RBD

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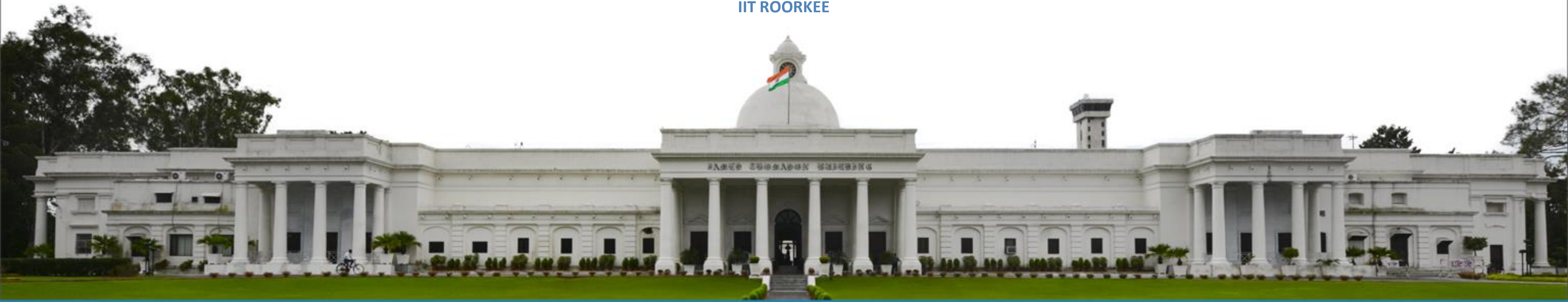

Learning Objectives

- Estimate variance components in an experiment involving random factors
- Understand the blocking principle and how it is used to isolate the effect of nuisance factors
- Design and conduct experiments involving the randomized complete block design

Randomized Block Design

- A completely randomized design (CRD) is useful when the experimental units are homogeneous
- If the experimental units are heterogeneous, **blocking** is often used to form homogeneous groups

Why RBD?

- A problem can arise whenever differences due to extraneous factors (ones not considered in the experiment) cause the MSE term in this ratio to become large.
- In such cases, the *F* value in equation can become small, signaling no difference among treatment means when in fact such a difference exists.

$$F = \frac{\text{MSTR}}{\text{MSE}}$$

Randomized block design

- Experimental studies in business often involve experimental units that are highly heterogeneous; as a result, randomized block designs are often employed.
- Blocking in experimental design is similar to stratification in sampling.

Randomized block design

- Its purpose is to control some of the extraneous sources of variation by removing such variation from the MSE term.
- This design tends to provide a better estimate of the true error variance and leads to a more powerful hypothesis test in terms of the ability to detect differences among treatment means.

Air Traffic Controller Stress Test

- A study measuring the fatigue and stress of air traffic controllers resulted in proposals for modification and redesign of the controller’s work station
- After consideration of several designs for the work station, three specific alternatives are selected as having the best potential for reducing controller stress
- The key question is: To what extent do the three alternatives differ in terms of their effect on controller stress?

Air Traffic Controller Stress Test

- In a completely randomized design, a random sample of controllers would be assigned to each work station alternative.
- However, controllers are believed to differ substantially in their ability to handle stressful situations.
- What is high stress to one controller might be only moderate or even low stress to another.
- Hence, when considering the within-group source of variation (MSE), we must realize that this variation includes both random error and error due to individual controller differences.
- In fact, managers expected controller variability to be a major contributor to the MSE term.

A randomized block design for the air traffic controller stress test

Treatments

Blocks

	System A	System B	System C
Controller 1	15	15	18
Controller 2	14	14	14
Controller 3	10	11	15
Controller 4	13	12	17
Controller 5	16	13	16
Controller 6	13	13	13

Solving this example using ANOVA in python

In [1]:
```
import pandas as pd
import numpy as np
import scipy
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

In [4]:
```
df = pd.read_excel('RBD.xlsx')
df
```

Out[4]:

	System A	System B	System C
0	15	15	18
1	14	14	14
2	10	11	15
3	13	12	17
4	16	13	16
5	13	13	13

Solving this example using ANOVA in python

ANOVA

In [20]:
```
data = pd.melt(df.reset_index(), id_vars=['index'], value_vars=['System A','System B','System C'])
data.columns = ['index', 'treatments', 'value']
```

In [21]:
```
model = ols('value ~ C(treatments)', data=data).fit()
anova_table = sm.stats.anova_lm(model, typ=1)
anova_table
```

Out[21]:

	df	sum_sq	mean_sq	F	PR(>F)
C(treatments)	2.0	21.0	10.500000	3.214286	0.068903
Residual	15.0	49.0	3.266667	NaN	NaN

In [9]:
```
# accept the null hypothesis
```

Summary of stress data for the air traffic controller stress test

Treatments → Blocks ↓	System A	System B	System C	Block total	Block means
Controller 1	15	15	18	48	$\overline{x_{1.}}$ =16
Controller 2	14	14	14	42	$\overline{x_{2.}}$ =14
Controller 3	10	11	15	36	$\overline{x_{3.}}$ =12
Controller 4	13	12	17	42	$\overline{x_{4.}}$ =14
Controller 5	16	13	16	45	$\overline{x_{5.}}$ =15
Controller 6	13	13	13	39	$\overline{x_{6.}}$ =13
Column Total	81	78	93	252	$\overline{\overline{x}}$ 252/18 = 14

Summary of stress data for the air traffic controller stress test

- Treatment means

$$\overline{x_{.1}} = 81/6 = 13.5$$

$$\overline{x_{.2}} = 78/6 = 13$$

$$\overline{x_{.3}} = 93/6 = 15.5$$

ANOVA TABLE FOR THE RANDOMIZED BLOCK DESIGN WITH *k* TREATMENTS AND *b* BLOCKS

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	P- value
Treatments	SS Treatments	k-1	MS Treatments = SSTR/k-1	MS Treatments / MSE	
Blocks	SS block	(b-1)	MSBL = SSBL/b-1		
Error	SSE	(k-1)(b-1)	MSE= SSE/(k-1)(b-1)		
Total	SST	n_T-1			

RBD Problem

$$
\begin{aligned}
x_{ij} &= \text{value of the observation corresponding to treatment } j \text{ in block } i \\
\bar{x}_{\cdot j} &= \text{sample mean of the } j\text{th treatment} \\
\bar{x}_{i\cdot} &= \text{sample mean for the } i\text{th block} \\
\bar{\bar{x}} &= \text{overall sample mean}
\end{aligned}
$$

RBD Problem

Step 1. Compute the total sum of squares (SST).

$$\text{SST} = \sum_{i=1}^{b}\sum_{j=1}^{k}(x_{ij} - \bar{\bar{x}})^2$$

Step 1. $\text{SST} = (15 - 14)^2 + (15 - 14)^2 + (18 - 14)^2 + \cdots + (13 - 14)^2 = 70$

Step 2. Compute the sum of squares due to treatments (SSTR).

$$\text{SSTR} = b\sum_{j=1}^{k}(\bar{x}_{\cdot j} - \bar{\bar{x}})^2$$

Step 2. $\text{SSTR} = 6[(13.5 - 14)^2 + (13.0 - 14)^2 + (15.5 - 14)^2] = 21$

RBD Problem

Step 3. Compute the sum of squares due to blocks (SSBL).

$$\text{SSBL} = k\sum_{i=1}^{b}(\bar{x}_{i\cdot} - \bar{\bar{x}})^2$$

Step 3. $\text{SSBL} = 3[(16 - 14)^2 + (14 - 14)^2 + (12 - 14)^2 + (14 - 14)^2 + (15 - 14)^2 + (13 - 14)^2] = 30$

Step 4. Compute the sum of squares due to error (SSE).

$$\text{SSE} = \text{SST} - \text{SSTR} - \text{SSBL}$$

Step 4. $\text{SSE} = 70 - 21 - 30 = 19$

ANOVA table for the air traffic controller stress test

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	P- value
Treatments	21	2	10.5	10.5/1.9 =5.53	0.024
Blocks	30	5	6.0		
Error	19	10	1.9		
Total	70	17			

$F_{.025} = 5.46$ and $F_{.01} = 7.56.$

Reject the null hypothesis

Solving RBD example using python

In [1]:
```
import pandas as pd
import numpy as np
import scipy
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

In [4]:
```
df = pd.read_excel('RBD.xlsx')
df
```

Out[4]:

	System A	System B	System C
0	15	15	18
1	14	14	14
2	10	11	15
3	13	12	17
4	16	13	16
5	13	13	13

Solving RBD example using python

In [20]:
```
data = pd.melt(df.reset_index(), id_vars=['index'], value_vars=['System A','System B','System C'])
data.columns = ['blocks', 'treatments', 'value']
```

In [22]:
```
model = ols('value ~ C(block)+ C(treatments)', data=data).fit()
anova_table = sm.stats.anova_lm(model, typ=1)
anova_table
```

Out[22]:

	df	sum_sq	mean_sq	F	PR(>F)
C(block)	5.0	30.0	6.0	3.157895	0.057399
C(treatments)	2.0	21.0	10.5	5.526316	0.024181
Residual	10.0	19.0	1.9	NaN	NaN

In [23]:
```
# reject the null hypothesis
```

Conclusion

- Finally, note that the ANOVA table shown in Table provides an *F* value to test for treatment effects but *not* for blocks.
- The reason is that the experiment was designed to test a single factor—work station design.
- The blocking based on individual stress differences was conducted to remove such variation from the MSE term.
- However, the study was not designed to test specifically for individual differences in stress.

Problem 2: RBD

- An experiment was performed to determine the effect of four different chemicals on the strength of a fabric.
- These chemicals are used as part of the permanent press finishing process.
- Five fabric samples were selected, and a randomized complete block design was run by testing each chemical type once in random order on each fabric sample.
- The data are shown in Table.
- We will test for differences in means using an ANOVA with alpha = 0.01.

Problem 2: RBD

- Table: Fabric Strength Data—Randomized Complete Block Design

Chemical Type	Fabric Sample 1	2	3	4	5	Treatment Totals $y_{i\cdot}$	Treatment Averages $\bar{y}_{i\cdot}$
1	1.3	1.6	0.5	1.2	1.1	5.7	1.14
2	2.2	2.4	0.4	2.0	1.8	8.8	1.76
3	1.8	1.7	0.6	1.5	1.3	6.9	1.38
4	3.9	4.4	2.0	4.1	3.4	17.8	3.56
Block totals $y_{\cdot j}$	9.2	10.1	3.5	8.8	7.6	39.2($y_{\cdot\cdot}$)	
Block averages $\bar{y}_{\cdot j}$	2.30	2.53	0.88	2.20	1.90		1.96($\bar{y}_{\cdot\cdot}$)

Anova using jupyter

In [3]:
```
df = pd.read_excel('rbd2.xlsx')
df
```

Out[3]:

	chem1	chem2	chem3	chem4
0	1.3	2.2	1.8	3.9
1	1.6	2.4	1.7	4.4
2	0.5	0.4	0.6	2.0
3	1.2	2.0	1.5	4.1
4	1.1	1.8	1.3	3.4

In [4]:
```
data = pd.melt(df.reset_index(), id_vars=['index'], value_vars=['chem1','chem2','chem3','chem4'])
data.columns = ['index', 'treatments', 'value']
```

In [6]:
```
model = ols('value ~ C(treatments)', data=data).fit()
aov_table = sm.stats.anova_lm(model, typ=1)
aov_table
```

Out[6]:

	df	sum_sq	mean_sq	F	PR(>F)
C(treatments)	3.0	18.044	6.014667	12.589569	0.000176
Residual	16.0	7.644	0.477750	NaN	NaN

Problem 2: RBD

- The sums of squares for the analysis of variance are computed as follows:

$$SS_T = \sum_{i=1}^{4}\sum_{j=1}^{5} y_{ij}^2 - \frac{y_{..}^2}{ab}$$

$$= (1.3)^2 + (1.6)^2 + \cdots + (3.4)^2 - \frac{(39.2)^2}{20} = 25.69$$

$$SS_{\text{Treatments}} = \sum_{i=1}^{4} \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{ab}$$

$$= \frac{(5.7)^2 + (8.8)^2 + (6.9)^2 + (17.8)^2}{5} - \frac{(39.2)^2}{20} = 18.04$$

Problem 2: RBD

$$SS_{\text{Blocks}} = \sum_{j=1}^{5} \frac{y_{\cdot j}^2}{a} - \frac{y_{\cdot\cdot}^2}{ab}$$

$$= \frac{(9.2)^2 + (10.1)^2 + (3.5)^2 + (8.8)^2 + (7.6)^2}{4} - \frac{(39.2)^2}{20} = 6.69$$

$$SS_E = SS_T - SS_{\text{Blocks}} - SS_{\text{Treatments}}$$

$$= 25.69 - 6.69 - 18.04 = 0.96$$

Problem 2: RBD

- Analysis of Variance for the Randomized Complete Block Experiment

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	P- value
Chemical types (Treatments)	18.04	3	6.01	75.13	4.79 E-8
Fabric samples (Blocks)	6.69	4	1.67		
Error	0.96	12	0.08		
Total	25.69	19			

Conclusion

- The ANOVA is summarized in the previous table
- Since $f_0 = 75.13 > f_{0.01,3,12} = 5.95$ (the *P*-value is 4.79 x E-8), we conclude that there is a significant difference in the chemical types so far as their effect on strength is concerned.

Python code for problem 2

In [2]:
```
import pandas as pd
import statsmodels.api as sm
from statsmodels.formula.api import ols
from statsmodels.stats.anova import anova_lm
```

In [3]:
```
df = pd.read_excel('RBD2.xlsx')
```

In [4]:
```
df
```

Out[4]:

	chem1	chem2	chem3	chem4
0	1.3	2.2	1.8	3.9
1	1.6	2.4	1.7	4.4
2	0.5	0.4	0.6	2.0
3	1.2	2.0	1.5	4.1
4	1.1	1.8	1.3	3.4

Python code for problem 2

In [7]:
```
data = pd.melt(df.reset_index(), id_vars=['index'], value_vars=['chem1','chem2','chem3','chem4'])
data.columns = ['Fabric samples', 'Chemical types', 'value']
data
```

Out[7]:

	Fabric samples	Chemical types	value
0	0	chem1	1.3
1	1	chem1	1.6
2	2	chem1	0.5
3	3	chem1	1.2
4	4	chem1	1.1
5	0	chem2	2.2
6	1	chem2	2.4
7	2	chem2	0.4
8	3	chem2	2.0
9	4	chem2	1.8
10	0	chem3	1.8
11	1	chem3	1.7
12	2	chem3	0.6
13	3	chem3	1.5
14	4	chem3	1.3
15	0	chem4	3.9
16	1	chem4	4.4
17	2	chem4	2.0
18	3	chem4	4.1

Python code for problem 2

In [11]:
```
model = ols('value ~ C(Fabric) + C(Chemical)', data=data).fit()
anova_table = sm.stats.anova_lm(model, typ=1)
anova_table
```

Out[11]:

	df	sum_sq	mean_sq	F	PR(>F)
C(Fabric)	4.0	6.693	1.673250	21.113565	2.318913e-05
C(Chemical)	3.0	18.044	6.014667	75.894848	4.518310e-08
Residual	12.0	0.951	0.079250	NaN	NaN