# Prime numbers

$10^{1}$	+1	+3	+7	+9	+13	+19	+21	+27	-3	-5	-7	-8
$10^{2}$	+1	+3	+7	+9	+13	+27	+31	+37	-3	-11	-17	-21
$10^{3}$	+9	+13	+19	+21	+31	+33	+39	+49	-3	-9	-17	-23
$10^{4}$	+7	+9	+37	+39	+61	+67	+69	+79	-27	-33	-51	-59
$10^{5}$	+3	+19	+43	+49	+57	+69	+103	+109	-9	-11	-29	-39
$10^{6}$	+3	+33	+37	+39	+81	+99	+117	+121	-17	-21	-39	-41
$10^{7}$	+19	+79	+103	+121	+139	+141	+169	+189	-9	-27	-29	-57
$10^{8}$	+7	+37	+39	+49	+73	+81	+123	+127	-11	-29	-41	-59
$10^{9}$	+7	+9	+21	+33	+87	+93	+97	+103	-63	-71	-107	-117
$10^{10}$	+19	+33	+61	+69	+97	+103	+121	+141	-33	-57	-71	-119
$10^{11}$	+3	+19	+57	+63	+69	+73	+91	+103	-23	-53	-57	-93
$10^{12}$	+39	+61	+63	+91	+121	+163	+169	+177	-11	-39	-41	-63
$10^{13}$	+37	+51	+99	+129	+183	+259	+267	+273	-29	-137	-201	-237
$10^{14}$	+31	+67	+97	+99	+133	+139	+169	+183	-27	-29	-41	-69
$10^{15}$	+37	+91	+159	+187	+223	+241	+249	+259	-11	-53	-117	-123
$10^{16}$	+61	+69	+79	+99	+453	+481	+597	+613	-63	-83	-113	-149
$10^{17}$	+3	+13	+19	+21	+49	+81	+99	+141	-3	-23	-39	-57
$10^{18}$	+3	+9	+31	+79	+177	+183	+201	+283	-11	-33	-123	-137

# **Primitive Roots**

mod	$12 \cdot 2^{10} + 1$	$13 \cdot 2^{10} + 1$	$15 \cdot 2^{10} + 1$	$57 \cdot 2^{10} + 1$	$58 \cdot 2^{10} + 1$	$60 \cdot 2^{10} + 1$	$148 \cdot 2^{10} + 1$
root	49	7	84	29	9	21	38
mod	$6 \cdot 2^{11} + 1$	$9 \cdot 2^{11} + 1$	$20 \cdot 2^{11} + 1$	$56 \cdot 2^{11} + 1$	$65 \cdot 2^{11} + 1$	$140 \cdot 2^{11} + 1$	$150 \cdot 2^{11} + 1$
root	7	19	32	16	39	106	91
mod	$3 \cdot 2^{12} + 1$	$10 \cdot 2^{12} + 1$	$15 \cdot 2^{12} + 1$	$66 \cdot 2^{12} + 1$	$70 \cdot 2^{12} + 1$	$75 \cdot 2^{12} + 1$	$127 \cdot 2^{12} + 1$
root	41	28	19	114	19	41	71
mod	$136 \cdot 2^{12} + 1$	$141 \cdot 2^{12} + 1$	$5 \cdot 2^{13} + 1$	$8 \cdot 2^{13} + 1$	$14 \cdot 2^{13} + 1$	$51 \cdot 2^{13} + 1$	$78 \cdot 2^{13} + 1$
root	66	114	12	13	2	67	87
mod	$90 \cdot 2^{13} + 1$	$113 \cdot 2^{13} + 1$	$4 \cdot 2^{14} + 1$	$7 \cdot 2^{14} + 1$	$9 \cdot 2^{14} + 1$	$63 \cdot 2^{14} + 1$	$69 \cdot 2^{14} + 1$
root	96	63	15	15	22	94	86
mod	$73 \cdot 2^{14} + 1$	$139 \cdot 2^{14} + 1$	$2 \cdot 2^{15} + 1$	$5 \cdot 2^{15} + 1$	$17 \cdot 2^{15} + 1$	$81 \cdot 2^{15} + 1$	$110 \cdot 2^{15} + 1$
root	31	20	9	7	19	89	117
mod	$114 \cdot 2^{15} + 1$	$135 \cdot 2^{15} + 1$	$1 \cdot 2^{16} + 1$	$12 \cdot 2^{16} + 1$	$18 \cdot 2^{16} + 1$	$55 \cdot 2^{16} + 1$	$88 \cdot 2^{16} + 1$
root	27	126	3	3	14	30	10
mod	$102 \cdot 2^{16} + 1$	$112 \cdot 2^{16} + 1$	$117 \cdot 2^{16} + 1$	$6 \cdot 2^{17} + 1$	$9 \cdot 2^{17} + 1$	$21 \cdot 2^{17} + 1$	$51 \cdot 2^{17} + 1$
root	51	83	15	8	74	83	43
mod	$53 \cdot 2^{17} + 1$	$63 \cdot 2^{17} + 1$	$104 \cdot 2^{17} + 1$	$108 \cdot 2^{17} + 1$	$123 \cdot 2^{17} + 1$	$3 \cdot 2^{18} + 1$	$22 \cdot 2^{18} + 1$
root	47	10	13	54	26	5	74
mod	$28 \cdot 2^{18} + 1$	$52 \cdot 2^{18} + 1$	$54 \cdot 2^{18} + 1$	$63 \cdot 2^{18} + 1$	$108 \cdot 2^{18} + 1$	$127 \cdot 2^{18} + 1$	$147 \cdot 2^{18} + 1$
root	79	4	25	70	108	99	34
mod	$11 \cdot 2^{19} + 1$	$14 \cdot 2^{19} + 1$	$26 \cdot 2^{19} + 1$	$54 \cdot 2^{19} + 1$	$57 \cdot 2^{19} + 1$	$71 \cdot 2^{19} + 1$	$134 \cdot 2^{19} + 1$
root	12	25	2	106	20	86	49
mod	$7 \cdot 2^{20} + 1$	$13 \cdot 2^{20} + 1$	$22 \cdot 2^{20} + 1$	$66 \cdot 2^{20} + 1$	$67 \cdot 2^{20} + 1$	$106 \cdot 2^{20} + 1$	$115 \cdot 2^{20} + 1$
root	5	3	50	54	7	85	138
mod	$148 \cdot 2^{20} + 1$	$11 \cdot 2^{21} + 1$	$33 \cdot 2^{21} + 1$	$39 \cdot 2^{21} + 1$	$53 \cdot 2^{21} + 1$	$54 \cdot 2^{21} + 1$	$63 \cdot 2^{21} + 1$
root	81	38	45	94	54	134	46
mod	$110 \cdot 2^{21} + 1$	$119 \cdot 2^{21} + 1$	$123 \cdot 2^{21} + 1$	$25 \cdot 2^{22} + 1$	$27 \cdot 2^{22} + 1$	$33 \cdot 2^{22} + 1$	$55 \cdot 2^{22} + 1$
root	68	135	95	21	66	30	63
mod	$90 \cdot 2^{22} + 1$	$99 \cdot 2^{22} + 1$	$20 \cdot 2^{23} + 1$	$56 \cdot 2^{23} + 1$	$77 \cdot 2^{23} + 1$	$107 \cdot 2^{23} + 1$	$119 \cdot 2^{23} + 1$
root	139	65	4	53	19	45	31
mod	$132 \cdot 2^{23} + 1$	$10 \cdot 2^{24} + 1$	$28 \cdot 2^{24} + 1$	$66 \cdot 2^{24} + 1$	$73 \cdot 2^{24} + 1$	$108 \cdot 2^{24} + 1$	$120 \cdot 2^{24} + 1$
root	64	2	40	8	149	126	21
mod	$148 \cdot 2^{24} + 1$						
root	25						

#### Misc

Gomory-Hu tree (Gusfield's algorithm): label nodes from 0 to (|V|-1) and set  $p_i=0 \forall i>0$ .  $\forall i>0$ : find min-cut (S,T) between i and  $p_i$ , where  $i \in S, p_i \in T$ ; for each node j, s.t.  $i < j, j \in S, p_j = p_i$  set  $p_j = i$ 

### **Suffix Tree**

```
// Ukkonen's algorithm O(n)
const int A = 27; // Alphabet size
struct SuffixTree {
 struct Node { // [1, r) !!!
    int 1, r, link, par;
    int nxt[A];
    Node(): l(-1), r(-1), link(-1), par(-1) {
      fill(nxt, nxt + A, -1);
    Node(int _l, int _r, int _link, int _par)
        : 1(_1), r(_r), link(_link), par(_par) {
      fill(nxt, nxt + A, -1);
    }
    int& next(int c) { return nxt[c]; }
    int get_len() const { return r - 1; }
 };
 struct State {
    int v, len;
 };
 vec<Node> t;
 State cur_state;
 vec<int> s;
 SuffixTree() : cur_state({0, 0}) {
    t.push_back(Node());
 // v \rightarrow v + s[l, r) !!!
```

```
State go(State st, int 1, int r) {
   while (1 < r) {
     if (st.len == t[st.v].get_len()) {
       State nx = State({t[st.v].next(s[1]),
→ 0});
       if (nx.v == -1) return nx;
       st = nx:
       continue;
     if (s[t[st.v].l + st.len] != s[l])
       return State({-1, -1});
     if (r - 1 < t[st.v].get_len() - st.len)</pre>
       return State({st.v, st.len + r - 1});
     1 += t[st.v].get_len() - st.len;
     st.len = t[st.v].get_len();
   }
   return st;
 }
 int get_vertex(State st) {
   if (t[st.v].get_len() == st.len) return st.v;
   if (st.len == 0) return t[st.v].par;
   Node& v = t[st.v];
   Node& pv = t[v.par];
   Node add(v.1, v.1 + st.len, -1, v.par);
   pv.next(s[v.1]) = (int)t.size();
   add.next(s[v.l + st.len]) = st.v;
```

```
// par
    v.par = (int)t.size();
    // [l, r)
    v.l += st.len;
    t.push_back(add); // !!!
    return (int)t.size() - 1;
 int get_link(int v) {
    if (t[v].link != -1) return t[v].link;
    if (t[v].par == -1) return 0;
    int to = get_link(t[v].par);
    to = get_vertex(
      go(State({to, t[to].get_len()}),
         t[v].1 + (t[v].par == 0), t[v].r));
    return t[v].link = to;
 }
 void add_symbol(int c) {
    assert(0 \le c \&\& c \le A);
    s.push_back(c);
    while (1) {
      State hlp = go(cur_state, (int)s.size() -
\hookrightarrow 1.
                      (int)s.size());
      if (hlp.v != -1) {
        cur_state = hlp;
        break;
      }
      int v = get_vertex(cur_state);
      Node add((int)s.size() - 1, +inf, -1, v);
      t.push_back(add);
      t[v].next(c) = (int)t.size() - 1;
      cur_state.v = get_link(v);
      cur_state.len = t[cur_state.v].get_len();
      if (!v) break;
    }
 }
};
```

## **Suffix Array**

```
const int LOG = 21;
struct SuffixArray {
 string s;
 int n;
 vec<int> p;
 vec<int> c[LOG];
 SuffixArray() : n(0) {}
 SuffixArray(string ss) : s(ss) {
    s.push_back(0);
    n = (int)s.size();
    vec<int> pn, cn;
    vec<int> cnt;
    p.resize(n);
    for (int i = 0; i < LOG; i++)</pre>
      c[i].resize(n);
    pn.resize(n);
    cn.resize(n);
    cnt.assign(300, 0);
    for (int i = 0; i < n; i++)
      cnt[s[i]]++;
    for (int i = 1; i < (int)cnt.size(); i++)</pre>
      cnt[i] += cnt[i - 1];
```

```
for (int i = n - 1; i >= 0; i--)
      p[--cnt[s[i]]] = i;
    for (int i = 1; i < n; i++) {
      c[0][p[i]] = c[0][p[i - 1]];
      if (s[p[i]] != s[p[i - 1]]) c[0][p[i]]++;
    for (int lg = 0, k = 1; k < n;
         k \ll 1, lg++ 
      for (int i = 0; i < n; i++) {
        if ((pn[i] = p[i] - k) < 0) pn[i] += n;
      cnt.assign(n, 0);
      for (int i = 0; i < n; i++)
        cnt[c[lg][pn[i]]]++;
      for (int i = 1; i < (int)cnt.size(); i++)</pre>
        cnt[i] += cnt[i - 1];
      for (int i = n - 1; i \ge 0; i--)
        p[--cnt[c[lg][pn[i]]] = pn[i];
      for (int 11, r1, 12, r2, i = 1; i < n;
           i++) {
        cn[p[i]] = cn[p[i - 1]];
        11 = p[i - 1];
        12 = p[i];
        if ((r1 = 11 + k) >= n) r1 -= n;
        if ((r2 = 12 + k) >= n) r2 -= n;
        if (c[lg][11] != c[lg][12] ||
            c[lg][r1] != c[lg][r2])
          cn[p[i]]++;
      c[lg + 1] = cn;
   p.erase(p.begin(), p.begin() + 1);
  int get_lcp(int i, int j) {
    int res = 0;
    for (int lg = LOG - 1; lg >= 0; lg--) {
      if (i + (1 << lg) > n || j + (1 << lg) > n)
        continue;
      if (c[lg][i] == c[lg][j]) {
        i += (1 << lg);
        j += (1 << lg);
        res += (1 << lg);
   return res;
  }
};
```

### **Suffix Automation**

```
const int ALPHSIZE = 26; // alphabet size
struct SuffixAutomaton {
   struct Node {
    int link, len;
    int next[ALPHSIZE];
   Node() {
      link = -1;
      len = 0;
      for (int i(0); i < ALPHSIZE; i++)
            next[i] = -1;</pre>
```

```
}
 };
 string s;
 vector<Node> sa;
 int last;
 SuffixAutomaton() {
    sa.emplace_back();
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    for (int i(0); i < ALPHSIZE; i++)</pre>
      sa[0].next[i] = -1;
 void add(const int& c) {
    s.push_back(c + 'a');
    int cur = (int)sa.size();
    sa.emplace_back();
    sa[cur].len = sa[last].len + 1;
    for (p = last; p != -1 && sa[p].next[c] ==
         p = sa[p].link) {
      sa[p].next[c] = cur;
    if (p == -1) {
      sa[cur].link = 0;
    } else {
      int q = sa[p].next[c];
      if (sa[p].len + 1 == sa[q].len) {
        sa[cur].link = q;
      } else {
        int clone = (int)sa.size();
        sa.emplace_back();
        sa[clone].len = sa[p].len + 1;
        sa[clone].link = sa[q].link;
        for (int i(0); i < ALPHSIZE; i++)</pre>
          sa[clone].next[i] = sa[q].next[i];
        sa[cur].link = sa[q].link = clone;
        for (; p != -1 && sa[p].next[c] == q;
             p = sa[p].link) {
          sa[p].next[c] = clone;
        }
      }
    last = cur;
 }
};
```

## LCP

#### Manacker

```
pair<vector<int>, vector<int>>
manacker(const string& s) {
  // -> {d0, d1}. RUN test!
  int n = (int)s.size();
  vector<int> d0(n), d1(n);
  for (int l = 0, r = -1, i = 0; i < n; i++) {
    d1[i] =
      i \le r ? min(r - i, d1[1 + r - i]) : 0;
    while (i >= d1[i] \&\& i + d1[i] < n \&\&
           s[i - d1[i]] == s[i + d1[i]])
      d1[i]++;
    d1[i]--;
    if (i + d1[i] > r)
      1 = i - d1[i], r = i + d1[i];
  for (int l = 0, r = -1, i = 0; i < n; i++) {
    d0[i] =
      i < r ? min(r - i, d0[1 + r - i - 1]) : 0;
    while (i >= d0[i] \&\& i + d0[i] + 1 < n \&\&
           s[i - d0[i]] == s[i + d0[i] + 1])
      d0[i]++;
    if (d0[i] > 0 \&\& i + d0[i] > r)
      1 = i - d0[i] + 1, r = i + d0[i];
 return {d0, d1};
```

### **Prefix Function**

```
vector<int> get_pi(const string& s) {
  int n = (int)s.length();
  vector<int> pr(n);
  for (int i = 1; i < n; i++) {
    int k = pr[i - 1];
    while (k && s[k] != s[i])
        k = pr[k - 1];
    if (s[k] == s[i]) k++;
    pr[i] = k;
  }
  return pr;
}</pre>
```

#### **Z**-Function

```
vector<int> get_z(const string& s) {
  int n = (int)s.length();
  vector<int> z(n);
  for (int i = 1, l = 0, r = 0; i < n; i++) {</pre>
```

## Tandem (Lorentz)

```
struct Tandem {
 int 1, r, k;
 // [l, l + 2 * k) [l + 1, l + 1 + 2 * k) [l
 // + 2, l + 2 + 2 * k, ..., [r, r + 2 * k)
vector<int> z_func(const string& s) {
 int n = (int)s.size();
 vector<int> z(n);
 for (int l = 0, r = -1, i = 1; i < n; i++) {
    int k = i > r ? 0 : min(z[i - 1], r - i + 1);
    while (i + k < n \&\& s[i + k] == s[k])
     k++;
    z[i] = k;
    if (i + k - 1 > r) {
     r = i + k - 1;
     1 = i;
 }
 return z;
const int SIZE = (1000006) * 30;
const int MAXL = (1000006) * 4;
Tandem tandems[SIZE], hlp[MAXL];
void rec(const string& s, int L, int R) {
 if (R - L + 1 <= 1) { return; }
 int M = (L + R) / 2;
 rec(s, L, M);
 rec(s, M + 1, R);
 int nu = M - L + 1;
 int nv = R - M;
 string vu =
    s.substr(M + 1, nv) + "#" + s.substr(L, nu);
 string urvr = vu;
 reverse(urvr.begin(), urvr.end());
 vector<int> z1 = z_func(urvr);
 vector<int> z2 = z_func(vu);
 for (int x = L; x \le R; x++) {
    if (x \le M) {
      int k = M + 1 - x;
      int k1 = L < x ? z1[nu - x + L] : 0;
      int k2 = z2[nv + 1 + x - L];
      int lsh = max(0, k - k2);
      int rsh = min(k1, k - 1);
      if (lsh <= rsh) {</pre>
       tandems[tsz++] = \{x - rsh, x - lsh, k\};
     }
    } else {
      int k = x - M;
      int k1 = x < R ? z2[x - M] : 0;
      int k2 = z1[nu + nv - x + M + 1];
```

```
int lsh = max(1, k - k1);
      int rsh = min(k2, k - 1);
      if (lsh <= rsh) {</pre>
        tandems[tsz++] = \{x - rsh + 1 - k,
                            x - lsh + 1 - k, k;
    }
 }
}
void compress() { // \ \mathcal{O}(n*log(n)*log(n)) can be
                   // replace with count sort
                   // (O(n*log(n))) BE careful
\hookrightarrow with
                   // ML !!!
  // O(n*log(n)) \longrightarrow O(n)
  sort(tandems, tandems + tsz,
       [](const Tandem& t1, const Tandem& t2) {
         return t1.k < t2.k ||
                 (t1.k == t2.k \&\& t1.1 < t2.1);
       });
  int hlp_sz = 0;
  for (int i = 0; i < tsz; i++) {
    int j = i;
    while (j + 1 < tsz \&\&
           tandems[i].k == tandems[j + 1].k &&
           tandems[j].r + 1 == tandems[j + 1].1)
    {
      j++;
    hlp[hlp_sz++] = {tandems[i].1, tandems[j].r,
                      tandems[j].k};
    i = j;
 memcpy(tandems, hlp, sizeof(Tandem) * hlp_sz);
  tsz = hlp_sz;
}
void main_lorentz(const string& s) {
  // n = 10^6 time = 1.8 sec MEM = nlog(n) * 12
  // bytes
  int n = (int)s.size();
  tsz = 0;
  rec(s, 0, n - 1);
  compress();
```

#### **Aho-Corasick**

```
const int A = 300; // alphabet size
struct Aho {
    struct Node {
        int nxt[A], go[A];
        int par, pch, link;
        int good;
        Node()
            : par(-1), pch(-1), link(-1), good(-1) {
            fill(nxt, nxt + A, -1);
            fill(go, go + A, -1);
        }
    };
    vec<Node> a;
    Aho() { a.push_back(Node()); }
```

```
void add_string(const string& s) {
    int v = 0;
    for (char c : s) {
      if (a[v].nxt[c] == -1) {
        a[v].nxt[c] = (int)a.size();
        a.push_back(Node());
        a.back().par = v;
        a.back().pch = c;
      v = a[v].nxt[c];
    a[v].good = 1;
 int go(int v, int c) {
    if (a[v].go[c] == -1) {
      if (a[v].nxt[c] != -1) {
        a[v].go[c] = a[v].nxt[c];
      } else {
        a[v].go[c] = v ? go(get_link(v), c) : 0;
    }
    return a[v].go[c];
 }
 int get_link(int v) {
    if (a[v].link == -1) {
      if (!v || !a[v].par)
        a[v].link = 0;
      else
        a[v].link =
          go(get_link(a[v].par), a[v].pch);
    return a[v].link;
 bool is_good(int v) {
    if (!v) return false;
    if (a[v].good == -1) {
     a[v].good = is_good(get_link(v));
    return a[v].good;
 bool is_there_substring(const string& s) {
    int v = 0;
    for (char c : s) {
     v = go(v, c);
      if (is_good(v)) { return true; }
    return false;
 }
};
```

#### Eertree

```
const int N = 2e6 + 5;
struct EerTree {
  char s[N];
  int n;
  int sz;
  int link[N];
  int len[N];
  map<char, int> nxt[N];
  int diff[N];
  int dp[N][2];
```

```
int slink[N];
  int max_suff;
  int ans[N]; // number of partitions into
              // palindromes of even length
  void clr() {
    fill(s, s + N, 0);
    fill(link, link + N, 0);
    fill(len, len + N, 0);
    fill(nxt, nxt + N, map<char, int>());
    fill(diff, diff + N, 0);
    fill((int*)dp, (int*)dp + N * 2, 0);
    fill(slink, slink + N, 0);
   n = 0;
    sz = 0;
   max_suff = 0;
    fill(ans, ans + N, 0);
  EerTree() {
   clr();
    s[0] = '#'; // not in alphabet
    link[0] = 1;
   link[1] = 0;
   len[0] = -1;
    sz = 2;
    ans[0] = 1;
  int get_link(int from) {
    while (s[n] != s[n - len[from] - 1]) {
      from = link[from];
   return from;
  }
  void add_symbol(char c) {
   s[++n] = c;
   max_suff = get_link(max_suff);
    if (!nxt[max_suff].count(c)) {
        int x = get_link(link[max_suff]);
        link[sz] =
          nxt[x].count(c) ? nxt[x][c] : 1;
      len[sz] = len[max_suff] + 2;
      diff[sz] = len[sz] - len[link[sz]];
      slink[sz] = diff[sz] == diff[link[sz]]
                    ? slink[link[sz]]
                    : link[sz];
      nxt[max_suff][c] = sz++;
   max_suff = nxt[max_suff][c];
    for (int x = max_suff; len[x] > 0;
         x = slink[x]) {
      dp[x][0] = dp[x][1] = 0;
      int j = n - (len[slink[x]] + diff[x]);
      _inc(dp[x][j & 1], ans[j]);
      if (diff[x] == diff[link[x]]) {
        _inc(dp[x][0], dp[link[x]][0]);
        _inc(dp[x][1], dp[link[x]][1]);
      _inc(ans[n], dp[x][n & 1]);
 }
};
```

## Components of Vertex Duality

```
struct Edge {
 int fr, to, id;
 int get(int v) { return v == fr ? to : fr; }
void dfs(const vector<vector<Edge>>& g,
         vector<int>& fup, vector<int>& tin,
         vector<int>& used, int& timer, int v,
         int par = -1) {
 tin[v] = fup[v] = timer++;
 used[v] = 1;
 for (Edge e : g[v]) {
    int to = e.get(v);
    if (to == par) continue;
    if (used[to]) {
      fup[v] = min(fup[v], tin[to]);
    } else {
      dfs(g, fup, tin, used, timer, to, v);
      fup[v] = min(fup[v], fup[to]);
 }
}
void paintEdges(const vector<vector<Edge>>& g,
                vector<int>& fup,
                vector<int>& tin.
                vector<int>& used,
                vector<int>& colors, int v,
                int curColor, int& maxColor,
                int par = -1) {
 used[v] = 1;
 for (Edge e : g[v]) {
    int to = e.get(v);
    if (to == par) continue;
    if (!used[to]) {
      if (tin[v] <= fup[to]) {</pre>
        int tmpColor = maxColor++;
        colors[e.id] = tmpColor;
        paintEdges(g, fup, tin, used, colors, to,
                   tmpColor, maxColor, v);
      } else {
        colors[e.id] = curColor;
        paintEdges(g, fup, tin, used, colors, to,
                   curColor, maxColor, v);
    } else if (tin[to] < tin[v]) {</pre>
      colors[e.id] = curColor;
 }
vector<vector<Edge>>
get2components(const vector<vector<Edge>>& g,
               int m, const vector<Edge>& es) {
 int n = (int)g.size();
 vector<int> fup(n), tin(n), used(n);
 vector<int> colors(m);
 int timer;
 used.assign(n, 0);
 timer = 0;
 for (int v = 0; v < n; v++) {
    if (used[v]) continue;
    dfs(g, fup, tin, used, timer, v);
 }
```

## Hungarian Algorihtm

```
vector<int>
Hungarian(const vector<vector<int>>&
            a) { // ALARM: INT everywhere
  int n = (int)a.size();
  vector<int> row(n), col(n), pair(n, -1),
    back(n, -1), prev(n, -1);
  auto get = [&](int i, int j) {
    return a[i][j] + row[i] + col[j];
  };
  for (int v = 0; v < n; v++) {
    vector<int> min_v(n, v), A_plus(n),
\rightarrow B_plus(n);
   A_{plus}[v] = 1;
    int jb;
    while (true) {
      int pos_i = -1, pos_j = -1;
      for (int j = 0; j < n; j++) {
        if (!B_plus[j] && (pos_i == -1 ||
                            get(min_v[j], j) <</pre>
                              get(pos_i, pos_j)))
← {
          pos_i = min_v[j], pos_j = j;
      int weight = get(pos_i, pos_j);
      for (int i = 0; i < n; i++)
        if (!A_plus[i]) row[i] += weight;
      for (int j = 0; j < n; j++)
        if (!B_plus[j]) col[j] -= weight;
      B_plus[pos_j] = 1, prev[pos_j] = pos_i;
      int x = back[pos_j];
      if (x == -1) {
        jb = pos_j;
        break;
      A_plus[x] = 1;
      for (int j = 0; j < n; j++)
        if (get(x, j) < get(min_v[j], j))</pre>
          min_v[j] = x;
    while (jb != -1) {
      back[jb] = prev[jb];
      swap(pair[prev[jb]], jb);
    }
  }
```

```
return pair;
}
```

## General Matching

```
struct GeneralMatching { // O(n^3)
 int n = 0, cc = 10; // [0, n]
 vector<vector<int>> g; // undirected
 vector<int> mt, used, base, p, color;
 queue<int> q;
 GeneralMatching(int nn)
      : n(nn), mt(n, -1), used(n), base(n), p(n),
        color(n), g(n) {}
 void add_edge(int u, int v) {
   g[u].push_back(v), g[v].push_back(u);
 }
 void add(int v) {
    if (!used[v]) used[v] = 1, q.push(v);
 }
 int get_lca(int u, int v) {
   cc++;
   while (1) {
      u = base[u], color[u] = cc;
      if (mt[u] == -1) break;
     u = p[mt[u]];
   }
   while (1) {
     v = base[v];
     if (color[v] == cc) break;
     v = p[mt[v]];
   return v;
 }
 void mark_path(int v, int child, int b) {
   while (base[v] != b) {
      color[base[v]] = color[base[mt[v]]] = cc;
      p[v] = child, child = mt[v], v = p[child];
 }
 int bfs(int root) {
   add(root);
   while (!q.empty()) {
      int v = q.front();
      q.pop();
      for (int to : g[v]) {
        if (base[v] == base[to] || mt[v] == to)
          continue;
        else if (used[to]) {
          int w = get_lca(v, to);
          cc++, mark_path(v, to, w),
            mark_path(to, v, w);
          for (int i = 0; i < n; i++)
            if (color[base[i]] == cc)
              base[i] = w, add(i);
        } else if (p[to] == -1) {
          p[to] = v;
          if (mt[to] == -1) return to;
          add(mt[to]);
     }
   }
   return -1;
```

```
void xor_path(int v) {
    while (v != -1) {
      int pv = p[v], ppv = mt[pv];
      mt[v] = pv, mt[pv] = v;
      v = ppv;
  }
  bool inc(int root) {
    used.assign(n, 0), p.assign(n, -1),
      iota(base.begin(), base.end(), 0);
    while (!q.empty())
      q.pop();
    int v = bfs(root);
    if (v == -1) return false;
    xor_path(v);
    return true;
  void match() {
    for (int i = 0; i < n; i++)
      if (mt[i] == -1) inc(i);
  }
};
```

## Hopcroft-Karp

```
struct HopcroftKarp {
  int n, m;
  vec<vec<int>> g;
  vec<int> pl, pr, dist;
  vec<bool> vis;
  \texttt{HopcroftKarp()} \; : \; \texttt{n(0), m(0)} \; \{ \}
  HopcroftKarp(int _n, int _m) : n(_n), m(_m) {
    g.resize(n);
  void add_edge(int u, int v) {
    g[u].push_back(v);
  bool bfs() {
    dist.assign(n + 1, inf);
    queue<int> q;
    for (int u = 0; u < n; u^{++}) {
      if (pl[u] < m) continue;</pre>
      dist[u] = 0;
      q.push(u);
    }
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      if (dist[u] >= dist[n]) continue;
      for (int v : g[u]) {
        if (dist[pr[v]] > dist[u] + 1) {
           dist[pr[v]] = dist[u] + 1;
           q.push(pr[v]);
        }
      }
    }
    return dist[n] < inf;</pre>
  }
  bool dfs(int v) {
    if (v == n) return 1;
```

```
vis[v] = true;
    for (int to : g[v]) {
      if (dist[pr[to]] != dist[v] + 1) continue;
      if (vis[pr[to]]) continue;
      if (!dfs(pr[to])) continue;
      pl[v] = to;
      pr[to] = v;
      return 1;
    }
    return 0;
 }
 int find_max_matching() {
    pl.resize(n, m);
    pr.resize(m, n);
    int result = 0;
    while (bfs()) {
      vis.assign(n + 1, false);
      for (int u = 0; u < n; u++) {
        if (pl[u] < m) continue;</pre>
        if (vis[u]) continue;
        result += dfs(u);
      }
    }
    return result;
 }
};
```

### Dinic

```
struct Dinic {
 struct Edge {
    int fr, to, cp, id, fl;
 };
 int n, S, T;
 vector<Edge> es;
 vector<vector<int>> g;
 vector<int> dist, res, ptr, used;
 Dinic(int n_, int S_, int T_)
      : n(n_), S(S_), T(T_) {
    g.resize(n);
 }
 void add_edge(int fr, int to, int cp, int id) {
    g[fr].push_back((int)es.size());
    es.push_back({fr, to, cp, id, 0});
    g[to].push_back((int)es.size());
    es.push_back({to, fr, 0, -1, 0});
 }
 bool bfs(int K) {
    dist.assign(n, inf);
    dist[S] = 0;
    queue<int> q;
    q.push(S);
    while (!q.empty()) {
      int v = q.front();
      q.pop();
      for (int ps : g[v]) {
        Edge& e = es[ps];
        if (e.fl + K > e.cp) continue;
        if (dist[e.to] > dist[e.fr] + 1) {
          dist[e.to] = dist[e.fr] + 1;
          q.push(e.to);
        }
```

```
}
 return dist[T] < inf;</pre>
int dfs(int v, int _push = INT_MAX) {
  if (v == T || !_push) return _push;
  for (int& iter = ptr[v];
       iter < (int)g[v].size(); iter++) {</pre>
    int ps = g[v][ptr[v]];
    Edge& e = es[ps];
    if (dist[e.to] != dist[e.fr] + 1) continue;
    int tmp =
      dfs(e.to, min(_push, e.cp - e.fl));
    if (tmp) {
      e.fl += tmp;
      es[ps ^ 1].fl -= tmp;
      return tmp;
    }
 }
 return 0;
11 find_max_flow() {
  ptr.resize(n);
  11 max_flow = 0, add_flow;
  for (int K = 1 \ll 30; K > 0; K >>= 1) {
    while (bfs(K)) {
      ptr.assign(n, 0);
      while ((add_flow = dfs(S))) {
        max_flow += add_flow;
    }
  }
 return max_flow;
void assign_result() {
  res.resize(es.size());
  for (Edge e : es)
    if (e.id != -1) res[e.id] = e.fl;
int get_flow(int id) { return res[id]; }
bool go(int v, vector<int>& F,
        vector<int>& path) {
  if (v == T) return 1;
  if (used[v]) return 0; used[v] = 1;
  for (int ps : g[v]) {
    if (F[ps] <= 0) continue;</pre>
    if (go(es[ps].to, F, path)) {
      path.push_back(ps);
      return 1;}}
 return 0;}
vector<pair<int, vector<int>>> decomposition()
{
  find_max_flow();
 vector<int> F((int)es.size()), path, add;
  vector<pair<int, vector<int>>> dcmp;
  for (int i = 0; i < (int)es.size(); i++)</pre>
    F[i] = es[i].fl;
  used.assign(n, 0);
 while (go(S, F, path)) {
    used.assign(n, 0);
    int mn = INT_MAX;
    for (int ps : path)
```

```
mn = min(mn, F[ps]);
for (int ps : path)
   F[ps] -= mn;
for (int ps : path)
   add.push_back(es[ps].id);
   reverse(add.begin(), add.end());
   dcmp.push_back({mn, add});
   add.clear();
   path.clear();
}
return dcmp;
}
```

#### **MCMF**

```
struct MCMF {
 struct Edge {
    int fr, to, cp, fl, cs, id;
 int n, S, T;
 vec<Edge> es;
 vec<vec<int>>> g;
 vec<ll> dist, phi;
 vec<int> from;
 MCMF(int _n, int _S, int _T)
      : n(_n), S(_S), T(_T) {
    g.resize(n);
 }
 void add_edge(int fr, int to, int cp, int cs,
                int id) {
    g[fr].push_back((int)es.size());
    es.push_back({fr, to, cp, 0, cs, id});
    g[to].push_back((int)es.size());
    es.push_back({to, fr, 0, 0, -cs, -1});
 void init_phi() {
    dist.assign(n, LLONG_MAX);
    dist[S] = 0;
    for (int any, iter = 0; iter < n - 1;</pre>
         iter++) { // Ford Bellman
      any = 0;
      for (Edge e : es) {
        if (e.fl == e.cp) continue;
        if (dist[e.to] - dist[e.fr] > e.cs) {
          dist[e.to] = dist[e.fr] + e.cs;
          any = 1;
        }
      7
      if (!any) break;
    phi = dist;
 }
 bool Dijkstra() {
    dist.assign(n, LLONG_MAX);
    from.assign(n, -1);
    dist[S] = 0;
    priority_queue<pair<11, int>,
                   vec<pair<11, int>>,
                   greater<pair<11, int>>>
    pq.push({dist[S], S});
```

```
while (!pq.empty()) {
     int v;
     ll di;
     tie(di, v) = pq.top();
     pq.pop();
      if (di != dist[v]) continue;
     for (int ps : g[v]) {
       Edge& e = es[ps];
       if (e.fl == e.cp) continue;
       if (dist[e.to] - dist[e.fr] >
            e.cs + phi[e.fr] - phi[e.to]) {
         dist[e.to] = dist[e.fr] + e.cs +
                       phi[e.fr] - phi[e.to];
         from[e.to] = ps;
         pq.push({dist[e.to], e.to});
       }
     }
   for (int v = 0; v < n; v++) {
     phi[v] += dist[v];
   return dist[T] < LLONG_MAX;</pre>
 }
 pll find_mcmf() {
   init_phi();
   ll flow = 0, cost = 0;
   while (Dijkstra()) {
      int mn = INT_MAX;
     for (int v = T; v != S;
           v = es[from[v]].fr) {
       mn = min(mn,
                 es[from[v]].cp -
\rightarrow es[from[v]].fl);
     }
     flow += mn;
     for (int v = T; v != S;
           v = es[from[v]].fr) {
       es[from[v]].fl += mn;
       es[from[v] ^ 1].fl = mn;
     }
   }
   for (Edge& e : es) {
     if (e.fl >= 0) cost += 111 * e.fl * e.cs;
   return make_pair(flow, cost);
 bool go(int v, vec<int>& F, vec<int>& path,
         vec<int>& used) {
   if (used[v]) return 0;
   used[v] = 1;
   if (v == T) return 1;
   for (int ps : g[v]) {
     if (F[ps] <= 0) continue;</pre>
     if (go(es[ps].to, F, path, used)) {
       path.push_back(ps);
       return 1;
     }
   }
   return 0;
 vec<pair<int, vec<int>>>
 decomposition(ll& _flow, ll& _cost) {
```

```
tie(_flow, _cost) = find_mcmf();
    vec<int> F((int)es.size()), path, add,
      used(n);
    vec<pair<int, vec<int>>> dcmp;
    for (int i = 0; i < (int)es.size(); i++)
      F[i] = es[i].fl;
    while (go(S, F, path, used)) {
      used.assign(n, 0);
      int mn = INT_MAX;
      for (int ps : path)
        mn = min(mn, F[ps]);
      for (int ps : path)
        F[ps] = mn;
      for (int ps : path)
        add.push_back(es[ps].id);
      reverse(ALL(add));
      dcmp.push_back({mn, add});
      add.clear();
      path.clear();
    return dcmp;
  }
};
```

## Algorithm of Two Chinese

```
struct Edge {
 int fr, to, w, id;
 bool operator<(const Edge& o) const {</pre>
   return w < o.w;
 }
};
// find oriented mst (tree)
// there are no edge --> root (root is 0)
// 0 .. n - 1, weights and vertices will be
// changed, but ids are ok
vector<Edge>
work(const vector<vector<Edge>>& graph) {
 int n = (int)graph.size();
 vector<int> color(n), used(n, -1);
 for (int i = 0; i < n; i++)
    color[i] = i;
 vector<Edge> e(n);
 for (int i = 0; i < n; i++) {
    if (graph[i].empty()) {
      e[i] = \{-1, -1, -1, -1\};
    } else {
      e[i] = *min_element(graph[i].begin(),
                          graph[i].end());
 vector<vector<int>>> cycles;
 used[0] = -2;
 for (int s = 0; s < n; s++) {
    if (used[s] != -1) continue;
    int x = s;
    while (used[x] == -1) {
     used[x] = s;
     x = e[x].fr;
    }
    if (used[x] != s) continue;
    vector<int> cycle = {x};
```

```
for (int y = e[x].fr; y != x; y = e[y].fr)
      cycle.push_back(y), color[y] = x;
    cycles.push_back(cycle);
  }
  if (cycles.empty()) return e;
  vector<vector<Edge>> next_graph(n);
  for (int s = 0; s < n; s++) {
    for (const Edge& edge : graph[s]) {
      if (color[edge.fr] != color[s])
        next_graph[color[s]].push_back(
          {color[edge.fr], color[s],
           edge.w - e[s].w, edge.id});
  vector<Edge> tree = work(next_graph);
  for (const auto& cycle : cycles) {
    int cl = color[cycle[0]];
   Edge next_out = tree[cl], out{};
    int from = -1;
   for (int v : cycle) {
      tree[v] = e[v];
      for (const Edge& edge : graph[v])
        if (edge.id == next_out.id)
          from = v, out = edge;
    tree[from] = out;
  }
  return tree;
}
```

#### **Dominator Tree**

```
struct Edge {
  int fr = -1;
  int to = -1;
  int id = -1;
};
struct DSU {
  int n = 0; // [0, n)
  vector<int> p, mn;
  DSU() = default;
 DSU(int nn) {
   n = nn;
    p.resize(n);
    mn.resize(n, inf);
    for (int v = 0; v < n; v++)
      p[v] = v;
  void set_value(int v, int x) { mn[v] = x; }
  int find(int v) {
    if (p[v] == v) return v;
    int pv = find(p[v]);
    mn[v] = min(mn[v], mn[p[v]]);
    p[v] = pv;
    return pv;
  void merge(int P, int S) { p[S] = P; }
struct DominatorTree {
  int n = 0; // [0, n)
  vector<Edge> edges;
```

```
vector<vector<int>> g, gr;
vector<int> used, tin, sdom, idom, order,
  depth;
DSU dsu;
vector<vector<int>> cost, parent;
DominatorTree() = default;
DominatorTree(int nn) { n = nn; }
void add_edge(Edge e) { edges.push_back(e); }
void dfs(int v) {
  used[v] = 1;
  tin[v] = (int)order.size();
  order.push_back(v);
  for (int eid : g[v]) {
    const auto& e = edges[eid];
    if (!used[e.to]) {
      depth[e.to] = depth[v] + 1;
      parent[0][e.to] = v;
      dfs(e.to);
    }
  }
}
void init_binary_jumps() {
  int LOG = 0;
  while ((1 \ll LOG) < n)
    LOG++:
  cost.resize(LOG, vector<int>(n, inf));
  parent.resize(LOG, vector<int>(n, -1));
}
void build_sdom(int s) {
  used.assign(n, 0);
  tin.assign(n, 0);
  depth.assign(n, 0);
  order.clear();
  dfs(s);
  sdom.assign(n, inf);
  idom.assign(n, inf);
  dsu = DSU(n);
  for (int it = (int)order.size() - 1; it >= 0;
       it--) {
    int v = order[it];
    for (int eid : gr[v]) {
      const auto& e = edges[eid];
      if (!used[e.fr]) continue;
      sdom[v] = min(sdom[v], tin[e.fr]);
      if (tin[e.fr] > tin[v]) {
        dsu.find(e.fr);
        sdom[v] = min(sdom[v], dsu.mn[e.fr]);
      }
    }
    dsu.set_value(v, sdom[v]);
    for (int eid : g[v]) {
      const auto& e = edges[eid];
      if (parent[0][e.to] == v) {
        dsu.merge(v, e.to);
      }
    }
  }
int get_min_on_path(int P, int S) {
  int res = inf;
  for (int j = (int)cost.size() - 1; j >= 0;
       j--) {
    int pS = parent[j][S];
```

```
if (pS == -1 \mid \mid depth[pS] < depth[P])
        continue;
      res = min(res, cost[j][S]);
      S = pS;
    }
    return res;
  void set_value(int v, int x) {
    cost[0][v] = x;
    for (int j = 1; j < (int)cost.size(); j++) {</pre>
      int pv = parent[j - 1][v];
      if (pv == -1) {
        cost[j][v] = cost[j - 1][v];
        parent[j][v] = pv;
      } else {
        cost[j][v] =
          min(cost[j - 1][v], cost[j - 1][pv]);
        parent[j][v] = parent[j - 1][pv];
      }
    }
  }
  void build_idom(int s) {
    for (int v : order) {
      if (v == s) continue;
      idom[v] = min(
        sdom[v], get_min_on_path(order[sdom[v]],
                                   parent[0][v]));
      set_value(v, idom[v]);
  }
  void build(int s) {
    init_binary_jumps();
    g.clear();
    g.resize(n);
    gr.clear();
    gr.resize(n);
    for (int i = 0; i < (int)edges.size(); i++) {</pre>
      const auto& e = edges[i];
      g[e.fr].push_back(i);
      gr[e.to].push_back(i);
    build_sdom(s);
    build_idom(s);
 }
};
```

#### **Factorization**

```
inline bool miller_rabin_det(ll n) {
 static const ll bases[] = {
   2, 3, 5, 7, 11,
   13, 17, 19, 23}; // works for n < 3.8e18
  // static const int bases[] = {2, 3, 5, 7, 11,
  // 13, 17, 19, 23, 29, 31, 37}; // n < 3.1e23
 if (n \le 2) return (n == 2);
 if (n \% 2 == 0) return false;
 11 d = n - 1;
 while (!(d & 1))
   d >>= 1;
 for (ll a : bases) {
   if (a == n) return true;
   a = mpow(a, d, n);
   if (a == 1) continue;
   for (11 nd = d; nd != n - 1 && a != n - 1;
        nd \ll 1
     a = mmul(a, a, n);
   if (a != n - 1) return false;
 }
 return true;
}
inline ll pollard(ll n) {
 static std::mt19937_64 gen;
 static const int LOG = 50;
 if (n <= 1 || miller_rabin_det(n)) return 1;</pre>
 if (!(n & 1)) return 2;
 auto f = [n](11 x) {
   return mmul(x, x, n) + 1;
 }; // it is ok if 0 == n
 for (int st = 2, lg = 0;; st = gen() % n) {
   11 cur = 1;
   for (ll x = st, y = f(st); x != y;
         x = f(x), y = f(f(y))) {
      if (ll c = mmul(cur, std::abs(x - y), n);
   c)
       cur = c;
      if (lg++ == LOG) {
       lg = 0;
        if (ll val = std::__gcd(cur, n); val !=
→ 1)
          return val;
 }
 return 1;
```

```
11 SUB(11 a, 11 b, 11 mod) {
  return a \ge b? a - b: a - b + mod;
11 BINPOW(11 x, 11 pw, 11 mod) {
  if (x == 0) return 0;
  ll res = 1 \% \mod, \mod = x;
  while (pw > 0) {
    if (pw & 1) res = MUL(res, tmp, mod);
    pw >>= 1;
    tmp = MUL(tmp, tmp, mod);
  return res;
ll DIV(ll a, ll b, ll mod) {
  return MUL(a, BINPOW(b, mod - 2, mod), mod);
11 find_sqrt_by_mod(
  11 A) { // x^2 = a \pmod{p}, x = ?, p is prime
  assert(Oll \leftarrow A \&\& A < p);
  if (A == 0 || p == 2) return A;
  if (BINPOW(A, (p - 1) / 2, p) != 1) return

→ -111;
  static mt19937_64 GEN(42);
  auto mult = [\&] (pll p1, pll p2) -> pll {
    auto [a, b] = p1;
    auto [c, d] = p2;
    11 k1 = SUM(MUL(a, d, p), MUL(b, c, p), p);
    11 k2 = SUM(MUL(MUL(a, c, p), A, p),
                MUL(b, d, p), p);
    return {k1, k2};
  };
  while (1) {
    11 i = GEN() \% (p - 1) + 1;
    11 pw = (p - 1) / 2;
    pll res = \{0, 1\}, tmp = \{1, i\};
    while (pw > 0) {
      if (pw & 1) res = mult(res, tmp);
      pw >>= 1;
      tmp = mult(tmp, tmp);
    if (res.first == 0) continue;
    res.second = SUB(res.second, 1, p);
    11 x = DIV(res.second, res.first, p);
    if (MUL(x, x, p) == A) return x;
  }
}
```

# Square Root in $\mathbb{Z}_p$ in $O(\log p)$

```
11 MUL(11 a, 11 b, 11 mod) {
    static __int128 xa = 1;
    static __int128 xb = 1;
    static __int128 xm = 1;
    xa = a;
    xb = b;
    xm = mod;
    return 11((xa * xb) % xm);
}
11 SUM(11 a, 11 b, 11 mod) {
    return a + b < mod ? a + b : a + b - mod;
}</pre>
```

# Euclid (??)

```
ll nxt_left_len = left_len % right_len;
 11 nxt_left_cost =
    (left_len / right_len) * right_cost +
   left_cost;
 if (nxt_left_len == 0) return pos;
   11 t = pos / nxt_left_len;
   if (t * nxt_left_cost > k)
     return pos -
            nxt_left_len * (k / nxt_left_cost);
   k -= t * nxt_left_cost;
   pos -= t * nxt_left_len;
 return rec(pos, nxt_left_len, nxt_left_cost,
             right_len % nxt_left_len,
             (right_len / nxt_left_len) *
                 nxt_left_cost +
               right_cost,
             k);
// finds (nw_st + step * x) \% mod --> min, 0 <= x
// <= bound
11 euclid(ll nw_st, ll step, ll mod, ll bound) {
 return rec(nw_st, mod, 0, step, 1, bound);
```

## **Primes on Segment**

```
vector<int> find_sum_of_primes(ll N) {
    // sum_{i=1}^{n} [i \in primes] * i
    // n in \{N // k, 1 \le k \le N\}
    // can be generalize to sum of i^T
    // O(n^{3/4}/log(N))
    vector<11> vals;
    for (11 x = 1; x < N; x = N / (N / (x + 1)))
        vals.push_back(N / x);
    reverse(vals.begin(), vals.end());
    int sz = (int) vals.size();
    vector<int> S(sz), nx(sz);
    for (int i = 0; i < sz; i++) {
        ll n = vals[i];
        // (2 + n) * (n - 1) / 2 = 2 + 3 + ... +
        S[i] = n \% 2 ?
                mul(((n - 1) / 2) \% mod, (n + 2)
   % mod) :
                mul(((n + 2) / 2) \% mod, (n - 1)
   % mod);
    unordered_map<11, int> pos_hm;
    for (int i = 0; i < sz; i++)
        pos_hm[vals[i]] = i;
    for (int fr = 0, j = 1;;j++) {
        int any = 0;
        for (int i = fr; i < sz; i++) {
            ll n = vals[i];
            ll need = n / ps[j];
            auto fnd = pos_hm.find(need);
            int pos = fnd == pos_hm.end() ? -1 :
   fnd->second;
            nx[i] = S[i];
            if (pos < 0 \mid | need < ps[j - 1]) {
```

### Pro Euclid

```
// ALL in Z-ring
// T, k > 0 66 return (T - k) + (T - 2 * k) + ...
// last, last > 0
ll f(ll T, ll k) {
  ll cnt = T / k;
  return T * cnt - k * cnt * (cnt + 1) / 2;
// A, B, C > 0
// |\{(x, y): x, y > 0 \& Ax + By <= C\}|
11 count_triangle(11 A, 11 B, 11 C) {
 if (A + B > C) return 0;
  if (A > B) swap(A, B);
 11 k = B / A;
 return f(k * C / B, k) +
         count_triangle(A, B - A * k,
                        C - A * (k * C / B));
// A, B, C, cx, cy > 0
// |{(x,y) : 1 <= x <= cx && 1 <= y <= cy && Ax +
// By <= C }/
11 count_solutions(11 A, 11 B, 11 C, 11 cx,
                   11 cy) {
  assert(A > 0);
  assert(B > 0);
  if (C \le 0 \mid | cx \le 0 \mid | cy \le 0) return 0;
  if (A * cx + B * cy \le C) return cx * cy;
  if (cx >= C / A \&\& cy >= C / B)
    return count_triangle(A, B, C);
  return count_triangle(A, B, C) -
         count_triangle(A, B, C - B * cy) -
         count_triangle(A, B, C - A * cx);
}
```

# FFT with prime mod

```
template<int mod, int root, int LOG>
struct FFT {
   //   const int mod = 998244353;
   //   const int root = 31;
   //   const int LOG = 23;
   vector<int> G[LOG + 1];
   vector<int> rev[LOG + 1];
   FFT() {
```

```
for (int start = root, lvl = LOG; lvl >=
  0; lvl--, start = mul(start, start)) {
            int tot = 1 << lvl;</pre>
            G[lvl].resize(tot);
            for (int cur = 1, i = 0; i < tot;
  i++, cur = mul(cur, start)) {
                G[lvl][i] = cur;
            }
       for (int lvl = 1; lvl <= LOG; lvl++) {</pre>
            int tot = 1 << lvl;</pre>
            rev[lvl].resize(tot);
            for (int i = 1; i < tot; i++) {
                rev[lvl][i] = ((i & 1) << (lvl -
→ 1)) | (rev[lvl][i >> 1] >> 1);
            }
       }
   void fft(vector<int>& a, int sz, bool invert)
       int n = 1 << sz;</pre>
       for (int j, i = 0; i < n; i++) {
            if ((j = rev[sz][i]) < i)
                swap(a[i], a[j]);
       for (int f1, f2, lv1 = 0, len = 1; len <
   n; len <<= 1, lvl++) {
            for (int i = 0; i < n; i+= (len <<
   1)) {
                for (int j = 0; j < len; j++) {
                    f1 = a[i + j];
                    f2 = mul(a[i + j + len],
\hookrightarrow G[lvl + 1][j]);
                    a[i + j] = sum(f1, f2);
                    a[i + j + len] = sub(f1, f2);
            }
       }
        if (invert) {
            reverse(a.begin() + 1, a.end());
            int rn = binpow(n, mod - 2);
            for (int i = 0; i < n; i++) {</pre>
                a[i] = mul(a[i], rn);
            }
       }
   vector<int> multiply(const vector<int>& a,
   const vector<int>& b) {
       vector<int> fa(a.begin(), a.end());
       vector<int> fb(b.begin(), b.end());
       int n = (int) a.size();
       int m = (int) b.size();
       int maxnm = max(n, m), sz = 0;
       while ((1 \ll sz) < maxnm)
            sz++;
       sz++;
       fa.resize(1 << sz);</pre>
       fb.resize(1 << sz);</pre>
       fft(fa, sz, false);
       fft(fb, sz, false);
       int SZ = 1 << sz;</pre>
       for (int i = 0; i < SZ; i++) {</pre>
            fa[i] = mul(fa[i], fb[i]);
```

```
fft(fa, sz, true);
        return fa;
    int sum(int x, int y) {
        return x + y < mod ? x + y : x + y - mod;
    int sub(int x, int y) {
        return x \ge y ? x - y : x - y + mod;
    int mul(int x, int y) {
        return (111 * x * y) % mod;
    int mul(const vector<int>& a) {
        int res = 1;
        for (const auto& x : a)
            res = mul(res, x);
        return res;
   }
    void inc(int& x, int y) {
        if ((x += y) >= mod)
            x = mod;
    void dec(int& x, int y) {
        if ((x -= y) < 0)
            x += mod;
    int binpow(int x, int pw) {
        int res = 1, tmp = x;
        while (pw > 0) {
            if (pw & 1) res = mul(res, tmp);
            tmp = mul(tmp, tmp);
            pw >>= 1;
        return res;
   }
};
```

## **Polynomial Division**

```
// let A = series and A[0] != 0 in Z/pZ, p is
// prime finds (A^{-1}) \% x^n
vector<int>
series_inverse(const vector<int>& series, int n,
               11 p) {
  vector<int> current = {_div(1, series[0], p)};
  vector<int> A = {};
  int 1 = 0;
  while ((int)current.size() < n) {</pre>
    while (1 < 2 * (int)current.size()) {</pre>
      A.push_back(
        1 < (int)series.size() ? series[1] : 0);</pre>
    }
    vector<int> next = multiply(A, current);
    for (int& x : next)
      x = (-x \% p + p) \% p;
    next[0] = _sum(2 % p, next[0], p);
    next = multiply(next, current);
    for (int& x : next)
      x = (x \% p + p) \% p;
```

```
next.resize(2 * current.size());
    current = next;
 }
 current.resize(n);
 return current;
// calculates a / b
vector<int> division(const vector<int>& a,
                     const vector<int>& b,
                     int p) {
 int n = (int)a.size() - 1; // deg(a)
 int m = (int)b.size() - 1; // deq(b)
 if (n < m) { return {0}; }
 vector<int> ar = a, br = b;
 reverse(ar.begin(), ar.end());
 reverse(br.begin(), br.end());
 ar.resize(n - m + 1);
 br.resize(n - m + 1);
 vector<int> qr =
    series_inverse(br, n - m + 1, p);
 qr = multiply(qr, ar);
 qr.resize(n - m + 1);
 for (int& x : qr)
    x = (x \% p + p) \% p;
 reverse(qr.begin(), qr.end()); // q = q^r
 return qr;
// calculates a - bQ
vector<int> module(const vector<int>& a,
                   const vector<int>& b,
                   const vector<int>& Q, int p) {
 vector<int> r = multiply(b, Q);
 r.resize(b.size());
 for (int i = 0; i < (int)r.size(); i++) {</pre>
    int ai = i < (int)a.size() ? a[i] : 0;</pre>
    int ri = (r[i] % p + p) % p;
   r[i] = _sub(ai, ri, p);
 }
 return r;
```

#### FFT

```
template <int LOG>
struct FFT {
   vector<int> rev[LOG + 1];
   vector<base> G[LOG + 1];
    FFT() {
        int N = 1 << LOG;
        base root(cosl(2 * pi / N), sinl(2 * pi / N))
   N));
        base start = root;
        for (int lvl = LOG; lvl >= 0; lvl--,
   start = start * start) {
            int tot = 1 << lvl;</pre>
            G[lvl].resize(tot);
            base cur = 1;
            for (int i = 0; i < tot; i++, cur *=
   start)
                G[lvl][i] = cur;
        for (int lvl = 1; lvl <= LOG; lvl++) {
```

```
int tot = 1 << lvl;</pre>
            rev[lvl].resize(tot);
            for (int i = 1; i < tot; i++) {
                rev[lvl][i] = ((i & 1) << (lvl -
→ 1)) | (rev[lvl][i >> 1] >> 1);
        }
    }
    void fft(vector<base>& a, int sz, bool
        int n = 1 << sz;</pre>
        for (int j, i = 0; i < n; i++) {
            if ((j = rev[sz][i]) < i) {
                swap(a[i], a[j]);
            }
        }
        base f1, f2;
        for (int lvl = 0, len = 1; len < n; len
   <<= 1, lvl++) {
            for (int i = 0; i < n; i += (len <<
   1)) {
                for (int j = 0; j < len; j++) {
                    f1 = a[i + j];
                    f2 = a[i + j + len] * G[lvl +

→ 1][j];

                     a[i + j] = f1 + f2;
                     a[i + j + len] = f1 - f2;
                }
            }
        if (invert) {
            reverse(a.begin() + 1, a.end());
            for (int i = 0; i < n; i++)
                a[i] /= n;
        }
    }
    vector<ld> multiply(const vector<ld>& a,

    const vector<ld>& b) {
        vector<base> fa(a.begin(), a.end());
        vector<base> fb(b.begin(), b.end());
        int n = (int) a.size();
        int m = (int) b.size();
        int maxnm = max(n, m), sz = 0;
        while ((1 \ll sz) < maxnm)
            sz++;
        sz++;
        fa.resize(1 \ll sz);
        fb.resize(1 \ll sz);
        fft(fa, sz, false);
        fft(fb, sz, false);
        int SZ = 1 << sz;</pre>
        for (int i = 0; i < SZ; i++)
            fa[i] = fa[i] * fb[i];
        fft(fa, sz, true);
        vector<ld> res(SZ);
        for (int i = 0; i < SZ; i++)
            res[i] = fa[i].real();
        return res;
    }
};
```

## Extrapolation

```
int fact[N];
int rfact[N];
void precalc2() {
 fact[0] = 1;
 for (int i = 1; i < N; i++) {
   fact[i] = _mul(fact[i - 1], i);
 }
 rfact[N - 1] = rev(fact[N - 1]);
 for (int i = N - 2; i >= 0; i--) {
   rfact[i] = _mul(rfact[i + 1], i + 1);
}
int getMulOnSegment(int 1, int r) {
 assert(1 <= r);</pre>
 if (1 == 0 && r == 0) return 1;
 if (r <= 0) {
   int res = getMulOnSegment(-r, -1);
   int cnt = r - 1 + 1;
   if (cnt % 2) {
     res = (-res \% mod + mod) \% mod;
   return res;
 }
 if (1 < 0) {
   int resl = getMulOnSegment(0, -1);
   if (1 % 2) {
     resl = (-resl % mod + mod) % mod;
   int resr = getMulOnSegment(0, r);
   return _mul(resl, resr);
 }
 assert(1 >= 0);
 int res = fact[r];
 if (1 > 0) { res = _mul(res, rfact[1 - 1]); }
 return res;
vector<int> extrapolate(vector<int> y, int m) {
 vector<int> yy = y;
 int n = (int)y.size() - 1;
 for (int i = 0; i <= n; i++) {
   yy[i] = _mul(
      y[i], _rev(getMulOnSegment(i - n, i - 0)));
 }
 vector<int> ff(n + m + 1);
 for (int i = 1; i <= n + m; i++) {
   ff[i] = _mul(fact[i - 1], rfact[i]);
 vector<int> ss = multiply(yy, ff);
 for (int i = 1; i <= m; i++) {
   int cc = getMulOnSegment(i, n + i);
   int Si = ss[n + i];
   y.push_back(_mul(cc, Si));
 }
 return y;
```

## Xor FWHT

```
// _sum, _sub, _mul - arithmetic operations
void xor_fwht(vector<int>& a,
```

```
bool inverse = false) {
  for (int x, y, len = 1; len < (int)a.size();</pre>
       len <<= 1) {
    for (int i = 0; i < (int)a.size();</pre>
         i += len << 1) {
      for (int j = 0; j < len; j++) {
        x = a[i + j], y = a[i + j + len];
        a[i + j] = _sum(x, y);
        a[i + j + len] = \_sub(x, y);
    }
  }
  if (inverse) {
    int rn = _binpow((int)a.size(), mod - 2);
    for (int& x : a)
      x = _{mul}(x, rn);
  }
void or_fwht(vector<int>& a,
             bool inverse = false) {
  for (int x, y, len = 1; len < (int)a.size();</pre>
       len <<= 1) {
    for (int i = 0; i < (int)a.size();</pre>
         i += len << 1) {
      for (int j = 0; j < len; j++) {
        x = a[i + j], y = a[i + j + len];
        a[i + j] = x,
               a[i + j + len] =
                 inverse ? _{sub}(y, x) : _{sum}(y,
\hookrightarrow x);
    }
  }
}
void and_fwht(vector<int>& a,
              bool inverse = false) {
  for (int x, y, len = 1; len < (int)a.size();</pre>
       len <<= 1) {
    for (int i = 0; i < (int)a.size();
         i += len << 1) {
      for (int j = 0; j < len; j++) {
        x = a[i + j], y = a[i + j + len];
        a[i + j] =
           inverse ? _{sub}(x, y) : _{sum}(x, y),
               a[i + j + len] = y;
      }
    }
  }
}
```

### CHT

```
return x < other.x;</pre>
    } else {
      return k < other.k;</pre>
    }
 }
 ld intersect(const Line& other) const {
    return ld(b - other.b) / ld(other.k - k);
 }
 ll get_func(ll x0) const { return k * x0 + b; }
};
struct CHT {
 set<Line> qs;
 set<Line>::iterator fnd, help;
 bool hasr(const set<Line>::iterator& it) {
    return it != qs.end() && next(it) !=

¬ qs.end();

 }
 bool hasl(const set<Line>::iterator& it) {
   return it != qs.begin();
 bool check(const set<Line>::iterator& it) {
    if (!hasr(it)) return true;
    if (!hasl(it)) return true;
    return it->intersect(*prev(it)) <</pre>
           it->intersect(*next(it));
 }
 void update_intersect(
    const set<Line>::iterator& it) {
    if (it == qs.end()) return;
    if (!hasr(it)) return;
    Line tmp = *it;
    tmp.x = tmp.intersect(*next(it));
    qs.insert(qs.erase(it), tmp);
 }
 void add_line(Line L) {
    if (qs.empty()) {
      qs.insert(L);
      return;
    }
      fnd = qs.lower_bound(L);
      if (fnd != qs.end() && fnd->k == L.k) {
        if (fnd->b >= L.b)
          return;
        else
          qs.erase(fnd);
      }
    }
    fnd = qs.insert(L).first;
    if (!check(fnd)) {
      qs.erase(fnd);
      return;
    while (hasr(fnd) &&
           !check(help = next(fnd))) {
      qs.erase(help);
    while (hasl(fnd) &&
           !check(help = prev(fnd))) {
      qs.erase(help);
    if (hasl(fnd)) {
      update_intersect(prev(fnd));
```

```
update_intersect(fnd);
  11 get_max(ld x0) {
    if (qs.empty()) return -inf64;
    fnd = qs.lower_bound(Line(0, 0, x0, 1));
    if (fnd == qs.end()) fnd--;
    11 res = -inf64;
    int i = 0;
    while (i < 2 && fnd != qs.end()) {
      res = max(res, fnd->get_func(x0));
      fnd++;
      i++;
    while (i-- > 0)
      fnd--;
    while (i < 2) {
     res = max(res, fnd->get_func(x0));
      if (hasl(fnd)) {
        fnd--;
        i++;
      } else {
        break;
      }
    }
    return res;
 }
};
```

### **Euler Tour Trees**

```
class EulerTourTrees {
  /*
  graph - forest
  1 \dots n
  qet = is connected?
  no memory leaks
  1 <= n, q <= 10^5
  0.7 sec
  */
private:
  struct Node {
    Node* 1, * r, * p;
    int prior, cnt, rev;
    Node() : l(nullptr), r(nullptr), p(nullptr),
          prior(rnd()), cnt(1), rev(0) {}
    ~Node() {
      delete 1; delete r;}};
  void do_rev(Node* v) {
    if (v) v \rightarrow rev = 1, swap(v \rightarrow 1, v \rightarrow r);
  int get_cnt(Node* v) const {
    return v ? v->cnt : 0;
  void update(Node* v) {
    if (!v) return;
    v->cnt = 1 + get_cnt(v->1) + get_cnt(v->r);
    v->p = nullptr;
    if (v->1) v->1->p = v;
    if (v->r) v->r->p = v;
  }
```

```
void push(Node* v) {
  if (!v) return;
  if (v->rev) {
    do_rev(v->1);
    do_rev(v->r);
    v->rev ^= 1;
}
void merge(Node*& v, Node* 1, Node* r) {
  if (!l || !r) {
    v = 1 ? 1 : r;
    return; }
  push(1); push(r);
  if (l->prior < r->prior) {
    merge(1->r, 1->r, r); v = 1;
  } else {
    merge(r->1, 1, r->1); v = r;
  update(v);
}
void split_by_cnt(Node* v, Node*& 1, Node*& r,
                  int x) {
  if (!v) {
    1 = r = nullptr; return;
  push(v);
  if (get_cnt(v->1) + 1 <= x) {
    split_by_cnt(v->r, v->r, r,
                 x - get_cnt(v->1) - 1);
    1 = v;
  } else {
    split_by_cnt(v->1, 1, v->1, x);
    r = v;
  update(1);
  update(r);
void push_path(Node* v) {
  if (!v) return;
  push_path(v->p);
  push(v);
}
int get_pos(Node* v) {
  push_path(v);
  int res = 0, ok = 1;
  while (v) {
    if (ok) res += get_cnt(v->1) + 1;
    ok = v - p \&\& v - p - r == v;
    v = v -> p;
  }
  return res;
Node* get_root(Node* v) const {
  while (v \&\& v->p)
    v = v - > p;
  return v:
}
Node* shift(Node* v) {
  if (!v) return v;
  int pos = get_pos(v);
  Node *nl = nullptr, *nr = nullptr;
  Node* root = get_root(v);
  split_by_cnt(root, nl, nr, pos - 1);
```

```
do_rev(nl);
    do_rev(nr);
    merge(root, nl, nr);
    do_rev(root);
    return root;
public:
  EulerTourTrees() = default;
  EulerTourTrees(int _n) : n(_n) {
    ptr.resize(_n + 1);
    where_edge.resize(_n + 1);
  7
  bool get(int u, int v) const {
    if (u == v) return true;
    Node* ru = get_root(
      ptr[u].empty() ? nullptr :
 → *ptr[u].begin());
    Node* rv = get_root(
      ptr[v].empty() ? nullptr :
 → *ptr[v].begin());
    return ru && ru == rv;}
  void link(int u, int v) {
    Node* ru = shift(
      ptr[u].empty() ? nullptr :
 → *ptr[u].begin());
    Node* rv = shift(
      ptr[v].empty() ? nullptr :
    *ptr[v].begin());
    Node* uv = new Node(), * vu = new Node();
    ptr[u].insert(uv); ptr[v].insert(vu);
    where_edge[u][v] = uv; where_edge[v][u] = vu;
    merge(ru, ru, uv); merge(ru, ru, rv);
    merge(ru, ru, vu);}
  void cut(int u, int v) {
    Node* uv = where_edge[u][v]6 * vu =
    where_edge[v][u];
    ptr[u].erase(uv); ptr[v].erase(vu);
    Node* root = shift(uv);
    Node *nl = nullptr, *nm = nullptr,
         *nr = nullptr;
    int pos1 = get_pos(uv), pos2 = get_pos(vu);
    if (pos1 < pos2) {
      split_by_cnt(root, nl, nr, pos2);
      split_by_cnt(nl, nl, vu, pos2 - 1);
      split_by_cnt(nl, nl, nm, pos1);
      split_by_cnt(nl, nl, uv, pos1 - 1);
      merge(nl, nl, nr);
    } else {
      split_by_cnt(root, nl, nr, pos1);
      split_by_cnt(nl, nl, uv, pos1 - 1);
      split_by_cnt(nl, nl, nm, pos2);
      split_by_cnt(nl, nl, vu, pos2 - 1);
      merge(nl, nl, nm);}
    delete uv; delete vu;}
  ~EulerTourTrees() {
    set<Node*> roots;
    for (int i = 1; i <= n; i++)
      for (Node* v : ptr[i])
        roots.insert(get_root(v));
    for (Node* root : roots)
      delete root;}
private:
```

```
int n = 0;
vec<set<Node*>> ptr;
vec<unordered_map<int, Node*>>
    where_edge; // ptr to node
};
```

## **Simplex**

```
template <class T>
vector<T> operator+(const vector<T>& a,
                     const vector<T>& b) {
 vector<T> res(a.size());
 for (int i = 0; i < (int)a.size(); i++)</pre>
   res[i] = a[i] + b[i];
 return res;
template <class T>
vector<T> operator*(const T& coef,
                     const vector<T>& a) {
 vector<T> res(a.size());
 for (int i = 0; i < (int)a.size(); i++)</pre>
    res[i] = coef * a[i];
 return res;
}
const ld eps = 1e-9;
struct Simplex {
 // Ax = b, x \ge 0, < c, x \ge - > max
 int m;
                         // the number of
\hookrightarrow equations
                         // the number of
 int n;
\hookrightarrow variables
 vector<vector<ld>> A; // (m + 2) x (n + 1)
 // (m + 1)-th row: primary c
 // (m + 2)-th row: seconday c (c')
 // (n + 1)-th col: column of b
 vector<int> basis;
 bool bounded = true;
 Simplex(const vector<vector<ld>>& mat,
          const vector<int>& _basis)
      : A(mat), basis(_basis) {
   m = (int)mat.size() - 2,
    n = (int)mat[0].size() - 1;
 /// make primary c under basis components zero
 void reset_c() {
    for (int i = 0; i < m; i++) {
      int j = basis[i];
      A[m] = A[m] + (-A[m][j]) * A[i];
      A[m + 1] = A[m + 1] + (-A[m + 1][j]) *
   A[i];
 void pivot(int i, int k) {
    A[k] = (1d(1) / 1d(A[k][i])) * A[k];
    for (int j = 0; j < (int)A.size(); j++) {</pre>
      if (j == k) continue;
      A[j] = A[j] + (-A[j][i]) * A[k];
    basis[k] = i;
 }
 void run() {
    while (true) {
```

```
int j = 0;
      while (j < n \&\& A[m][j] \le eps)
        j++;
      if (j == n) break;
      int k = -1;
      for (int i = 0; i < m; i++)
        if (A[i][j] > eps &&
            (k == -1 || (A[i][n] / A[i][j] <
                         A[k][n] / A[k][j]))
          k = i;
      if (k == -1) {
        bounded = false;
        break;
     pivot(j, k);
    }
  }
  vector<ld> get_solution() {
    vector<ld> res(n);
    for (int i = 0; i < m; i++)
      res[basis[i]] = A[i][n];
    return res;
  void reset_column(int j) {
    for (int i = 0; i < (int)A.size(); i++)</pre>
      A[i][j] = 0;
  ld get_max_value() { return -A[m][n]; }
  void swap_primary_c() { swap(A[m], A[m + 1]); }
  void flip_task_type() {
    A[m] = ld(-1) * A[m];
    A[m + 1] = ld(-1) * A[m + 1];
};
struct Response {
 bool bounded = true;
  bool exist = true;
 ld value = 0;
 vector<ld> solution = {};
};
// aa * x \le bb, <cc, x > --- > max
Response solve(const vector<vector<ld>>>& aa,
               const vector<ld>& bb,
               const vector<ld>& cc) {
  int m = (int)aa.size();
  int n = (int)aa[0].size();
  vector<vector<ld>>> a(m,
                       vector<ld>(n + m + 1 +

→ 1));
 for (int i = 0; i < m; i++) {
    for (int j = 0; j < n; j++)
      a[i][j] = aa[i][j];
    a[i][n + i] = +1;
    a[i][n + m] = -1;
    a[i][n + m + 1] = bb[i];
  vector<ld> c(n + m + 1 + 1), c2(n + m + 1 + 1);
  for (int i = 0; i < n; i++)
    c[i] = cc[i];
  c2[n + m] = -1;
  vector<int> basis(m);
  for (int j = 0; j < m; j++)
```

```
basis[j] = n + j;
a.push_back(c2);
a.push_back(c);
Simplex simplex(a, basis);
simplex.reset_c();
  int k = 0;
  for (int i = 1; i < m; i++)
    if (a[i][n + m + 1] < a[k][n + m + 1])
      k = i;
  if (a[k][n + m + 1] < -eps)
    simplex.pivot(n + m, k);
simplex.run();
if (!simplex.bounded ||
    -simplex.get_max_value() > eps) {
  return Response{true, false, 0, {}};
}
{
  vector<int> in_basis(n + m + 1, -1);
  for (int i = 0; i < m; i++)</pre>
    in_basis[simplex.basis[i]] = i;
  int k = in_basis[n + m];
  if (k != -1) {
    for (int i = 0; i < n + m; i++) {
      if (in_basis[i] != -1) continue;
      if (std::abs(simplex.A[k][i]) <= eps)</pre>
        continue;
      simplex.pivot(i, k);
      break;
    }
  }
  simplex.reset_column(n + m);
}
simplex.swap_primary_c();
simplex.run();
if (!simplex.bounded) {
  return Response{false, true, 0, {}};
}
Response response;
response.value = simplex.get_max_value();
response.solution = simplex.get_solution();
response.solution.resize(n);
return response;
```

### **Fast Allocator**

#endif

## **Angle Comparator**

```
struct comparator {
  pll center;
  comparator(pll p) : center(p) {}
  bool operator()(const pll& p,
                  const pll& q) const {
    pll start(1, 0);
    if (p == q) return false;
    auto op = vect(center, p);
    auto oq = vect(center, q);
    if (cp(op, oq) == 0 \&\& dp(op, oq) > 0)
      return false;
    11 sop = cp(start, op), soq = cp(start, oq);
    if (sop == 0) {
      if (dp(start, op) > 0) return true;
      return soq < 0;</pre>
    }
    if (soq == 0) {
      if (dp(start, oq) > 0) return false;
      return sop > 0;
    }
    if ((sop > 0 && soq > 0) ||
        (sop < 0 \&\& soq < 0)) {
      return cp(op, oq) > 0;
    }
    return sop > 0;
};
```

## **Common Tangents**

```
struct circle {
   pt c;
    ld r = 0;
    ld area() const {
        return pi * r * r;
    }
};
vector<pair<pt, pt>> get_tangents(circle w1,

    circle w2) {

    if (w1.r > w2.r) // IMPORTANT!!!
        swap(w1, w2);
    1d D = (w1.c - w2.c).norm();
    if (D < abs(w1.r - w2.r) + eps) // ONE INSIDE
→ ANOTHER!!!
        return {};
   ld cos_alpha = abs(w1.r - w2.r) / D;
    ld sin_alpha = sqrtl(max(ld(0), 1 - cos_alpha

    * cos_alpha));
   vector<pair<pt, pt>> ts;
   pt v1 = (w1.c - w2.c).rotate(cos_alpha,
  +sin_alpha);
   pt v2 = (w1.c - w2.c).rotate(cos_alpha,
   -sin_alpha);
```

```
ts.emplace_back(w1.c + (v1 * (w1.r / v1.norm())), w2.c + (v1 * (w2.r / v1.norm())));
ts.emplace_back(w1.c + (v2 * (w1.r / v2.norm())), w2.c + (v2 * (w2.r / v2.norm()));
return ts;
}
```

## Find Tangents in polygon $O(\log n)$

```
pair<pt, pt>
→ vector<pt>& P, pt q) {
   // q is strictly outside P
   // P in counter-clockwise order
    // P.size >= 3
    // P is strictly convex
   // return {leftmost visible, rightmost
→ visible}
   // be careful with corner cases
   // (when two different points may be
   // leftmost/rightmost visible)
   int n = (int) P.size();
   auto is_visible_edge = [&](int i) -> bool {
       return (P[i] - q).vector_mul(P[(i + 1) %
\rightarrow n] - P[i]) < -eps;
   };
   auto is_visible = [&](int i) -> bool {
       return is_visible_edge(i) ||
   is_visible_edge((i - 1 + n) \% n);
   };
   auto is_on_right = [&](int i) -> bool {
       return (P[0] - q).vector_mul(P[i] - q) <</pre>
   -eps;
   };
   int bl, br, bm;
   int A = -1, B = -1;
   if (is_visible(0)) {
       bl = 0, br = n;
        while (br - bl > 1) {
           bm = (bl + br) >> 1;
           if (is_visible(bm) &&
   is_on_right(bm)) bl = bm;
           else br = bm;
       }
       B = b1;
       bl = 0, br = n;
        while (br - bl > 1) {
           bm = (bl + br) >> 1;
           int i = (B + n - bm) \% n;
           if (is_visible(i)) bl = bm;
           else br = bm;
       }
       A = (B + n - b1) \% n;
   } else {
       bl = 0, br = n;
       while (br - bl > 1) {
           bm = (bl + br) >> 1;
           if (!is_visible(bm) &&
   !is_on_right(bm)) bl = bm;
```

```
else br = bm;
}
A = (bl + 1) % n;

bl = 0, br = n;
while (br - bl > 1) {
    bm = (bl + br) >> 1;
    int i = (A + bm) % n;
    if (is_visible(i)) bl = bm;
    else br = bm;
}
B = (A + bl) % n;
}
return {P[A], P[B]};
}
```

## Minkowsky Polygon Sum

```
vector<pt> minkowski_polygons_sum(vector<pt> a,
                                   vector<pt> b) {
  // a and b have counter-clock wise order
  auto cmp = [](const pt& p1,
                const pt& p2) -> bool {
    return make_pair(p1.x, p1.y) <</pre>
           make_pair(p2.x, p2.y);
  };
  rotate(a.begin(),
         min_element(a.begin(), a.end(), cmp),
         a.end());
  rotate(b.begin(),
         min_element(b.begin(), b.end(), cmp),
         b.end());
  pt q = a[0] + b[0];
  int n = (int)a.size();
  int m = (int)b.size();
  vector<pt> result = {q};
  for (int i = 0, j = 0; i < n \mid \mid j < m;) {
    pt vi, vj;
    if (i < n)
      vi = a[i + 1 < n ? i + 1 : 0] - a[i];
    if (j < m)
     vj = b[j + 1 < m ? j + 1 : 0] - b[j];
    if (i < n &&
        (j == m || vi.vector_mul(vj) > eps))
      q = q + vi, i++;
    else
      q = q + vj, j++;
    result.push_back(q);
  result.pop_back();
  return result;
}
```

## Halfplanes Intersection $O(n \log n)$

```
const ld eps = 1e-9;
struct pt {
  ld x = 0, y = 0;
  pt operator+(const pt% o) const {
```

```
return \{x + o.x, y + o.y\};
 }
 pt operator-(const pt& o) const {
    return \{x - o.x, y - o.y\};
 pt operator*(ld coef) const {
    return {x * coef, y * coef};
 }
 ld vector_mul(const pt& o) const {
    return x * o.y - o.x * y;
 }
 ld scalar_mul(const pt& o) const {
    return x * o.x + y * o.y;
 ld sqr_norm() const {
    return scalar_mul(*this);
 }
 ld norm() const {
    return sqrtl(max(ld(0), sqr_norm()));
 }
 int quadrant() const {
    if (x \ge eps && y \ge -eps)
      return 1;
    else if (x < eps && y >= eps)
      return 2;
    else if (x \le -eps \&\& y \le eps)
      return 3;
    else
      return 4;
 }
 bool operator<(const pt& o) const {</pre>
    int q1 = quadrant();
    int q2 = o.quadrant();
    if (q1 != q2) return q1 < q2;
    return vector_mul(o) >= eps;
 }
};
struct Line {
 pt a, b;
 pt dir() const { return b - a; }
};
pair < bool, pt > intersect_lines(const Line & L1,
                                const Line& L2) {
 ld vm = L1.dir().vector_mul(L2.dir());
 if (abs(vm) < eps) return {false, pt{}};</pre>
 ld t = L2.dir().vector_mul(L1.a - L2.a) / vm;
 return {true, L1.a + L1.dir() * t};
struct Response {
 enum TYPE { EMPTY, INF, FINITE };
 TYPE type;
 vector<Line> halfs;
};
bool is_line_good(Line L, Line L1, Line L2,
                  bool strictly = false) {
 int any_colinear = 0;
 for (Line L_hat : {L1, L2}) {
    ld vm = L.dir().vector_mul(L_hat.dir());
    ld sm = L.dir().scalar_mul(L_hat.dir());
    if (abs(vm) < eps) {</pre>
      any_colinear = 1;
      if (sm >= eps) {
        if (L.dir().vector_mul(L_hat.b - L.a) >=
```

```
(strictly ? eps : -eps))
          return false;
      } else {
        if (L.dir().vector_mul(L_hat.b - L.a) <=</pre>
            -eps)
          return false;
      }
    }
  }
  if (any_colinear) return true;
  ld vm1 = L.dir().vector_mul(L1.dir());
  ld vm2 = L.dir().vector_mul(L2.dir());
  int t1 = vm1 >= eps ? +1 : -1;
  int t2 = vm2 >= eps ? +1 : -1;
  if (t1 == t2) return true;
  if (t1 > t2)
    swap(t1, t2), swap(vm1, vm2), swap(L1, L2);
 pt p1 = intersect_lines(L, L1).second;
 pt p2 = intersect_lines(L, L2).second;
  return (p2 - p1).scalar_mul(L.dir()) > -eps;
bool check_empty(Line L1, Line L2, Line L3) {
 return !is_line_good(L1, L2, L3, true) &&
         !is_line_good(L2, L1, L3, true) &&
         !is_line_good(L3, L1, L2, true);
Response intersect_halfs(vector<Line> halfs) {
  sort(halfs.begin(), halfs.end(),
       [](const Line% 11, const Line% 12) {
         return 11.dir() < 12.dir();
       });
  int n = (int)halfs.size(), is_inf = 0,
      any_positive_vm = 0;
  deque<Line> hull;
  Line L1, L2, L3;
  ld vm, sm;
  for (int i = 0; i < n; i++) {
    vm = halfs[i].dir().vector_mul(
     halfs[i + 1 < n ? i + 1 : 0].dir());
    sm = halfs[i].dir().scalar_mul(
     halfs[i + 1 < n ? i + 1 : 0].dir());
    any_positive_vm |= vm >= eps;
    if (vm <= -eps || (vm < eps && sm <= -eps))
      is_inf = 1;
    hull.push_back(halfs[i]);
    for (int sz; (sz = (int)hull.size()) >= 3;) {
      L1 = hull[0], L2 = hull[1],
      L3 = hull[sz - 1];
      if (check_empty(L1, L2, L3))
        return Response{Response::TYPE::EMPTY,
                        {}};
      if (!is_line_good(L1, L2, L3)) {
        hull.pop_front();
        continue;
      }
     L1 = hull[sz - 1], L2 = hull[0],
     L3 = hull[sz - 2];
      if (check_empty(L1, L2, L3))
        return Response { Response : : TYPE : : EMPTY,
                         {}};
      if (!is_line_good(L1, L2, L3)) {
        hull.pop_back();
```

```
continue;
      }
     L1 = hull[sz - 2], L2 = hull[sz - 1],
     L3 = hull[(2 * sz - 3) \% sz];
      if (check_empty(L1, L2, L3))
        return Response { Response : : TYPE : : EMPTY,
                        {}};
      if (!is_line_good(L1, L2, L3)) {
        swap(hull[sz - 1], hull[sz - 2]);
        hull.pop_back();
        continue;
     }
     break;
    if ((int)hull.size() == 2 &&
        check_empty(hull[0], hull[1], hull[1]))
      return Response{Response::TYPE::EMPTY, {}};
 }
 is_inf |= !any_positive_vm;
 vector<Line> res(hull.begin(), hull.end());
 return Response{is_inf ? Response::TYPE::INF
  Response::TYPE::FINITE,
                  res};
ld calculate_area(Response response) {
 assert(response.type != Response::TYPE::INF);
 if (response.type == Response::TYPE::EMPTY)
    return 0;
 const auto& halfs = response.halfs;
 int n = (int)halfs.size();
 ld area = 0;
 vector<pt> ps;
 for (int i = 0; i < n; i++) {
    int j = i + 1 < n ? i + 1 : 0;
    if (abs(halfs[i].dir().vector_mul()
          halfs[j].dir())) < eps)
      continue;
    ps.push_back(
      intersect_lines(halfs[i],
→ halfs[j]).second);
 }
 n = (int)ps.size();
 for (int i = 0; i < n; i++) {
    int j = i + 1 < n ? i + 1 : 0;
    area += ps[i].vector_mul(ps[j]);
 }
 return abs(area) / 2;
Line get_line(ld A, ld B,
              1d C) { // Ax + By + C >= 0
 pt v = \{B, -A\}, a, b;
 assert(A * A + B * B > eps);
 if (abs(v.x) >= eps) {
    a = \{0, -C / B\};
 } else {
    a = \{-C / A, 0\};
 }
 b = a + v;
 return Line{a, b};
```

#### Fenwick Descent

```
struct Processor {
  int n = 0; // [0, n)
  vector<int> a;
  Processor() = default;
  Processor(int nn) {
    n = nn;
    a.assign(n, 0);
  void increase(int i, int x) {
    for (int cur = i; cur < n; cur |= (cur + 1))</pre>
      a[cur] += x;
  int descent(int lb) {
    int pos = 0;
    for (int pw = 1 << 19; pw > 0; pw >>= 1) {
      if (pos + pw \le n \&\& a[pos + pw - 1] \le 1b)
← {
        lb -= a[pos + pw - 1];
        pos += pw;
    }
    return pos;
 }
};
```

### STL Tree

# Convex Hull 3d $O(n^2)$

```
vector<plane> convex_hull_3d(vector<pt> a) {
    int n = (int) a.size(), timer = 10;
    vector<plane> hull;
    { /** NB: initial corner cases (find first 4
→ no coplonar points) **/ }
    auto in_hull = [&](const pt& q) -> bool {
        for (const auto& w : hull) if (sign(w, q)
    <= -eps) return false;
        return true;
   };
   vector<vector<int>> last_see(n,
→ vector<int>(n)), last_not_see(n,

    vector<int>(n));
    auto add = [&](int nid) {
        pt q = a[nid]; timer++;
        for (const auto& w : hull) {
           auto\& ar = sign(w, q) <= -eps ?
   last_see : last_not_see;
            for (int i = 0; i < 3; i++) {
                int j = (i + 1) \% 3, id_i =
  w.ps[i], id_j = w.ps[j];
```

```
ar[id_i][id_j] = ar[id_j][id_i] = |
timer;
    vector<plane> next_hull;
    for (const auto& w : hull) {
         if (sign(w, q) \le -eps) \{
             const auto& ar = w.ps;
             int sz = (int) ar.size();
             assert(sz == 3);
             for (int i = 0; i < sz; i++) {
                 int j = (i + 1) % sz, id_i =
ar[i], id_j = ar[j], id_k = ar[3 ^ i ^ j];
                 if (last_see[id_i][id_j] ==
timer && last_not_see[id_i][id_j] == timer) {
                     plane add_plane =
get_plane(q, a[id_i], a[id_j]);
                     add_plane.ps = {nid,
id_i, id_j};
                     if (sign(add_plane,
a[id_k]) <= -eps) add_plane.v = -add_plane.v;</pre>
next_hull.push_back(add_plane);
             }
        } else {
             next_hull.push_back(w);
    swap(hull, next_hull);
};
for (int i = 0; i < n; i++) if
(!in_hull(a[i])) add(i);
for (auto& w : hull) for (auto& id : w.ps) id
= a[id].id;
return hull;
```

# Sums of two squares $O(ANS \cdot \log^k(n))$

```
pll find_sums_of_two_squares_for_prime(ll p) {
    // O(poly log p) a^2 + b^2 = p, 1 <= a < b <=
\hookrightarrow sqrt(p)
    assert(p % 4 == 1); // p mod 4 = 1 (Ferma th)
    11 x = 1; // find x: x^2=-1 \pmod{p}
    while (BINPOW(x, (p - 1) / 2, p) != p - 1)
\hookrightarrow x++;
    x = BINPOW(x, (p - 1) / 4, p);
    assert(MUL(x, x, p) == p - 1);
    11 y = BINPOW(x, p - 2, p), sqrt_p =

    find_sqrt(p);

    // 0 \le b' \le sqrt_p ; (y + y * b') \% p \rightarrow
\rightarrow min; b = b' + 1
    11 a = euclid(y, y, p, sqrt_p - 1);
    11 b = (_int128(a) * x) \% p;
    assert(a != b); if (a > b) swap(a, b);
    return {a, b};}
vector<pll> find_all_sums_of_two_squares(ll n) {
    assert(n >= 1);
    if (n == 1) return \{\{0, 1\}\};
    // find all (0 <= a <= b <= n: a^2+b^2=n^2)
```

```
// gaussian numbers multiplication
// (complex values with integer coordinates)
vector<pll> ds = factorize(n);
for (auto& [p, c] : ds)
    if (p \% 4 == 3 \&\& c \% 2 == 1) return \{\};
vector<pll> res, add, nxt;
for (auto [p, c] : ds) {
    add.clear();
    if (p == 2) while (c--)
add.emplace_back(1, 1);
    else if (p \% 4 == 3) {
        c \neq 2; while (c--)
add.emplace_back(0, p); } else {
        auto [a, b]
find_sums_of_two_squares_for_prime(p);
        while (c--) add.emplace_back(a, b); }
    for (auto [aa, bb] : add) {
        if (res.empty()) {
res.emplace_back(aa, bb);
        } else { nxt.clear(); for (auto [cc,
dd]: res)
                for (int it = 0; it < 2;</pre>
it++, swap(cc, dd)) {
                    11 A = abs(aa * cc - bb *
dd), B = abs(aa * dd + bb * cc);
                    if (A > B) swap(A, B);
nxt.emplace_back(A, B);}
            for (auto [cc, dd] : nxt)
res.emplace_back(cc, dd);
            swap(res, nxt); sort(res.begin(),
res.end());
            res.erase(unique(res.begin(),
res.end()), res.end());
        }}return res;}
```

## check point in non-convex polygon

```
bool check_point_in_polygon(vector<pt> P, pt q) {
    1d sm = 0;
    int n = (int) P.size();
    for (int i = 0; i < n; i++) {
        if ((P[i] - q).norm() < eps) return true;</pre>
        int j = (i + 1) \% n;
        if (check_point_on_segment(P[i], P[j],
→ q)) return true;
    int cnt = 0;
    for (int i = 0; i < n; i++) {
        int j = (i + 1) \% n;
        pt A = P[i];
        pt B = P[j];
        if (A.y > B.y) swap(A, B);
        if (abs(A.y - B.y) < eps) continue;</pre>
        if (q.y + eps \le A.y \mid \mid q.y + eps > B.y)
   continue;
        if ((B - A).vector_mul(q - A) + eps \le 0)
   continue;
        cnt ^= 1;
    return cnt == 1;
```