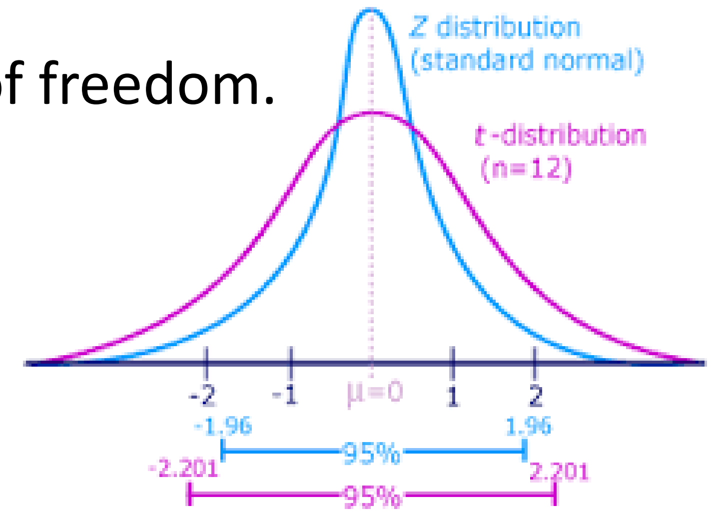
Hypothesis Testing-III

Tests About a Population Mean: σ Unknown

- Test Statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

This test statistic has a *t* distribution with *n* - 1 degrees of freedom.



Tests About a Population Mean: σ Unknown

Rejection Rule: p -Value Approach

Reject H0 if p –value $\leq \alpha$

Rejection Rule: Critical Value Approach

H0: $\mu \geq \mu_0$ Reject H0 if t $\leq -t_\alpha$

H0: $\mu \leq \mu_0$ Reject H0 if t $\geq t_\alpha$

H_0: $\mu = \mu_0$ Reject H0 if t $\leq -\ t_{\alpha/2}$ or $t \geq t_{\alpha/2}$

In [10]:
```
from scipy import stats
import numpy as np
```

In [11]:
```
x=[10,12,20,21,22,24,18,15]
stats.ttest_1samp(x,15)
```

Out[11]:
```
Ttest_1sampResult(statistic=1.5623450931857947, pvalue=0.16217875605928894)
```

One-Tailed Test About a Population Mean: σ Unknown

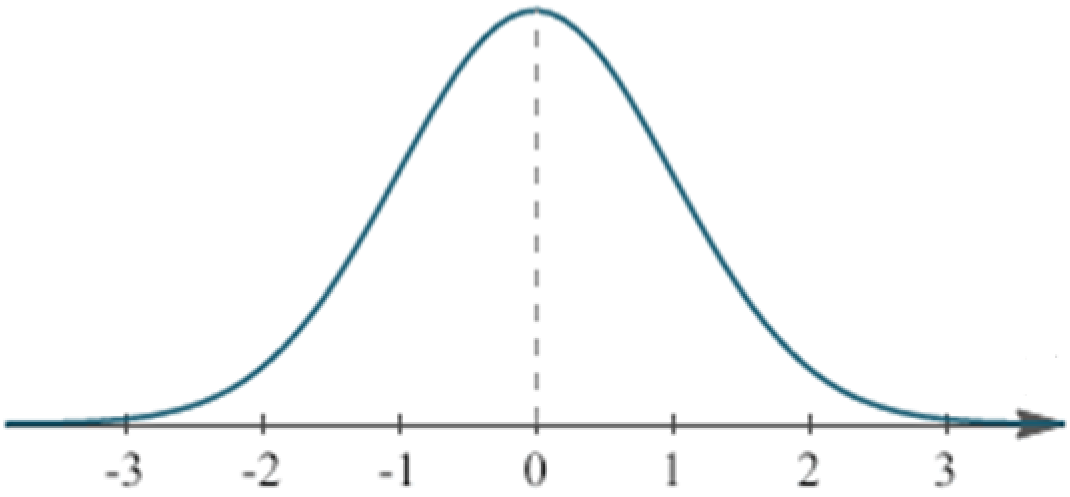
Example: Ice Cream Demand

- In a ice cream parlor at IIT Roorkee, the following data represent the number of ice-creams sold in 20 days

- Test hypothesis H_0: $\mu \leq 10$
- Use α = .05 to test the hypothesis.

Day	No. of Ice-cream Sold	Day	No. of Ice-cream Sold
1	13	11	12
2	8	12	11
3	10	13	11
4	10	14	12
5	8	15	10
6	9	16	12
7	10	17	7
8	11	18	10
9	6	19	11
10	8	20	8

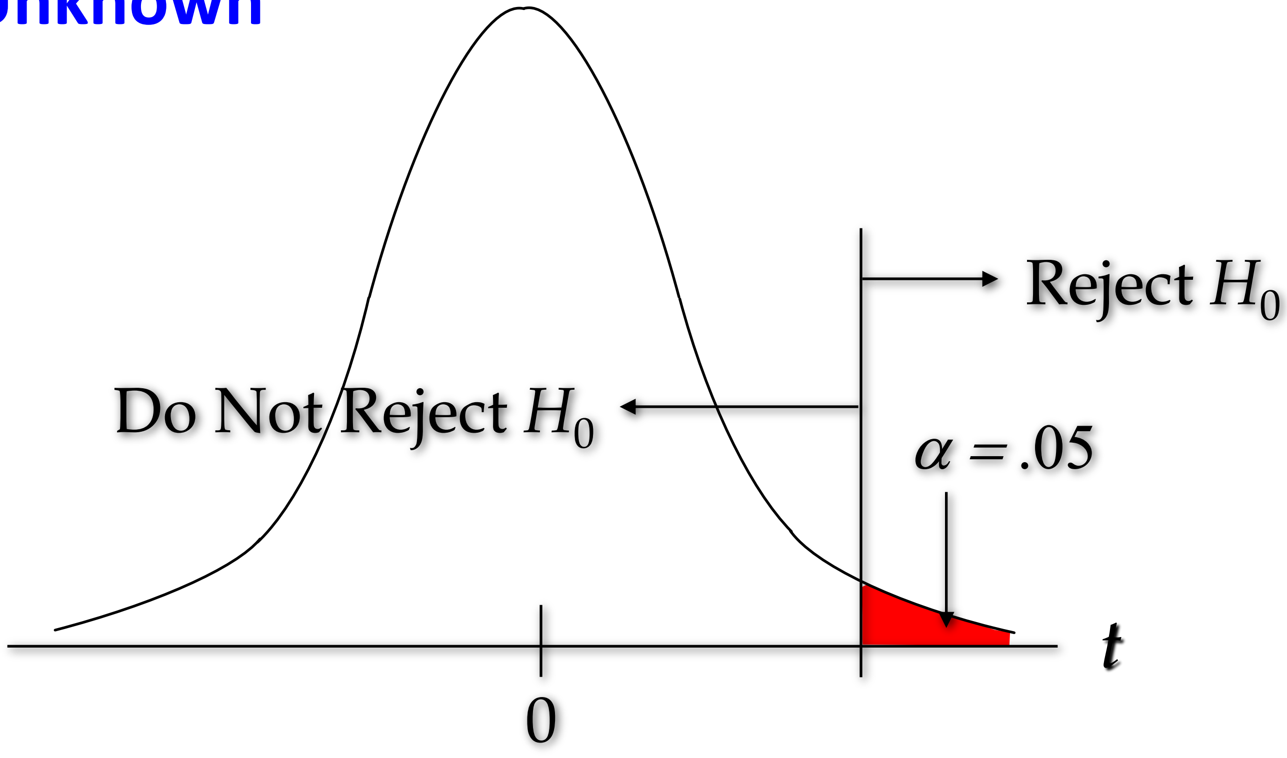
Given Data



```
In [8]: x=[13,8,10,10,8,9,10,11,6,8,12,11,11,12,10,12,7,10,11,8]

In [9]: stats.ttest_1samp(x,10)

Out[9]: Ttest_1sampResult(statistic=-0.35843385854878496, pvalue=0.7239703579964252)
```

```
In [10]: 0.7239703579964252/2

Out[10]: 0.3619851789982126
```

One-Tailed Test About a Population Mean: σ Unknown




```
In [3]:  stats.t.cdf(-0.384,19)
Out[3]:  0.35262102566795583
```

```
[2]:  stats.t.ppf(0.05,19)
Out[2]:  -1.7291328115213678
```

Hypothesis Testing – proportion

Null and Alternative Hypotheses: Population Proportion

- The equality part of the hypotheses always appears in the null hypothesis.
- In general, a hypothesis test about the value of a population proportion p must take one of the following three forms (where p0 is the hypothesized value of the population proportion).

One-tailed (lower tail)	One-tailed (upper tail)	Two-tailed
$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$

Tests About a Population Proportion

Test Statistic

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}$$

where: $$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

assuming $np \geq 5$ and $n(1 - p) \geq 5$

Tests About a Population Proportion

Rejection Rule: *p* –Value Approach

$$\text{Reject } H_0 \text{ if } p \text{ –value} \leq \alpha$$

Rejection Rule: Critical Value Approach

$H_0\!: \; p \leq p_0$ Reject H_0 if $z \geq z_\alpha$

$H_0\!: \; p \geq p_0$ Reject H_0 if $z \leq -z_\alpha$

$H_0\!: \; p = p_0$ Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$

Two-Tailed Test About a Population Proportion

Example: City Traffic Police

For a New Year's week, the City Traffic Police claimed that 50% of the accidents would be caused by drunk driving.

A sample of 120 accidents showed that 67 were caused by drunk driving. Use these data to test the Traffic Police's claim with $\alpha = .05$.

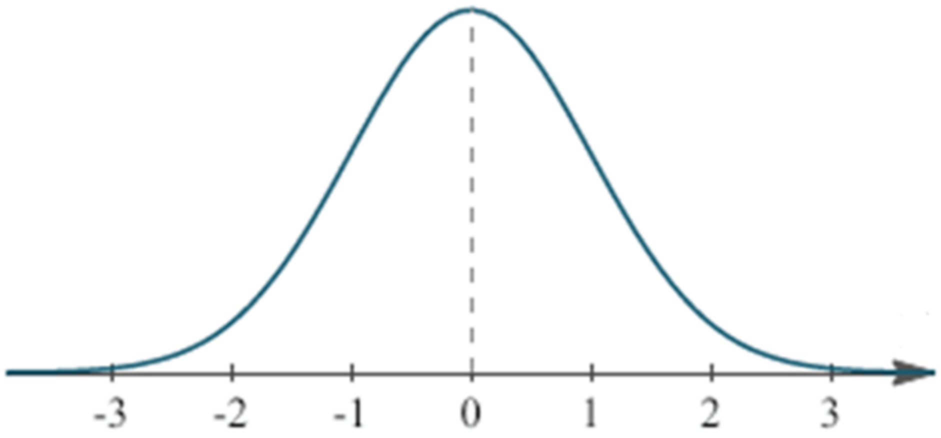
p –Value Approach

Two-Tailed Test About a Population Proportion

1. Determine the hypotheses.

$$H_0: p = .5$$

$$H_a: p \neq .5$$



2. Specify the level of significance. $\alpha = .05$

3. Compute the value of the test statistic.

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.5(1-.5)}{120}} = .045644$$

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{(67/120) - .5}{.045644} = 1.28$$

Two-Tailed Test About a Population Proportion

4. Compute the p -value.

For z = 1.28, cumulative probability = .8997 p–value = 2(1 - .8997) = .2006

5. Determine whether to reject H0.

Because p–value = .2006 > α = .05, we cannot reject H0.

```
In [13]: from statsmodels.stats.proportion import proportions_ztest

In [14]: count=67

In [16]: samplesize = 120

In [17]: P=0.5

In [18]: proportions_ztest(count, samplesize,P)

Out[18]: (1.286806739751111, 0.19816165722384555)
```

Critical Value Approach

Two-Tailed Test About a Population Proportion

4. Determine the critical value and rejection rule.

For $\alpha/2 = .05/2 = .025$, $z_{.025} = 1.96$

Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$

5. Determine whether to reject H0.

Because 1.278 > -1.96 and < 1.96, we cannot reject H0.