

14.11.22

UNIT 1:
MATRICES

$$\begin{bmatrix} 3 & 1 & 0 \\ 4 & 6 & 1 \\ 2 & 5 & 1 \end{bmatrix}$$

A p value will be found.

Ansible.

Rank of Matrix:

$$\begin{bmatrix} 8 & 1 & 1 & 7 \\ 4 & 8 & 1 & 5 \end{bmatrix}$$

Highest order determinant that may be found from

The order non zero determinant that may be found from the element of matrix by selecting by arbitrarily by equal number of rows and columns from it.

$$180 - g^2 \leftarrow g^2 \quad 11 - 2 \leftarrow 0$$

Problem 1:

Find the ranks of A :

$$\begin{bmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ 3 & -6 & -1 \end{bmatrix}$$

solution:

$$\text{Let } A = \begin{bmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ 3 & -6 & -1 \end{bmatrix} \quad |(-4+6)$$

Definition of rank of matrix is maximum number of linearly independent rows.

$$|A| = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \quad \text{as } 4-4=0 \text{ which is a relation}$$

$$|A| = \begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix} \quad \text{as } 12-12=0 \text{ which is a relation}$$

$$|A| = \begin{bmatrix} 4 & 1 \\ -6 & -1 \end{bmatrix} \quad \text{as } -4+6=2 \neq 0$$

Rank of A is 2.

2) Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 4 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 4 \end{bmatrix}$ where P does not change until rank known as

also write

from text, row interchanges

can't affect rank

so we can write

rank answer

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -11 \end{bmatrix} R_2 \rightarrow R_2 + 2R_1, \text{ swap } R_2$$

$$R_3 \rightarrow R_3 - 5R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & -7 \end{bmatrix} \text{ now } R_3 \neq 0$$

$$R_3 \rightarrow R_3 + 2R_2$$

\therefore Number of non zero rows = 3. A lot

\Rightarrow The rank of A is 2.

consistency system of simultaneous equation:

consider a system of m linear equations

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = k_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = k_2$$

$$a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n = k_3$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = k_m$$

where $n = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ P does not

$$\rightarrow \textcircled{2}$$

$$B = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

(1)
 3 = x_1 + k_1 + k_2
 4 = x_1 + k_1 + k_3
 5 = x_1 + k_2 + k_3

to solve the equations of system

The matrix $[A, B] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1m} & k_1 \\ a_{21} & a_{22} & a_{23} & \dots & a_{2m} & k_2 \\ a_{31} & a_{32} & a_{33} & \dots & a_{3m} & k_3 \end{bmatrix}$

is called augmented matrix

Rouché's theorem:

The system of equations (1) is consistent if and only if the coefficient matrix A and the augmented matrix $[A, B]$ are of the same rank. Otherwise the system is inconsistent.

Let Rank of A be r and $[A, B]$ be r'

* If $r = r'$, the equations are consistent and there is unique solution.

* If $r = r' < n$, the equations are consistent and there are infinite number of solution.

* If $r \neq r'$, the equations are inconsistent, there is no solution.

The system is always inconsistent. If the coefficient matrix A is non-singular, the system has unique solution namely $x_1 = x_2 = \dots = x_n = 0$.

The unique solution is called trivial solution.

If coefficient matrix A is singular, the system has an infinitely no. of non zero solution, this solution is called non trivial solution.

1) Test whether the equation

$$2x + y + z = 5$$

$$x + y + 2z = 4$$

$$x - y + 2z = 1$$
 are

consistent, if consistent and solve it.

$[A, B]$ system soln

solution

The system of equations can be written in the form
Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$

$$AX = B$$

method of
elimination

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$
 method of
elimination

Since now we have $[B, A]$ system between them

The Augment matrix is

$$[A, B] = \begin{bmatrix} 2 & 1 & 1 & 5 \\ 1 & 1 & 2 & 4 \\ 1 & -1 & 2 & 1 \end{bmatrix}$$
 as A is not in
row echelon form

To prove: required unique or not

$$f(A) = f(A, B)$$
 unique soln

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 1 & 5 \end{bmatrix} R_1 \leftrightarrow R_3$$
 row echelon form, L.R.G.F

using L.R.G.F

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 2 & -1 & 3 & 3 \\ 0 & 2 & -1 & 3 & 3 \end{bmatrix} R_2 \rightarrow R_2 - R_1$$
 row echelon form

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 2 & -1 & 3 & 3 \\ 0 & 3 & -3 & 3 & 3 \end{bmatrix} R_3 \rightarrow R_3 - QR_1$$
 row echelon form

unique soln, unique in a row echelon form

if unique then P on manifold

$$\text{matrix form} \begin{bmatrix} 1 & -1 & 2 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & +3 & | & -3 \end{bmatrix} \text{ for reduction } 2(1) - 3(2) \\ R_3 \rightarrow R_3 - 3R_2 \quad 2 \\ 6 - 6 = 0 \\ 8 = 3(2) - 3(-1) \quad 2(2) - 3(3) \\ 8 = 6 + 3 \quad 6 - 9 \\ 8 = 9 \quad 6 + 3$$

$$\delta(A) = 3 \quad \delta(A_1, B) = 3$$

$$f(A) = f(A_1, B) = 3 = n$$

\Rightarrow The system is consistent.

To find x, y, z

By Back Substitution method

$$x+y \quad x-y+\cancel{z} = 1 \rightarrow ①$$

$$2y - z = 3 \rightarrow ②$$

$$-3z = -3 \rightarrow ③$$

$$\boxed{z=1}$$

$$8 = xA$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & -1 & | & -3 \end{bmatrix}$$

$$8 = xA$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & -1 & | & -3 \end{bmatrix}$$

$$z=1 \text{ sub in } ②$$

$$2y - 1 = 3$$

$$2y = 4$$

$$\boxed{y=2}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & -1 & | & -3 \end{bmatrix} = [B, A]$$

$$y=2, z=1 \text{ sub in } ①$$

solving for

~~$$x - 2 + 2 = 1 \quad x - 2 + 2(1) = [A]7 = (A)7$$~~

~~$$x - 0 = 1 \quad x - 0 = 1$$~~

~~$$x = 2$$~~

$$x - 2 + 2 = 1$$

$$x + 0 = 1$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & -1 & | & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & -1 & | & -3 \end{bmatrix}$$

$$x=1, y=2 \text{ and } z=1.$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 2 & -1 & | & 3 \\ 0 & 0 & -1 & | & -3 \end{bmatrix}$$

2. Test the consistency of system of equation

$$\begin{aligned} x+2y+6z &= 5 & \text{LHS} \\ -x+y+2z &= 3 & \text{RHS} \\ x-4y-2z &= 1 & \end{aligned}$$

$\therefore L = (3, A)T \quad R = (3, B)$

Solution.

$$AX \neq B$$

Introducing in making art of
so, we bring of
books in multiplying and get

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 5 \\ -1 & 1 & -2 & 3 \\ 1 & -4 & -2 & 1 \end{array} \right]$$

$$AX = B$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 5 \\ -1 & 1 & -2 & 3 \\ 1 & -4 & -2 & 1 \end{array} \right] \left[\begin{array}{c} x \\ y \\ z \end{array} \right] = \left[\begin{array}{c} 5 \\ 3 \\ 1 \end{array} \right]$$

$\therefore L = R = B$

The Augment Matrix is

$$[A, B] = \left[\begin{array}{ccc|c} 1 & 2 & 6 & 5 \\ -1 & 1 & -2 & 3 \\ 1 & -4 & -2 & 1 \end{array} \right]$$

② in LHS $L = R$
 $R = I - B$
 $H = B$
 $L = B$

To prove

$$f(A) = f[A, B] = (1)B + R - L = I + B - R$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 5 \\ 0 & 3 & 4 & 8 \\ 0 & -6 & -8 & -4 \end{array} \right]$$

$\therefore L = R = B$, $I = R$

$R_2 \rightarrow R_2 + R_1$
 $R_3 \rightarrow R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 5 \\ 0 & 3 & 4 & 8 \\ 0 & 0 & 0 & 12 \end{array} \right]$$

$-4 + 16$
 -8

$R_3 \rightarrow R_3 + 2R_2$

$$\text{Rank}(A) = 2$$

$$\text{Rank}(A, B) = 3$$

3. Discuss the consistency of the following equation and if consistent find the solution.

$$x + 2y + 2z = 1$$

$$2x + y + z = 2$$

$$3x + 2y + 2z = 3$$

- reduces from $y + z = 6$ and multiply 2 in \therefore

solution

test 93. 4 form to get answer

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

$P = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 0 + 0 \cdot 0 = 5$ which is not equal to 8
 $Q = 2 \cdot 1 + 1 \cdot 2 + 1 \cdot 0 + 0 \cdot 1 = 4$ which is not equal to 8
 $R = 3 \cdot 2 + 2 \cdot 3 + 2 \cdot 0 + 0 \cdot 1 = 12$ which is not equal to 8
 $S = 0 \cdot 1 + 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$ which is not equal to 6

reduces to inconsistent due to self error.

The Augment Matrix is given below

$$[A|B] = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 3 & 2 & 2 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$P = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 0 + 0 \cdot 0 = 5$ which is not equal to 8
 $Q = 2 \cdot 1 + 1 \cdot 2 + 1 \cdot 0 + 0 \cdot 1 = 4$ which is not equal to 8
 $R = 3 \cdot 2 + 2 \cdot 3 + 2 \cdot 0 + 0 \cdot 1 = 12$ which is not equal to 8
 $S = 0 \cdot 1 + 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$ which is not equal to 6

To reduce

$$f(A) = f(A|B)$$

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 0 & -4 & -4 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_3 - 3R_1$ \rightarrow $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 0 & -4 & -4 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = [B, A]$

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 0 & -4 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 4R_4$ \rightarrow $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$Y \times Y + B \neq 0.$$

By back solution

$$-4y - 4x = 0$$

$$y + x = 0$$

$$y = -x$$

$$\text{Rank}(A) = 3$$

$$\text{Rank}(A, B) = 3$$

This system is consistent

$$\delta = \delta' \alpha = 3 \alpha 4$$

$$\text{Let } x = k \text{ assuming}$$

$$2x + y + z = 2$$

$$2k + y + z = 2$$

$$2k = 2$$

$$k = 1$$

$$y = 2k + y + z = 2$$

\therefore The system has infinitely many solutions.

4. Investigate the value of λ and μ , so that the equation $2x + 3y + 5z = 9$

$$4x + 3y - 2z = 8$$

$$8x + 3y + \lambda z = \mu$$

have first subdivision no solution

ii) unique solution iii) infinite number of solution

solution

$$AX = B$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 3 & -2 \\ 8 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 8 & 8 \end{bmatrix} \stackrel{\text{R}_1 \rightarrow R_1 - 4R_2}{\sim} \begin{bmatrix} 1 & 3 & 5 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 8 & 8 \end{bmatrix} \stackrel{\text{R}_3 \rightarrow R_3 - 8R_2}{\sim} \begin{bmatrix} 1 & 3 & 5 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since, $\text{R}_3 = 0$

$$(A, B) \perp \sim (A) \perp$$

The Augment Matrix is

$$[A, B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 4 & 3 & -2 & 8 \\ 8 & 3 & \lambda & \mu \end{array} \right] \xrightarrow{\text{R}_1 \rightarrow R_1 - 2R_2, \text{R}_2 \rightarrow R_2 - 4R_1} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -3 & -12 & -16 \\ 8 & 3 & \lambda & \mu \end{array} \right] \xrightarrow{\text{R}_3 \rightarrow R_3 - 4R_1} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -3 & -12 & -16 \\ 0 & 15 & \lambda-20 & \mu-36 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -3 & -12 & -16 \\ 0 & 15 & \lambda-20 & \mu-36 \end{array} \right] \xrightarrow{\text{R}_2 \rightarrow R_2 \cdot (-1/3)} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & 1 & 4 & 16/3 \\ 0 & 15 & \lambda-20 & \mu-36 \end{array} \right] \xrightarrow{\text{R}_3 \rightarrow R_3 - 15R_2} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & 1 & 4 & 16/3 \\ 0 & 0 & \lambda-44 & \mu-104/3 \end{array} \right]$$

Rank of $(A) = 3$, Rank of $(A, B) = 3$

∴ The system is consistent and unique.

To find

case 1:

If $\lambda = 5, \mu = 9$

Augment matrix is

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\delta(A) = \delta(A, B) = 2 \times 3$$

∴ The system is consistent but infinitely many solutions.

$$\left[\begin{array}{c|cc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{Row operations}}$$

case 2:

If $\lambda = 5, \mu \neq 9$

Augment matrix is

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 0 & 0 & 0 & \mu-9 \end{array} \right]$$

$$\left[\begin{array}{c|cc|c} 1 & 3 & 2 & 9 \\ 1 & 3 & -1 & 8 \\ 0 & 0 & 0 & \mu-9 \end{array} \right] \xrightarrow{\text{Row operations}}$$

$$\delta(A) = 2, \delta(A, B) = 3$$

∴ The system is inconsistent and no solution.

case 3:

If $\lambda \neq 5, \mu = 9$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 0 & 0 & \lambda-5 & 0 \end{array} \right]$$

$$\delta(A) = 3, \delta(A, B) = 3$$

$$O = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$(1-1)(1+(1-1)) + (1-1)(1-1)$$

$$O = 1-1+1+1-1-(1-1) = 1$$

$$O = 1-1+1+1-1-1 = 1$$

consistent, but many solutions.

- 5) Find the value of k to the system of equation has i) unique ii) many solution
 iii) no solution

$$kx + y + z = 1$$

$$x + ky + z = 1$$

$$x + y + kz = 1$$

brief of

: L 200

$$P: A + B = I$$

ii) neither unique

solution.

The system of equation can be written in the form as

$$AX = B$$

$$Ex: (A, B) \rightarrow (I, f)$$

$$\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Augment matrix is

$$[A, B] = \begin{bmatrix} k & 1 & 1 & 1 \\ 1 & k & 1 & 1 \\ 1 & 1 & k & 1 \end{bmatrix}$$

Augment matrix

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & k & 1 \\ 1 & k & 1 & 1 \\ k & 1 & 1 & 1 \end{bmatrix}$$

$R_2 - R_1$

$$\xrightarrow{R_3 - kR_1} \begin{bmatrix} 1 & 1 & k & 1 \\ 0 & k-1 & 1-k & 0 \\ 0 & 1-k & 1-k^2 & 0 \end{bmatrix}$$

$R_3 - (k-1)R_2$

To find k by choose $|A| = 0$, $B = (A)^{-1}$

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$$

$$k(k^2 - 1) - 1(k-1) + 1(1-k)$$

$$k(k^2 - 1) - k + 1 + 1 - k = 0$$

$$k^3 - k^2 - k + 1 + 1 - k = 0$$

$$k^3 - 3k + 2 = 0$$

$$\begin{bmatrix} P & 1 & 1 & 1 \\ 1 & k-1 & 1-k & 0 \\ 0 & 1-k & 1-k^2 & 0 \end{bmatrix}$$

$$B = (A)^{-1}, B = (A)^{-1}$$

$$\left| \begin{array}{cccc|c} 1 & 0 & -3 & 2 & 1 \\ 1 & 1 & 1 & -2 & 0 \\ 1 & 1 & -2 & 0 & 0 \end{array} \right|$$

$$k^2 + k - 2 = 0$$

$$k^2 + k - 2 = 0$$

$$(k+2)(k-1) = 0$$

$$k = -2 \quad k = 1$$

\wedge

\wedge

so either $k = -2$ or $k = 1$ makes matrix singular.

neither of them

$$k = 1, -2$$

Augmented matrix is

$$\left[\begin{array}{ccc|c} k & 1 & 1 & 1 \\ 1 & k & 1 & 1 \\ 1 & 1 & k & 1 \end{array} \right]$$

case 1:

$$k \neq 1, k \neq 2$$

$$[A, B] = \left[\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 \end{array} \right]$$

$$\text{if } k \neq 1, \text{ then } R_1 \rightarrow R_1 + R_2$$

$$\text{if } k \neq 2, \text{ then } R_2 \rightarrow R_2 + R_3$$

$$\gamma(A) = \gamma(A, B) = 3$$

so it has unique solution.

case 2:

$$k = 1, k = 2$$

$$[A, B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \right]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array}$$

$$r(A) = r(A|B) = 1 < 3$$

\therefore The system is consistent (and it has many solution)

base 3:

Put $k \neq 1, k \neq 2$

\therefore The system is inconsistent and it has no solution.

linear dependence and independence of vectors.

The vectors x_1, x_2, \dots, x_n are said to be linearly dependent if there exist a number $\lambda_1, \lambda_2, \dots, \lambda_n$ (not all zero) such that $\lambda_1 x_1 + \lambda_2 x_2 + \dots + \lambda_n x_n = 0$.

If such number other than zero exists and it has non-zero exist the vector are said to linearly independent

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = [B|A]$$

1. Show that the vector $\mathbf{g}_1 = [1 \ 1 \ 2]$, $\mathbf{g}_2 = [1 \ 2 \ 5]$ and $\mathbf{g}_3 = [5 \ 3 \ 4]$ are linearly dependent also express each vectors as a linear combination of the other two.

Solution:

The given vector can be written in the form as

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 5 & 3 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -6 \end{bmatrix} R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - 5R_1$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} R_3 \rightarrow R_3 + 2R_2$$

$$\rho(A) = 2 < 3$$

∴ The given vectors are linearly dependent.

To find the expression

$$R_3 + 2R_2 = 0$$

$$R_3 + 2(R_2 - R_1) = 0$$

$$R_3 + 2R_2 - 2R_1 = 0$$

$$R_3 + 2R_2 - 5R_1 = 0$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 5 & 3 & 4 \end{bmatrix} = A$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_3 + 2R_2 - 7R_1 = 0$$

$$x_3 + 2x_2 - 7x_1 = 0$$

$$\boxed{-7x_1 + 2x_2 + x_3 = 0}$$

$$-7x_1 = -2x_2 - x_3 \Rightarrow 0$$

$$\boxed{x_1 = \frac{2}{7}x_2 + \frac{1}{7}x_3}$$

$$2x_2 = 7x_1 - x_3$$

$$\boxed{x_2 = \frac{7}{2}x_1 - \frac{1}{2}x_3}$$

$$\begin{bmatrix} 8 & 1 & 1 \\ 0 & 8 & 1 \\ 0 & 0 & 0 \end{bmatrix} = A$$

$$\boxed{x_3 = 7x_1 - 2x_2}$$

$$\begin{bmatrix} 8 & 1 & 1 \\ 0 & 8 & 0 \\ 0 & 0 & 0 \end{bmatrix} = A$$

2.

Show that the vector $\alpha_1 = [2 -2 1]$, $\alpha_2 = [1 4 -1]$, $\alpha_3 = [4 6 -3]$ are linearly independent also find the expression for each vectors.

Solution

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 4 & -1 \\ 4 & 6 & -3 \end{bmatrix}$$

The given vector can be written in the form

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 4 & -1 \\ 4 & 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 1 \\ 0 & 10 & -3 \\ 0 & 10 & 6 \end{bmatrix}$$

$$\begin{aligned} R_2 &\rightarrow R_2 + R_1 - 2R_3 \\ R_3 &\rightarrow R_3 - 2R_1 \end{aligned}$$

$$= \begin{bmatrix} 2 & -2 & 1 \\ 0 & 10 & -3 \\ 0 & 10 & -2 \end{bmatrix} \quad \text{makes with mixed. t. r. 3 \rightarrow R_3 - R_2}$$

$$0 = 8E + 8F - 1E - 1$$

To find expression

$$0 = 8E - 8F + 1E$$

$$0 = 8CE + 8CF + 1CE$$

$$R_3 - R_2 = 0$$

$$R_1 - 2R_2$$

$$R_3 - 2R_1 - (2R_2 + R_1) = 0$$

reduces

$$\text{with } R_3 - 2R_1 + 2R_2 + R_1 = 0 \quad \text{now with } R_1$$

$$R_3 - 3R_1 + 2R_2 = 0$$

now, reduces

$$R_3 + 2R_2 - 3R_1 = 0$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix} = 16A$$

$$-3x_1 + 2x_2 + x_3 = 0$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2x_2 = 3x_1 - x_3$$

$$x_2 = \frac{3}{2}x_1 - \frac{1}{2}x_3 \quad (I+ (C+E)I + (A+F)I) =$$

$$x_3 = 3x_1 - 2x_2 \quad (C-E+F)I =$$

$$E - C + F =$$

$$P =$$

$$O + P = |A|$$

Homogeneous system of linear equation.

Basic problem based on trivial, non-

trivial, orthogonal matrix. etc used to
interpret in vector problems. etc used to

$$\begin{bmatrix} \sin \theta & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & \cos \theta \\ \sin \theta & \cos \theta & \sin \theta \end{bmatrix} = A$$

1. Examine the system has trivial or non-trivial solution.

$$x_1 - x_2 + x_3 = 0$$

$$x_1 + 2x_2 - x_3 = 0$$

$$2x_1 + x_2 + 3x_3 = 0$$

trivial (non singular) $\neq 0$.

solution

The given equation of solution in the matrix form

$$0 = 8x_1 + 18x_2 - 8x_3$$

$$|A| = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$0 = 18x_1 + 8x_2 + 8x_3$$

$$0 = 8x_1 + 18x_2 - 18x_3$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$8x_1 - 8x_2 - 18x_3 = 0$$

$$\frac{8}{8}x_1 + \frac{8}{8}x_2 - \frac{18}{8}x_3 = 0$$

$$= 1(6+1) + 1(3+2) + 1(1-4)$$

$$= 1(7) + 1(5) + 1(-2)$$

$$= 7 + 5 - 3$$

$$= 9$$

$$|A| = 9 \neq 0$$

$$8x_1 - 8x_2 - 8x_3 = 0$$

$$\frac{8}{8}x_1 - \frac{8}{8}x_2 - \frac{8}{8}x_3 = 0$$

$\therefore A$ is non-trivial solution

2.

Prove the following matrix is orthogonal

$$A = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

solution vector rigid p. linearized

To prove A is orthogonal

that is to prove

$$AA^T = I$$

vector a vector rigid p. rigid est

$$A = \begin{bmatrix} -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}$$

or vector in vector after taking transpose

$$A^T = \begin{bmatrix} -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \end{bmatrix}$$

we want AA^T to be a vector rigid

$$AA^T = \frac{1}{3} \begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} &= \frac{1}{3} \begin{bmatrix} 4+1+4 & -4+2+2 & -2-2+4 \\ -4+2+2 & 4+4+1 & 2-4+2 \\ -2-2+4 & 2-4+2 & 1+4+4 \end{bmatrix} \text{ est} \\ &\text{from } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ p. unitary} \end{aligned}$$

$$= \frac{1}{3} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \text{ p. unitary}$$

rigid in $\frac{1}{3}$ next $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ rigid as in I.

$$= \begin{bmatrix} \frac{9}{3} & 0 & 0 \\ 0 & \frac{9}{3} & 0 \\ 0 & 0 & \frac{9}{3} \end{bmatrix} \text{ p. unitary}$$

system decomposed by rigid no. 3 if

$$LHS = 3I$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ est. in 3rd next}$$

Properties of Eigen values and eigen vectors.

Property 1:-

The sum of Eigen values of matrix is equal to sum of elements of main diagonals.

Product of the eigen values is equal to determinant of matrix.

Property 2 :-

Square matrix A and its A^T have the same Eigen values. (characteristic value)

Property 3 :-

The characteristic roots (Eigen values) of a triangular are just the diagonal elements of the matrix.

Property 4:-

If λ is an Eigen value of A then $\frac{1}{\lambda}$ is Eigen value of A^{-1}

Property 5:-

If λ is an Eigen value of orthogonal matrix then λ is also its eigen values.

Property 6:

If $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ are eigen value of matrix A then A^m has eigen value of $\lambda_1^m, \lambda_2^m, \dots, \lambda_n^m$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Property 7:

The eigen value of the real symmetric matrix are real numbers.

प्रैक्टिस एवं प्रॉफ

Property 8:

The similar matrices have same eigen value.

Property 9: $|A| = \text{सेलर रूपीज ग्रा. त्रिकोर्ड}$

The eigen corresponding to distinct eigen value of a real symmetric matrix are orthogonal.

Property 10:

The eigen vector x_i^{out} of matrix A is not unique.

Property 11: क्रिट. द्वारा

$$\begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix} \rightarrow A$$

Two eigen vector x_1, x_2 are called orthogonal vectors. If $x_1^T x_2 = 0$.

प्रैक्टिस एवं प्रॉफ

$|A| = \text{सेलर रूपीज एवं प्रौद्योगिकी}$

Problems :-

P value ratio are $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$ check it

Ex. 1. Find the sum and product of Eigen values & matrix

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Let's solve it P value ratio are $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$

Solution. Eigen values are known

By the property.

Sum of the Eigen values = sum of main diagonal

$$= -1 - 1 + 1 = -1$$

Product of Eigen values = $|A|$

$$|A| = (-1)(1+1) - 1(-1-1) + 1(1+1) = 4$$

$$\text{corresponds to } \lambda_1 = 1(0) - 1(-2) + 1(2) = 4$$

$$= +2 + 2$$

$$= 4$$

(ii) Q. The product of two Eigen values of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

is 16. Find the 3rd eigen value.

corresponds to $\lambda_1 = 6(1) - 1(-2) + 1(2) = 10$

Solution.

By the property.

Product of the Eigen values = $|A|$.

Given that $\lambda_1, \lambda_2 = 16$ therefore λ_3

Product of eigen value = $|A|$
 $(\lambda_1 \cdot \lambda_2) \cdot \lambda_3 = |A|$

$$16 \cdot \lambda_3 = \begin{bmatrix} 6 & -6 & 2 \\ -2 & 3 & -4 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\begin{aligned}|A| &= 6(9+1) + 2(-6-2) + 2(6-6) \\&= 6(10) + 2(-8) + 2(-4) \\&= 60 + 24 - 8 \\&= 60 - 32 \\&= 32\end{aligned}$$

$$16 \cdot \lambda_3 = 32$$

$$\therefore \lambda_3 = \frac{32}{16} \text{ i.e. minor ratio is 2}$$

$$\lambda_3 = 2$$

3. If 3 and 15 are two eigen values of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 3 & 4 \\ 2 & -4 & 3 \end{bmatrix}$

find $|A|$ without expand the determinant.

Solution.

By the property

Sum of eigen values = sum of diagonals

Given that $\lambda_1 = 3, \lambda_2 = 15$ fit with sum zero

Sum of eigen values = sum of diagonals

$$\lambda_1 + \lambda_2 + \lambda_3 = 8 + 7 + 3$$

$$3 + 15 + \lambda_3 = 18$$

$$\lambda_3 = 18 - 18$$

$$\lambda_3 = 0$$

By property

Product of eigen value = $|A|$

$$\lambda_1 \lambda_2 \lambda_3$$

$$18 \cdot 15 \cdot 10$$

$$= |A| = d \cdot (d - 1)$$

$$S = |A| = d^2 = d \cdot d$$

$$D = |A|$$

$$S = D - D$$

4. If α, β, γ are the eigen values of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$
find the eigen value of A^T .

Solution.

By the property

A square matrix A and its A^T have same eigen values.

That is eigen values of A^T are $2, 2, 3$

5. Find the eigen value of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
without using characteristic equation.

Solution

By the property,

the eigen value of triangular matrix
are just the diagonal elements.
 \therefore eigen values are $2, 2, 0$.

- b. Two of the eigen values of $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 8 \end{bmatrix}$
 are 3 & 6, find eigen values of A^{-1} .
 solution.

By the property of eigen values
 sum of the eigen values = sum of main diagonals

$$\lambda_1 + \lambda_2 + \lambda_3 = 3 + 5 + 8$$

$$3 + 6 + \lambda_3 = 18$$

$$9 + \lambda_3 = 18$$

$$\lambda_3 = 18 - 9 = 9$$

$$\lambda_3 = 2$$

By the property = scalar multiple of matrix

If λ is an eigen value of A then

$\frac{1}{\lambda}$ is an eigen value of A^{-1}

\Rightarrow If 2, 3, 6 are eigen value of A .

then $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ are eigen value of A^{-1} .

7. Find the eigen value of inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}$

solution.

By the property,
 the eigen value of triangular matrix are just the diagonal elements.

\therefore eigen values are 2, 3, 4

By the property.

If λ is eigen value of A then eigen value of A^{-1} .

2, 3, 4 are Eigen value of A and A^{-1}

$\lambda_2, \lambda_3, \lambda_4$ are Eigen value of A^{-1} .

8. Two eigen value of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find eigen value of A^{-1} .

Solution.

$$\lambda_1 = \lambda_2 = 1, \lambda_3 = ?$$

By the property:

sum of eigen values = sum of main diagonal

$$\lambda_1 + \lambda_2 + \lambda_3 = 2 + 3 + 2 = 7$$

$$1 + 1 + \lambda_3 = 1 + 1 + \lambda_3 = 7$$

$$2 + \lambda_3 = 7$$

$$\lambda_3 = 5$$

A^{-1} is also eigen value of A .

1, 1, 5 are eigen value of A .

By the property

If λ is eigen value of A then $\frac{1}{\lambda}$ is eigen value of A^{-1} .

1, 1, 5 are eigen value of A^{-1} .

1, 1, 5 are eigen value of A^{-1} .

1, 1, 5 are eigen value of A^{-1} .

10.

Find the eigen values of A^3 , $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -7 \\ 0 & 0 & 3 \end{bmatrix}$

Solution:

Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -7 \\ 0 & 0 & 3 \end{bmatrix}$ p. e. v. r. n. g. i. s.

By the property (3)

The eigen values of triangular matrix are the just the diagonal elements.

The eigen values of A are $1, 2, 3$ v. o.

By the property (A³)

If λ is an eigen value of A^3 then λ^3 is an eigen value of A .
 \Rightarrow The eigen values of A^3 are $1, 8, 27$.

Q. If $1, 1, 5$ are the eigen values of A , find the eigen values of $5A$.

Solution:

Given that, eigen values of A are $1, 1, 5$

By the property.

The eigen value of $5A$ are $5, 5, 25$.

Q. If $2, -1, -3$ are the eigen values of A . Find the eigen values of matrix $A^2 - 8I$.

Solution:

$$\text{Ans} = \frac{1}{|A|} A$$

$$\text{Ans} = |A|^{-1} A$$

$$|A| = 2 \cdot (-1) \cdot (-3) = 6$$

$$|A| = 6$$

$$|A| = 6$$

$\begin{bmatrix} 8 & 2 & 1 \\ 4 & 5 & 0 \\ 8 & 0 & 0 \end{bmatrix}$ Given that,
Eigen value of A are $2, -1, -3$

Eigen value of A^2 are $4, 1, 9$.

Eigen value of $A^2 - 8I$ are $-4, -2, 1 - 2, 9 - 2 = 7, 7, 7$.

What are the eigen values of the matrix $A^2 + 3I$. If the eigen values of matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ are $2, 6, 9 - 1$.

Solution.

Eigen values of A are $2, 6, 9 - 1$.

Eigen values of A^2 are $36, 1$.

Eigen values of $A^2 + 3I = 39, 4$.

5. If the eigen values of A are of order order 3×3 are $2, 3$ and 1 the find the eigen values of $\text{adj } A$.

Solution.

The eigen values of A are $2, 3, 1$.

We know that eigen value of $A^{-1} \Rightarrow \frac{1}{2}, \frac{1}{3}, \frac{1}{1}$

To find the eigen of $\text{adj } A$.

$$A^{-1} = \frac{1}{|A|} \text{adj } A.$$

$$A^{-1} \cdot |A| = \text{adj } A \rightarrow \text{①}$$

By the property.

$$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |A|$$

$$2 \cdot 3 \cdot 1 = |A|$$

$$6 = |A|.$$

Product \Rightarrow

Eigen values of $\text{adj}^0 A = 6(1/2, 1/3, 1)$ ($0 = PAA'$)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{adj}} \begin{bmatrix} 1/2 & 1/3 & 1 \\ 1/2 & 1/3 & 1 \\ 1/2 & 1/3 & 1 \end{bmatrix} \xrightarrow{\text{adj}^0} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1/3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\text{adj}^0 A = (2, 2, 6)$

6. Two eigen values λ_1, λ_2 are equal and third one is $1/2$ times to third. Find them.
- Solution:

$$\lambda_1 = \frac{1}{5} \lambda_3, \lambda_2 = \frac{1}{5} \lambda_3, \lambda_3 = ?$$

Sum of eigen values = sum of all diagonals.

$$\lambda_1 + \lambda_2 + \lambda_3 = 2 + 3 + 2$$

$$\frac{1}{5} \lambda_3 + \frac{1}{5} \lambda_3 + \lambda_3 = 7.$$

$\lambda_3 \left(\frac{1}{5} + \frac{1}{5} + 1 \right)$ will give us result of

$$\lambda_3 \left(\frac{7}{5} \right) = 7.$$

$$\lambda_3 = \frac{7}{\frac{7}{5}} = 5$$

$$\lambda_3 = 7 \times \frac{5}{7} = 5$$

$$\lambda_3 = 5$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 5 & 1 \\ 5 & 5 & 5 \end{bmatrix}$$

Similarly

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 5 & 1 \\ 5 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\lambda_1 = \frac{1}{5}(5) = 1, \lambda_2 = \frac{1}{5}(5) = 1$$

$$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 5.$$

7. Prove that the eigen vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ are orthogonal pairs.

solution

$$(AA^T = 0)$$

Let

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$x_1 x_2^T = 0, x_2 x_3^T = 0, x_3 x_1^T = 0.$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T = 0$$

also x_1 is eigenvector of $A^T A$ with eigenvalue 0.

$$= \begin{bmatrix} 1+0-1 \\ 1-2+1 \\ 1+0+1 \end{bmatrix} = 0 \cdot \text{most likely zero.}$$

$$S = \lambda_1 + \lambda_2 + \lambda_3 = 0 + 1 + 1 = 2$$

• store eigen values and eigen vectors

Problems based on non-symmetric matrix with non-repeated eigen values.

$$8\lambda + 8\lambda \frac{1}{\lambda} + 8\lambda \frac{1}{\lambda}$$

1. Find the eigen values and eigen vectors of

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

solution.

$$\text{Let } A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

Step 1:

$$(A - \lambda I) = 0 \quad (\lambda - 1) \frac{1}{\lambda} = 0$$

To find the 1st characteristic eigen value, the general term is $\lambda^3 - 8\lambda^2 + 5\lambda - 5 = 0$.

S_1 = sum of diagonal elements etc.

$$S_1 = 1+2+3$$

$$S_1 = 6$$

$$0 = \lambda + 6 - \frac{6}{\lambda}$$

$$0 = 6 + \lambda - \frac{6}{\lambda}$$

S_2 = sum of determinant of principle minors.

$$S_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= (6-2) + (3+2) + (2-0)$$

$$= 4 + 5 + 2$$

$$S_2 = 11$$

$$S_3 = |A|$$

$$= \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix} \xrightarrow{\text{R}_1 \leftrightarrow \text{R}_2} \begin{vmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} \xrightarrow{\text{R}_2 - 2\text{R}_1} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix} \xrightarrow{\text{R}_3 - 2\text{R}_1} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 1(6-2) - 0(3-2) - 1(2-4) \\ = 1(4) + 0 - 1(-2) \\ = 4 + 2$$

$$S_3 = 6$$

$$\Rightarrow \text{the characteristic equation of } |A - \lambda I| \text{ is } S_3 = 0 \\ \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0.$$

Step 2:

To find the characteristic roots (Eigen values).

By synthetic division.

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ \downarrow & 1 & -5 & 6 & 0 \\ \hline & 1 & -5 & 6 & 0 \end{array} \quad \begin{array}{l} 0 = \lambda^3 - 6\lambda^2 + 11\lambda - 6 \\ 0 = \lambda^3 - 5\lambda^2 + 6\lambda + 0 \end{array}$$
$$\lambda^2 - 5\lambda + 6 = 0.$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$8+6+1 = 6$$

$$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$$

$$\begin{array}{r} \wedge \\ 8 \\ 6 \\ 3-2 \end{array}$$

$$(\lambda-3)(\lambda-2) = 0.$$

$$\boxed{\lambda=3} \quad \boxed{\lambda=2} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 2 & 2 & 3 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$$

Step 3:

$$(0-8) + (8+8) + (8-8) =$$

To find eigen vector, by

$$S+F+H =$$

$$(A - \lambda I)x = 0.$$

$$11 = 82$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 2 & 3 & 0 \end{array} \right] - \lambda \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \Rightarrow \begin{cases} x_1 - x_3 = 0 \\ 2x_1 + 2x_2 + x_3 = 0 \\ 2x_1 + 2x_2 + 3x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 2 & 3 & 0 \end{array} \right] - \left[\begin{array}{ccc|c} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \Rightarrow \begin{cases} (\lambda-1)x_1 - x_3 = 0 \\ 2(\lambda-1)x_1 + 2x_2 + x_3 = 0 \\ 2(\lambda-1)x_1 + 2x_2 + 3x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1-\lambda & 0 & -1 & 0 \\ 1 & 2-\lambda & 1 & 0 \\ 2 & 2 & 3-\lambda & 0 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \Rightarrow \text{①} \quad d = 82$$

case (i) put $\lambda = 1$.

$$0 = d - \lambda(11 + 8\lambda - 8)$$

$$0x_1 + 0x_2 x_3 = 0 \Rightarrow \text{①}$$

$$x_1 + 1x_2 + x_3 = 0 \Rightarrow \text{②} \quad \text{works with brief or}$$

$$2x_1 + 2x_2 + 2x_3 = 0 \Rightarrow \text{③} \quad \text{otherwise identical pg}$$

$$x_1 + x_2 + x_3 = 0 \Rightarrow \text{④}$$

consider ① and ② (different).

$$\frac{x_1}{0+1} = \frac{x_2}{-1+0} = \frac{x_3}{0} \Rightarrow$$

$$\left[\begin{array}{ccc} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\text{eigen vectors } \alpha_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

case(2) :

$$\text{Put } \lambda = 2$$

$$\text{matrix } -1x_1 + 0x_2 - 1x_3 = 0 \Rightarrow \text{second equation}$$

$$(0+2)x_1 + 0x_2 + 1x_3 = 0 \Rightarrow \text{3rd equation}$$

$$2x_1 + 0x_2 + 1x_3 = 0 \Rightarrow ③$$

matrix with basis vector equal with basis

$$① \rightarrow x \text{ by } (-) = x_1 + 0x_2 + x_3 = 0 \Rightarrow ①$$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ = A matrix with P

consider ② & ③

$$\frac{x_1}{0+2} = \frac{x_2}{2-1} = \frac{x_3}{2-0}$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 2 & 1 & 2 & 2 \end{bmatrix}$$

matrix

$$x_2 = \begin{bmatrix} +2 \\ 1 \\ 2 \end{bmatrix}$$

first value is always (+) ve.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

: E resp

case 3 :- $\lambda = 3$

matrix eigenvectors with basis or

$$0 = 2x_1 + 0x_2 - 1x_3 \Rightarrow ① \text{ second eqn with}$$

$$x_1 - 1x_2 + x_3 = 0 \Rightarrow ②$$

$$2x_1 + 2x_2 + 0x_3 = 0 \Rightarrow ③ \text{ basis p. mult = 12}$$

$$0+1+8 =$$

$$1 = 3$$

consider ① & ②.

$$\text{matrix with basis } \begin{bmatrix} 0 & -1 & -2 & 0 \\ 2 & 0 & 2 & 2 \end{bmatrix} = 8$$

$$\frac{x_1}{0+2} = \frac{x_2}{-2+0} = \frac{x_3}{3+0}$$

$$\frac{x_1}{x_1} = \frac{x_2}{-2} = \frac{x_3}{3} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} =$$

$$2/8/7$$

$$\text{eigen vector } \alpha_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad (8+4) + (8-0) + (8+6) = 8$$

$$d \neq 8 - 8 = 0$$

The eigen values of A are $\lambda = 1, 2, 3$ with the corresponding eigen vectors.

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \text{ repis}$$

: Class
S-1 b/w,

Problem based on non-symmetric matrix with repeated eigen values ($A \neq A^T, \lambda_i$ same),

1. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

solution

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \text{ eular kris}$$

$$0 = \lambda^3 + \lambda^2 + 1$$

$$0 = \lambda^3 + \lambda^2 + 1 = (\lambda + 1)^2(\lambda - 1)$$

$$\lambda = -1, -1, 1$$

$$\text{solution}$$

$$\lambda^3 = \lambda^2 = \lambda$$

$$0 = \lambda^3 - \lambda^2 - \lambda$$

$$0 = \lambda^2(\lambda - 1) - \lambda$$

$$0 = \lambda(\lambda - 1)^2 - \lambda$$

$$0 = \lambda(\lambda - 1)^2 - \lambda$$

Step 1:

To find the characteristic eigen value the general form is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \text{sum of diagonal}$

$$= -2 + 1 + 0$$

$$S_1 = -1$$

$S_2 = \text{sum of determinant of principle minors}$

$$= \begin{vmatrix} 1 & -6 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 2 & -2 \end{vmatrix} = -12 - 3 + 4 = -11$$

$$= (0 \cdot 1 - 12) + (0 - 3) + (4 - 2)$$

$$= -12 - 3 + 4 = -11$$

$$S_3 = -2(2) = -4$$

euler kris

$$S_3 = |A|$$

$$\rightarrow \begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \xrightarrow{\text{C1} - 2\text{C2}, \text{C2} + \text{C3}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -3 \\ 0 & 0 & -6 \\ -1 & -2 & 0 \end{bmatrix} \xrightarrow{\text{C1} - \frac{1}{2}\text{C3}, \text{C2} - \frac{1}{2}\text{C3}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -2 & 0 \end{bmatrix}$$

$$\text{①} - 0 = gR^E - gR^F + 1R$$

$$S_3 = 0 \quad |A| = 45$$

$$\text{②} - 6R^E - 6R^F + 1R - 0 = 6R^E + 6R^F - 1R -$$

The CE of $|A - \lambda I|$ is

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\begin{array}{c} 3 \\ \left[\begin{array}{cccc} 1 & 1 & -21 & -45 \\ 0 & 3 & 12 & -27 \\ 0 & 8 & 16 & -54 \\ 0 & 6 & 12 & -9 \end{array} \right] \xrightarrow{\text{R3} - 2\text{R1}, \text{R4} - \text{R2}} \begin{array}{c} \text{retired} \\ \downarrow -3 \\ \left[\begin{array}{cccc} 1 & 1 & -21 & -45 \\ 0 & 3 & 6 & -27 \\ 0 & 1 & -2 & -15 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \\ \text{③} - 0 = gR^E - gR^F + 1R \end{array}$$

$$gR^E = gR^F$$

$$\frac{gR^E}{g} = \frac{gR^F}{g}$$

$$\begin{array}{c} 4 \\ \left[\begin{array}{cccc} 1 & 1 & -21 & -45 \\ 0 & 4 & 20 & 4 \\ 0 & 5 & 25 & 41 \end{array} \right] \xrightarrow{\text{R3} - 5\text{R1}} \begin{array}{c} \text{reduces} \\ \downarrow -5 \\ \left[\begin{array}{cccc} 1 & 1 & -21 & -45 \\ 0 & 4 & 20 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \\ \text{④} - 0 = gR^E \end{array}$$

$$0 = 0 + gR^E + 1R$$

$$gR^E = 1R$$

$$\frac{gR^E}{1} = \frac{1R}{1}$$

$$\text{Let } \lambda = -3$$

$$\lambda^2 - 2\lambda - 15 = 0 \quad \begin{array}{c} -15 \\ \wedge \\ -5 \quad 3 \end{array} \quad \text{(ii) case.}$$

$$(\lambda - 5)(\lambda + 3) = 0$$

$$\boxed{\lambda = 5} \quad \boxed{\lambda = -3}$$

\therefore the eigen values $\lambda = -3, 5$. (λ values are same repeated).

$$\text{① in } \lambda = 5$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

② bus ③ retired

Step 3:

$$\begin{bmatrix} -2-\lambda & 1 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{array}{l} \text{④} = \frac{gR^E}{1} = 1R \\ \text{⑤} = \frac{gR^E}{4} = \frac{1R}{4} \end{array}$$

case (1)

put $\lambda = -3$ in ④.

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = gR^E \text{ retired repis}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 - 3x_3 = 0 \rightarrow ①$$

$$2x_1 + 4x_2 - 6x_3 = 0 \div 2 \Rightarrow x_1 + 2x_2 - 3x_3 \rightarrow ②$$

$$-x_1 - 2x_2 + 3x_3 = 0. (x) \Rightarrow x_1 + 2x_2 - 3x_3 \rightarrow ③$$

all the three equation are same.

consider $x_1 + 2x_2 - 3x_3 = 0$
choose $x_1 = 0$.

$$0 + 2x_2 - 3x_3 = 0.$$

$$2x_2 = 3x_3$$

$$\frac{x_2}{3} = \frac{x_3}{2}$$

\therefore Eigen vector of $x_1 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

choose $x_3 = 0$.

$$x_1 + 2x_2 + 0 = 0$$

$$x_1 = -2x_2$$

$$\frac{x_1}{-2} = \frac{x_2}{1}$$

\therefore Eigen vector of $x_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

case ii)

$\lambda = 5$ in ①.

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ 1 & -2 & -5 \end{bmatrix}$$

$$-7x_1 + 2x_2 - 3x_3 = 0 \rightarrow ①$$

$$2x_1 - 4x_2 - 6x_3 = 0 \rightarrow ②$$

$$1x_1 - 2x_2 - 5x_3 = 0 \rightarrow ③$$

consider ② and ③

$$\frac{x_1}{20-12} = \frac{x_2}{-6+10} = \frac{x_3}{-4+4}$$

$$\frac{x_1}{8} = \frac{x_2}{4} = \frac{x_3}{0}$$

$$\begin{bmatrix} -4 & -6 & 2 & -4 \\ -2 & -5 & 1 & -2 \\ 1 & -2 & 5 & 1 \end{bmatrix}$$

Eigen vector of $x_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

Problems based on symmetric matrices
with non repeated eigen values.

1. Find the eigen values and eigen vectors of the matrix
- $$\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

Solution

Step 1: To find the equation
The characteristic equation of $(A - \lambda I)$ is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

S_1 = sum of diagonals.

$$7+6+5 = 18.$$

S_2 = sum of determinant of diagonals.

$$\begin{vmatrix} 6 & -2 \\ -2 & 5 \end{vmatrix} + \begin{vmatrix} 7 & 0 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix}$$

$$= (30-4) + (35-0) + (42-4)$$

$$= 26 + 35 + 28$$

$$= 99$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \frac{26}{184} = 0$$

$$S_3 = |A|$$

$$= 7(30-4) + 2(-10+0) + 0(0)$$

$$= 7(26) + 2(-10)$$

$$= 182 - 20$$

$$S_3 = 162$$

The characteristic equation is $\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$.

③ how ④ review

Step 2:

To find eigen values.

~~cofactors of elements of 2nd row~~ $\frac{89 \times 2}{89}$

$$\begin{array}{c} 1 -18 +99 -162 \\ \downarrow 8 -80 178 \\ 1 -10 89 \end{array} \quad \begin{array}{c} -8 \\ \downarrow 178 \\ 1 -18 +99 -162 \\ -8 \\ 1 -10 89 \end{array}$$

~~cofactors of 2nd row~~ $\frac{178}{-8}$

row 1 \downarrow row 2 \downarrow row 3 \downarrow

row 1 \downarrow row 2 \downarrow row 3 \downarrow

row 1 \downarrow row 2 \downarrow row 3 \downarrow

$$\frac{2^2}{3 \times 69} =$$

$$\frac{99}{45} = \frac{54}{54}$$

$$3 \begin{array}{c} 1 -18 99 -162 \\ \downarrow 3 -45 -80 162 \\ 8 -15 69 0 \end{array}$$

neither

$$\lambda^2 - 15\lambda + 54 = 0$$

$$\lambda = \frac{54}{\lambda} + \lambda_1 + \lambda_2 - \lambda$$

$$(\lambda - 9)(\lambda - 6) = 0$$

$$\lambda_1 = 9, \lambda_2 = 6$$

$\lambda = 3, 6, 9$ are the eigen values of matrix A

$$\boxed{\lambda = 9} \quad \boxed{\lambda = 6}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 15$$

Step 3: To find the eigen values.

By $(A - \lambda I)x = 0$

$$\begin{bmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(7-\lambda) + (6-\lambda) + (5-\lambda) = 18 - 3\lambda$

base i)

$$\text{Put } \lambda = 3$$

$$\begin{bmatrix} 4 & 6 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$18 - 3\lambda = 18$

$$4x_1 - 2x_2 + 0x_3 = 0 \rightarrow ①$$

$$-2x_1 + 3x_2 - 2x_3 = 0 \rightarrow ②$$

$$0x_1 - 2x_2 + 2x_3 = 0 \rightarrow ③$$

consider ② and ③

$$\frac{x_1}{6-4} = \frac{x_2}{0+4} = \frac{x_3}{4-0} = \frac{8}{8} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$\frac{x_1}{2_1} = \frac{x_2}{4_2} = \frac{x_3}{4_2} \quad \text{①} \rightarrow 0 = 8x_1 + 8x_2 + 16x_3$$

$$\text{Eigen vector } x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad 0 = 8x_1 + 8x_2 + 16x_3$$

case i) from ① revision

base ii)

$$\lambda = 6 \text{ in ①.} \quad \begin{bmatrix} 6 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{ex: } \frac{x_1}{6} = \frac{x_2}{-2} = \frac{x_3}{-1} = 1$$

$$x_1 - 2x_2 + 0x_3 = 0 \rightarrow ① \quad \text{ex: ① solution neigie}$$

$$-2x_1 + 4x_2 - 2x_3 = 0 \rightarrow ②$$

$$0x_1 - 2x_2 - 1x_3 = 0 \rightarrow ③ \quad \text{euler}$$

we have $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = R$ zero solution problem

consider ① and ③

$$\frac{x_1}{2+0} = \frac{x_2}{0+1} = \frac{x_3}{-2+0} = \frac{1}{1} \quad \begin{bmatrix} -2 & 0 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

so $x_1 = x_2 = x_3 = 1$ no zero solution

with eigen vector $x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ Euler neigie with basis 1

case iii)

put $\lambda = 9$ in equ ①.

result

$$\begin{bmatrix} -2 & -2 & 0 \\ -2 & -3 & -2 \\ 0 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$E = C E = I$$

$$-2x_1 - 2x_2 + 0x_3 = 0 \rightarrow ①$$

$$-2x_1 - 3x_2 - 2x_3 = 0 \rightarrow ②$$

$$0x_1 - 2x_2 - 4x_3 = 0 \rightarrow ③$$

consider ① and ②.

$$\frac{x_1}{4+0} = \frac{x_2}{0-4} = \frac{x_3}{6-4}$$

$$\begin{bmatrix} -2 & 0 & -2 \\ -3 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{R2} \leftarrow R2 - 3R1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{x_1}{4+0} = \frac{x_2}{0-4} = \frac{x_3}{2}$$

$$\begin{bmatrix} -2 & 0 & -2 \\ -3 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{R3} \leftarrow R3 - R2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Eigen vector of x_3

The eigen value of A are $1, 6, 96$ with cooresponding vectors are $x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $x_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, $x_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} 50-1 & 0 & 5 \\ 50-0 & 1 & 5 \end{bmatrix} \xrightarrow{\text{R2} \leftarrow R2 - R1} \begin{bmatrix} 50 & 0 & 5 \\ 50 & 1 & 5 \end{bmatrix} \xrightarrow{\text{R2} \leftarrow R2 - R1} \begin{bmatrix} 50 & 0 & 5 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow{\text{R2} \leftarrow R2 - 50R1} \begin{bmatrix} 50 & 0 & 5 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{R1} \leftarrow R1 - 50R2} \begin{bmatrix} 0 & 0 & 5 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{R1} \leftarrow R1 - 5R2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Problems based on symmetric matrices with repeated eigen values.

1. Find the eigen values and vector of the matrix $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(iii) soln.

solution.

① we get $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Step 1:

To find characteristic equation $|A - \lambda I| = 0$
 the general equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3$

S_1 = sum of diagonal.

$$S_1 = 0+0+0$$

$$S_1 = 0$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

① in $\lambda = 0$ (i.e. 0)

S_2 = sum of determinant of diagonal.

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= (0-1) + (0-1) + (0-1)$$

$$S_2 = -3$$

$$\textcircled{1} \leftarrow 0 = g^0 + g^R + 1g^B$$

$$\textcircled{2} \leftarrow 0 = g^R + g^B - 1R$$

$$S_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\textcircled{3} \leftarrow 0 = g^Rg^B - g^R + 1R$$

② from ① relation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 1(0-1) + 1(1-0) \frac{g^R}{g+1} = \frac{g^R}{g+1} = \frac{1R}{g+1}$$

$$= 1+1$$

$$S_3 = 2$$

$$\frac{g^R}{g} = \frac{1R}{g} = \frac{1R}{1g}$$

$$\lambda^3 - 0\lambda^2 - 3\lambda - 2 = 0$$

, R solve negi?

Step 2:-

To find the eigen values.

$$\rightarrow \begin{vmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

(ii) now

$$\downarrow \begin{vmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

① in due to -R

$$\lambda^2 - \lambda - 2 = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(\lambda+1)(\lambda-2) = 0$$

$$\boxed{\lambda = -1} \quad \boxed{\lambda = 2}$$

$$\lambda = -1, -1, 2.$$

$$\textcircled{1} \leftarrow 0 = g^0 + g^R + g^B$$

$$\textcircled{2} \leftarrow 0 = g^R + g^B - 1R$$

$$\textcircled{3} \leftarrow 0 = g^R - g^B + 1R$$

Step 3:

To find the eigen vector By solving $(A - \lambda I)x = 0$.

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(for its a result $\Rightarrow 0$)
 $0+0+0=0$
 $0=0$

case(i) $\lambda = 1$ in ①

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 1 & 1 & 1-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = 0$$

$(-1) + (-1) + (1) = 0$

$$-2x_1 + x_2 + x_3 = 0 \rightarrow ①$$

$$x_1 - 2x_2 + x_3 = 0 \rightarrow ②$$

$$x_1 + x_2 - 2x_3 = 0 \rightarrow ③$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = 0$$

consider ① and ②.

$$\frac{x_1}{1+2} = \frac{x_2}{1+2} = \frac{x_3}{1+2} \quad (0-1)(1+2) = 2 \quad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\frac{x_1}{3} = \frac{x_2}{3} = \frac{x_3}{3}$$

* eigen vector $x_1 =$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

• vector might not be unique

case ii)

$\lambda = -1$ sub in ①.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0 = 0 - 1 - 1$$

$$x_1 + x_2 + x_3 = 0 \rightarrow ①$$

$$x_1 + x_2 + x_3 = 0 \rightarrow ②$$

$$x_1 + x_2 + x_3 = 0 \rightarrow ③$$

$$0 = (2-\lambda)(1+\lambda)$$

$$2-\lambda = 0 \rightarrow \lambda = 2$$

$$1+\lambda = 0 \rightarrow \lambda = -1$$

All the above equations are equal.

choose $x_3 = 0$.

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$\frac{x_1}{-1} = \frac{x_2}{1}$$

$$0 \cdot \begin{bmatrix} l \\ m \\ n \end{bmatrix} = [0 \ 0 \ 0]$$

$$0 \leftarrow 0 = 100 \text{ m-l}$$

$$\frac{l}{-1} = \frac{m}{0} = \frac{n}{1}$$

Eigen vector of $x_2 = \begin{bmatrix} +1 \\ -1 \\ 0 \end{bmatrix}$

$$d = \frac{m}{l} = \frac{\lambda}{1}$$

case iii)

choose $x_2 = 0$

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

= 6th vector regis.

$$\text{Orthogonal} = x_1^T x_2$$

base vector $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is a vector regis with

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \text{6th vector part}$$

Eigen vector $x_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

The eigen

case iii)

choose $x_3 = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$ is orthogonal to x_1^T, x_2^T vectors

$$\rightarrow x_1^T x_3 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$$l+m+n=0 \rightarrow 0.$$

$\Rightarrow x_2^T x_3$ no solution made with 2/1

$$\begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$$l-m+n=0 \rightarrow ②$$

$$\frac{l}{0+1} = \frac{m}{+1-0} = \frac{n}{-1-1}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{R2} \leftrightarrow \text{R1}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{R3} \leftarrow \text{R3} + \text{R1}}$$

$$\frac{l}{1} = \frac{m}{1} = \frac{n}{-2}$$

eigen vectors $x_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ (iii) ex 2

0 = 6x 2nd row

The eigen values of $A = -1, -1, 2$ with correspond
ing vectors are $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $x_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6x \text{ satar. repis}$$

repie est

water level first component in $\begin{bmatrix} x \\ m \\ n \end{bmatrix} = 6x$ ex 2

$$\begin{bmatrix} x \\ m \\ n \end{bmatrix}$$

(iii) ex 2

0 = 6x 1st row

$$0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [1 \ 1]$$

① & ② 0 = 16m+2

Unit - II

Diagonalisation of Matrices.

Cayley Hamilton theorem - Reduction of quadratic form to canonical form through orthogonal transformation.

Cayley

Statement: Every square satisfies its own characteristic equation.

Uses of Cayley Hamilton theorem:

To calculate inverse of non singular square matrix A ($|A| \neq 0$).

To calculate the positive integral powers of A (highest power of A).

Now if we have a system except zero problem:

1. Verify Cayley Hamilton theorem, find A^4 and inverse of the matrix A (A^{-1}). When $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Solution:

Step 1:
To find the characteristic equation $(A - \lambda I)$
general equation $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

S_1 = sum of diagonal.

$$= 2+2+2 = \begin{bmatrix} H+H+H & S_0-S_0-S_0 & S_0+H+H \\ S_0-S_0-S_0 & H+H+H & 1-S_0-S_0 \\ H+H+H & S_0-S_0-1 & S_0+H+H \end{bmatrix}$$

$$S_1 = 6.$$

S_2 = sum of determinant of diagonal.

$$\left| \begin{array}{cc} 2 & -1 \\ -1 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & 2 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & -1 \\ 1 & -2 \end{array} \right|$$

calculated - result obtained below
 $= (4-1) + (4-2) + (-2+1)$ } or matrix operations
 $= 3+2-1$, determinant is negative.
 $S_2 = 48$

$S_3 = |A|$.

now $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ we can prove it by expanding along the 3rd row.

 $= 2(4-1) + 1(-2+1) + 2(1-2)$
 $= 6 \neq 1-2$

resultant = 3 or it is equal to its box or

$$\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0$$

Step 2:

Now let's verify left to right or
By Cayley Hamilton theorem

"every square matrix A satisfies its own C.E."

The characteristic equation is .

$$\begin{bmatrix} S_0 & 1 & -1 \\ 1 & S_1 & 1 \\ 1 & 1 & S_2 \end{bmatrix} \quad A^3 - 6A^2 + 8A - 3I = 0$$

Verification

$$A^2 = AXA.$$

$(A^2 - A)$ position of 2nd & 3rd row

$$\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

with left or
interchange of rows

$$\rightarrow \begin{bmatrix} 4+1+2 & -2-2-2 & 4+1+4 \\ -2-2-1 & 1+4+1 & -2-2-2 \\ 2+1+2 & -1-2-2 & 2+1+4 \end{bmatrix}$$

is true - 18
 $5+6+6 =$
 $d = 18$

$$A^2 = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+6+9 & -12-9+18 & 14+6+18 \\ -10-6-6 & 5+12+6 & -10-6-12 \\ 10+5+7 & -5-10-7 & 10+5+14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix}$$

$$A^3 - 6A^2 + 8A - 3I = 0$$

$$18 + Ad - A = 18$$

$$\begin{bmatrix} P & d-F \\ d & d-A \\ F & d-G \end{bmatrix}$$

$$\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 6 \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 8 \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$-3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 8 & 0 & 8 \\ 0 & 8 & 1 \\ 8 & 1 & 8 \end{bmatrix} = 18$$

$$\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - \begin{bmatrix} 42 & -36 & 54 \\ -30 & 36 & -36 \\ 30 & -30 & 42 \end{bmatrix} + \begin{bmatrix} 16 & -8 & 16 \\ -8 & 16 & -8 \\ 8 & -8 & 16 \end{bmatrix}$$

$$- \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 8 & 8 \\ 8 & 8 & 8 \\ 8 & 8 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 8 & 8 \\ 1 & 8 & 8 \end{bmatrix} = 18$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} P & d & f \\ d & d & g \\ F & G & E \end{bmatrix}$$

Hence verified.

Step 3:

$$\text{To find } A^{-1} \quad \begin{bmatrix} S & I - S \\ I - S & I \end{bmatrix} \quad \begin{bmatrix} P & d & f \\ d & d & g \\ F & G & E \end{bmatrix}$$

The characteristic equation is $P + 8I + A^3 - 6A^2 + 18A - 3I = 0$

$$\therefore A^3 - 6A^2 + 8I - 3I = 0 \quad d + d + d \quad d - d - d$$

$$A^3 - 6A^2 + 8I - 3I = 0 \quad F + G + H \quad F - G - H \quad F + G + H$$

$$3A^3 = A^3 - 6A^2 + 8I$$

$$= \begin{bmatrix} 7 & -6 & 9 \\ 5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 6 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -6 & 9 \\ 5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} - \begin{bmatrix} 12 & 6 & 12 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$3A^3 = \begin{bmatrix} 3 & 0 & -3 \\ 1 & 2 & 0 \\ -1 & 1 & 3 \end{bmatrix} \quad 0 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \therefore$$

$$A^3 = \begin{bmatrix} 3/3 & 0/3 & -3/3 \\ 1/3 & 2/3 & 0/3 \\ -1/3 & 1/3 & 3/3 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1/3 & 2/3 & 0 \\ -1/3 & 1/3 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 8 \\ 0 & 8 & 0 \\ 8 & 0 & 0 \end{bmatrix}$$

Step 4:- To find A^4

The characteristic equation is $\lambda^3 - 6\lambda^2 + 18\lambda - 31 = 0$.

$$(XA) \Rightarrow A^4 - 6A^3 + 18A^2 - 31A = 0.$$

$$A^4 = 6A^3 - 8A^2 + 3A.$$

$$\begin{aligned} &= 6 \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & 22 & 49 \end{bmatrix} + 8 \begin{bmatrix} 7 & -6 & 9 & 8 \\ -5 & 6 & -6 & 8 \\ 5 & -5 & 7 & 8 \end{bmatrix} + 3 \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 174 & -168 & 228 \\ -132 & 138 & -168 \\ 132 & 132 & 174 \end{bmatrix} - \begin{bmatrix} 56 & -48 & 72 \\ -40 & 48 & -48 \\ 40 & -40 & 56 \end{bmatrix} + \begin{bmatrix} 6 & -3 & 6 \\ 3 & 6 & -3 \\ 3 & -3 & 6 \end{bmatrix} \\ A^4 &= \begin{bmatrix} 124 & -123 & 162 \\ -95 & 96 & -123 \\ 95 & -95 & 124 \end{bmatrix} \end{aligned}$$

2) Verify Cayley Hamilton theorem and find its
inverses of A^4 .

$$\begin{bmatrix} 2 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

Solution.

Step 1:

To find the characteristic equation $(A - \lambda I)$ is

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

S_1 = sum of diagonal.

$$S_2 = 5 \begin{bmatrix} 7 & -1 & -1 \\ 2 & 4 & 4 & -1 & -1 & -1 \\ 2 & -6 & -2 & 4 & 1 & -1 \\ 1 & 4 & 1 & -1 & -1 & -1 \end{bmatrix} =$$

S_2 : sum of determinant of all diagonal.

$$S_2 = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -6 & 1 \end{vmatrix} + \begin{vmatrix} 7 & 2 \\ -6 & -1 \end{vmatrix}$$

+ 1st col of
either 1st or 2nd row

$$\therefore (-4) + (-7+12) + (-7+12)$$

$A = A^2 - A^2 + A^2 \rightarrow A = (A^2)$
 $A^2 + A^2 - A^2 = A$

$$\begin{bmatrix} S_1 & S_2 \\ 1 & S_3 \end{bmatrix} = \begin{bmatrix} -3+5+5 \\ 7 \cdot 1 - 1 \cdot 7 \end{bmatrix}$$

$$S_2 = 7 \cdot 1 - 1 \cdot 7 = 8 \rightarrow \begin{bmatrix} 88 & 88 & 88 \\ 88 & 88 & 88 \\ 88 & 88 & 88 \end{bmatrix}$$

$S_3 = 1A)$

$$1A) = \begin{bmatrix} 7 & 2 & -2 \\ -6 & 2 & 2 \\ 6 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 88 & 88 & 88 \\ 88 & 88 & 88 \\ 88 & 88 & 88 \end{bmatrix}$$

$$\begin{aligned} &= 7(1-4) - 2(6-12) - 2(-12+6) \\ &= 7(-3) - 2(-6) - 2(-6) \\ &= -21 + 12 + 12 \end{aligned}$$

$S_3 = 3.$

Step 2:

By using Hamilton theorem.

The characteristic equation is

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0.$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0.$$

verification

$$A^2 = A \cdot A$$

$$A^2 = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 49-12-12 & 14-2-4 & -14+4+12 \\ -42+6+12 & -12+1+4 & 12-2-2 \\ 42-12-6 & 12-2-2 & -12+4+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix}$$

$0 = 18 - A + F/A - A$ non homogeneous eqns

$$\begin{array}{r} 18 \\ -24 \\ \hline 18 \\ -12 \\ \hline 5 \\ -7 \\ \hline 24 \\ -24 \\ \hline 0 \end{array}$$

$$0 = A^3 - 7AF + 18A - A^2$$

$$A^3 = A^2 \times A$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} \times \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ -6 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 175 & 50 & -50 \\ -168 & -168 & 168 \\ 168 & -48 & -42 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 175 - 48 - 48 & 50 - 8 - 16 & -50 + 16 + 8 \\ -168 + 42 + 48 & -48 + 7 + 16 & 8 + 48 - 14 - 8 \\ 168 - 48 - 42 & -48 - 8 - 14 & 8 - 48 + 16 + 7 \end{bmatrix} = \begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix} \begin{bmatrix} 6 & 8 & 8 \\ 6 & 8 & 8 \\ 6 & 8 & 8 \end{bmatrix} = A^3$$

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$\begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix} - 5 \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} + 7 \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 8 & -1 \end{bmatrix} - 3 \begin{bmatrix} 6 & 8 & 8 \\ 6 & 8 & 8 \\ 6 & 8 & 8 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$0 = 18 - A + F/A - A$ non homogeneous eqns

Hence verified.

$A^3 - A \leftarrow H(X)$

Step 3:

To find the A^{-1} .

The characteristic equation is $A^3 - 5A^2 + 7A - 8I = 0$

$$\therefore A \Rightarrow A^2 - 5A + 7I - 8A^{-1} = 0$$

$$3A^{-1} = A^2 - 5A + 7I.$$

$$= \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} - 5 \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\xrightarrow{\text{R1} + \text{R2}, \text{R2} - 3\text{R3}}$ $\xrightarrow{\text{R1} - 8\text{R2}}$ $\xrightarrow{\text{R1} - 8\text{R3}}$

$$= \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} \xrightarrow{\text{R1} + 8\text{R2}} \begin{bmatrix} 35 & 10 & -16 \\ -24 & -30 & -5 \\ 24 & 8 & -7 \end{bmatrix} \xrightarrow{\text{R1} - 5\text{R2}} \begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$3A^{-1} = \begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix} \xrightarrow{\text{R1} + 2\text{R2}, \text{R2} + \text{R3}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3A$$

$$A^{-1} = \begin{bmatrix} -3/3 & -2/3 & 2/3 \\ 6/3 & 5/3 & -2/3 \\ -6/3 & -2/3 & 5/3 \end{bmatrix} = I_3 - A^{-1} + A^{-3} - 8A$$

$$A^{-1} = \begin{bmatrix} -1 & -2/3 & 2/3 \\ 2 & 5/3 & -2/3 \\ -2 & -2/3 & 5/3 \end{bmatrix} \xrightarrow{\text{R1} + 2\text{R2}, \text{R2} + \text{R3}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 4:

To find A^4 .

The characteristic equation $A^3 - 5A^2 + 7A - 8I = 0$,

$$(X)A \Rightarrow A^4 - 5A$$

before over