ll x, ll pw, ll mod) {
    if (x == 0) return 0;
    ll res = 1 % mod, tmp = x;
    while (pw > 0) {
        if (pw & 1) res = MUL(res, tmp, mod);
        pw >>= 1;
        tmp = MUL(tmp, tmp, mod);
    }
    return res;
}

ll DIV(ll a, ll b, ll mod) {
    return MUL(a, BINPOW(b, mod - 2, mod), mod);
}

ll find_sqrt_by_mod(
    ll p,
    ll A) { // x^2 = a (mod p), x = ?, p is prime
    assert(0ll <= A && A < p);
    if (A == 0 || p == 2) return A;
    if (BINPOW(A, (p - 1) / 2, p) != 1) return
        ↪ -1ll;
    static mt19937_64 GEN(42);
    auto mult = [&](pll p1, pll p2) -> pll {
        auto [a, b] = p1;
        auto [c, d] = p2;
        ll k1 = SUM(MUL(a, d, p), MUL(b, c, p), p);
        ll k2 = SUM(MUL(MUL(a, c, p), A, p),
            MUL(b, d, p), p);
        return {k1, k2};
    };
    while (1) {
        ll i = GEN() % (p - 1) + 1;
        ll pw = (p - 1) / 2;
        pll res = {0, 1}, tmp = {1, i};
        while (pw > 0) {
            if (pw & 1) res = mult(res, tmp);
            pw >>= 1;
            tmp = mult(tmp, tmp);
        }
        if (res.first == 0) continue;
        res.second = SUB(res.second, 1, p);
        ll x = DIV(res.second, res.first, p);
        if (MUL(x, x, p) == A) return x;
    }
}

```

Square Root in \mathbb{Z}_p in $O(\log p)$

```

ll MUL(ll a, ll b, ll mod) {
    static __int128 xa = 1;
    static __int128 xb = 1;
    static __int128 xm = 1;
    xa = a;
    xb = b;
    xm = mod;
    return ll((xa * xb) % xm);
}

ll SUM(ll a, ll b, ll mod) {
    return a + b < mod ? a + b : a + b - mod;
}

```

Euclid (??)

```

ll rec(ll pos, ll left_len, ll left_cost,
    ll right_len, ll right_cost, ll k) {
    if (!k || !right_len) return pos;
    if (pos >= right_len) {
        ll t = (left_len - pos + right_len - 1) /
            right_len;
        if (t * right_cost + left_cost > k)
            return pos;
        pos += t * right_len - left_len;
        k -= (t * right_cost + left_cost);
    }
}

```

```

ll nxt_left_len = left_len % right_len;
ll nxt_left_cost =
    (left_len / right_len) * right_cost +
    left_cost;
if (nxt_left_len == 0) return pos;
{
    ll t = pos / nxt_left_len;
    if (t * nxt_left_cost > k)
        return pos -
            nxt_left_len * (k / nxt_left_cost);
    k -= t * nxt_left_cost;
    pos -= t * nxt_left_len;
}
return rec(pos, nxt_left_len, nxt_left_cost,
    right_len % nxt_left_len,
    (right_len / nxt_left_len) *
        nxt_left_cost +
        right_cost,
    k);
}
// finds (nw_st + step * x) % mod --> min, 0 <= x
// <= bound
ll euclid(ll nw_st, ll step, ll mod, ll bound) {
    return rec(nw_st, mod, 0, step, 1, bound);
}

```

Primes on Segment

```

vector<int> find_sum_of_primes(ll N) {
    // sum_{i=1}^{n} [i \in primes] * i
    // n in {N // k, 1 <= k <= N}
    // can be generalize to sum of i^T
    // O(n^{3/4}/log(N))
    vector<ll> vals;
    for (ll x = 1; x < N; x = N / (N / (x + 1)))
        vals.push_back(N / x);
    reverse(vals.begin(), vals.end());
    int sz = (int) vals.size();
    vector<int> S(sz), nx(sz);
    for (int i = 0; i < sz; i++) {
        ll n = vals[i];
        // (2 + n) * (n - 1) / 2 = 2 + 3 + ... +
        S[i] = n % 2 ?
            mul(((n - 1) / 2) % mod, (n + 2)
        % mod) :
            mul(((n + 2) / 2) % mod, (n - 1)
        % mod);
    }
    unordered_map<ll, int> pos_hm;
    for (int i = 0; i < sz; i++)
        pos_hm[vals[i]] = i;
    for (int fr = 0, j = 1;; j++) {
        int any = 0;
        for (int i = fr; i < sz; i++) {
            ll n = vals[i];
            ll need = n / ps[j];
            auto fnd = pos_hm.find(need);
            int pos = fnd == pos_hm.end() ? -1 :
            fnd->second;
            nx[i] = S[i];
            if (pos < 0 || need < ps[j - 1]) {

```

```

                fr++;
                continue;
            }
            any = 1;
            int X = sub(S[pos], pr_sum[ps[j] -
            1]);
            dec(nx[i], mul(ps[j], X));
        }
        swap(S, nx);
        if (!any)
            break;
    }
    return S;
}

```

Pro Euclid

```

// ALL in Z-ring
// T, k > 0 && return (T - k) + (T - 2 * k) + ...
// last, last > 0
ll f(ll T, ll k) {
    ll cnt = T / k;
    return T * cnt - k * cnt * (cnt + 1) / 2;
}
// A, B, C > 0
// |{(x, y): x, y > 0 && Ax + By <= C}|
ll count_triangle(ll A, ll B, ll C) {
    if (A + B > C) return 0;
    if (A > B) swap(A, B);
    ll k = B / A;
    return f(k * C / B, k) +
        count_triangle(A, B - A * k,
            C - A * (k * C / B));
}
// A, B, C, cx, cy > 0
// |{(x, y) : 1 <= x <= cx && 1 <= y <= cy && Ax +
// By <= C}|
ll count_solutions(ll A, ll B, ll C, ll cx,
    ll cy) {
    assert(A > 0);
    assert(B > 0);
    if (C <= 0 || cx <= 0 || cy <= 0) return 0;
    if (A * cx + B * cy <= C) return cx * cy;
    if (cx >= C / A && cy >= C / B)
        return count_triangle(A, B, C);
    return count_triangle(A, B, C) -
        count_triangle(A, B, C - B * cy) -
        count_triangle(A, B, C - A * cx);
}

```

FFT with prime mod

```

template<int mod, int root, int LOG>
struct FFT {
    // const int mod = 998244353;
    // const int root = 31;
    // const int LOG = 23;
    vector<int> G[LOG + 1];
    vector<int> rev[LOG + 1];
    FFT() {

```

```

    for (int start = root, lvl = LOG; lvl >=
→ 0; lvl--, start = mul(start, start)) {
        int tot = 1 << lvl;
        G[lvl].resize(tot);
        for (int cur = 1, i = 0; i < tot;
→ i++, cur = mul(cur, start)) {
            G[lvl][i] = cur;
        }
    }
    for (int lvl = 1; lvl <= LOG; lvl++) {
        int tot = 1 << lvl;
        rev[lvl].resize(tot);
        for (int i = 1; i < tot; i++) {
            rev[lvl][i] = ((i & 1) << (lvl -
→ 1)) | (rev[lvl][i >> 1] >> 1);
        }
    }
}
void fft(vector<int>& a, int sz, bool invert)
→ {
    int n = 1 << sz;
    for (int j, i = 0; i < n; i++) {
        if ((j = rev[sz][i]) < i)
            swap(a[i], a[j]);
    }
    for (int f1, f2, lvl = 0, len = 1; len <
→ n; len <= 1, lvl++) {
        for (int i = 0; i < n; i += (len <<
→ 1)) {
            for (int j = 0; j < len; j++) {
                f1 = a[i + j];
                f2 = mul(a[i + j + len],
→ G[lvl + 1][j]);
                a[i + j] = sum(f1, f2);
                a[i + j + len] = sub(f1, f2);
            }
        }
    }
    if (invert) {
        reverse(a.begin() + 1, a.end());
        int rn = binpow(n, mod - 2);
        for (int i = 0; i < n; i++) {
            a[i] = mul(a[i], rn);
        }
    }
}
vector<int> multiply(const vector<int>& a,
→ const vector<int>& b) {
    vector<int> fa(a.begin(), a.end());
    vector<int> fb(b.begin(), b.end());
    int n = (int) a.size();
    int m = (int) b.size();
    int maxnm = max(n, m), sz = 0;
    while ((1 << sz) < maxnm)
        sz++;
    sz++;
    fa.resize(1 << sz);
    fb.resize(1 << sz);
    fft(fa, sz, false);
    fft(fb, sz, false);
    int SZ = 1 << sz;
    for (int i = 0; i < SZ; i++) {
        fa[i] = mul(fa[i], fb[i]);

```

```

    }
    fft(fa, sz, true);
    return fa;
}
int sum(int x, int y) {
    return x + y < mod ? x + y : x + y - mod;
}
int sub(int x, int y) {
    return x >= y ? x - y : x - y + mod;
}
int mul(int x, int y) {
    return (1ll * x * y) % mod;
}
int mul(const vector<int>& a) {
    int res = 1;
    for (const auto& x : a)
        res = mul(res, x);
    return res;
}
void inc(int& x, int y) {
    if ((x += y) >= mod)
        x -= mod;
}
void dec(int& x, int y) {
    if ((x -= y) < 0)
        x += mod;
}
int binpow(int x, int pw) {
    int res = 1, tmp = x;
    while (pw > 0) {
        if (pw & 1) res = mul(res, tmp);
        tmp = mul(tmp, tmp);
        pw >>= 1;
    }
    return res;
}
};

```

Polynomial Division

```

// let A = series and A[0] != 0 in Z/pZ, p is
// prime finds (A^{-1}) % x^n
vector<int>
series_inverse(const vector<int>& series, int n,
               ll p) {
    vector<int> current = {_div(1, series[0], p)};
    vector<int> A = {};
    int l = 0;
    while ((int)current.size() < n) {
        while (1 < 2 * (int)current.size()) {
            A.push_back(
                1 < (int)series.size() ? series[l] : 0);
            l++;
        }
        vector<int> next = multiply(A, current);
        for (int& x : next)
            x = (-x % p + p) % p;
        next[0] = _sum(2 % p, next[0], p);
        next = multiply(next, current);
        for (int& x : next)
            x = (x % p + p) % p;
    }
}

```



```

    next.resize(2 * current.size());
    current = next;
}
current.resize(n);
return current;
}
// calculates a / b
vector<int> division(const vector<int>& a,
                    const vector<int>& b,
                    int p) {
    int n = (int)a.size() - 1; // deg(a)
    int m = (int)b.size() - 1; // deg(b)
    if (n < m) { return {0}; }
    vector<int> ar = a, br = b;
    reverse(ar.begin(), ar.end());
    reverse(br.begin(), br.end());
    ar.resize(n - m + 1);
    br.resize(n - m + 1);
    vector<int> qr =
        series_inverse(br, n - m + 1, p);
    qr = multiply(qr, ar);
    qr.resize(n - m + 1);
    for (int& x : qr)
        x = (x % p + p) % p;
    reverse(qr.begin(), qr.end()); // q = q~r
    return qr;
}
// calculates a - bQ
vector<int> module(const vector<int>& a,
                  const vector<int>& b,
                  const vector<int>& Q, int p) {
    vector<int> r = multiply(b, Q);
    r.resize(b.size());
    for (int i = 0; i < (int)r.size(); i++) {
        int ai = i < (int)a.size() ? a[i] : 0;
        int ri = (r[i] % p + p) % p;
        r[i] = _sub(ai, ri, p);
    }
    return r;
}

```

FFT

```

template <int LOG>
struct FFT {
    vector<int> rev[LOG + 1];
    vector<base> G[LOG + 1];
    FFT() {
        int N = 1 << LOG;
        base root(cosl(2 * pi / N), sinl(2 * pi /
→ N));
        base start = root;
        for (int lvl = LOG; lvl >= 0; lvl--,
→ start = start * start) {
            int tot = 1 << lvl;
            G[lvl].resize(tot);
            base cur = 1;
            for (int i = 0; i < tot; i++, cur *=
→ start)
                G[lvl][i] = cur;
        }
        for (int lvl = 1; lvl <= LOG; lvl++) {

```

```

            int tot = 1 << lvl;
            rev[lvl].resize(tot);
            for (int i = 1; i < tot; i++) {
                rev[lvl][i] = ((i & 1) << (lvl -
→ 1)) | (rev[lvl][i >> 1] >> 1);
            }
        }
    }
    void fft(vector<base>& a, int sz, bool
→ invert) {
        int n = 1 << sz;
        for (int j, i = 0; i < n; i++) {
            if ((j = rev[sz][i]) < i) {
                swap(a[i], a[j]);
            }
        }
        base f1, f2;
        for (int lvl = 0, len = 1; len < n; len
→ <= 1, lvl++) {
            for (int i = 0; i < n; i += (len <<
→ 1)) {
                for (int j = 0; j < len; j++) {
                    f1 = a[i + j];
                    f2 = a[i + j + len] * G[lvl +
→ 1][j];

                    a[i + j] = f1 + f2;
                    a[i + j + len] = f1 - f2;
                }
            }
        }
        if (invert) {
            reverse(a.begin() + 1, a.end());
            for (int i = 0; i < n; i++)
                a[i] /= n;
        }
    }
    vector<ld> multiply(const vector<ld>& a,
→ const vector<ld>& b) {
        vector<base> fa(a.begin(), a.end());
        vector<base> fb(b.begin(), b.end());
        int n = (int) a.size();
        int m = (int) b.size();
        int maxnm = max(n, m), sz = 0;
        while ((1 << sz) < maxnm)
            sz++;
        sz++;
        fa.resize(1 << sz);
        fb.resize(1 << sz);
        fft(fa, sz, false);
        fft(fb, sz, false);
        int SZ = 1 << sz;
        for (int i = 0; i < SZ; i++)
            fa[i] = fa[i] * fb[i];
        fft(fa, sz, true);
        vector<ld> res(SZ);
        for (int i = 0; i < SZ; i++)
            res[i] = fa[i].real();
        return res;
    }
};

```


Extrapolation

```

int fact[N];
int rfact[N];
void precalc2() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = _mul(fact[i - 1], i);
    }
    rfact[N - 1] = _rev(fact[N - 1]);
    for (int i = N - 2; i >= 0; i--) {
        rfact[i] = _mul(rfact[i + 1], i + 1);
    }
}
int getMulOnSegment(int l, int r) {
    assert(l <= r);
    if (l == 0 && r == 0) return 1;
    if (r <= 0) {
        int res = getMulOnSegment(-r, -1);
        int cnt = r - 1 + 1;
        if (cnt % 2) {
            res = (-res % mod + mod) % mod;
        }
        return res;
    }
    if (l < 0) {
        int resl = getMulOnSegment(0, -1);
        if (l % 2) {
            resl = (-resl % mod + mod) % mod;
        }
        int resr = getMulOnSegment(0, r);
        return _mul(resl, resr);
    }
    assert(l >= 0);
    int res = fact[r];
    if (l > 0) { res = _mul(res, rfact[l - 1]); }
    return res;
}
vector<int> extrapolate(vector<int> y, int m) {
    vector<int> yy = y;
    int n = (int)y.size() - 1;
    for (int i = 0; i <= n; i++) {
        yy[i] = _mul(
            y[i], _rev(getMulOnSegment(i - n, i - 0)));
    }
    vector<int> ff(n + m + 1);
    for (int i = 1; i <= n + m; i++) {
        ff[i] = _mul(fact[i - 1], rfact[i]);
    }
    vector<int> ss = multiply(yy, ff);
    for (int i = 1; i <= m; i++) {
        int cc = getMulOnSegment(i, n + i);
        int Si = ss[n + i];
        y.push_back(_mul(cc, Si));
    }
    return y;
}

```

Xor FWHT

```

// _sum, _sub, _mul - arithmetic operations
void xor_fwht(vector<int>& a,

```

```

        bool inverse = false) {
    for (int x, y, len = 1; len < (int)a.size();
        len <= 1) {
        for (int i = 0; i < (int)a.size();
            i += len << 1) {
            for (int j = 0; j < len; j++) {
                x = a[i + j], y = a[i + j + len];
                a[i + j] = _sum(x, y);
                a[i + j + len] = _sub(x, y);
            }
        }
    }
    if (inverse) {
        int rn = _binpow((int)a.size(), mod - 2);
        for (int& x : a)
            x = _mul(x, rn);
    }
}
void or_fwht(vector<int>& a,
        bool inverse = false) {
    for (int x, y, len = 1; len < (int)a.size();
        len <= 1) {
        for (int i = 0; i < (int)a.size();
            i += len << 1) {
            for (int j = 0; j < len; j++) {
                x = a[i + j], y = a[i + j + len];
                a[i + j] = x,
                a[i + j + len] =
                    inverse ? _sub(y, x) : _sum(y,
→ x);
            }
        }
    }
}
void and_fwht(vector<int>& a,
        bool inverse = false) {
    for (int x, y, len = 1; len < (int)a.size();
        len <= 1) {
        for (int i = 0; i < (int)a.size();
            i += len << 1) {
            for (int j = 0; j < len; j++) {
                x = a[i + j], y = a[i + j + len];
                a[i + j] =
                    inverse ? _sub(x, y) : _sum(x, y),
                a[i + j + len] = y;
            }
        }
    }
}

```

CHT

```

struct Line {
    ll k, b;
    int type;
    ld x;
    Line() : k(0), b(0), type(0), x(0) {}
    Line(ll _k, ll _b, ld _x = 1e18, int _type = 0)
        : k(_k), b(_b), x(_x), type(_type) {}
    bool operator<(const Line& other) const {
        if (type + other.type > 0) {

```

```

        return x < other.x;
    } else {
        return k < other.k;
    }
}
ld intersect(const Line& other) const {
    return ld(b - other.b) / ld(other.k - k);
}
ll get_func(ll x0) const { return k * x0 + b; }
};

struct CHT {
    set<Line> qs;
    set<Line>::iterator fnd, help;
    bool hasr(const set<Line>::iterator& it) {
        return it != qs.end() && next(it) !=
→ qs.end();
    }
    bool hasl(const set<Line>::iterator& it) {
        return it != qs.begin();
    }
    bool check(const set<Line>::iterator& it) {
        if (!hasr(it)) return true;
        if (!hasl(it)) return true;
        return it->intersect(*prev(it)) <
            it->intersect(*next(it));
    }
    void update_intersect(
        const set<Line>::iterator& it) {
        if (it == qs.end()) return;
        if (!hasr(it)) return;
        Line tmp = *it;
        tmp.x = tmp.intersect(*next(it));
        qs.insert(qs.erase(it), tmp);
    }
    void add_line(Line L) {
        if (qs.empty()) {
            qs.insert(L);
            return;
        }
        fnd = qs.lower_bound(L);
        if (fnd != qs.end() && fnd->k == L.k) {
            if (fnd->b >= L.b)
                return;
            else
                qs.erase(fnd);
        }
        fnd = qs.insert(L).first;
        if (!check(fnd)) {
            qs.erase(fnd);
            return;
        }
        while (hasr(fnd) &&
            !check(help = next(fnd))) {
            qs.erase(help);
        }
        while (hasl(fnd) &&
            !check(help = prev(fnd))) {
            qs.erase(help);
        }
        if (hasl(fnd)) {
            update_intersect(prev(fnd));

```

```

    }
    update_intersect(fnd);
}
ll get_max(ld x0) {
    if (qs.empty()) return -inf64;
    fnd = qs.lower_bound(Line(0, 0, x0, 1));
    if (fnd == qs.end()) fnd--;
    ll res = -inf64;
    int i = 0;
    while (i < 2 && fnd != qs.end()) {
        res = max(res, fnd->get_func(x0));
        fnd++;
        i++;
    }
    while (i-- > 0)
        fnd--;
    while (i < 2) {
        res = max(res, fnd->get_func(x0));
        if (hasl(fnd)) {
            fnd--;
            i++;
        } else {
            break;
        }
    }
    return res;
}
};

```

Euler Tour Trees

```

class EulerTourTrees {
    /*
    graph - forest
    1 .. n
    get = is connected?
    no memory leaks
    1 <= n, q <= 10^5
    0.7 sec
    */
private:
    struct Node {
        Node* l;
        Node* r;
        Node* p;
        int prior;
        int cnt;
        int rev;
        Node()
            : l(nullptr), r(nullptr), p(nullptr),
              prior(rnd()), cnt(1), rev(0) {}
        ~Node() {
            delete l;
            delete r;
        }
    };
    void do_rev(Node* v) {
        if (v) v->rev ^= 1, swap(v->l, v->r);
    }
    int get_cnt(Node* v) const {
        return v ? v->cnt : 0;
    }

```

```

}
void update(Node* v) {
    if (!v) return;
    v->cnt = 1 + get_cnt(v->l) + get_cnt(v->r);
    v->p = nullptr;
    if (v->l) v->l->p = v;
    if (v->r) v->r->p = v;
}
void push(Node* v) {
    if (!v) return;
    if (v->rev) {
        do_rev(v->l);
        do_rev(v->r);
        v->rev ^= 1;
    }
}
void merge(Node*& v, Node* l, Node* r) {
    if (!l || !r) {
        v = l ? l : r;
        return;
    }
    push(l);
    push(r);
    if (l->prior < r->prior) {
        merge(l->r, l->r, r);
        v = l;
    } else {
        merge(r->l, l, r->l);
        v = r;
    }
    update(v);
}
void split_by_cnt(Node* v, Node*& l, Node*& r,
                  int x) {
    if (!v) {
        l = r = nullptr;
        return;
    }
    push(v);
    if (get_cnt(v->l) + 1 <= x) {
        split_by_cnt(v->r, v->r, r,
                     x - get_cnt(v->l) - 1);
        l = v;
    } else {
        split_by_cnt(v->l, l, v->l, x);
        r = v;
    }
    update(l);
    update(r);
}
void push_path(Node* v) {
    if (!v) return;
    push_path(v->p);
    push(v);
}
int get_pos(Node* v) {
    push_path(v);
    int res = 0, ok = 1;
    while (v) {
        if (ok) res += get_cnt(v->l) + 1;
        ok = v->p && v->p->r == v;
        v = v->p;
    }
}

```

```

    return res;
}
Node* get_root(Node* v) const {
    while (v && v->p)
        v = v->p;
    return v;
}
Node* shift(Node* v) {
    if (!v) return v;
    int pos = get_pos(v);
    Node *nl = nullptr, *nr = nullptr;
    Node* root = get_root(v);
    split_by_cnt(root, nl, nr, pos - 1);
    do_rev(nl);
    do_rev(nr);
    merge(root, nl, nr);
    do_rev(root);
    return root;
}
public:
    EulerTourTrees() = default;
    EulerTourTrees(int _n) : n(_n) {
        ptr.resize(_n + 1);
        where_edge.resize(_n + 1);
    }
    bool get(int u, int v) const {
        if (u == v) return true;
        Node* ru = get_root(
            ptr[u].empty() ? nullptr :
→ *ptr[u].begin());
        Node* rv = get_root(
            ptr[v].empty() ? nullptr :
→ *ptr[v].begin());
        return ru && ru == rv;
    }
    void link(int u, int v) {
        Node* ru = shift(
            ptr[u].empty() ? nullptr :
→ *ptr[u].begin());
        Node* rv = shift(
            ptr[v].empty() ? nullptr :
→ *ptr[v].begin());
        Node* uv = new Node();
        Node* vu = new Node();
        ptr[u].insert(uv);
        ptr[v].insert(vu);
        where_edge[u][v] = uv;
        where_edge[v][u] = vu;
        merge(ru, ru, uv);
        merge(ru, ru, rv);
        merge(ru, ru, vu);
    }
    void cut(int u, int v) {
        Node* uv = where_edge[u][v];
        Node* vu = where_edge[v][u];
        ptr[u].erase(uv);
        ptr[v].erase(vu);
        Node* root = shift(uv);
        Node *nl = nullptr, *nm = nullptr,
              *nr = nullptr;
        int pos1 = get_pos(uv);
        int pos2 = get_pos(vu);
    }
}

```

```

    if (pos1 < pos2) {
        split_by_cnt(root, nl, nr, pos2);
        split_by_cnt(nl, nl, vu, pos2 - 1);
        split_by_cnt(nl, nl, nm, pos1);
        split_by_cnt(nl, nl, uv, pos1 - 1);
        merge(nl, nl, nr);
    } else {
        split_by_cnt(root, nl, nr, pos1);
        split_by_cnt(nl, nl, uv, pos1 - 1);
        split_by_cnt(nl, nl, nm, pos2);
        split_by_cnt(nl, nl, vu, pos2 - 1);
        merge(nl, nl, nm);
    }
    delete uv;
    delete vu;
}

~EulerTourTrees() {
    set<Node*> roots;
    for (int i = 1; i <= n; i++) {
        for (Node* v : ptr[i]) {
            roots.insert(get_root(v));
        }
    }
    for (Node* root : roots) {
        delete root;
    }
}

private:
    int n = 0;
    vec<set<Node*>> ptr;
    vec<unordered_map<int, Node*>>
        where_edge; // ptr to node
};

```

Simplex

```

template <class T>
vector<T> operator+(const vector<T>& a,
                    const vector<T>& b) {
    vector<T> res(a.size());
    for (int i = 0; i < (int)a.size(); i++)
        res[i] = a[i] + b[i];
    return res;
}

template <class T>
vector<T> operator*(const T& coef,
                    const vector<T>& a) {
    vector<T> res(a.size());
    for (int i = 0; i < (int)a.size(); i++)
        res[i] = coef * a[i];
    return res;
}

const ld eps = 1e-9;
struct Simplex {
    // Ax = b, x >= 0, <c, x> -> max
    int m; // the number of
    ↪ equations
    int n; // the number of
    ↪ variables
    vector<vector<ld>> A; // (m + 2) x (n + 1)
    // (m + 1)-th row: primary c
    // (m + 2)-th row: secondary c (c')

```

```

    // (n + 1)-th col: column of b
    vector<int> basis;
    bool bounded = true;
    Simplex(const vector<vector<ld>>& mat,
            const vector<int>& _basis)
        : A(mat), basis(_basis) {
        m = (int)mat.size() - 2,
        n = (int)mat[0].size() - 1;
    }

    /// make primary c under basis components zero
    void reset_c() {
        for (int i = 0; i < m; i++) {
            int j = basis[i];
            A[m] = A[m] + (-A[m][j]) * A[i];
            A[m + 1] = A[m + 1] + (-A[m + 1][j]) *
    ↪ A[i];
        }
    }

    void pivot(int i, int k) {
        A[k] = (ld(1) / ld(A[k][i])) * A[k];
        for (int j = 0; j < (int)A.size(); j++) {
            if (j == k) continue;
            A[j] = A[j] + (-A[j][i]) * A[k];
        }
        basis[k] = i;
    }

    void run() {
        while (true) {
            int j = 0;
            while (j < n && A[m][j] <= eps)
                j++;
            if (j == n) break;
            int k = -1;
            for (int i = 0; i < m; i++)
                if (A[i][j] > eps &&
                    (k == -1 || (A[i][n] / A[i][j] <
                                A[k][n] / A[k][j])))
                    k = i;
            if (k == -1) {
                bounded = false;
                break;
            }
            pivot(j, k);
        }
    }

    vector<ld> get_solution() {
        vector<ld> res(n);
        for (int i = 0; i < m; i++)
            res[basis[i]] = A[i][n];
        return res;
    }

    void reset_column(int j) {
        for (int i = 0; i < (int)A.size(); i++)
            A[i][j] = 0;
    }

    ld get_max_value() { return -A[m][n]; }
    void swap_primary_c() { swap(A[m], A[m + 1]); }
    void flip_task_type() {
        A[m] = ld(-1) * A[m];
        A[m + 1] = ld(-1) * A[m + 1];
    }
};

```

```

struct Response {
    bool bounded = true;
    bool exist = true;
    ld value = 0;
    vector<ld> solution = {};
};
// aa * x <= bb, <cc, x> ---> max
Response solve(const vector<vector<ld>>& aa,
               const vector<ld>& bb,
               const vector<ld>& cc) {
    int m = (int)aa.size();
    int n = (int)aa[0].size();
    vector<vector<ld>> a(m,
                       vector<ld>(n + m + 1 +
→ 1));
    for (int i = 0; i < m; i++) {
        for (int j = 0; j < n; j++)
            a[i][j] = aa[i][j];
        a[i][n + i] = +1;
        a[i][n + m] = -1;
        a[i][n + m + 1] = bb[i];
    }
    vector<ld> c(n + m + 1 + 1), c2(n + m + 1 + 1);
    for (int i = 0; i < n; i++)
        c[i] = cc[i];
    c2[n + m] = -1;
    vector<int> basis(m);
    for (int j = 0; j < m; j++)
        basis[j] = n + j;
    a.push_back(c2);
    a.push_back(c);
    Simplex simplex(a, basis);
    simplex.reset_c();
    {
        int k = 0;
        for (int i = 1; i < m; i++)
            if (a[i][n + m + 1] < a[k][n + m + 1])
                k = i;
        if (a[k][n + m + 1] < -eps)
            simplex.pivot(n + m, k);
    }
    simplex.run();
    if (!simplex.bounded ||
        -simplex.get_max_value() > eps) {
        return Response{true, false, 0, {}};
    }
    {
        vector<int> in_basis(n + m + 1, -1);
        for (int i = 0; i < m; i++)
            in_basis[simplex.basis[i]] = i;
        int k = in_basis[n + m];
        if (k != -1) {
            for (int i = 0; i < n + m; i++) {
                if (in_basis[i] != -1) continue;
                if (std::abs(simplex.A[k][i]) <= eps)
                    continue;
                simplex.pivot(i, k);
                break;
            }
        }
        simplex.reset_column(n + m);
    }
    simplex.swap_primary_c();

```

```

simplex.run();
if (!simplex.bounded) {
    return Response{false, true, 0, {}};
}
Response response;
response.value = simplex.get_max_value();
response.solution = simplex.get_solution();
response.solution.resize(n);
return response;
}

```

Fast Allocator

```

#define FAST_ALLOCATOR_MEMORY 4e8
#ifdef FAST_ALLOCATOR_MEMORY
int allocator_pos = 0;
char
→ allocator_memory[(int)FAST_ALLOCATOR_MEMORY];
inline void* operator new(size_t n) {
    char* res = allocator_memory + allocator_pos;
    allocator_pos += n;
    assert(allocator_pos <=
        (int)FAST_ALLOCATOR_MEMORY);
    return (void*)res;
}
inline void operator delete(void*) noexcept {}
#endif

```

Angle Comparator

```

struct comparator {
    pll center;
    comparator(pll p) : center(p) {}
    bool operator()(const pll& p,
                   const pll& q) const {
        pll start(1, 0);
        if (p == q) return false;
        auto op = vect(center, p);
        auto oq = vect(center, q);
        if (cp(op, oq) == 0 && dp(op, oq) > 0)
            return false;
        ll sop = cp(start, op), soq = cp(start, oq);
        if (sop == 0) {
            if (dp(start, op) > 0) return true;
            return soq < 0;
        }
        if (soq == 0) {
            if (dp(start, oq) > 0) return false;
            return sop > 0;
        }
        if ((sop > 0 && soq > 0) ||
            (sop < 0 && soq < 0)) {
            return cp(op, oq) > 0;
        }
        return sop > 0;
    }
};

```

Common Tangents

```

struct circle {
    pt c;
    ld r = 0;

    ld area() const {
        return pi * r * r;
    }
};

vector<pair<pt, pt>> get_tangents(circle w1,
→ circle w2) {
    if (w1.r > w2.r) // IMPORTANT!!!
        swap(w1, w2);
    ld D = (w1.c - w2.c).norm();
    if (D < abs(w1.r - w2.r) + eps) // ONE INSIDE
→ ANOTHER!!!
        return {};
    ld cos_alpha = abs(w1.r - w2.r) / D;
    ld sin_alpha = sqrtl(max(ld(0), 1 - cos_alpha
→ * cos_alpha));
    vector<pair<pt, pt>> ts;
    pt v1 = (w1.c - w2.c).rotate(cos_alpha,
→ +sin_alpha);
    pt v2 = (w1.c - w2.c).rotate(cos_alpha,
→ -sin_alpha);
    ts.emplace_back(w1.c + (v1 * (w1.r /
→ v1.norm()))), w2.c + (v1 * (w2.r /
→ v1.norm())));
    ts.emplace_back(w1.c + (v2 * (w1.r /
→ v2.norm()))), w2.c + (v2 * (w2.r /
→ v2.norm())));
    return ts;
}

```

Find Tangents in polygon $O(\log n)$

```

pair<pt, pt>
→ find_tangents_on_convex_polygon(const
→ vector<pt>& P, pt q) {
    // q is strictly outside P
    // P in counter-clockwise order
    // P.size >= 3
    // P is strictly convex
    // return {leftmost visible, rightmost
→ visible}
    // be careful with corner cases
    // (when two different points may be
    // leftmost/rightmost visible)
    int n = (int) P.size();
    auto is_visible_edge = [&](int i) -> bool {
        return (P[i] - q).vector_mul(P[(i + 1) %
→ n] - P[i]) < -eps;
    };
    auto is_visible = [&](int i) -> bool {
        return is_visible_edge(i) ||
→ is_visible_edge((i - 1 + n) % n);
    };
    auto is_on_right = [&](int i) -> bool {
        return (P[0] - q).vector_mul(P[i] - q) <
→ -eps;
    };
}

```

```

};
int bl, br, bm;
int A = -1, B = -1;
if (is_visible(0)) {
    bl = 0, br = n;
    while (br - bl > 1) {
        bm = (bl + br) >> 1;
        if (is_visible(bm) &&
→ is_on_right(bm)) bl = bm;
        else br = bm;
    }
    B = bl;

    bl = 0, br = n;
    while (br - bl > 1) {
        bm = (bl + br) >> 1;
        int i = (B + n - bm) % n;
        if (is_visible(i)) bl = bm;
        else br = bm;
    }
    A = (B + n - bl) % n;
} else {
    bl = 0, br = n;
    while (br - bl > 1) {
        bm = (bl + br) >> 1;
        if (!is_visible(bm) &&
→ !is_on_right(bm)) bl = bm;
        else br = bm;
    }
    A = (bl + 1) % n;

    bl = 0, br = n;
    while (br - bl > 1) {
        bm = (bl + br) >> 1;
        int i = (A + bm) % n;
        if (is_visible(i)) bl = bm;
        else br = bm;
    }
    B = (A + bl) % n;
}
return {P[A], P[B]};
}

```

Minkowsky Polygon Sum

```

vector<pt> minkowski_polygons_sum(vector<pt> a,
→ vector<pt> b) {
    // a and b have counter-clock wise order
    auto cmp = [] (const pt& p1,
        const pt& p2) -> bool {
        return make_pair(p1.x, p1.y) <
            make_pair(p2.x, p2.y);
    };
    rotate(a.begin(),
        min_element(a.begin(), a.end(), cmp),
        a.end());
    rotate(b.begin(),
        min_element(b.begin(), b.end(), cmp),
        b.end());
    pt q = a[0] + b[0];
    int n = (int)a.size();
}

```

```

int m = (int)b.size();
vector<pt> result = {q};
for (int i = 0, j = 0; i < n || j < m; ) {
    pt vi, vj;
    if (i < n)
        vi = a[i + 1 < n ? i + 1 : 0] - a[i];
    if (j < m)
        vj = b[j + 1 < m ? j + 1 : 0] - b[j];
    if (i < n &&
        (j == m || vi.vector_mul(vj) > eps))
        q = q + vi, i++;
    else
        q = q + vj, j++;
    result.push_back(q);
}
result.pop_back();
return result;
}

```

Halfplanes Intersection $O(n \log n)$

```

const ld eps = 1e-9;
struct pt {
    ld x = 0, y = 0;
    pt operator+(const pt& o) const {
        return {x + o.x, y + o.y};
    }
    pt operator-(const pt& o) const {
        return {x - o.x, y - o.y};
    }
    pt operator*(ld coef) const {
        return {x * coef, y * coef};
    }
    ld vector_mul(const pt& o) const {
        return x * o.y - o.x * y;
    }
    ld scalar_mul(const pt& o) const {
        return x * o.x + y * o.y;
    }
    ld sqr_norm() const {
        return scalar_mul(*this);
    }
    ld norm() const {
        return sqrtl(max(ld(0), sqr_norm()));
    }
    int quadrant() const {
        if (x >= eps && y > -eps)
            return 1;
        else if (x < eps && y >= eps)
            return 2;
        else if (x <= -eps && y < eps)
            return 3;
        else
            return 4;
    }
    bool operator<(const pt& o) const {
        int q1 = quadrant();
        int q2 = o.quadrant();
        if (q1 != q2) return q1 < q2;
        return vector_mul(o) >= eps;
    }
};

```

```

struct Line {
    pt a, b;
    pt dir() const { return b - a; }
};
pair<bool, pt> intersect_lines(const Line& L1,
                              const Line& L2) {
    ld vm = L1.dir().vector_mul(L2.dir());
    if (abs(vm) < eps) return {false, pt{}};
    ld t = L2.dir().vector_mul(L1.a - L2.a) / vm;
    return {true, L1.a + L1.dir() * t};
}
struct Response {
    enum TYPE { EMPTY, INF, FINITE };
    TYPE type;
    vector<Line> halves;
};
bool is_line_good(Line L, Line L1, Line L2,
                  bool strictly = false) {
    int any_colinear = 0;
    for (Line L_hat : {L1, L2}) {
        ld vm = L.dir().vector_mul(L_hat.dir());
        ld sm = L.dir().scalar_mul(L_hat.dir());
        if (abs(vm) < eps) {
            any_colinear = 1;
            if (sm >= eps) {
                if (L.dir().vector_mul(L_hat.b - L.a) >=
                    (strictly ? eps : -eps))
                    return false;
            } else {
                if (L.dir().vector_mul(L_hat.b - L.a) <=
                    -eps)
                    return false;
            }
        }
    }
    if (any_colinear) return true;
    ld vm1 = L.dir().vector_mul(L1.dir());
    ld vm2 = L.dir().vector_mul(L2.dir());
    int t1 = vm1 >= eps ? +1 : -1;
    int t2 = vm2 >= eps ? +1 : -1;
    if (t1 == t2) return true;
    if (t1 > t2)
        swap(t1, t2), swap(vm1, vm2), swap(L1, L2);
    pt p1 = intersect_lines(L, L1).second;
    pt p2 = intersect_lines(L, L2).second;
    return (p2 - p1).scalar_mul(L.dir()) > -eps;
}
bool check_empty(Line L1, Line L2, Line L3) {
    return !is_line_good(L1, L2, L3, true) &&
        !is_line_good(L2, L1, L3, true) &&
        !is_line_good(L3, L1, L2, true);
}
Response intersect_halfs(vector<Line> halves) {
    sort(halves.begin(), halves.end(),
        [](const Line& l1, const Line& l2) {
            return l1.dir() < l2.dir();
        });
    int n = (int)halves.size(), is_inf = 0,
        any_positive_vm = 0;
    deque<Line> hull;
    Line L1, L2, L3;
    ld vm, sm;
}

```



```

for (int i = 0; i < n; i++) {
    vm = halves[i].dir().vector_mul(
        halves[i + 1 < n ? i + 1 : 0].dir());
    sm = halves[i].dir().scalar_mul(
        halves[i + 1 < n ? i + 1 : 0].dir());
    any_positive_vm |= vm >= eps;
    if (vm <= -eps || (vm < eps && sm <= -eps))
        is_inf = 1;
    hull.push_back(halves[i]);
    for (int sz; (sz = (int)hull.size()) >= 3;) {
        L1 = hull[0], L2 = hull[1],
        L3 = hull[sz - 1];
        if (check_empty(L1, L2, L3))
            return Response{Response::TYPE::EMPTY,
                            {}};
        if (!is_line_good(L1, L2, L3)) {
            hull.pop_front();
            continue;
        }
        L1 = hull[sz - 1], L2 = hull[0],
        L3 = hull[sz - 2];
        if (check_empty(L1, L2, L3))
            return Response{Response::TYPE::EMPTY,
                            {}};
        if (!is_line_good(L1, L2, L3)) {
            hull.pop_back();
            continue;
        }
        L1 = hull[sz - 2], L2 = hull[sz - 1],
        L3 = hull[(2 * sz - 3) % sz];
        if (check_empty(L1, L2, L3))
            return Response{Response::TYPE::EMPTY,
                            {}};
        if (!is_line_good(L1, L2, L3)) {
            swap(hull[sz - 1], hull[sz - 2]);
            hull.pop_back();
            continue;
        }
        break;
    }
    if ((int)hull.size() == 2 &&
        check_empty(hull[0], hull[1], hull[1]))
        return Response{Response::TYPE::EMPTY, {}};
    is_inf |= !any_positive_vm;
    vector<Line> res(hull.begin(), hull.end());
    return Response{is_inf ? Response::TYPE::INF
                    :
    → Response::TYPE::FINITE,
                    res};
}

ld calculate_area(Response response) {
    assert(response.type != Response::TYPE::INF);
    if (response.type == Response::TYPE::EMPTY)
        return 0;
    const auto& halves = response.halves;
    int n = (int)halves.size();
    ld area = 0;
    vector<pt> ps;
    for (int i = 0; i < n; i++) {
        int j = i + 1 < n ? i + 1 : 0;
        if (abs(halves[i].dir().vector_mul(
            halves[j].dir())) < eps)

```

```

        continue;
        ps.push_back(
            intersect_lines(halves[i],
    → halves[j]).second);
    }
    n = (int)ps.size();
    for (int i = 0; i < n; i++) {
        int j = i + 1 < n ? i + 1 : 0;
        area += ps[i].vector_mul(ps[j]);
    }
    return abs(area) / 2;
}

Line get_line(ld A, ld B,
               ld C) { // Ax + By + C >= 0
    pt v = {B, -A}, a, b;
    assert(A * A + B * B > eps);
    if (abs(v.x) >= eps) {
        a = {0, -C / B};
    } else {
        a = {-C / A, 0};
    }
    b = a + v;
    return Line{a, b};
}

```

Fenwick Descent

```

struct Processor {
    int n = 0; // [0, n)
    vector<int> a;
    Processor() = default;
    Processor(int nn) {
        n = nn;
        a.assign(n, 0);
    }
    void increase(int i, int x) {
        for (int cur = i; cur < n; cur |= (cur + 1))
            a[cur] += x;
    }
    int descent(int lb) {
        int pos = 0;
        for (int pw = 1 << 19; pw > 0; pw >= 1) {
            if (pos + pw <= n && a[pos + pw - 1] < lb)
    → {
                lb -= a[pos + pw - 1];
                pos += pw;
            }
        }
        return pos;
    }
};

```

STL Tree

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<pair<int, int>, null_type,
    → less<pair<int, int>>, rb_tree_tag,
        tree_order_statistics_node_update>

```



```
stat_set;
```

```
return hull;
}
```

Convex Hull 3d $O(n^2)$

```
vector<plane> convex_hull_3d(vector<pt> a) {
    int n = (int) a.size(), timer = 10;
    vector<plane> hull;
    { /** NB: initial corner cases (find first 4
    → no coplanar points) **/ }
    auto in_hull = [&](const pt& q) -> bool {
        for (const auto& w : hull) if (sign(w, q)
    → <= -eps) return false;
        return true;
    };
    vector<vector<int>> last_see(n,
    → vector<int>(n)), last_not_see(n,
    → vector<int>(n));
    auto add = [&](int nid) {
        pt q = a[nid]; timer++;
        for (const auto& w : hull) {
            auto& ar = sign(w, q) <= -eps ?
    → last_see : last_not_see;
            for (int i = 0; i < 3; i++) {
                int j = (i + 1) % 3, id_i =
    → w.ps[i], id_j = w.ps[j];
                ar[id_i][id_j] = ar[id_j][id_i] =
    → timer;
            }
        }
        vector<plane> next_hull;
        for (const auto& w : hull) {
            if (sign(w, q) <= -eps) {
                const auto& ar = w.ps;
                int sz = (int) ar.size();
                assert(sz == 3);
                for (int i = 0; i < sz; i++) {
                    int j = (i + 1) % sz, id_i =
    → ar[i], id_j = ar[j], id_k = ar[3 ^ i ^ j];
                    if (last_see[id_i][id_j] ==
    → timer && last_not_see[id_i][id_j] == timer) {
                        plane add_plane =
    → get_plane(q, a[id_i], a[id_j]);
                        add_plane.ps = {nid,
    → id_i, id_j};
                        if (sign(add_plane,
    → a[id_k]) <= -eps) add_plane.v = -add_plane.v;
                        next_hull.push_back(add_plane);
                    }
                }
            } else {
                next_hull.push_back(w);
            }
        }
        swap(hull, next_hull);
    };
    for (int i = 0; i < n; i++) if
    → (!in_hull(a[i])) add(i);
    for (auto& w : hull) for (auto& id : w.ps) id
    → = a[id].id;
```

Sums of two squares $O(ANS \cdot \log^k(n))$

```
pll find_sums_of_two_squares_for_prime(ll p) {
    //  $O(\text{poly } \log p) \ a^2 + b^2 = p, 1 \leq a < b \leq$ 
    →  $\sqrt{p}$ 
    assert(p % 4 == 1); //  $p \bmod 4 = 1$  (Ferma th)
    ll x = 1; // find  $x: x^2 \equiv -1 \pmod p$ 
    while (BINPOW(x, (p - 1) / 2, p) != p - 1)
    → x++;
    x = BINPOW(x, (p - 1) / 4, p);
    assert(MUL(x, x, p) == p - 1);
    ll y = BINPOW(x, p - 2, p), sqrt_p =
    → find_sqrt(p);
    //  $0 \leq b' < \sqrt{p} \ ; \ (y + y * b') \% p \rightarrow$ 
    →  $\min \ ; \ b = b' + 1$ 
    ll a = euclid(y, y, p, sqrt_p - 1);
    ll b = (__int128(a) * x) % p;
    assert(a != b); if (a > b) swap(a, b);
    return {a, b};
}
vector<pll> find_all_sums_of_two_squares(ll n) {
    assert(n >= 1);
    if (n == 1) return {{0, 1}};
    // find all  $(0 \leq a \leq b \leq n: a^2 + b^2 = n^2)$ 
    // gaussian numbers multiplication
    // (complex values with integer coordinates)
    vector<pll> ds = factorize(n);
    for (auto& [p, c] : ds)
        if (p % 4 == 3 && c % 2 == 1) return {};
    vector<pll> res, add, nxt;
    for (auto [p, c] : ds) {
        add.clear();
        if (p == 2) while (c--)
    → add.emplace_back(1, 1);
        else if (p % 4 == 3) {
            c /= 2; while (c--)
    → add.emplace_back(0, p); } else {
            auto [a, b] =
    → find_sums_of_two_squares_for_prime(p);
            while (c--) add.emplace_back(a, b); }
        for (auto [aa, bb] : add) {
            if (res.empty()) {
    → res.emplace_back(aa, bb);
            } else { nxt.clear(); for (auto [cc,
    → dd] : res)
                for (int it = 0; it < 2;
    → it++, swap(cc, dd)) {
                    ll A = abs(aa * cc - bb *
    → dd), B = abs(aa * dd + bb * cc);
                    if (A > B) swap(A, B);
    → nxt.emplace_back(A, B); }
                for (auto [cc, dd] : nxt)
    → res.emplace_back(cc, dd);
                swap(res, nxt); sort(res.begin(),
    → res.end());
                res.erase(unique(res.begin(),
    → res.end()), res.end());
            } } return res;
```