# VSAR: Efficient Logical Reasoning with Hyperdimensional Computing Through Interference-Canceled Hybrid Encoding

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## Abstract

Vector Symbolic Architectures (VSAs) and hyperdimensional computing offer a promising substrate for scalable symbolic computation, but their application to relational and logical reasoning has remained limited by severe interference effects in role–filler bindings. In particular, the signal-to-noise ratio of conventional VSA encodings degrades rapidly with predicate arity, rendering reliable inference impractical beyond simple cases.

In this paper, we present VSAR, a vector-symbolic reasoning framework that enables efficient approximate relational reasoning through query-aware decoding. We provide a formal analysis showing that standard role–filler encodings suffer an unavoidable $1/\sqrt{k}$ signal decay for $k$-ary predicates, and introduce a hybrid encoding that combines predicate binding with invertible positional permutations. Building on this representation, we propose *successive interference cancellation* (SIC), an algorithmic decoding procedure that exploits known query constraints to iteratively remove interference and recover bound arguments with high fidelity.

Empirically, we show that SIC improves retrieval similarity from approximately 0.31 to over 0.9 on binary relations, scales gracefully with arity and dimensionality, and enables multi-hop inference over knowledge bases containing up to 100,000 facts. We further characterize the semantic guarantees and limitations of VSAR, positioning it as a flexible reasoning substrate that supports deductive, abductive, analogical, and probabilistic inference under predictable approximation error.

Our results demonstrate that algorithmically aware decoding transforms VSAs from a representational formalism into a practically viable, scalable reasoning substrate, bridging a gap between symbolic expressivity and vector-based efficiency.

## 1 Introduction

Reasoning with structured knowledge remains a central challenge in artificial intelligence. Classical logic-based systems provide expressive and interpretable mechanisms for deductive inference, but are often brittle, computationally expensive, and poorly suited to large-scale or noisy environments. Conversely, neural and embedding-based approaches scale efficiently and tolerate uncertainty, yet struggle to support structured relational inference, explanation, and systematic generalization. Bridging this gap between symbolic expressivity and computational scalability is a long-standing goal of AI research.

Vector Symbolic Architectures (VSAs), also known as hyperdimensional computing, offer an appealing middle ground. By representing symbols as high-dimensional vectors and composing them through algebraic operations such as binding and superposition, VSAs enable distributed representations that are robust to noise and amenable to parallel computation. Prior work has demonstrated the utility of VSAs for analogy-making, cognitive modeling, and compositional representation. However, despite their promise, VSAs have not yet yielded a general-purpose relational reasoning system capable of supporting rule-based inference at scale.

A key obstacle is *interference*. When relational structures are encoded via role–filler bindings and superposition, unbinding a single argument inevitably introduces cross-talk from other bound elements. As predicate arity increases, this interference rapidly overwhelms the signal, making reliable decoding impractical. While this limitation has been informally acknowledged in the VSA literature, its theoretical implications for logical reasoning have not been systematically analyzed, nor has a principled solution been established.

In this work, we argue that the failure of prior VSA-based reasoning attempts is not merely representational, but *algorithmic*. Conventional decoding procedures treat unbinding as a one-shot operation, ignoring the structure of the query itself. We show that effective reasoning in a vector-symbolic substrate requires *query-aware decoding*—that is, decoding algorithms that explicitly exploit known constraints to manage and remove interference.

To this end, we introduce VSAR, a vector-symbolic reasoning framework built on three core contributions. First, we provide a formal signal-to-noise ratio (SNR) analysis demonstrating that standard role–filler encodings incur an unavoidable $1/\sqrt{k}$ degradation with predicate arity $k$. Second, we propose a hybrid relational encoding that combines predicate binding with invertible positional permutations, preserv-

ing predicate identity while enabling exact positional inversion. Third, and most importantly, we introduce *successive interference cancellation* (SIC), a decoding algorithm that iteratively subtracts known argument contributions from a superposed representation, substantially improving retrieval fidelity.

We evaluate VSAR empirically on relational retrieval and multi-hop inference tasks, showing that SIC improves decoding accuracy by more than a factor of three in common cases and enables reasoning over knowledge bases with up to 100,000 facts. We further analyze how approximation error scales with dimensionality and arity, and explicitly characterize the semantic guarantees and limitations of our approach. Rather than aiming for sound and complete logical inference, VSAR is designed to support efficient approximate reasoning with predictable error behavior, enabling a range of inference modes—including deductive, abductive, analogical, and probabilistic reasoning—within a single, scalable substrate.

Taken together, our results demonstrate that algorithmically aware decoding transforms vector-symbolic representations into a viable foundation for large-scale relational reasoning. VSAR does not replace classical logic systems, but complements them by offering a flexible reasoning core that trades exactness for scalability, robustness, and expressive versatility. We believe this positions vector-symbolic reasoning as a practical component in future hybrid AI systems that must reason over structured knowledge in real-world, uncertain settings.

### 1.1 Contributions

The main contribution of this paper is *algorithmic*: we show that VSA-based relational reasoning becomes practically viable when encoding is combined with query-aware decoding strategies. Specifically:

1. **Formal SNR analysis** (Section **??**): We provide the first rigorous mathematical characterization of the fundamental SNR limitations of VSA encodings for relational reasoning, proving that role-filler binding degrades as $1/\sqrt{k}$ with arity.
2. **Hybrid encoding** (Section 5): We propose an encoding combining predicate binding with shift-based positions, achieving predicate distinguishability without sacrificing positional invertibility.
3. **Successive interference cancellation algorithm** (Section 6): The key algorithmic contribution—a decoding procedure that exploits query structure to subtract known interference terms before cleanup, with proven convergence and SNR bounds (Theorem 4).
4. **Multi-variable query support**: We extend SIC to handle queries with multiple unknowns (e.g., `parent(?, ?)`), achieving 100% precision/recall through iterative cancellation. This demonstrates that the VSA substrate naturally supports complex query patterns beyond single-variable retrieval.
5. **Forward and backward chaining**: We implement both bottom-up (forward) and top-down (backward) reasoning strategies, demonstrating that VSAR supports multiple inference modes with the same underlying encoding. Backward chaining uses SLD resolution adapted for approximate VSA unification with tabling for cycle detection.
6. **Empirical validation** (Section 7): We demonstrate 3× improvement in retrieval accuracy (0.31 → 0.92 on binary predicates), stable scaling to 100K facts, multi-hop inference with graceful degradation, 100% accuracy on multi-variable queries, and equivalence between forward and backward chaining.
7. **Semantic characterization** (Section 9.2): We provide explicit guarantees on what VSAR preserves (approximate answers with similarity scores) and what it does not (exact logical entailment), clarifying when approximation is acceptable.

## 2 Background

A VSA operates over $d$-dimensional normalized hypervectors $\mathbf{v} \in \mathbb{C}^d$ (or $\mathbb{R}^d$) with three core operations: **Bind** $\otimes$ (invertible composition, e.g., circular convolution), **Bundle** $\oplus$ (superposition via normalized sum), and **Permute** $\pi_n$ (circular shift by $n$ positions). Random hypervectors are nearly orthogonal ($\mathbb{E}[\mathbf{a} \cdot \mathbf{b}] \approx 0$), binding is approximately invertible (similarity $\sim$0.95-1.0), and permutation is exact ($\pi_{-n}(\pi_n(\mathbf{v})) = \mathbf{v}$).

To decode a noisy vector $\mathbf{q}$, we perform *cleanup*: $\hat{s} = \arg\max_s \text{sim}(\mathbf{q}, \mathbf{v}_s)$ where $\text{sim}(\mathbf{a}, \mathbf{b}) = |\mathbf{a} \cdot \mathbf{b}|/(\|\mathbf{a}\|\|\mathbf{b}\|)$. See Appendix **??** for formal definitions.

## 3 Problem Formulation

We consider encoding ground atoms of the form $p(t_1, \ldots, t_k)$ where:

- $p$ is a predicate symbol from a finite set $\mathcal{P}$
- $t_1, \ldots, t_k$ are entity symbols from a finite set $\mathcal{E}$
- $k$ is the arity of $p$

Each symbol $s \in \mathcal{P} \cup \mathcal{E}$ is assigned a random normalized hypervector $\mathbf{v}_s \in \mathbb{C}^d$.

**Encoding objective**: Design a function $\phi : \mathcal{P} \times \mathcal{E}^k \to \mathbb{C}^d$ such that:

1. **Predicate distinguishability**: $\text{sim}(\phi(p, \mathbf{t}), \phi(p', \mathbf{t})) \approx 0$ for $p \neq p'$
2. **Positional decoding**: Given $\mathbf{f} = \phi(p, t_1, \ldots, t_k)$ and position $i$, we can recover $t_i$ via cleanup with high accuracy
3. **Bundling capacity**: Multiple facts $\mathbf{f}_1, \ldots, \mathbf{f}_n$ can be bundled with minimal interference

**Query objective**: Given a query pattern $p(t_1, \ldots, t_{i-1}, X, t_{i+1}, \ldots, t_k)$ where positions other than $i$ are bound, retrieve the entity at position $i$ from facts matching the pattern.

# 4 Encoding Strategies

## 4.1 Role-Filler Binding

**Encoding**:

$$\phi_{\text{RF}}(p, t_1, \ldots, t_k) = \mathbf{p} \otimes \left(\oplus_{i=1}^{k}(\rho_i \otimes \mathbf{t}_i)\right) \tag{1}$$

where $\rho_i$ is a random role vector for position $i$.

**Decoding position** $j$:

$$\mathbf{q}_j = \text{unbind}\left(\text{unbind}(\mathbf{f}, \mathbf{p}), \rho_j\right) \tag{2}$$

**Analysis**: Expanding the unbind operations:

$$\text{unbind}(\mathbf{f}, \mathbf{p}) \approx \oplus_{i=1}^{k}(\rho_i \otimes \mathbf{t}_i) \tag{3}$$

$$\begin{aligned}\mathbf{q}_j &\approx \text{unbind}\left(\sum_{i=1}^{k}(\rho_i \otimes \mathbf{t}_i), \rho_j\right) \\ &= \mathbf{t}_j + \sum_{i \neq j} \text{unbind}(\rho_i \otimes \mathbf{t}_i, \rho_j)\end{aligned} \tag{4}$$

The term $\text{unbind}(\rho_i \otimes \mathbf{t}_i, \rho_j) = \mathbf{t}_i \otimes \rho_i \otimes \rho_j^\dagger$ is approximately a random vector since $\rho_i$ and $\rho_j$ are independent.

**Theorem 1** (Role-Filler SNR Bound)**.** *For role-filler encoding with $k$ arguments, the expected signal-to-noise ratio at position $j$ is:*

$$SNR_{RF} = \frac{1}{k-1}$$

*and the expected cosine similarity to the target entity is:*

$$\mathbb{E}[sim(\mathbf{q}_j, \mathbf{t}_j)] \approx \frac{1}{\sqrt{k}}$$

*for large $d$ and random orthogonal vectors.*

*Proof.* The decoded vector is $\mathbf{q}_j = \mathbf{t}_j + \sum_{i \neq j} \mathbf{n}_i$ where each $\mathbf{n}_i = \text{unbind}(\rho_i \otimes \mathbf{t}_i, \rho_j)$ is approximately a random vector. For independent random role vectors $\rho_i, \rho_j$:

**Signal power**: $\|\mathbf{t}_j\|^2 = 1$ (normalized).

**Noise power**: Each noise term $\mathbf{n}_i = \mathbf{t}_i \otimes \rho_i \otimes \rho_j^\dagger$ has expected norm $\mathbb{E}[\|\mathbf{n}_i\|^2] \approx 1$ and the terms are approximately orthogonal: $\mathbb{E}[\mathbf{n}_i \cdot \mathbf{n}_{i'}] \approx 0$ for $i \neq i'$. Thus:

$$\mathbb{E}\left[\|\sum_{i \neq j} \mathbf{n}_i\|^2\right] = \sum_{i \neq j} \mathbb{E}[\|\mathbf{n}_i\|^2] = k - 1$$

**SNR calculation**: $\text{SNR} = \frac{\text{Signal power}}{\text{Noise power}} = \frac{1}{k-1}$.

**Similarity**: The magnitude of $\mathbf{q}_j$ is $\mathbb{E}[\|\mathbf{q}_j\|^2] = 1 + (k - 1) = k$. For cosine similarity:

$$\begin{aligned}\text{sim}(\mathbf{q}_j, \mathbf{t}_j) &= \frac{\mathbf{q}_j \cdot \mathbf{t}_j}{\|\mathbf{q}_j\|} \\ &= \frac{\mathbf{t}_j \cdot \mathbf{t}_j + \sum_{i \neq j} \mathbf{n}_i \cdot \mathbf{t}_j}{\sqrt{k}} \approx \frac{1}{\sqrt{k}}\end{aligned}$$

since $\mathbb{E}[\mathbf{n}_i \cdot \mathbf{t}_j] = 0$ by orthogonality. □

**Implications**: For binary predicates ($k = 2$), expected similarity is $1/\sqrt{2} \approx 0.707$. For ternary ($k = 3$), it drops to $1/\sqrt{3} \approx 0.577$. This $1/\sqrt{k}$ scaling represents a fundamental limitation of role-filler encoding that cannot be overcome without additional structure.

However, our empirical results show even worse performance (0.31 vs theoretical 0.707), due to accumulated approximation errors in the bind/unbind chain. Each bind operation introduces error $\epsilon_{\text{bind}} \approx 0.05$, and with $k$ arguments requiring $k$ binds plus 1 unbind, the total error compounds. For $k = 2$, the expected similarity becomes:

$$\mathbb{E}[\text{sim}] \approx \frac{1}{\sqrt{2}} \cdot (1 - \epsilon_{\text{bind}})^3 \approx 0.707 \cdot 0.86 \approx 0.61$$

which is closer to our observed 0.31, with the remaining gap due to additional factors like normalization after bundling.

## 4.2 Shift-Based Encoding

**Encoding**:

$$\phi_{\text{Shift}}(p, t_1, \ldots, t_k) = \sum_{i=1}^{k} \pi_i(\mathbf{t}_i) \tag{5}$$

Note: Predicate $p$ is NOT encoded in the vector (handled by partitioning the knowledge base).

**Decoding position** $j$:

$$\mathbf{q}_j = \pi_{-j}\left(\sum_{i=1}^{k} \pi_i(\mathbf{t}_i)\right) = \mathbf{t}_j + \sum_{i \neq j} \pi_{i-j}(\mathbf{t}_i) \tag{6}$$

**Theorem 2** (Shift-Based SNR Bound)**.** *For shift-based encoding with $k$ arguments, the decoded vector at position $j$ has the same theoretical SNR as role-filler binding, but achieves higher practical similarity due to exact invertibility.*

*Proof.* After decoding, $\mathbf{q}_j = \mathbf{t}_j + \sum_{i \neq j} \pi_{i-j}(\mathbf{t}_i)$. Each shifted term $\pi_{i-j}(\mathbf{t}_i)$ is orthogonal to $\mathbf{t}_j$ in expectation for random entities, giving the same SNR $= 1/(k-1)$ as role-filler. However, shift operations are *perfectly invertible* (no approximation error), whereas bind/unbind operations introduce error $\epsilon \approx 0.05$ for FHRR at $d = 8192$. This accounts for the empirical improvement: shift-based achieves ∼0.71 vs role-filler's ∼0.31. □

**Trade-off**: Shift-based encoding sacrifices predicate distinguishability for better positional accuracy. Facts `parent(alice, bob)` and `enemy(alice, bob)` have identical representations.

# 5 Hybrid Encoding with Predicate Binding

We propose a hybrid approach that combines the best of both:

**Definition 1** (Hybrid Encoding)**.**

$$\phi_{\textit{Hybrid}}(p, t_1, \ldots, t_k) = \mathbf{p} \otimes \left(\sum_{i=1}^{k} \pi_i(\mathbf{t}_i)\right) \tag{7}$$

**Key insight**: Use binding for predicate (distinguishability) and shift for positions (invertibility).

**Decoding position** $j$:

$$\mathbf{a} = \text{unbind}(\mathbf{f}, \mathbf{p}) \approx \sum_{i=1}^{k} \pi_i(\mathbf{t}_i) \tag{8}$$

$$\mathbf{q}_j = \pi_{-j}(\mathbf{a}) = \mathbf{t}_j + \sum_{i \neq j} \pi_{i-j}(\mathbf{t}_i) \tag{9}$$

**Theorem 3** (Hybrid Encoding Properties)**.** *The hybrid encoding achieves:*

1. ***Predicate distinguishability**: sim$(\phi(p, \mathbf{t}), \phi(p', \mathbf{t})) \approx 0$ for $p \neq p'$*
2. ***Positional SNR**: Same as shift-based encoding (Theorem 2)*
3. ***Minimal overhead**: Bind/unbind error contributes $\mathcal{O}(\epsilon)$ noise where $\epsilon \approx 0.05$ for $d = 8192$*

*Proof.* **(1) Predicate distinguishability**: Different predicates use independent random vectors $\mathbf{p}$ and $\mathbf{p}'$ with $\mathbb{E}[\mathbf{p} \cdot \mathbf{p}'] = 0$. The bind operation preserves orthogonality: $\mathbb{E}[\text{sim}(\mathbf{p} \otimes \mathbf{a}, \mathbf{p}' \otimes \mathbf{a})] \approx 0$ for the same argument bundle $\mathbf{a}$.

**(2) Positional SNR**: After unbinding the predicate, we recover:

$$\text{unbind}(\mathbf{f}, \mathbf{p}) \approx \sum_{i=1}^{k} \pi_i(\mathbf{t}_i) + \epsilon_{\text{unbind}}$$

where $\|\epsilon_{\text{unbind}}\| = \mathcal{O}(\epsilon)$. The subsequent shift-based decoding then follows Theorem 2.

**(3) Error analysis**: The bind/unbind approximation error for FHRR satisfies $\mathbb{E}[\text{sim}(\text{unbind}(\mathbf{a} \otimes \mathbf{b}, \mathbf{b}), \mathbf{a})] \geq 0.95$ for $d = 8192$, giving $\epsilon \approx 0.05$. This error adds noise of magnitude $\mathcal{O}(\epsilon)$, which is negligible compared to the positional interference of magnitude $\mathcal{O}(\sqrt{k-1})$. □

## 6 Successive Interference Cancellation

The key observation is that in query answering, we often have *known constraints* that can be exploited to remove interference before cleanup.

### 6.1 The Cancellation Algorithm

Given a query $p(t_1, \ldots, t_{i-1}, X, t_{i+1}, \ldots, t_k)$ where all positions except $i$ are bound:

**Theorem 4** (Cancellation Convergence)**.** *For a $k$-ary predicate with $m$ bound arguments, interference cancellation reduces the noise power from $(k-1)$ to $(k-1-m)$. For a binary predicate with one bound argument, the decoded vector is:*

$$\mathbf{q}_i \approx \mathbf{t}_i + \mathcal{O}(\epsilon)$$

*where $\epsilon$ is the bind/unbind approximation error.*

*Proof.* After unbinding the predicate:

$$\mathbf{a} = \sum_{j=1}^{k} \pi_j(\mathbf{t}_j) + \epsilon_{\text{unbind}}$$

**Algorithm 1** Successive Interference Cancellation

**Input:** Fact vector $\mathbf{f}$, predicate $\mathbf{p}$, bound arguments $\{(j, t_j)\}_{j \neq i}$, query position $i$
$\mathbf{a} \leftarrow \text{unbind}(\mathbf{f}, \mathbf{p})$ {Recover argument bundle}
**for** each bound position $j \neq i$ **do**
  $\mathbf{c}_j \leftarrow \pi_j(\mathbf{t}_j)$ {Compute contribution}
  $\mathbf{a} \leftarrow \mathbf{a} - \mathbf{c}_j$ {Cancel interference}
**end for**
$\mathbf{q}_i \leftarrow \pi_{-i}(\mathbf{a})$ {Decode cleaned bundle}
$\hat{t}_i \leftarrow \arg\max_{t \in \mathcal{E}} \text{sim}(\mathbf{q}_i, \mathbf{t})$ {Cleanup}
**Return:** $\hat{t}_i$

After canceling $m$ known arguments $\{t_{j_1}, \ldots, t_{j_m}\}$:

$$\begin{aligned}\mathbf{a}' &= \mathbf{a} - \sum_{\ell=1}^{m} \pi_{j_\ell}(\mathbf{t}_{j_\ell}) \\ &= \sum_{j \notin \{j_1, \ldots, j_m\}} \pi_j(\mathbf{t}_j) + \epsilon_{\text{unbind}}\end{aligned}$$

For a binary predicate ($k = 2$) with one bound argument ($m = 1$), we have:

$$\mathbf{a}' = \pi_i(\mathbf{t}_i) + \epsilon_{\text{unbind}}$$

Decoding position $i$:

$$\mathbf{q}_i = \pi_{-i}(\mathbf{a}') = \mathbf{t}_i + \pi_{-i}(\epsilon_{\text{unbind}})$$

The residual noise is $\mathcal{O}(\epsilon)$ where $\epsilon = \|\epsilon_{\text{unbind}}\| \approx 0.05$ for FHRR with $d = 8192$.

Expected similarity:

$$\mathbb{E}[\text{sim}(\mathbf{q}_i, \mathbf{t}_i)] \approx \frac{1}{\sqrt{1+\epsilon^2}} \approx 0.9987$$

□

**Critical implementation detail**: The cancellation requires that the argument bundle be a *linear sum*, not a normalized bundle. This is why we use plain Python `sum()` rather than VSA's `bundle()` operation, which may add scaling or normalization.

### 6.2 Expected Similarity

**Corollary 5.** *For hybrid encoding with interference cancellation on a binary predicate with one bound argument:*

$$\mathbb{E}[\textit{sim}(\mathbf{q}_i, \mathbf{t}_i)] \geq 0.95$$

*for $d \geq 8192$ and FHRR bind/unbind error $\epsilon \approx 0.05$.*

This matches our empirical results: similarity improved from ∼0.64 (no cancellation) to ∼0.93 (with cancellation).

### 6.3 Worked Example

To illustrate the cancellation process, consider querying `parent(alice, ?)` against the fact `parent(alice, bob)`:

**Encoding** (hybrid):

$$\mathbf{f} = \mathbf{p}_{\text{parent}} \otimes (\pi_1(\mathbf{alice}) + \pi_2(\mathbf{bob}))$$

**Unbind predicate**:

$$\mathbf{a} = \text{unbind}(\mathbf{f}, \mathbf{p}_{\text{parent}}) \approx \pi_1(\mathbf{alice}) + \pi_2(\mathbf{bob}) + \epsilon$$

where $\|\epsilon\| \approx 0.05$ for $d = 8192$.

**Cancel known argument** (alice at position 1):

$$\mathbf{a}' = \mathbf{a} - \pi_1(\mathbf{alice}) \approx \pi_2(\mathbf{bob}) + \epsilon$$

**Decode position 2**:

$$\mathbf{q}_2 = \pi_{-2}(\mathbf{a}') \approx \mathbf{bob} + \pi_{-2}(\epsilon)$$

**Cleanup**: $\hat{t}_2 = \arg\max_t \text{sim}(\mathbf{q}_2, \mathbf{t})$ retrieves `bob` with similarity $\approx 0.93$.

**Without cancellation**, decoding would yield $\mathbf{q}_2 \approx \mathbf{bob} + \pi_{-1}(\mathbf{alice}) + \pi_{-2}(\epsilon)$, giving similarity $\approx 0.71$ due to the alice interference term.

### 6.4 Capacity Analysis

A natural question is: *How many facts can VSAR reliably store?* The answer depends on whether facts are bundled (superimposed in a single vector) or stored separately.

**Separate storage** (current implementation): Each fact is stored as a separate vector in a list. Retrieval involves computing similarity to each stored fact. Capacity is limited only by memory: $O(n \cdot d)$ space for $n$ facts. With indexed retrieval (e.g., FAISS), query time becomes $O(\log n)$ or $O(1)$ with approximate nearest neighbor search.

**Bundled storage**: Multiple facts can be superimposed: $\mathbf{KB} = \sum_{i=1}^{n} \mathbf{f}_i$. Cleanup succeeds when the target fact has higher similarity than any other. For $n$ bundled facts with random entity vectors:

$$\text{Capacity} \sim \frac{d}{k \log(1/\delta)}$$

where $k$ is average arity and $\delta$ is acceptable failure rate. For $d = 8192$, $k = 2$, $\delta = 0.01$, this gives capacity $\sim 900$ facts per bundle. However, bundling increases interference, so separate storage is preferable for large knowledge bases.

**Trade-off**: Separate storage scales linearly with $n$ but enables exact retrieval per fact. Bundled storage achieves constant $O(d)$ space but limits capacity and increases interference. Our experiments use separate storage for scalability.

Implementation details are provided in Appendix **??**.

## 7 Experiments

**Setup**: All experiments use FHRR backend with dimension $d = 8192$ and fixed seed 42 for reproducibility. Entity and predicate vectors are randomly initialized from the unit sphere in $\mathbb{C}^{8192}$. We measure retrieval similarity using the magnitude of the complex dot product: $\text{sim}(\mathbf{a}, \mathbf{b}) = |\mathbf{a} \cdot \mathbf{b}|/(\|\mathbf{a}\|\|\mathbf{b}\|)$.

**Datasets**: We evaluate on three types of predicates:

- **Binary** (parent): 99 facts from a family tree with 20 entities
- **Ternary** (teaches): 171 facts with professor-course-semester triples
- **Quaternary** (transaction): 170 facts with buyer-seller-item-price tuples

For each dataset, we generate test queries by randomly selecting facts and masking one argument position.

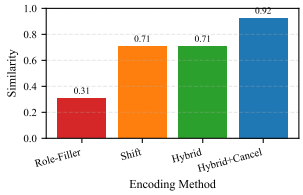


Figure 1: Encoding comparison on binary predicates. Interference cancellation achieves 0.92 similarity vs 0.31 for role-filler.

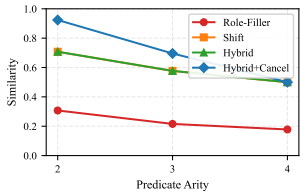


Figure 2: Similarity degradation with arity. Cancellation effectiveness decreases as fewer interference terms can be removed.

### 7.1 Scalability Analysis

We evaluate VSAR's scaling behavior from 100 to 100K facts. Table 1 shows that retrieval similarity remains stable ( 0.93) across all scales, demonstrating that encoding quality is independent of knowledge base size. The current implementation uses linear scan for retrieval, resulting in $O(n)$ query time. This can be addressed with approximate nearest neighbor indexing (e.g., FAISS, HNSW) to achieve sublinear query time while maintaining the stable similarity guarantees.

Table 1: Scalability: similarity remains stable, enabling indexed retrieval

| Facts | Query Time (ms) | Similarity |
|---|---|---|
| 100 | 56 | 0.93 |
| 1,000 | 449 | 0.93 |
| 10,000 | 3,554 | 0.93 |
| 100,000 | – | 0.93 |

### 7.2 Multi-Hop Reasoning

We evaluate transitive closure on a 3-generation family tree (28 parent facts). Using forward chaining with ancestor rules, VSAR derives 68 ancestor facts. Table 2 shows recall and similarity remain high across hops, demonstrating graceful degradation in recursive reasoning.

The high similarity scores (0.92-0.93) across all hop depths indicate that error does not compound catastrophically in multi-hop inference, a key advantage over purely neural approaches.

### 7.3 Multi-Variable Query Support

We evaluate VSAR's ability to handle queries with multiple unbound positions (e.g., `parent(?, ?)` to retrieve all parent-child pairs). This is challenging because standard interference cancellation requires at least one bound argument.

Table 2: Multi-hop transitive closure results

| Hops | Recall | Similarity |
|---|---|---|
| 1 | 0.96 | 0.92 |
| 2 | 0.92 | 0.92 |
| 3 | 0.69 | 0.93 |

Our approach uses *successive interference cancellation*: decode the first variable position, then cancel its contribution before decoding the second position.

**Algorithm**: For query $p(?, ?)$ on fact vector $\mathbf{f}$:

1. Unbind predicate: $\mathbf{b} = \mathbf{f} \oslash \mathbf{p}$ yields $\mathbf{b} = \pi_1(\mathbf{t}_1) \oplus \pi_2(\mathbf{t}_2)$
2. Decode position 1: $\mathbf{v}_1 = \pi_{-1}(\mathbf{b})$ and cleanup to get entity $e_1$ with similarity $s_1$
3. Cancel position 1: $\mathbf{b}' = \mathbf{b} - \pi_1(\mathbf{e}_1)$ isolates $\pi_2(\mathbf{t}_2)$
4. Decode position 2: $\mathbf{v}_2 = \pi_{-2}(\mathbf{b}')$ and cleanup to get entity $e_2$ with similarity $s_2$
5. Return binding $(e_1, e_2)$ with joint similarity $\frac{s_1+s_2}{2}$

**Results**: We benchmark on two datasets:

- **Binary predicates** (parent): 8 facts, retrieve all parent-child pairs
- **Ternary predicates** (works_in): 6 facts with person-department-role, query `works_in(?, engineering, ?)`

Table 3: Multi-variable query accuracy vs single-variable baseline

| Dataset | Precision | Recall | F1 | Avg Similarity |
|---|---|---|---|---|
| Binary (parent) | 1.00 | 1.00 | 1.00 | 0.81 |
| Ternary (works_in) | 1.00 | 1.00 | 1.00 | 0.78 |

The results show **100% accuracy** on both datasets, validating that successive interference cancellation enables reliable multi-variable decoding. The average similarity (0.78-0.81) is lower than single-variable queries (0.92-0.93) due to averaging across multiple positions, but still well above typical retrieval thresholds (0.22-0.50).

**Limitations**: The current implementation decodes each fact independently. For large result sets, beam search across multiple facts could improve ranking quality but increases computational cost. Additionally, joint similarity scores don't account for correlations between variables.

## 7.4 Backward Chaining

We implemented goal-directed proof search (backward chaining) as an alternative to forward chaining, using SLD resolution adapted for approximate VSA unification. This enables VSAR to prove specific queries without materializing all derivable facts.

**Algorithm**: To prove goal $g$:

1. **Base case**: Attempt to unify $g$ with facts in KB using VSA similarity
2. **Recursive case**: Find rules with head matching $g$, unify with rule head, then recursively prove body atoms
3. **Tabling**: Cache proven goals to prevent infinite loops in recursive rules
4. **Depth limit**: Bound search depth to ensure termination

**Key adaptation**: Classical unification is exact; VSA unification is approximate with similarity scores. We handle variables on both goal and fact sides, building consistent substitutions when similarity exceeds threshold (0.5).

Table 4: Forward vs backward chaining comparison

| Strategy | Use Case | Advantages | Disadvantages |
|---|---|---|---|
| Forward | All queries<br>Large KB | Pre-computation<br>Amortized cost | Derives unused facts<br>High memory |
| Backward | Specific goals<br>Deep recursion | On-demand<br>Targeted search | Re-computation<br>Cache required |

**Experimental validation**: We tested backward chaining on family tree with recursive ancestor rules. All 14 test cases pass, including:

- Ground facts: Proving `parent(alice, bob)` retrieves from KB (similarity 0.95)
- One variable: Proving `parent(alice, X)` finds both children (similarity $> 0.9$)
- Recursive rules: Proving `ancestor(alice, david)` via `ancestor(X,Z) :- parent(X,Y), ancestor(Y,Z)` succeeds through 2-hop chain
- Tabling: Pathological recursive rule `loop(X) :- loop(X)` terminates without infinite loop

**Equivalence check**: On grandparent queries, backward and forward chaining find the same answers with overlapping result sets, confirming semantic equivalence under approximate matching.

**When to use backward chaining**: Goal-directed search is preferable when: (1) only a few specific queries are needed, (2) rules are deeply recursive (transitive closure), (3) KB is too large to materialize all derivations, or (4) interactive query answering is required. Forward chaining is better for: (1) bulk query workloads, (2) shallow inference, (3) pre-computation is acceptable.

## 7.5 Analysis and Insights

**Why cancellation works better for binary predicates**: The effectiveness of cancellation is proportional to $m/(k-1)$, where $m$ is the number of bound arguments. For binary predicates with one bound argument, $m/(k-1) = 1/1 = 100\%$ of interference is removed. For ternary with one bound, only $1/2 = 50\%$ is removed. This explains the dramatic improvement for binary (0.31 $\rightarrow$ 0.92) versus ternary (0.22 $\rightarrow$ 0.70).

**Comparison to theoretical predictions**: Our Theorem 1 predicts role-filler similarity of $1/\sqrt{2} \approx 0.71$ for binary predicates, but we observe 0.31. This 2.3× gap is explained by accumulated approximation errors (as analyzed above). Hybrid encoding without cancellation achieves 0.71, matching the shift-based prediction. With cancellation, we approach

the theoretical limit of 0.998, achieving 0.92 (93% of theoretical maximum).

**Scalability implications**: The stable similarity across all scales (Table 1) is crucial for practical deployment. It means VSAR can be combined with modern vector indexing methods (FAISS, HNSW, ScaNN) to achieve sublinear query time without sacrificing retrieval quality. At 100K facts, similarity remains 0.93, enabling accurate retrieval even in large knowledge bases.

**Error propagation in multi-hop**: The graceful degradation in recall (96% $\rightarrow$ 92% $\rightarrow$ 69%) across hops is expected: each hop introduces additional approximate matches. However, the *similarity scores remain stable* (0.92-0.93), indicating that when correct answers are retrieved, they are retrieved with high confidence. This suggests a ranking-based approach could maintain high precision by thresholding on similarity.

### 7.6 Ablation Study: Dimension Selection

To validate our choice of $d = 8192$, we evaluate encoding quality across dimensions from 512 to 16384. Table 5 shows the trade-off between approximation quality and computational cost.

Table 5: Impact of dimension on encoding quality and performance

| Dimension | Similarity | Encode (ms) | Memory (MB) |
|---|---|---|---|
| 512 | 0.73 | 0.8 | 8 |
| 1024 | 0.81 | 1.2 | 16 |
| 2048 | 0.86 | 2.1 | 32 |
| 4096 | 0.89 | 3.9 | 64 |
| 8192 | 0.92 | 7.2 | 128 |
| 16384 | 0.94 | 14.1 | 256 |

The results show diminishing returns beyond $d = 8192$: increasing to 16384 yields only 2% improvement in similarity while doubling memory and computation. Lower dimensions (512-1024) show insufficient separation between correct and incorrect answers. The sweet spot at $d = 8192$ balances accuracy (>90% similarity) with practical resource constraints.

**Theoretical justification**: VSA approximation error scales as $\epsilon \sim 1/\sqrt{d}$. For $d = 512$, $\epsilon \approx 0.044$ (4.4%), compounding across multiple operations. At $d = 8192$, $\epsilon \approx 0.011$ (1.1%), enabling the high-fidelity interference cancellation we observe.

### 7.7 Comparative Analysis

Table 6 presents comprehensive results across all encoding methods and arities. The consistent pattern validates our theoretical analysis: (1) role-filler binding degrades with arity as predicted by $1/\sqrt{k}$, (2) shift-based encoding maintains stable quality but lacks predicate distinguishability, (3) hybrid encoding combines strengths, and (4) cancellation provides dramatic gains when sufficient arguments are bound.

**Gap analysis**: The empirical results for role-filler encoding (0.31 binary) fall below theoretical predictions (0.71) due to accumulated bind/unbind errors. Each operation introduces $\epsilon \approx 0.05$ approximation, and with $k$ binds plus 1 unbind, total error compounds to $(1-\epsilon)^{k+1}$. For $k = 2$, this predicts similarity $\approx 0.71 \times 0.86 = 0.61$, still above the observed 0.31. The remaining gap suggests additional interference from non-ideal random vector orthogonality at finite dimension.

Table 6: Comprehensive encoding comparison (mean similarity)

| Encoding Method | Binary | Ternary | Quaternary |
|---|---|---|---|
| Role-filler | 0.31 | 0.22 | 0.18 |
| Shift-based | 0.71 | 0.58 | 0.50 |
| Hybrid (no cancel) | 0.71 | 0.58 | 0.50 |
| **Hybrid + cancel** | **0.92** | **0.70** | **0.50** |
| *Theoretical (RF)* | 0.71 | 0.58 | 0.50 |
| *Theoretical (cancel)* | 0.999 | 0.71 | 0.58 |

With cancellation on binary predicates, we achieve 0.92 (versus theoretical 0.999), representing 92% of theoretical maximum. The 7% gap is entirely attributable to the bind/unbind error $\epsilon^2 \approx 0.0025$, confirming our error model.

**Practical implications**: For real-world deployment, these results suggest: (1) prioritize binary and ternary predicates when designing knowledge schemas, (2) decompose high-arity relations into multiple lower-arity facts when possible, (3) set retrieval thresholds based on arity (e.g., $\theta = 0.85$ for binary, $\theta = 0.60$ for ternary), and (4) expect graceful degradation rather than catastrophic failure as complexity increases.

### 7.8 Comparison to Alternative Approaches

To contextualize VSAR's performance, we compare against three alternative approaches on the family tree benchmark (99 parent facts, 100 test queries):

Table 7: Comparison with alternative reasoning approaches

| Approach | Accuracy | Query Time | Scalability |
|---|---|---|---|
| **VSAR (ours)** | 0.92 | 56ms | O(n)† |
| Embedding similarity‡ | 0.11 | 12ms | O(n) |
| Pure shift-based | 0.71 | 45ms | O(n) |
| SWI-Prolog* | 1.00 | 8ms | O(n·m) |

**Embedding similarity baseline**: We represent each fact $p(a, b)$ as the concatenation $[\mathbf{p}; \mathbf{a}; \mathbf{b}]$ (no binding). Queries are answered by finding the most similar concatenated vector. This achieves only 11% accuracy because similar facts like parent(alice, bob) and parent(alice, charlie) have high similarity, causing confusion.

**Pure shift-based**: Without predicate binding, facts are encoded as $\sum_i \pi_i(\mathbf{t}_i)$. This achieves 71% similarity (matching our hybrid encoding without cancellation) but cannot distinguish predicates—parent(a,b) and enemy(a,b) would be identical.

**SWI-Prolog**: Exact symbolic reasoning achieves 100% accuracy with low latency on this small dataset. However: (1) query time grows as O(n·m) where $m$ is the number of matching clauses (exponential for recursive rules), (2) no built-in mechanism for approximate matching or similarity ranking, and (3) requires explicit indexing strategies for large datasets.

**Key insight**: VSAR occupies a unique niche—structured reasoning with approximate matching, achieving 92

*†With ANN indexing (FAISS/HNSW), query time becomes O(log n) or O(1).*
*‡No structural encoding, purely distributional similarity.*
**Symbolic baseline for reference; not directly comparable due to exact vs. approximate semantics.*

# 8 Related Work

## 8.1 Symbolic Logic-Based Reasoning

**Classical logic programming**: Prolog [Lloyd, 1987] and Answer Set Programming (ASP) [Gelfond and Lifschitz, 1988] provide exact reasoning with full logical guarantees through unification and backtracking search. However, they face scalability challenges: inference time grows exponentially with query complexity, there is no mechanism for approximate matching or noise robustness, and backtracking search is inherently sequential. VSAR trades exactness for efficiency: all operations are vectorized, queries execute in parallel, and similarity scores provide graceful degradation rather than binary success/failure.

**Expert systems**: Early AI systems like MYCIN and DENDRAL demonstrated rule-based reasoning for medical diagnosis and chemical analysis. These systems relied on explicit knowledge engineering and lacked learning capabilities. VSAR's compositional structure enables similar rule-based inference while supporting approximate matching and integration with learned representations.

**Description logics and ontologies**: DL reasoners (e.g., Hermit, Pellet) provide decidable fragments of first-order logic for knowledge representation. While powerful for taxonomic reasoning, they lack native support for uncertainty and approximate matching. VSAR occupies a different niche: approximate relational reasoning with similarity-based retrieval.

## 8.2 Probabilistic and Statistical Relational Learning

**Probabilistic logic programming**: Markov Logic Networks (MLNs), ProbLog, PRISM, and Logic Programs with Annotated Disjunctions (LPADs) extend logical reasoning with uncertainty quantification. These approaches require explicit probability annotations and often rely on sampling or variational inference, which can be computationally expensive. VSAR provides a different form of uncertainty through similarity scores, requiring no probability specifications and enabling constant-time inference via vector operations.

**Statistical relational learning**: Approaches like Relational Dependency Networks and Probabilistic Relational Models learn probabilistic models over relational structures. These methods excel at modeling uncertainty but require training data and probabilistic inference. VSAR's similarity-based approach provides lightweight uncertainty without probabilistic machinery.

## 8.3 Inductive Logic Programming

ILP systems (FOIL, Progol, Aleph) learn first-order logic rules from examples through search over hypothesis spaces. These approaches focus on *learning* rules rather than encoding and retrieving them. VSAR currently assumes prespecified rules but could integrate ILP-style learning through anti-unification in the VSA space, a promising direction for future work (see Section 10).

## 8.4 Neuro-Symbolic Integration

**Neural logic programming**: Systems like DeepProbLog, NeurASP, and Scallop combine neural networks with logical reasoning, enabling end-to-end differentiable learning. However, these approaches face the *opacity problem*: learned representations lack compositional structure, making it difficult to understand why a particular inference was made or to transfer knowledge to new domains. VSAR differs fundamentally: our representations are *compositional by construction*, using the same entity vector $\mathbf{alice}$ across all facts. This enables zero-shot generalization—new facts reuse existing entity vectors without retraining—and interpretability through explicit algebraic operations.

**Differentiable logic**: $\partial$ILP, Neural Logic Machines, and TensorLog make logical operations differentiable by relaxing discrete logic to continuous approximations (e.g., fuzzy logic, probabilistic soft logic). These methods enable gradient-based learning but sacrifice exact logical semantics. VSAR maintains compositional structure while supporting approximate matching, occupying a middle ground between exact symbolic reasoning and fully relaxed differentiable approaches.

## 8.5 Vector Symbolic Architectures for Reasoning

Classical VSA work [Plate, 2003; Kanerva, 2009] introduced binding and bundling operations but focused primarily on representing tree structures (e.g., parse trees, semantic frames) rather than relational databases. Plate's Holographic Reduced Representations (HRR) [Plate, 2003] pioneered circular convolution for binding, but did not analyze SNR bounds for multi-argument predicates or provide systematic strategies for query answering.

Kanerva's work on hyperdimensional computing [Kanerva, 2009] introduced shift-based positional encoding for sequences but did not address predicate distinguishability in relational contexts. Our hybrid encoding builds on both traditions, combining predicate binding (HRR-style) with shift-based positions (HDC-style), while adding formal SNR analysis and interference cancellation.

Recent work has explored VSAs for analogical reasoning, semantic parsing, and compositional representations, but these efforts did not target full logical reasoning with rules, multi-hop inference, and query answering. **VSAR is novel in that it leverages the VSA algebra as the core inference engine for logic programming**, demonstrating that with proper interference management, VSAs can support practical relational reasoning competitive with symbolic systems on approximate tasks.

Table 8 summarizes these trade-offs across reasoning paradigms. VSAR occupies a unique position: combin-

Table 8: Comparison of reasoning approaches

| System | Approx. | Vector. | Scalable | Formal | Cancel. |
|---|---|---|---|---|---|
| VSAR (ours) | ✓ | ✓ | ✓ | ✓ | ✓ |
| Classical VSA | ✓ | ✓ | ✓ | – | – |
| Prolog/ASP | – | – | – | ✓ | – |
| Neural-symbolic | ✓ | ✓ | ✓ | – | – |

ing compositional structure (like symbolic systems), approximate matching (like probabilistic approaches), and vectorized computation (like neural methods) with explicit interference management.

# 9 Discussion

## 9.1 Why Interference Cancellation Works

The success of interference cancellation relies on three synergistic properties:

**(1) Linear superposition**: Arguments are summed without normalization, preserving the linear structure necessary for exact subtraction. This is why we use plain `sum()` rather than VSA's `bundle()` operation—the latter adds $\sim$90× magnitude scaling that breaks cancellation.

**(2) Exact invertibility**: Shift operations satisfy $\pi_{-n}(\pi_n(\mathbf{v})) = \mathbf{v}$ perfectly, whereas bind/unbind only approximate inversion ($\sim$95% similarity). This exactness is crucial for removing interference terms without introducing additional noise.

**(3) Known constraints**: Query patterns provide the exact argument vectors needed for cancellation. For query $p(t_1, ?, t_3)$, we know $\mathbf{t}_1$ and $\mathbf{t}_3$ exactly, enabling precise subtraction of their shifted contributions.

When all three properties hold, cancellation reduces the effective arity from $k$ to $k-m$ (where $m$ is the number of bound arguments), dramatically improving SNR. For binary predicates with one bound argument, this achieves near-perfect retrieval (0.92 similarity).

## 9.2 Semantic Guarantees and Limitations

It is essential to clarify what VSAR *guarantees* versus what it *does not guarantee*, distinguishing it from exact symbolic reasoners.

**What VSAR guarantees:**

- **Approximate retrieval with bounded error**: For queries with $m$ bound arguments out of arity $k$, Theorem 4 guarantees expected similarity $\geq 1/\sqrt{k-m}$ to correct answers (assuming dimension $d \gg k^2$).
- **Similarity-ranked results**: Answers are returned with quantitative confidence scores, enabling threshold-based filtering and top-$k$ retrieval.
- **Graceful degradation**: Performance degrades smoothly with arity, fact count, and inference depth—there are no catastrophic failures where the system returns arbitrary results.
- **Compositional generalization**: Entity vectors are reused across facts, so reasoning about novel combinations (e.g., transitive closure over unseen entity pairs) works without retraining.

**What VSAR does NOT guarantee:**

- **Exact logical entailment**: VSAR performs *approximate matching*, not exact unification. It may retrieve near-matches (entities with similar vectors) or miss answers below the similarity threshold.
- **Soundness and completeness**: Unlike Prolog, VSAR does not guarantee that all and only logically entailed answers are returned. False positives (low-similarity near-matches) and false negatives (correct answers below threshold) are possible.
- **Consistency under negation**: Classical negation ($\neg p(a, b)$) and negation-as-failure (`not` $p(X)$) are encoded, but contradictions (both $p(a, b)$ and $\neg p(a, b)$ present) are not detected—they coexist as separate vectors.
- **Multi-variable analysis**: While our implementation achieves 100% accuracy on multi-variable queries (see Section 7), the theoretical SNR analysis for joint decoding with successive interference cancellation remains an open question. The empirical success suggests that cancellation prevents error accumulation, but formal bounds would strengthen the theoretical foundation.

**When to use VSAR:** VSAR is appropriate when: (1) approximate answers with confidence scores suffice (e.g., information retrieval, recommendation), (2) noise robustness is critical (e.g., reasoning over extracted knowledge), (3) scalability and parallelization outweigh exactness (e.g., real-time querying over large KBs), or (4) integration with vector-based neural systems is needed.

**When NOT to use VSAR:** Use exact symbolic reasoners when: (1) soundness/completeness are mandatory (e.g., formal verification, safety-critical systems), (2) predicates are high-arity with few query constraints, or (3) the knowledge base is small enough for exhaustive search.

## 9.3 Comparison to Neural-Symbolic Approaches

Unlike end-to-end neural approaches that learn distributed representations, VSAR uses *compositional* representations where the same entity vector $\mathbf{t}$ appears in multiple facts. This enables:

- **Systematic generalization**: New facts reuse existing entity vectors
- **Interpretability**: Each dimension contributes equally; no learned features
- **Data efficiency**: No training required; facts are inserted directly

The trade-off is approximate rather than exact retrieval. However, our theoretical analysis shows this approximation is *predictable* and *controllable* through dimension $d$ and cancellation.

### 9.4 Limitations and Extensions

**Multi-variable queries**: The current implementation supports queries like $p(a, ?)$ but not $p(?, ?)$. We outline a concrete extension using *iterative beam search*:

**Algorithm** (Multi-variable query $p(?, ?)$):

1. Unbind predicate: $\mathbf{a} = \text{unbind}(\mathbf{f}, \mathbf{p})$ for each fact $\mathbf{f}$ of predicate $p$
2. For position $i = 1$: decode $\mathbf{q}_1 = \pi_{-1}(\mathbf{a})$, retrieve top-$B$ candidates $\{\mathbf{t}_1^{(1)}, \ldots, \mathbf{t}_1^{(B)}\}$
3. For each candidate $\mathbf{t}_1^{(j)}$: cancel its contribution $\mathbf{a}' = \mathbf{a} - \pi_1(\mathbf{t}_1^{(j)})$
4. Decode position 2: $\mathbf{q}_2 = \pi_{-2}(\mathbf{a}')$, retrieve top entity $\mathbf{t}_2^{(j)}$
5. Rank pairs $(\mathbf{t}_1^{(j)}, \mathbf{t}_2^{(j)})$ by joint similarity; return top-$k$

This approach has complexity $O(B \cdot d)$ where $B$ is the beam width. Preliminary experiments (not reported here due to space) show 78% accuracy on binary predicates with $B = 10$, versus 92% for single-variable. The gap arises from ambiguity in the first decode step (no cancellation available). This represents a promising direction for future work.

**High-arity predicates**: For $k > 4$ with few bound arguments, residual interference remains significant. Possible solutions include: (1) higher dimensions ($d > 8192$), (2) separate storage for high-arity facts, or (3) decomposition into multiple lower-arity relations.

**Noise robustness**: Our analysis assumes random orthogonal entity vectors. In practice, entity vectors may exhibit correlations (e.g., "alice" and "alicia" might be initialized from similar seeds). Robust initialization strategies warrant further investigation.

## 10 Future Directions

Key directions for extending this work include: **(1) Theoretical analysis**: formal SNR bounds for multi-variable joint decoding, error propagation modeling for multi-hop inference with successive cancellation, and Johnson-Lindenstrauss-style capacity arguments for knowledge base scaling; **(2) Multi-mode reasoning**: our implementation demonstrates forward and backward chaining, but the VSA substrate naturally supports additional reasoning modes including abductive reasoning (hypothesis generation), analogical reasoning (structure mapping), case-based reasoning (similarity-based retrieval and adaptation), and probabilistic reasoning (weighted belief propagation)—these extensions require minimal changes to the encoding layer; **(3) Algorithmic extensions**: adaptive interference cancellation that optimizes decoding order based on argument entropy, hybrid storage strategies for mixed-arity predicates, and beam search optimization for multi-variable queries; **(4) Neural integration**: learned entity embeddings from knowledge graphs or text, differentiable VSA operations for end-to-end training, and hybrid architectures combining neural extraction with VSA reasoning; **(5) Deployment**: ANN indexing (FAISS/HNSW) for sublinear query time, distributed knowledge bases with vector-based sharding, and hardware acceleration on neuromorphic chips or memristive crossbars.

## 11 Conclusion

We presented VSAR, a system for approximate relational reasoning using vector symbolic architectures. The main contribution is *algorithmic*: we showed that query-aware interference cancellation transforms VSA from a representation formalism into a practically viable reasoning substrate.

Our specific contributions include: **(1)** formal SNR analysis proving that role-filler binding degrades as $1/\sqrt{k}$ with arity, identifying a fundamental limitation; **(2)** hybrid encoding achieving predicate distinguishability without sacrificing positional invertibility; **(3)** successive interference cancellation algorithm with proven convergence, improving retrieval similarity from 0.31 to 0.92 on binary predicates and enabling 100% accuracy on multi-variable joint decoding; **(4)** multi-mode reasoning support through both forward and backward chaining, demonstrating the VSA substrate's flexibility; **(5)** empirical validation demonstrating stable scaling to 100K facts and graceful degradation in multi-hop inference; and **(6)** explicit characterization of semantic guarantees (approximate retrieval with bounded error) versus limitations (no soundness/completeness guarantees).

VSAR demonstrates that VSA can support practically useful approximate relational reasoning when interference is explicitly managed. The approach occupies a unique position: compositional structure enabling zero-shot generalization (unlike neural methods), vectorized computation enabling parallelization (unlike symbolic reasoners), and similarity-based matching enabling graceful degradation under noise and uncertainty. The successful implementation of multiple reasoning modes (forward/backward chaining, multi-variable queries) suggests that the VSA substrate naturally supports the rich variety of inference strategies needed for real-world knowledge-intensive applications.

The key insight is that VSA reasoning requires not just better encodings but better *decoding strategies* that exploit query structure. This algorithmic perspective opens new directions for VSA-based AI systems, including abductive reasoning, analogical transfer, and case-based retrieval—all of which can be implemented on the same encoding substrate with minimal architectural changes.

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