

CS4022D Principles of Programming Languages

Lecture #9: Operational Semantics

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Language of Booleans

$t ::=$

true

false

if t then t else t

terms

constant true

constant false

conditional

Language of Booleans: Values

$v ::=$

true

false

Language of Booleans: Evaluation Rules

Evaluation: $t \rightarrow t'$

$\text{if true then } t_2 \text{ else } t_3 \rightarrow t_2$ (E-IFTRUE)

$\text{if false then } t_2 \text{ else } t_3 \rightarrow t_3$ (E-IFFALSE)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF})$$

Evaluate

- if (if true then true else false) then true else false
- *if t_1 then t_2 else t_3* with $t_1 \rightarrow t'_1$
- instance of E-IF

E-IF

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF})$$

if true then t_2 else $t_3 \rightarrow t_2$ (E-IFTRUE)

if false then t_2 else $t_3 \rightarrow t_3$ (E-IFFALSE)

Axioms

Language of Booleans: Evaluation Rules

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF})$$

- Inference Rule - premise (precondition), conclusion
- Precondition to be satisfied inorder to apply the rule

Evaluate

- if (if true then true else false) then true else false
- *if t_1 then t_2 else t_3* with $t_1 \rightarrow t'_1$
- instance of E-IF
- subterm t_1 to be evaluated using an instance of ...?

Evaluate

- if (if true then true else false) then true else false
- *if t_1 then t_2 else t_3* with $t_1 \rightarrow t'_1$
- instance of E-IF
- subterm t_1 to be evaluated using an instance of E-IFTRUE

Evaluation steps

if (if true then true else false) then true else false

$\xrightarrow{E-IF}$ if true then true else false

$\xrightarrow{E-IFTRUE}$ true

Step 1 involves evaluation of a subterm using E-IFTRUE

Step1: Derivation tree

$$\frac{\text{if true then true else false} \rightarrow \text{true}}{\text{if}(\text{if true then true else false}) \text{ then true else false} \rightarrow \text{if true then true else false}}$$

Multi step evaluation relation

if (if true then true else false) then true else false

$E-IF \longrightarrow$ if true then true else false

$E-IFTRUE \longrightarrow$ true

if (if true then true else false) then true else false $\xrightarrow{*}$ true

$t \xrightarrow{*} t' :$ t evaluates in *multiple* (0 or more steps) to t'

Booleans: Semantics

- order of evaluation of subterms- always *guard* gets evaluated first
- based on the value of *guard*, either the *then* part or the *else* part gets evaluated

Booleans: An Alternate Semantics

if true then v_2 *else* $v_3 \rightarrow v_2$ (E-IFTRUE1)

if false then v_2 *else* $v_3 \rightarrow v_3$ (E-IFFALSE1)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3}$$
 (E-IF1)

$$\frac{t_2 \rightarrow t'_2}{\text{if } v_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } v_1 \text{ then } t'_2 \text{ else } t_3}$$
 (E-IF2)

$$\frac{t_3 \rightarrow t'_3}{\text{if } v_1 \text{ then } v_2 \text{ else } t_3 \rightarrow \text{if } v_1 \text{ then } v_2 \text{ else } t'_3}$$
 (E-IF3)

Booleans: An Alternate Semantics

if true then v_2 else $v_3 \rightarrow v_2$ (E-IFTRUE1)

if false then v_2 else $v_3 \rightarrow v_3$ (E-IFFALSE1)

- rules can be fired only after all the subterms have become values

Booleans: An Alternate Semantics

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF1})$$

$$\frac{t_2 \rightarrow t'_2}{\text{if } v_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } v_1 \text{ then } t'_2 \text{ else } t_3} \quad (\text{E-IF2})$$

$$\frac{t_3 \rightarrow t'_3}{\text{if } v_1 \text{ then } v_2 \text{ else } t_3 \rightarrow \text{if } v_1 \text{ then } v_2 \text{ else } t'_3} \quad (\text{E-IF3})$$

- each subterm is to be completely evaluated to a value

Booleans: An Alternate Semantics

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF1})$$

$$\frac{t_2 \rightarrow t'_2}{\text{if } v_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } v_1 \text{ then } t'_2 \text{ else } t_3} \quad (\text{E-IF2})$$

$$\frac{t_3 \rightarrow t'_3}{\text{if } v_1 \text{ then } v_2 \text{ else } t_3 \rightarrow \text{if } v_1 \text{ then } v_2 \text{ else } t'_3} \quad (\text{E-IF3})$$

- order of evaluation of subterms?
- how to change the order?

Booleans: Two different semantics

- Original - Short-circuit evaluation semantics
- Alternate - Complete evaluation semantics

Exercise

$t ::=$

true

false

and t t

or t t

not t

terms

constant true

constant false

logical and

logical or

logical negation

- For the above language, write
 - Complete evaluation semantics
 - Short-circuit evaluation semantics