Machine Learning

Name: Bhukya Vasanth Kumar
Roll: B180441CS
CS - A Batch (sem 7)

1) Applying Baye's theorem,

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$P(A) \rightarrow$ probability of event A
$P(B) \rightarrow$ probability of event B
$P(A|B) \rightarrow$ probability of observing event A if B is true
$P(B|A) \rightarrow$ probability of observing event B if A is true

	Diseased	Non diseased
Test +	0.98	0.03
Test −	0.02	0.97

Given
$P(D) = 0.008$
$P(P/D) = 0.98$
$P(N/ND) = 0.97$

let D be disease
ND be Non disease
P be positive
N be negative

Hence,
$P(ND) = 1 - 0.008 = 0.992$
$P(N/D) = 0.02$
$P(P/ND) = 0.03.$

$$P(D|P) = \frac{P(P|D) \times P(D)}{P(P|D) \times P(D) + P(ND) \times P(P|ND)} \quad \text{---} (1)$$

Finding (1)

$$P(+ve) = P(P|D) \times P(D) + P(ND) \times P(P|ND)$$
$$= 0.98 \times 0.008 + (1 - 0.97) \times 0.992$$
$$= 0.00784 + 0.02976$$
$$= 0.0376$$

put (1)

$$P(D|P) = \frac{0.98 \times 0.008}{0.0376} = \underline{0.2085} \quad \text{---} (1)$$

clearly, $P(D/+ve) = \frac{0.00784}{P(+ve)}$

$P(ND/+ve) = \frac{0.0298}{P(+ve)}$

$P(ND/+ve) > P(D/+ve).$ ——— (3)

using ②, ③

We can say that the patient most likely does not have the disease.

⑤ $W_{max}(x)$ be the state of nature for

$$P(w_{max}|x) \geq P(w_i|x) \text{ for all } i, i=1,2,\ldots c.$$

~~i.e.~~ Also, $P(\frac{w_1}{x}) + P(\frac{w_2}{x}) + \cdots P(\frac{w_c}{x}) = 1.$

(since, the total sum of all probability $= 1$).

Given,

$P(w_{max}|x) \geq P(w_i/x)$,

the least value of $P(W_{max}|x)$ is when it is equal to all $P(w_i|x)$.

$$\therefore P(\frac{w_1}{x}) = P(\frac{w_2}{x}) = \cdots = P(\frac{w_c}{x}) = P(\frac{w_{max}}{x}).$$

$$\Rightarrow c \times \left(P(\frac{w_{max}}{x})\right) = 1$$

$$P(w_{max}|x) = \frac{1}{c}.$$

since, $\frac{1}{c}$ is the least value it could take

$$P(w_{max}|x) \geq 1/c.$$

③ Given $P(w_1|x) = 0.01$

$P(w_2|x) = 0.99$

$$R(\alpha_i|x) = \sum_{j=1}^{c} \lambda(\alpha_i|w_j) \times P(w_j/x)$$

$$R(\alpha_1|x) = \lambda(\alpha_1|w_1) \times P(w_1|x) + \lambda(\alpha_1|w_2) \times P(w_2|x)$$

$$= 5 \times 0.01 + 60 \times 0.99$$

$$= 0.05 + 59.4 = 59.45$$

$$R(\alpha_2|X) = \lambda(\alpha_2|w_1) \times P(w_1|X) + \lambda(\alpha_2|w_2) \times P(w_2|X)$$
$$= 50 \times 0.01 + 3 \times 0.99$$
$$= 0.5 + 2.97 = 3.47$$

similarly,

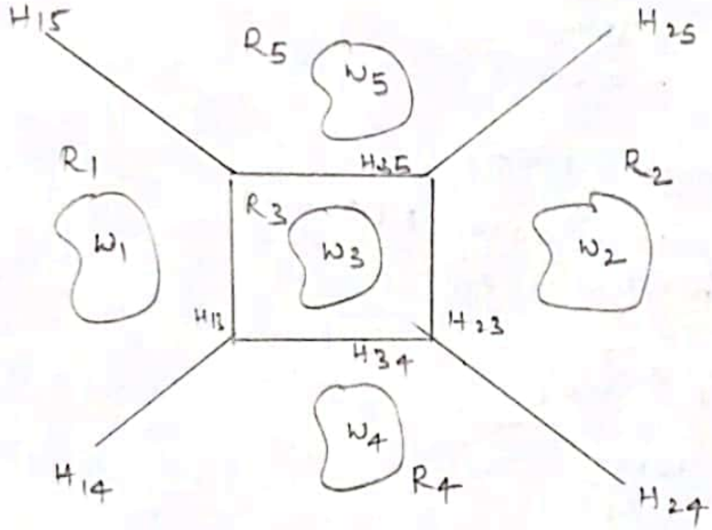
$$R(\alpha_3|X) = \lambda(\alpha_3|w_1) \times P(w_1|X) + \lambda(\alpha_3|w_2) \times P(w_2|X)$$
$$= 10000 \times 0.01 + 0 \times 0.99$$
$$= 100$$

out of all $R(\alpha_2|X)$ is least.

$\therefore$ Action α_2 = Treatement B takes least <u>loss</u>.

(6) Given we have 5 classes.

Decision Regions that will be formed by linear machine classifier:



Hxy
↓
Hyper plane between Region x, y.

Hyperplanes: H14, H15, H24, H25, H35, H34, H13, H23

Regions: R1, R2, R3, R4, R5.

(4) Baye's' rule is optimal if the true prior probabilities $P(w_i)$, probability density function $P(x|w_i)$ are known.

$\Rightarrow$ Mimimum conditional risk means max. discriminant

$$g_i(x) = P(w_i|x) = \frac{P(w_i)P(x|w_i)}{P(x)}.$$

where,

$$P(x) = \sum_{j=1}^{c} P(w_j) \cdot P(x|w_j).$$

2. Build Machine learning model inorder to predict whether a vehicle costs 50,000 or not, given height of vehicle.

Given, 1209 vehicles.

221 vehicles with cost > 50,000.

1209 − 221 = 998 with cost < 50,000.

Given height = 1.05m, whose price to be checked if greater than 50,000.

Given, 46 vehicles with price > 50,000. $\big\}$ vehicle height = 1.05m
59 vehicles with price < 50,000.

$$P(>50{,}000) = \frac{221}{1209} = 0.183$$

$$P(<50{,}000) = \frac{998}{1209} = 0.817$$

$$P(>50{,}000 \mid height = 1.05m)$$

$$= \frac{P(height = 1.05 \mid >50{,}000)\ P(>50000)}{P(height = 1.05)}$$

$$= \frac{\frac{46}{221} \times \frac{221}{1209}}{\frac{46+59}{1209}}$$

$$= \frac{0.208 \times 0.182}{0.087}$$

$$= 0.438$$

∴ vehicle's price is not greater than 50,000.

(since, the other probability i.e. $P(<50{,}000 \mid height = 1.05) = 1 - 0.438 > 0.438.$)

This vehicle's price $\ngtr$ 50,000.

Scanned with CamScanner

(7) Consider there are 2 classes, ω_1 and ω_2.
2 Actions, α_1 and α_2.

where, α_1 when true class is ω_1
α_2 when true class is ω_2.

From conditional risks, we get

$$R(\alpha_1|x) = \lambda_1 (P(\omega_1|x)) + \lambda_{12} P(\omega_2|x) \quad \text{——} (1)$$
$$R(\alpha_2|x) = \lambda_2 P(\omega_2|x) + \lambda_{22} P(\omega_2|x) \quad \text{——} (2)$$

From fundamental rule, we decide ω_1 if

$$R(\alpha_1|x) < R(\alpha_2|x)$$

[$\because$ conditional risk is minimum].

From (1), (2)

$$\Rightarrow \lambda_{11} P(\omega_1|x) + \lambda_{12} P(\omega_2|x) < \lambda_{21} P(\omega_1|x) + \lambda_{22} P(\omega_2/x)$$

$$\Rightarrow (\lambda_{11} - \lambda_{22}) P(\omega_1|x) < (\lambda_{22} - \lambda_{12}) P(\omega_2|x)$$

$$\Rightarrow \frac{P(\omega_1|x)}{P(\omega_2|x)} < \frac{\lambda_{22} - \lambda_{12}}{\lambda_{11} - \lambda_{21}}$$

$$\Rightarrow \frac{P(x|\omega_1)\, P(\omega_1)}{P(x|\omega_2)\, P(\omega_2)} < \frac{\lambda_{22} - \lambda_{12}}{\lambda_{11} - \lambda_{21}}$$

$$\Rightarrow \frac{P(x|\omega_1)}{P(x|\omega_2)} > \frac{\lambda_{12} - \lambda_{22}}{\lambda_{21} - \lambda_{11}} \cdot \frac{P(\omega_2)}{P(\omega_1)}$$

(Assuming $\lambda_{21} > \lambda_{11}$).

$\therefore$ Likelihood ratio $= \dfrac{P(x|\omega_1)}{P(x|\omega_2)}$.

We can say the Bayes Decision Rule says to decide ω_1 if likelihood ratio exceeds a threshold that is independent of observation x.