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ASSIGNMENT 1

Show that for any integer k>= 1, if a>b>= 1 and b<$F_{\{k+1\}}$ (where F_i is the i-th Fibonacci number), then Euclid's algorithm on a,b takes fewer than k iterations of the while loop. (Ignore swaps or use 2k instead.)

Pre condition:- k>=1, a>b>=1, b<F_{k+1}

Basis Case:- gcd(a,b) = gcd(b,r) , where r =rem(a,b)
We can say that the value of r independent of a will always lie between 0<=r<b, as b is the quotient.

gcd(b,r) = gcd(r,r_1) where r_1 = rem(b,r)

So we check the possible values of r_1 for all possible values of r.

If b>r>=F_k then r_1<F_{k-1},
If 0<r<b/2 then r_1<F_{k-1},
If b/2=<r<F_k then r_1>F_{k-1} but r_2<F_{k-2}
Because in division, for a value of r as the r_1 become large the value of r_2 gets smaller.

Termination:- This process keeps on going until when the $F_{(k=2)}$=1 and r_n must be smaller than F_2 which is 0. I.e the gcd process will terminate at or before F_2.

Correctness- We prove that it requires less than k steps of the Fibonacci series to complete gcd(a,b).

Mathematical Example

The value of a does not matter.
Let $K=12$, so $F_{k+1=13}=233$,
As $b<F_{k+1}$, we take upper bound for b which is $F_{k+1}-1=232$, so $b=232$,
Now r can be in the following range $0<=r<b$,

If $b>r>F_k$ i.e $232>r=>144$	then $r_1<89$
If $0<r<b/2$ i.e $0<r<116$	then $r_1<89$,
If $b/2<r<F_k$ i.e $116=<r<144$	then $r_1>89$ but $r_2<55$

Following with that process, $r_n<F_2=1$, or $r_n=0$, thus $r_n=0$ at $k=2$, thus gcd is calculated before k steps.