IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Confusion matrix and ROC - I

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Agenda

- Confusion matrix
- Receiver operating characteristics curve

Why Evaluate?

- Multiple methods are available to classify or predict
- For each method, multiple choices are available for settings
- To choose best model, need to assess each model's performance

Accuracy Measures (Classification)

Misclassification error

(0, 1)

- Error = classifying a record as belonging to one class when it belongs to another class.
- Error rate = percent of misclassified records out of the total records in the validation data

Confusion Matrix

Classification Confusion Matrix		
	Predicted Class	
Actual Class	1	0
1	201	85
0	25	2689

201 1's correctly classified as "1"

85 1's incorrectly classified as "0"

25 0's incorrectly classified as "1"

2689 0's correctly classified as "0"

Error Rate

Classification Confusion Matrix		
	Predicted Class	
Actual Class	1	0
1	201	85
0	25	2689

Overall error rate = (25+85)/3000 = 3.67%

Accuracy = 1 – err = (201+2689) = 96.33%

If multiple classes, error rate is:

(sum of misclassified records)/(total records)

Cutoff for classification

Most algorithms classify via a 2-step process:

For each record,

1. Compute **probability of belonging to class "1"**
2. Compare to cutoff value, and classify accordingly

- Default cutoff value is 0.50
  - If >= 0.50, classify as "1"
  - If < 0.50, classify as "0"
- Can use different cutoff values
- Typically, error rate is lowest for cutoff = 0.50

Cutoff Table

Actual Class	Prob. of "1"	Actual Class	Prob. of "1"
1	0.996	1	0.506
1	0.988	0	0.471
1	0.984	0	0.337
1	0.980	1	0.218
1	0.948	0	0.199
1	0.889	0	0.149
1	0.848	0	0.048
0	0.762	0	0.038
1	0.707	0	0.025
1	0.681	0	0.022
1	0.656	0	0.016
0	0.622	0	0.004

- If cutoff is 0.50: 11 records are classified as "1"
- If cutoff is 0.80: seven records are classified as "1"

Confusion Matrix for Different Cutoffs

Cut off Prob.Val. for Success (Updatable)	0.25

Classification Confusion Matrix		
	Predicted Class	
Actual Class	(1) owner	(0) non-owner
owner (1)	11	1
non-owner (0)	4	8

Cut off Prob.Val. for Success (Updatable)	0.75

Classification Confusion Matrix		
	Predicted Class	
Actual Class	owner	non-owner
owner	7	5
non-owner	1	11

Compute Outcome Measures

Confusion Matrix:

	Predicted Class = 0	Predicted Class = 1
Actual Class = 0	True Negatives (TN)	False Positives (FP)
Actual Class = 1	False Negatives (FN)	True Positives (TP)

N = number of observations

Overall accuracy = (TN + TP)/N

Overall error rate = (FP + FN)/N

Sensitivity = TP/(TP + FN) (1)

Specificity = TN/(TN + FP) (0)

False Negative Error Rate = FN/(TP + FN)

False Positive Error Rate = FP/(TN + FP)

When One Class is More Important

In many cases it is more important to identify members of one class

- Tax fraud
- Credit default
- Response to promotional offer (0, 1)
- Detecting electronic network intrusion
- Predicting delayed flights (1, 0)

In such cases, we are willing to tolerate greater overall error, in return for better identifying the important class for further attention

ROC curves

- *ROC* = *Receiver Operating Characteristic*
- Started in electronic signal detection theory (1940s - 1950s)
- Has become very popular in biomedical applications, particularly radiology and imaging
- Also used in machine learning applications to assess classifiers
- Can be used to compare tests/procedures

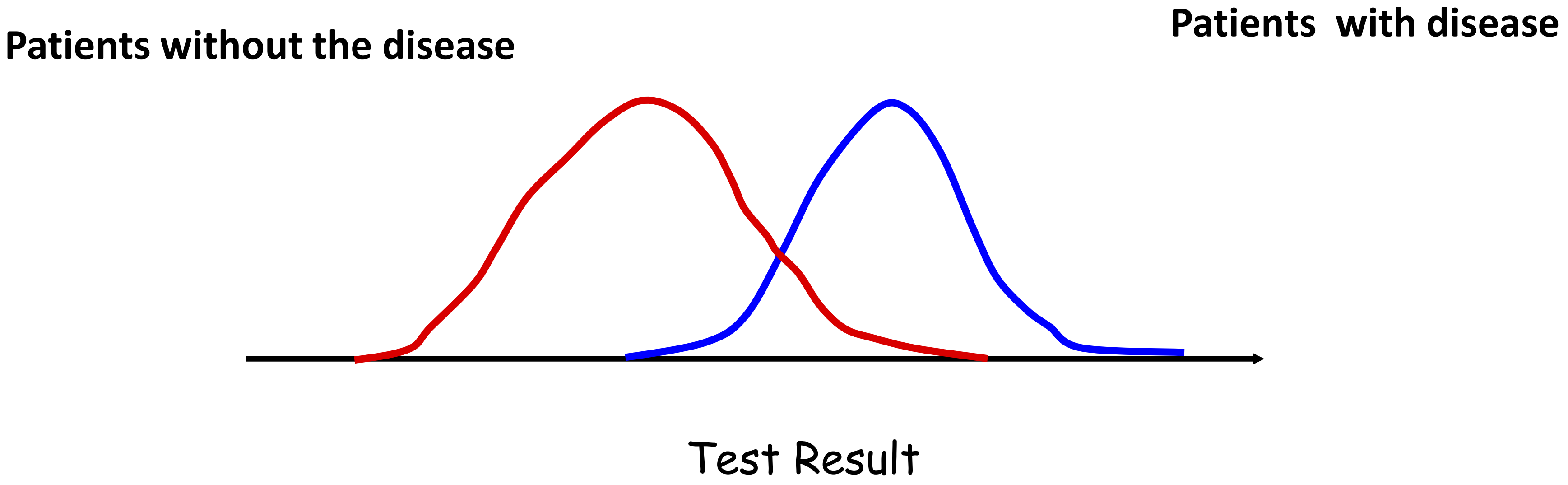
ROC curves: simplest case

- Consider diagnostic test for a disease
- Test has 2 possible outcomes:
  - ‘positive’ = suggesting presence of disease
  - ‘negative’
- An individual can test either positive or negative for the disease

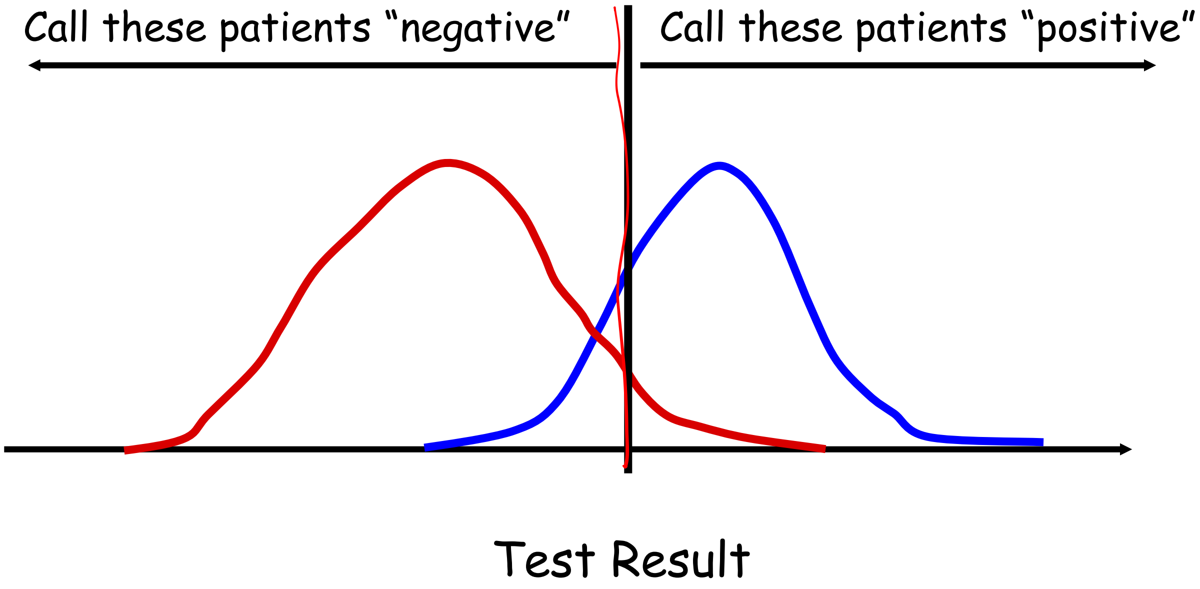
ROC Analysis

- True Positives = Test states you have the disease when you do have the disease
- True Negatives = Test states you do not have the disease when you do not have the disease
- False Positives = Test states you have the disease when you do not have the disease
- False Negatives = Test states you do not have the disease when you do

Specific Example

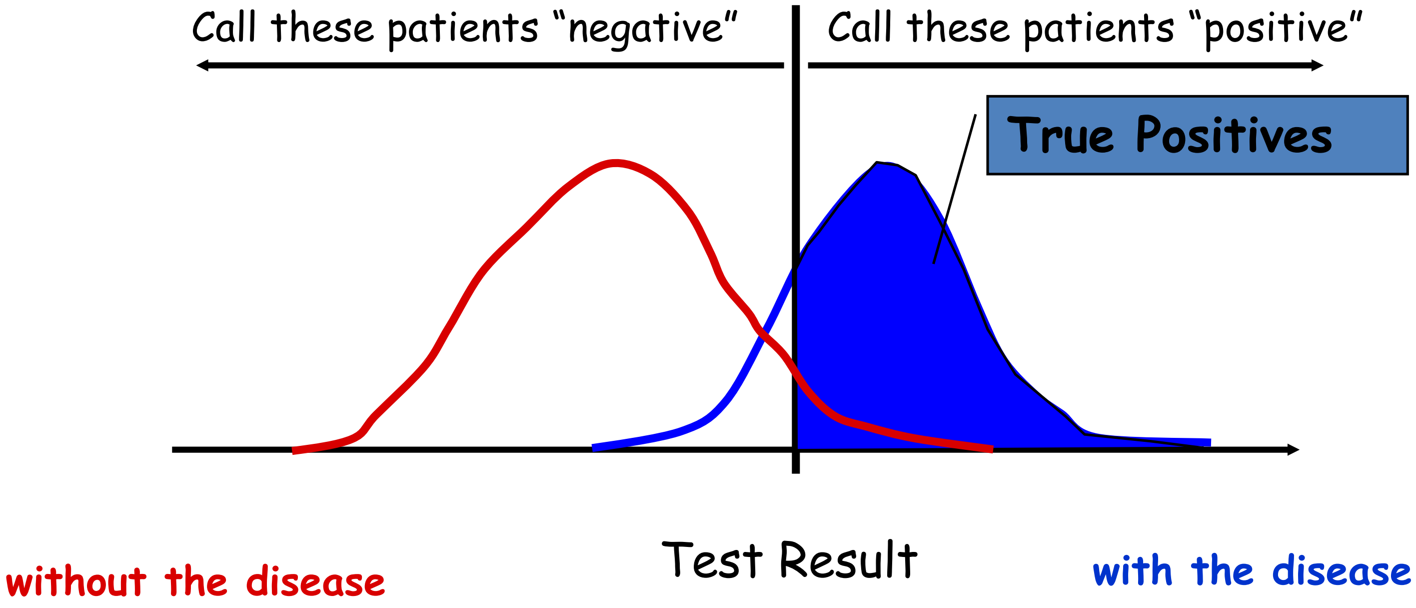


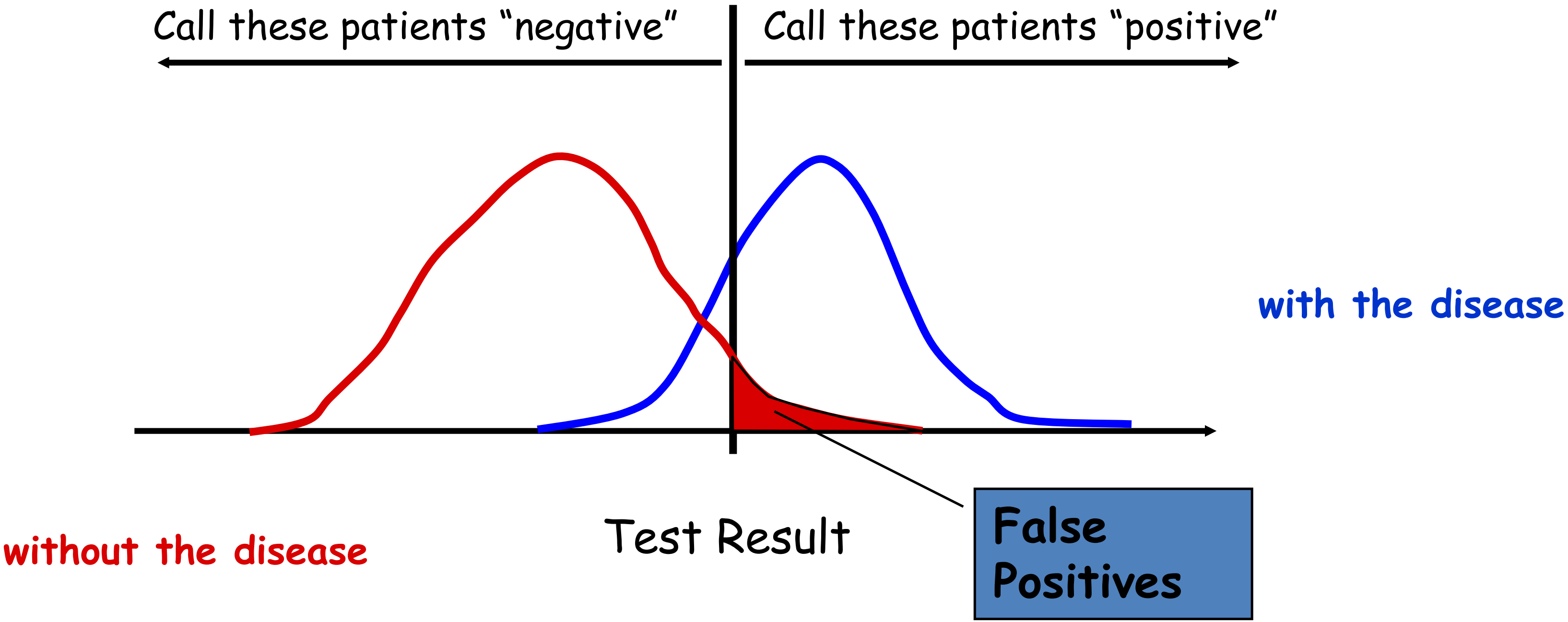
Threshold

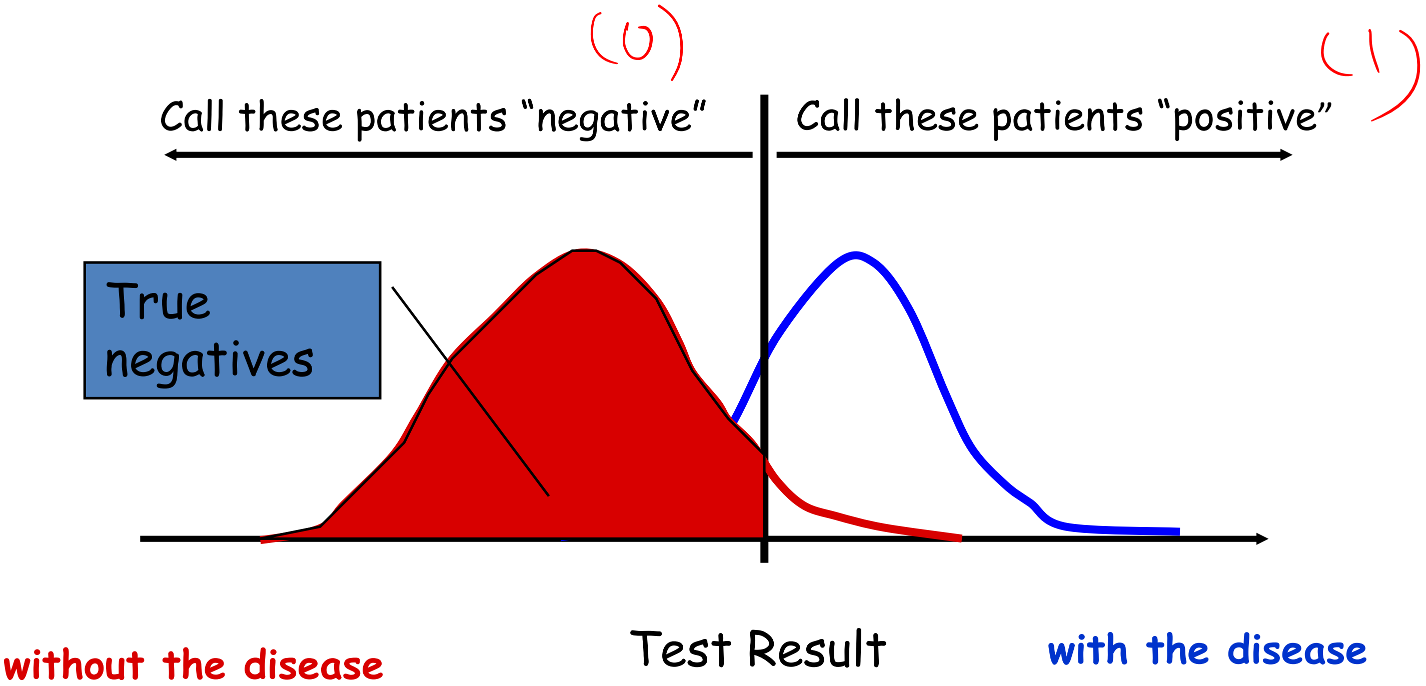


Call these patients "negative"
Call these patients "positive"
Test Result

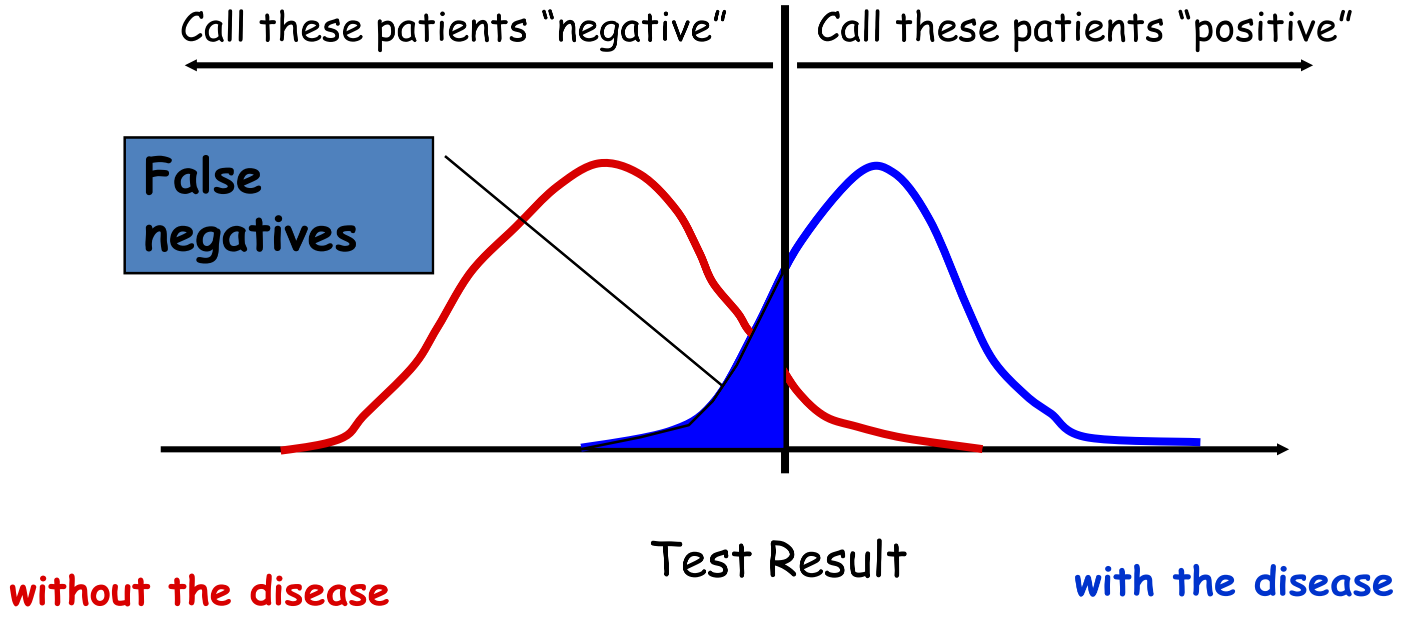
Some definitions ...



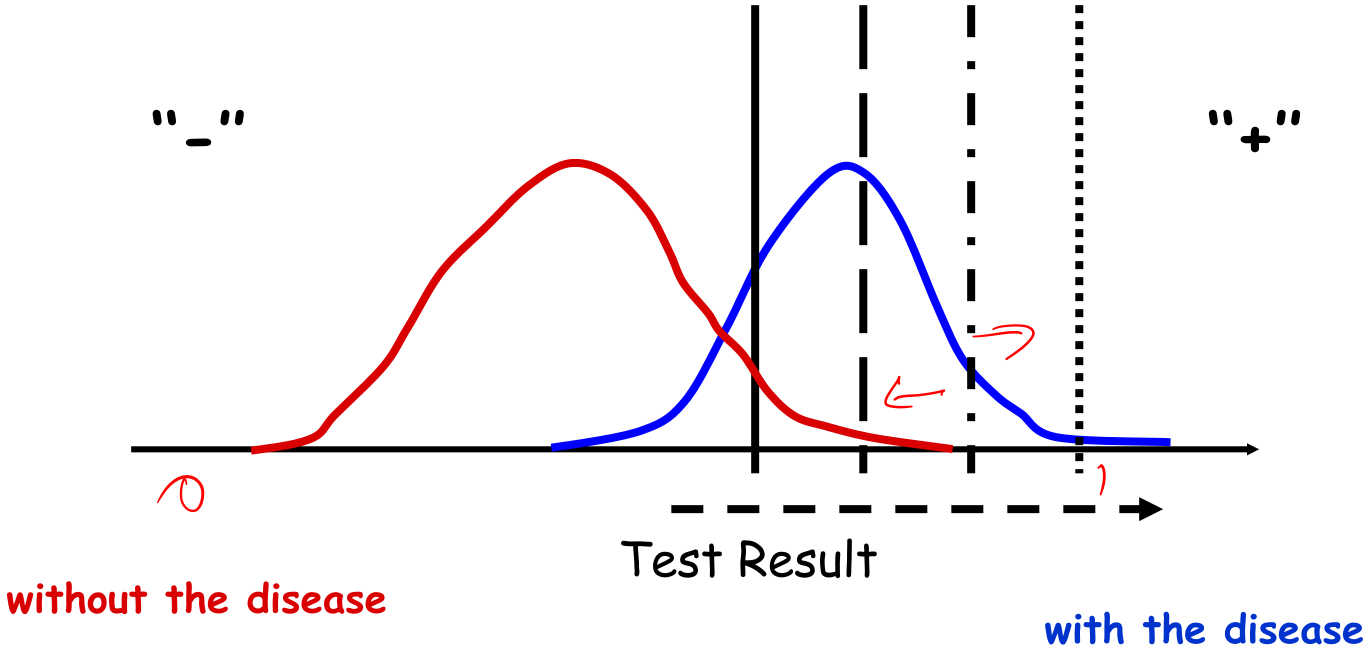

Call these patients "negative"
Call these patients "positive"
with the disease
without the disease
Test Result
False Positives



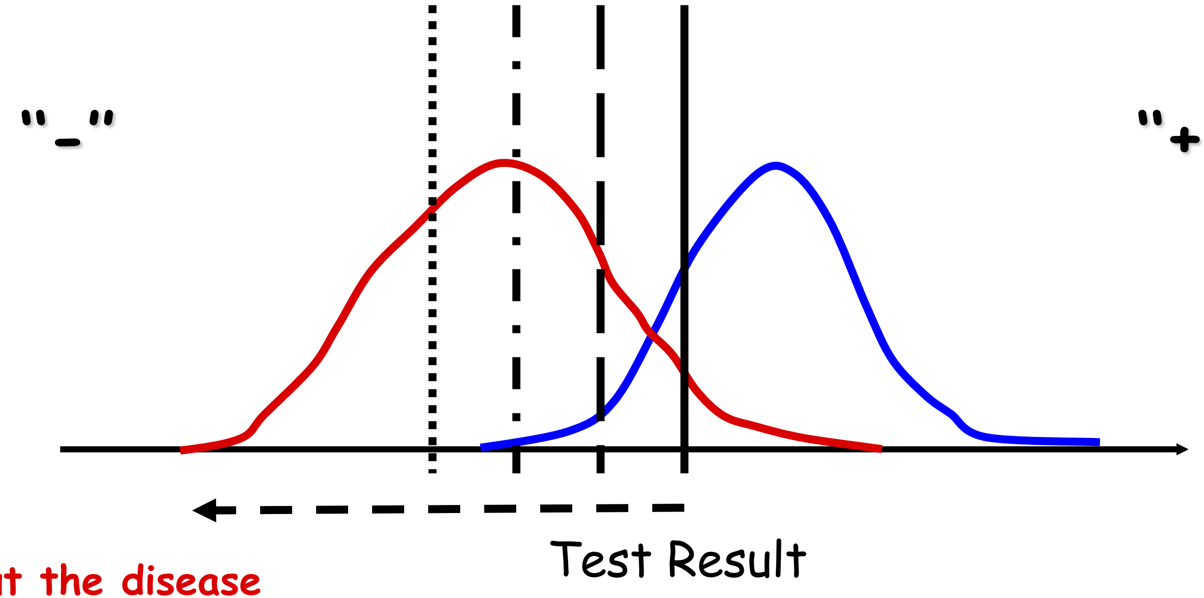
(0)
(1)
Call these patients "negative"
Call these patients "positive"
True negatives
without the disease
Test Result
with the disease


Call these patients "negative"
Call these patients "positive"
False negatives
Test Result
without the disease
with the disease

Moving the Threshold: right



Moving the Threshold: left



without the disease

with the disease

Threshold Value

- The outcome of a logistic regression model is a probability
- Often, we want to make a binary prediction
- We can do this using a *threshold value* t
- If $P(y = 1) \geq t$, predict positive
  - If $P(y = 1) < t$, predict negative
  - What value should we pick for t?

Threshold Value

- Often selected based on which errors are “better”
- If t is **large**, predict positive rarely (when P(y=1) is large)
  - More errors where we say negative , but it is actually positive
  - Detects patients who are negative
- If t is **small**, predict negative rarely (when P(y=1) is small)
  - More errors where we say positive, but it is actually negative
  - Detects all patients who are positive
- With no preference between the errors, select t = 0.5
  - Predicts the more likely outcome

Selecting a Threshold Value

- Compare actual outcomes to predicted outcomes using a *confusion matrix (classification matrix)*

	Predicted = 0	Predicted = 1
Actual = 0	True Negatives (TN)	False Positives (FP)
Actual = 1	False Negatives (FN)	True Positives (TP)

True disease state vs. Test result

Test / Disease	not rejected/accepted	rejected
No disease (D = 0)	☺ specificity	X Type I error (False +) α
Disease (D = 1)	X Type II error (False -) β	☺ Power $1 - \beta$; sensitivity

Classification matrix: Meaning of each cell

Actual Class	Predicted Class C_0	Predicted Class C_1
C_0	$n_{0.0}$ = number of C_0 cases classified correctly	$n_{0.1}$ = number of C_0 cases classified incorrectly as C_1
C_1	$n_{1.0}$ = number of C_1 cases classified incorrectly as C_0	$n_{1.1}$ = number of C_1 cases classified correctly

Alternate Accuracy Measures

If "C_1" is the important class,

Sensitivity = % of "C_1" class correctly classified

Sensitivity = $n_{1,1} / (n_{1,0} + n_{1,1})$

Specificity = % of "C_0" class correctly classified

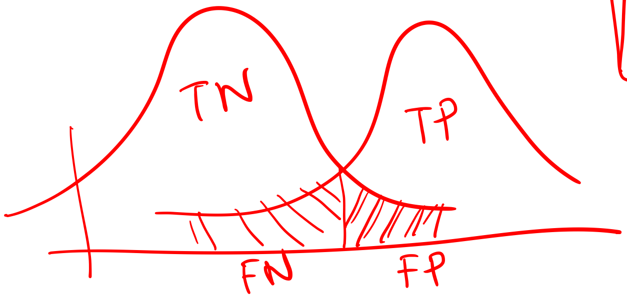
Specificity = $n_{0,0} / (n_{0,0} + n_{0,1})$

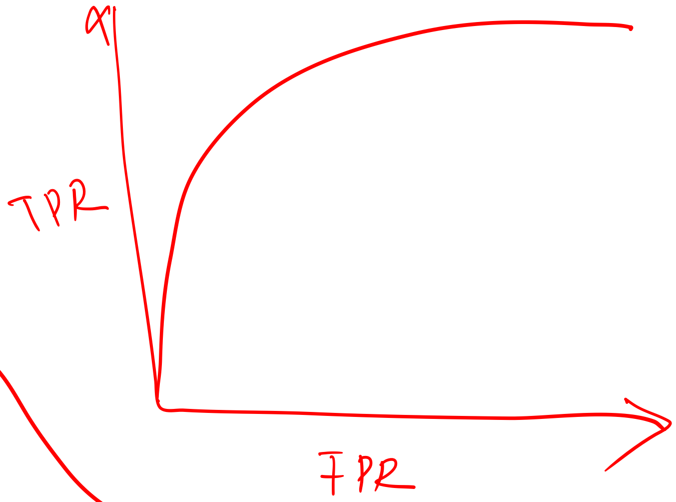
False positive rate = % of predicted "C_1's" that were not "C_1's"

False negative rate = % of predicted "C_0's" that were not "C_0's"

Receiver Operator Characteristic (ROC) Curve

- True positive rate (sensitivity) on y-axis
  - Proportion of positive
- False positive rate (1-specificity) on x-axis
  - Proportion of negative labelled as positive
- **Low Threshold**
  - Low specificity
  - High sensitivity





Selecting a Threshold using ROC

- Captures all thresholds simultaneously
- **High threshold**
  - High specificity
  - Low sensitivity
- **Low Threshold**
  - Low specificity
  - High sensitivity

Thank You