Group n.o: 4
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COMP 554: Algorithms
Assignment 4

Q. This is a theoretical assignment, so please only submit a PDF. Show the following: given a graph G, and given any MCST T for G, there is a valid ordering of the edges of G (according to cost) so that on input G, Kruskal's algorithm yields exactly T.

Ans..

Basis:

T* is a spanning tree and T is a minimum weight spanning tree.

Proof:

We are proving using induction that T*=T, that would mean T* is a minimum spanning tree and if not then there is exists a minimum weight edge in T which does not exist in T*.

Termination condition:

Since it is given that we are having MCST so there should be no cycles formed and the looping will end when edges = n-1.

Induction:

If T*=T -> minimum spanning tree.

If T* $\neq$ T -> edge e belongs to T and not in T*.

T*$\cup$ e is a cycle C such that:

-> every other edge in C contains weight < wt(e)

-> And there exists some edge f in C which does belong to T

Now, let's consider tree T_2 = T* \ ({e} $\cup$ {f}) which is a spanning tree:

-> Since we subtracted the sets, so now we have T_2 which has more edges in common with T than T had with T*

-> Here, we exchanged an edge which is less expensive, therefore, we have

$$wt(T_2) \geq wt(T^*)$$

Considering spanning tree T_3 with more edges common with T by using the same steps that were used for T_2.

By induction, the process continues until T is obtained. Thus from above we have:

$$wt(T) \geq \ldots \geq wt(T_3) \geq wt(T_2) \geq wt(T)^* \qquad \ldots \text{Eq}(4.1)$$

We can thus say that T is a minimum spanning tree and so the inequalities in Equation 4.1 must be equalities, hence, it can be concluded that T* is a minimum weight spanning tree.