B180441CS

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Q4)

Given function

$$f(x) = f(x_1, x_2, x_3)$$
$$= 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$$

$$g(x) \Rightarrow x_1 + x_2 + 2x_3 = 3$$
$$\Rightarrow x_1 + x_2 + 2x_2 - 3 = 0.$$

The lagrange functions given by

$$L(x_1, x_2, x_3, \lambda) = f + \lambda g$$
$$= (9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3)$$
$$+ \lambda(x_1 + x_2 + 2x_3 - 3)$$

The necessary conditions

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow -8 + 4x_1 + 2x_2 + 2x_3 + \lambda = 0 \quad \text{—— (1)}$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow -6 + 4x_2 + 2x_1 + \lambda = 0 \quad \text{—— (2)}$$

$$\frac{\partial L}{\partial x_3} = 0 \Rightarrow -4 + 2x_3 + 2x_1 + \lambda = 0 \quad \text{—— (3)}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow x_1 + x_2 + 2x_3 - 3 = 0 \quad \text{—— (4)}$$

From (1), (2), (3), (4)

$$x_1 = \frac{4}{3}$$
$$x_2 = \frac{7}{9}$$
$$x_3 = \frac{4}{9}$$
$$\lambda = 2/9$$

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Evaluate L_{ij} and g_{ij} at point (x, y, z)
$= (4/3, 7/9, 4/9) = X$

$$L_{11} = \left[\frac{\partial^2 L}{\partial x_1^2}\right]_X = 4$$

$$L_{12} = L_{21} = \left[\frac{\partial^2 L}{\partial x_1 x_2}\right]_X = 2$$

$$L_{13} = L_{31} = \left[\frac{\partial^2 L}{\partial x_1 x_3}\right]_X = 2$$

$$L_{22} = \left[\frac{\partial^2 L}{\partial x_2^2}\right]_X = 4$$

$$L_{23} = L_{32} = \left[\frac{\partial^2 L}{\partial x_2 x_3}\right]_X = 0$$

$$L_{33} = \left[\frac{\partial^2 L}{\partial x_3^2}\right]_X = 2$$

$$g_{11} = \left[\frac{\partial g}{\partial x_1}\right]_X = 1$$

$$g_{12} = \left[\frac{\partial g}{\partial x_2}\right]_X = 1$$

$$g_{13} = \left[\frac{\partial g}{\partial x_3}\right]_X = 2.$$

considering determinatour equaton,

$$\begin{vmatrix} L_{11} - z & L_{12} & L_{13} & g_{11} \\ L_{21} & L_{22} - z & L_{23} & g_{12} \\ L_{31} & L_{32} & L_{33} - z & g_{13} \\ g_{11} & g_{12} & g_{13} & 0 \end{vmatrix} = 0.$$

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$$\Rightarrow \begin{vmatrix} 4-z & 2 & 2 & 1 \\ 2 & 4-z & 0 & 1 \\ 2 & 0 & 2-z & 2 \\ 1 & 1 & 2 & 0 \end{vmatrix} = 0.$$

splitting the determinant by Row, col methods

$$\Rightarrow \begin{vmatrix} -2 & -2 & -1 \\ z-4 & 0 & -1 \\ 0 & z-2 & -2 \end{vmatrix} + \begin{vmatrix} 4-z & 2 & 2 \\ 2 & 0 & 2-z \\ 1 & 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 4-z & 2 & 0 \\ 2 & 4-z & 0 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow -1 \begin{vmatrix} 2 & 2 & 1 \\ 4-z & 0 & 1 \\ 0 & 2-z & 2 \end{vmatrix} + 1 \begin{vmatrix} 4-z & 2 & 2 \\ 2 & 0 & 2-z \\ 1 & 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 4-z & 2 & 0 \\ 2 & 4-z & 0 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad z^2 - 6z + 9 = 0$

$\Rightarrow \quad (z-3)^2 = 0$

$\Rightarrow \quad z = 3, 3.$

Given root 3, 3 are both positive.

$(x, y, z) = (4/3, 7/9, 4/9)$ is relative minimum of the function.

$\Rightarrow$ Given that the right hand side value of constraint is increased by 0.01.

$\Rightarrow$ Changes in right hand side of binding constraints always changes the soluton.

$S(x) \Rightarrow x_1 + x_2 + 2x_3 = 3.01.$

$g(x) \Rightarrow x_1 + x_2 + 2x_3 - 3.01 = 0.$

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$\Rightarrow \frac{\partial L}{\partial x_1}, \frac{\partial L}{\partial x_2}, \frac{\partial L}{\partial x_3}$ remains unchanged.

$\Rightarrow \frac{\partial L}{\partial \lambda}$ changes.

$x_1 + x_2 + 2x_3 - 3.01$

$$f + \lambda g = L = (9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3)$$
$$+ \lambda(x_1 + x_2 + 2x_3 - 3.01)$$

will be the changed function.

This should be maximised.

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