B18044 ICS

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03) Given that

$$f(x) = x^3 + px^2 + qx + r$$

We have to find the condition for the function to have

(i) extreme values

(ii) no extreme values.

$$f(x) = x^3 + px^2 + qx + r$$

$$f'(x) = 3x^2 + 2px + q.$$

(i)

In order to have extreme values,

$$f'(x) = 0.$$

$$3x^2 + 2px + q = 0. \quad \text{——— (1)}$$

$$D \geq 0$$

since, the equation should have roots.

$$(2p)^2 - 4(3)q \geq 0$$

$$4p^2 \geq 4.(3)q$$

$$p^2 \geq 3q.$$

$\therefore$ Inorder to have extreme values, $\underline{\underline{p^2 \geq 3q}}$.

(ii) For no extreme values, the equation should not have any real roots.

$$\Rightarrow \quad D < 0.$$

$$\Rightarrow \quad (2p)^2 - 4(3)q < 0 \quad (\text{From (1)})$$

$$\Rightarrow \quad 4p^2 < 4.3.q$$

$\Rightarrow \quad \underline{\underline{p^2 < 3q}}$ This should be satisfied, if there shouldn't be extreme values.

At $p=q=r=3$

$D = 2p^2 - 12q$

$= (2(3))^2 - 12(3)$

$= 6^2 - 12(3)$

$= 36 - 36$

$= 0. \geq 0.$

i.e. $p^2 = 3q$

$9 = 9$.

If $D = 0$, the equation ① has real roots.

$\therefore$ The function has extreme values at $p=q=r=3$

The equation:

$3x^2 + 2px + q = 0$

put $p=3, q=3$

$3x^2 + 6x + 3 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1, -1.$

are the roots.