Hypothesis Testing

Class Objectives

- Developing Null and Alternative Hypotheses
- Type I and Type II Errors- Explanation
- Population Mean: Sigma Known
- Population Mean: Sigma Unknown
- Population Proportion

Hypothesis Testing

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The null hypothesis, denoted by H_0 , is a tentative assumption about a population parameter
- The alternative hypothesis, denoted by Ha, is the opposite of what is stated in the null hypothesis
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H0 and Ha.

Developing Null and Alternative Hypotheses

- It is not always obvious how the null and alternative hypotheses should be formulated
- Care must be taken to structure the hypotheses appropriately so that the test conclusion provides the information the researcher wants
- The context of the situation is very important in determining how the hypotheses should be stated
- In some cases it is easier to identify the alternative hypothesis first. In other cases the null is easier
- Correct hypothesis formulation will take practice

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Many applications of hypothesis testing involve an attempt to gather evidence in support of a research hypothesis
- In such cases, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support
- The conclusion that the research hypothesis is true is made if the sample data provide sufficient evidence to show that the null hypothesis can be rejected

Developing Null and Alternative Hypotheses

Alternative Hypothesis as a Research Hypothesis

- Example: A new manufacturing method is believed to be better than the current method.
- Alternative Hypothesis:
  - The new manufacturing method is better.
- Null Hypothesis:
  - The new method is no better than the old method.

Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
- Example: A new bonus plan, that is developed in an attempt to increase sales
- Alternative Hypothesis:
  - The new bonus plan increase sales
- Null Hypothesis:
  - The new bonus plan does not increase sales

Developing Null and Alternative Hypotheses

- Alternative Hypothesis as a Research Hypothesis
- Example:
  - A new drug is developed with the goal of lowering Cholesterol-level more than the existing drug
- Alternative Hypothesis:
  - The new drug lowers Cholesterol-level more than the existing drug
- Null Hypothesis:
  - The new drug does not lower Cholesterol-level more than the existing drug

Developing Null and Alternative Hypotheses

- Null Hypothesis as an assumption to be challenged
- We might begin with a belief or assumption that a statement about the value of a population parameter is true
- We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect
- In these situations, it is helpful to develop the null hypothesis first

Developing Null and Alternative Hypotheses

- Null Hypothesis as an Assumption to be Challenged
- Example:
  - The label on a milk bottle states that it contains 1000 ml
- Null Hypothesis:
  - The label is correct. $\mu \geq 1000$ ml
- Alternative Hypothesis:
  - The label is incorrect. $\mu < 1000$ ml

Null and Alternative Hypotheses about a Population Mean μ

- The equality part of the hypotheses always appears in the null hypothesis
- In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean)

$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu = \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$	$H_a: \mu \neq \mu_0$
One-tailed (lower-tail)	One-tailed (upper-tail)	Two-tailed

Null and Alternative Hypotheses

- A major hospital in Chennai provides one of the most comprehensive emergency medical services in the world
- Operating in a multiple hospital system with approximately 10 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 8 minutes or less
- The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 8 minutes or less is being achieved.

Null and Alternative Hypotheses

H_0: $\mu \leq 8$

The emergency service is meeting the response goal; no follow-up action is necessary.

H_a: $\mu > 8$

The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

where: μ = mean response time for the population of medical emergency requests

Type I Error

- Because hypothesis tests are based on sample data, we must allow for the possibility of errors
- A Type I error is rejecting H_0 when it is true
- The probability of making a Type I error when the null hypothesis is called the level of significance
- Applications of hypothesis testing that only control the Type I error are often called significance tests

Type II Error

- A Type II error is accepting H_0 when it is false.
- It is difficult to control for the probability of making a Type II error.
- Statisticians avoid the risk of making a Type II error by using “do not reject H_0” and not “accept H_0”.

Type I and Type II Errors

Conclusion	Population Condition: H0 True ($\mu \le 8$)	Population Condition: H0 False ($\mu > 8$)
Accept H0 (Conclude $\mu \le 8$)	Correct Decision	Type II Error
Reject H0 (Conclude $\mu > 8$)	Type I Error	Correct Decision

Three Approaches for Hypothesis Testing

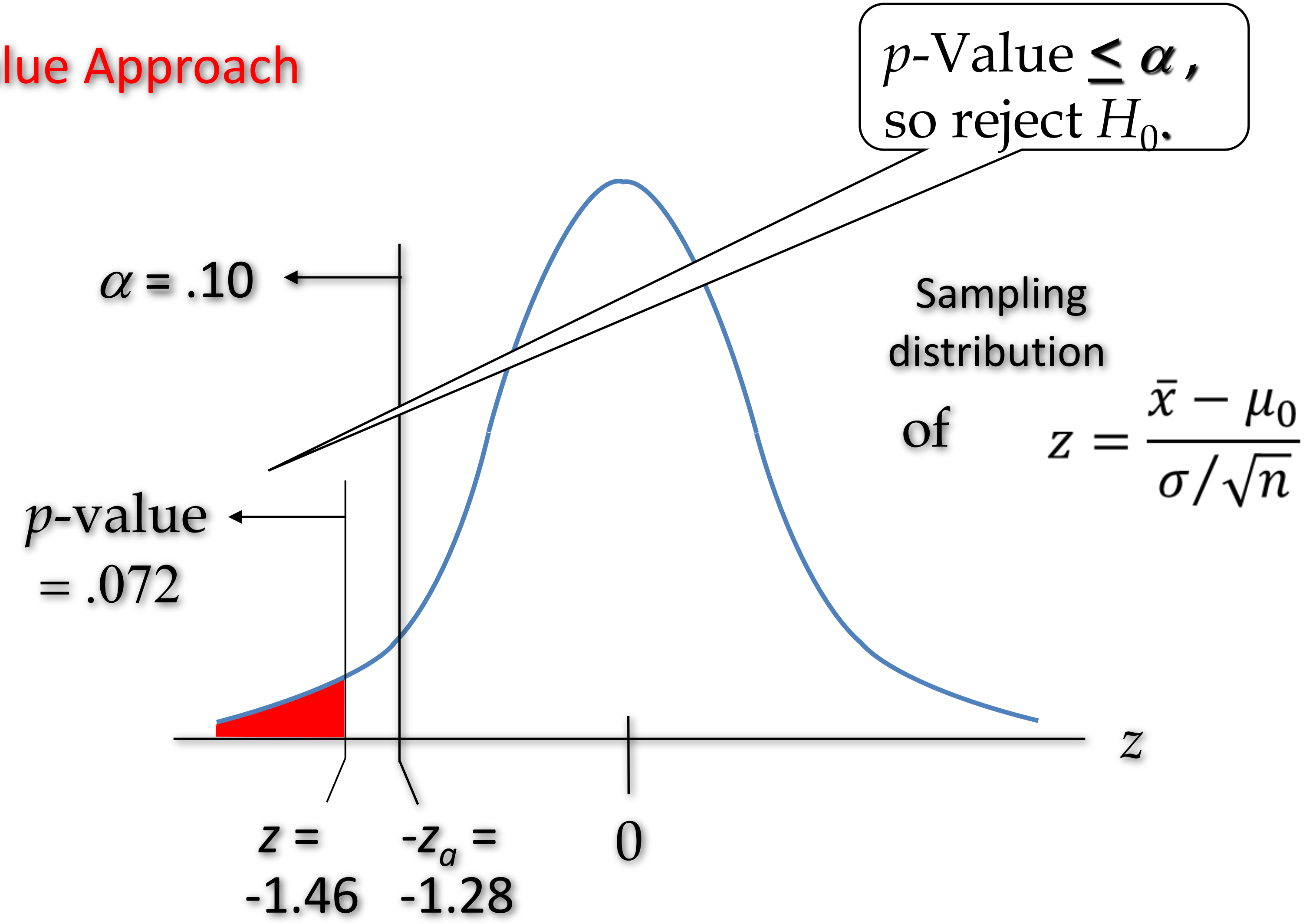
- P- Value
- Critical Value
- Confidence Interval Value

p-Value Approach to One-Tailed Hypothesis Testing

- The p-value is the probability, computed using the test statistic, that measures the support (or lack of support) provided by the sample for the null hypothesis
- If the p-value is less than or equal to the level of significance α, the value of the test statistic is in the rejection region
- Reject H_0 if the p-value $\leq \alpha$

Lower-Tailed Test About a Population Mean: σ Known

p-Value Approach



p-Value Approach

Finding P Value

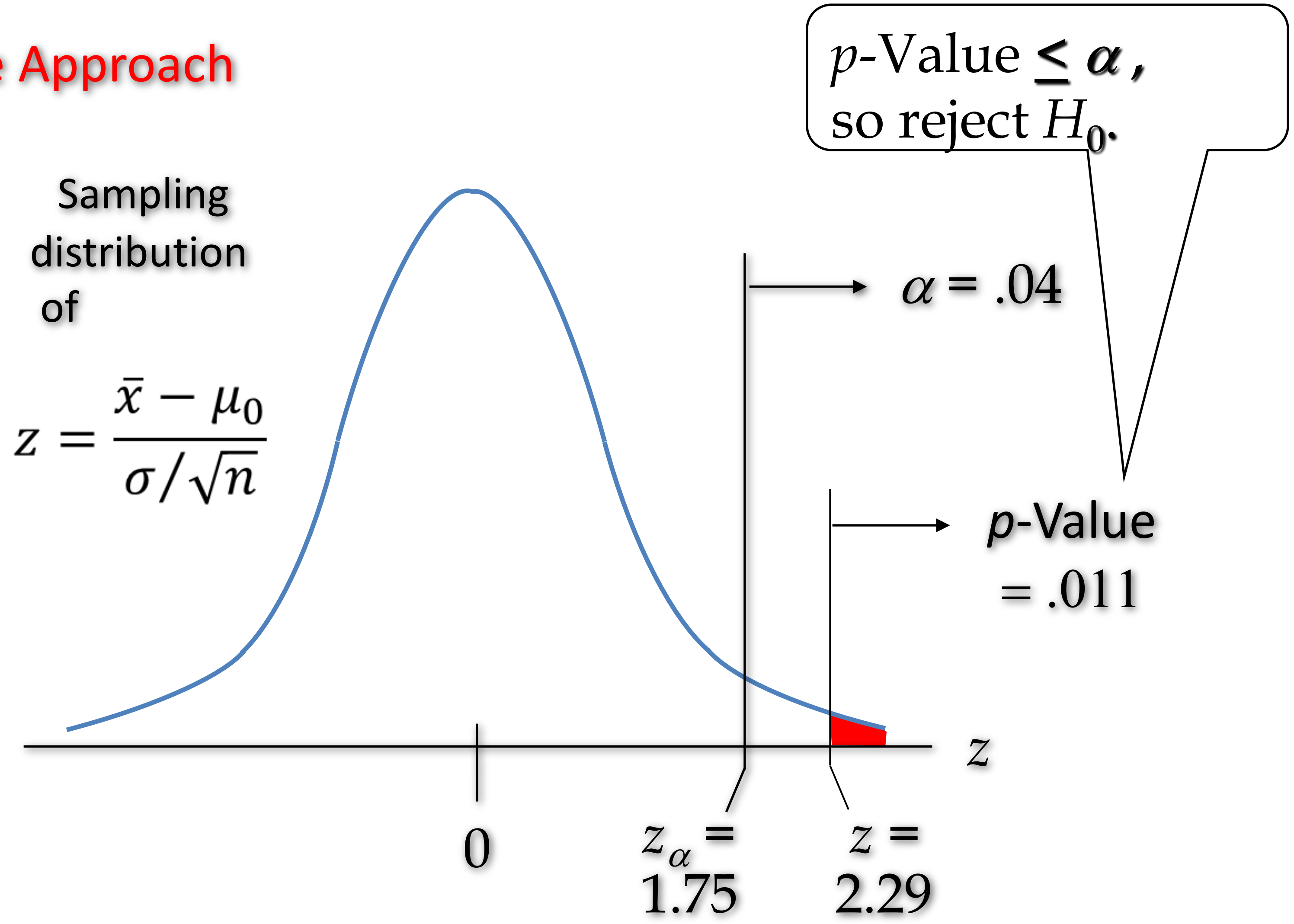
```
In [3]: stats.norm.cdf(-1.46)
Out[3]: 0.07214503696589378
```

Finding Z Value

```
In [5]: stats.norm.ppf(0.1)
Out[5]: -1.2815515655446004
```

Upper-Tailed Test About a Population Mean : σ Known

p-Value Approach



p-Value Approach

```
In [4]: 1-stats.norm.cdf(1.75)

Out[4]: 0.040059156863817114
```

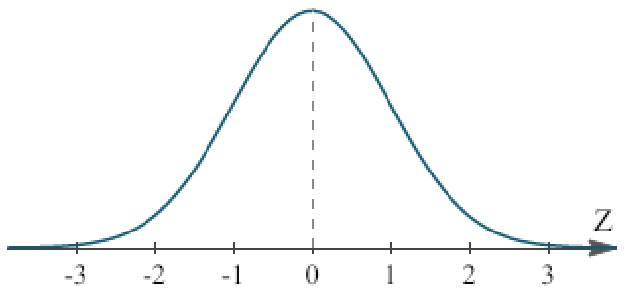
```
In [5]: 1-stats.norm.cdf(2.29)

Out[5]: 0.011010658324411393
```

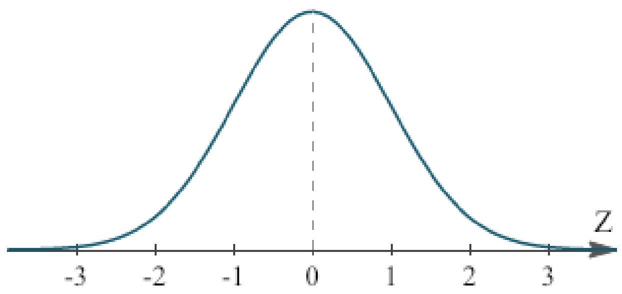
Critical Value Approach to One-Tailed Hypothesis Testing

- The test statistic z has a standard normal probability distribution.
- We can use the standard normal probability distribution table to find the z-value with an area of α in the lower (or upper) tail of the distribution.
- The value of the test statistic that established the boundary of the rejection region is called the critical value for the test.
- The rejection rule is:

  Lower tail: Reject H_0 if $z \leq -z_\alpha$

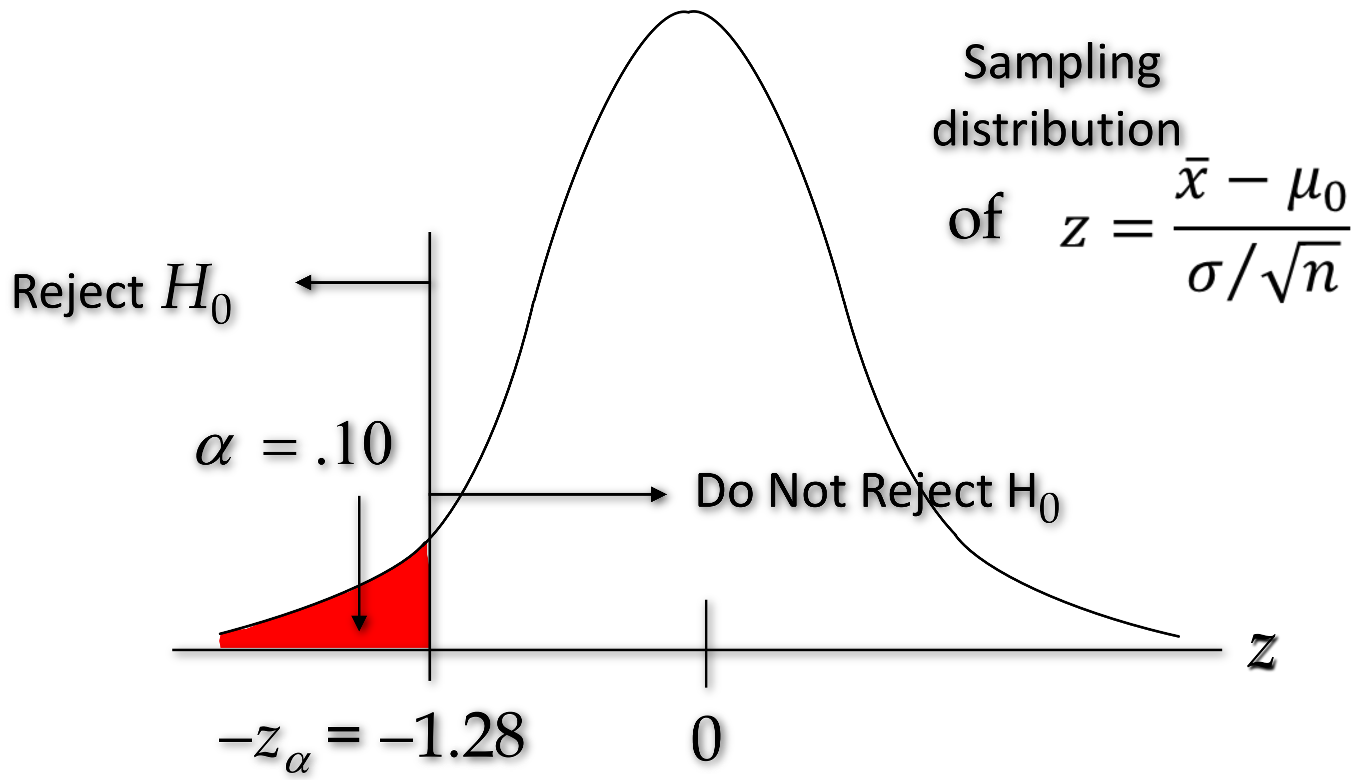
  Upper tail: Reject H_0 if $z \geq z_\alpha$





Lower-Tailed Test About a Population Mean: σ Known

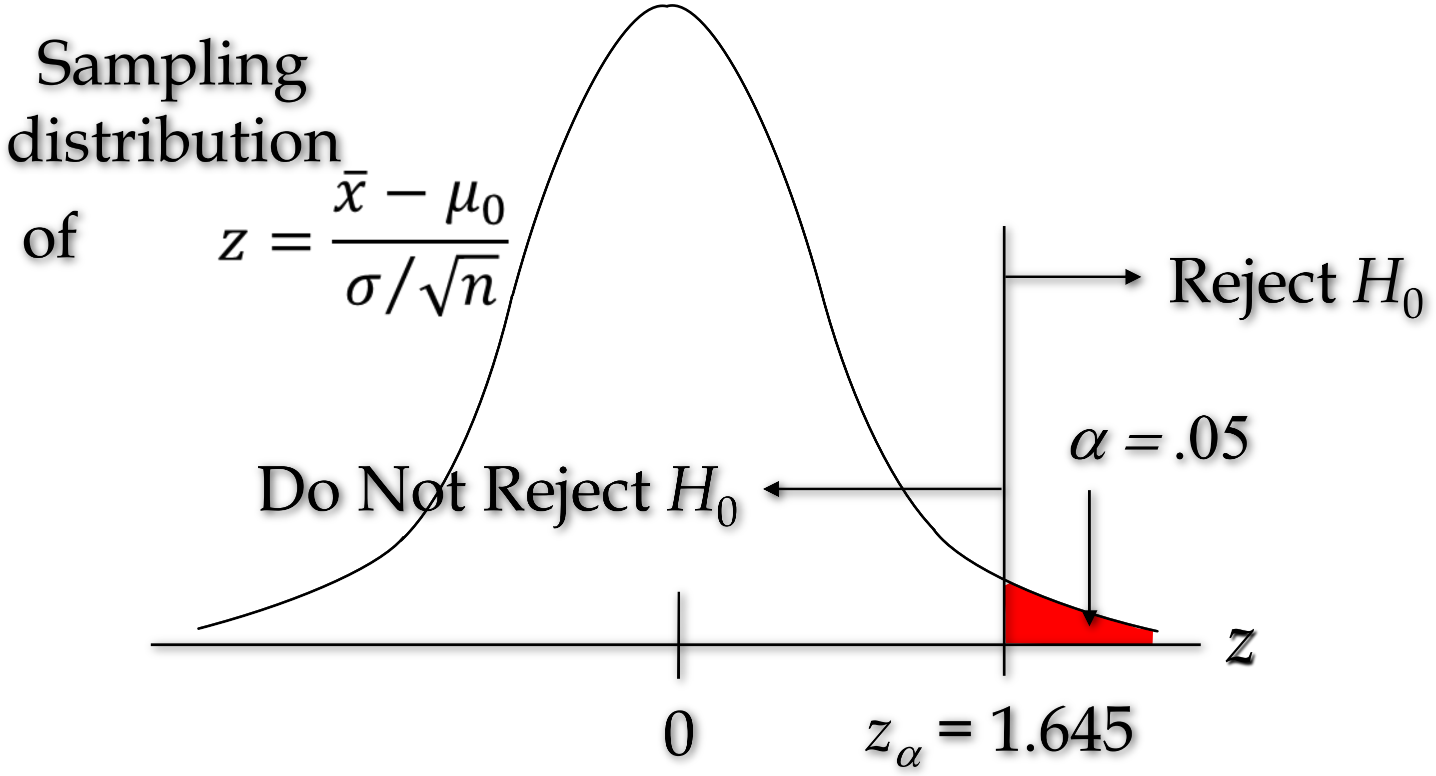
Critical Value Approach



```
In [6]: stats.norm.ppf(0.1)

Out[6]: -1.2815515655446004
```

Upper-Tailed Test About a Population Mean: σ Known

Critical Value Approach



```
In [7]: stats.norm.ppf(0.95)

Out[7]: 1.6448536269514722
```

Steps of Hypothesis Testing – P value approach

- Step 1. Develop the null and alternative hypotheses.
- Step 2. Specify the level of significance α.
- Step 3. Collect the sample data and compute the test statistic.
- <u>*p*-Value Approach</u>
- Step 4. Use the value of the test statistic to compute the *p*-value.
- Step 5. Reject H_0 if *p*-value $\leq \alpha$.

Steps of Hypothesis Testing

Critical Value Approach

- Step 4. Use the level of significance α to determine the critical value and the rejection rule.

- Step 5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0.