Hypothesis Testing

Class Objectives

- Population Mean: Sigma Known –Example

One-Tailed Tests About a Population Mean: σ Known

- Example: The mean response times for a random sample of 30 Pizza Deliveries is 32 minutes
- The population standard deviation is believed to be 10 minutes.
- The pizza delivery services director wants to perform a hypothesis test, with α =0.05 level of significance, to determine whether the service goal of 30 minutes or less is being achieved.

Given Values

- Sample
- Sample mean = 32 Min
- Sample size = 30

- Population
- α =0.05
- Population mean = 30 Min

p -Value Approach

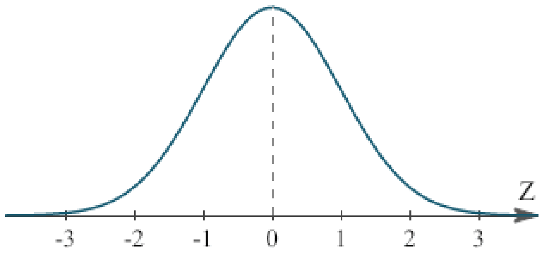
One-Tailed Tests About a Population Mean: σ Known

1. Develop the hypotheses.
2. Specify the level of significance.
3. Compute the value of the test statistic.

$$H_0:\ \mu \le 30$$
$$H_a:\ \mu > 30$$

$$\alpha = .05$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{32 - 30}{10 / \sqrt{30}} = 1.09$$



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In [8]: 1-stats.norm.cdf(1.09)

Out[8]: 0.1378565720320355
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One-Tailed Tests About a Population Mean: σ Known

p –Value Approach

4. Compute the p –value.

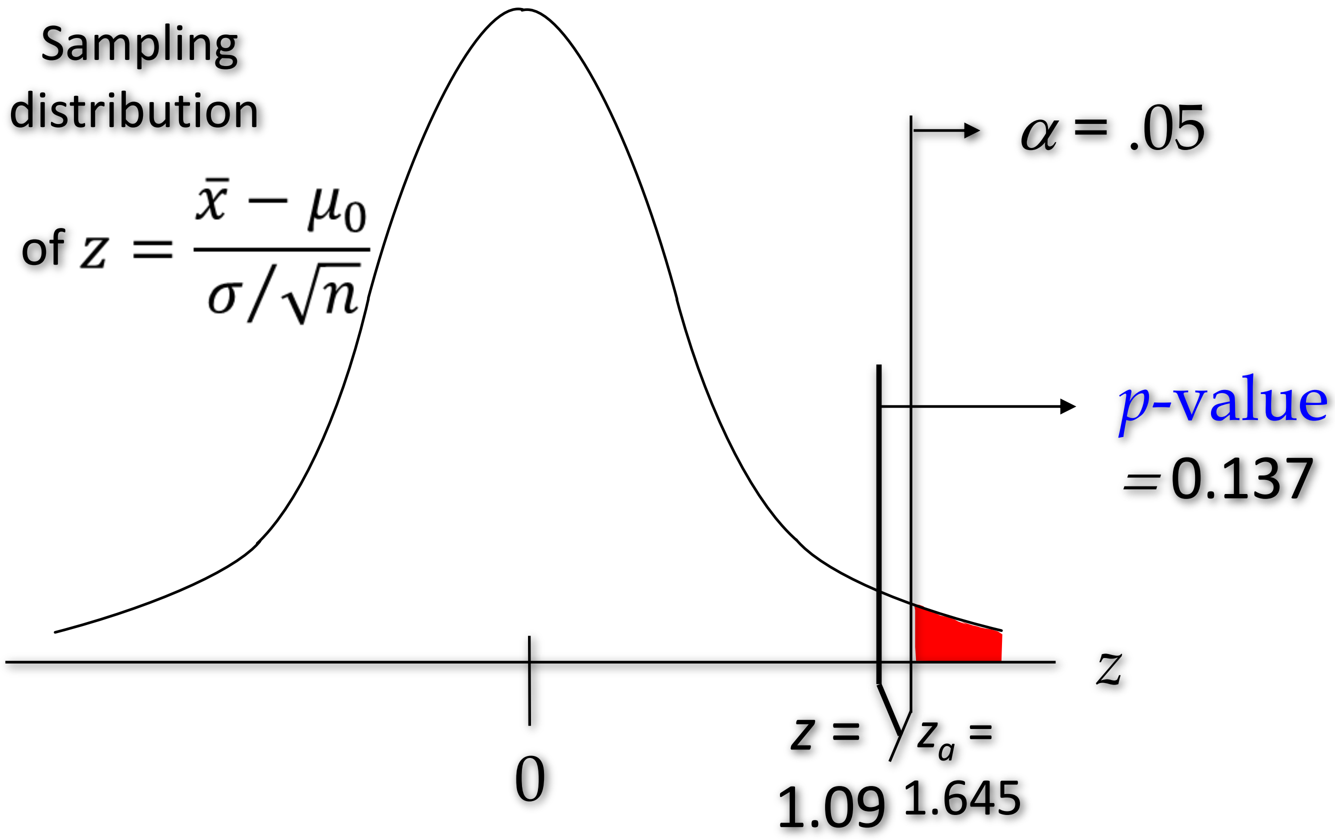
For z = 1.09, p–value = = 0.137

5. Determine whether to reject H0.

- Because p–value = 0.137 > $\alpha = .05$, we do not reject H_0.
- There are not sufficient statistical evidence to infer that Pizza delivery services is not meeting the response goal of 30 minutes.

One-Tailed Tests About a Population Mean: σ Known

p –Value Approach



Critical Value Approach

One-Tailed Tests About a Population Mean: σ Known

Critical Value Approach

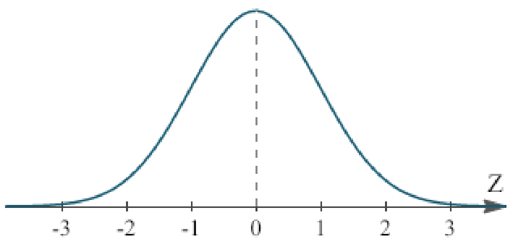
4. Determine the critical value and rejection rule.

   – For $\alpha = .05$, $z_{.05} = 1.645$

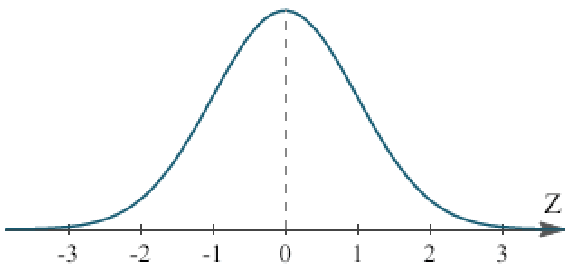
   – Reject H_0 if $z \geq 1.645$

5. Determine whether to reject H_0.

   – Because $1.645 \geq 1.05$, we do not reject H_0.



p-Value Approach to Two-Tailed Hypothesis Testing



Compute the p-value using the following three steps:

1. Compute the value of the test statistic z.
2. If z is in the upper tail $(z > 0)$, find the area under the standard normal curve to the right of z.
3. If z is in the lower tail $(z < 0)$, find the area under the standard normal curve to the left of z.
4. Double the tail area obtained in step 2 to obtain the p –value.

The rejection rule:

Reject H_0 if the p-value $\leq \alpha$.

Critical Value Approach to Two-Tailed Hypothesis Testing

- The critical values will occur in both the lower and upper tails of the standard normal curve.
- Use the standard normal probability distribution table to find $z_{\alpha/2}$ (the z-value with an area of $\alpha/2$ in the upper tail of the distribution).
- The rejection rule is:

Reject H_0 if $z \leq -z_{\alpha/2}$ **or** $z \geq z_{\alpha/2}$.

Two-Tailed Tests About a Population Mean: σ Known

- Example: Milk Carton
- Assume that a sample of 30 milk carton provides a sample mean of 505 ml.
- The population standard deviation is believed to be 10 ml.
- Perform a hypothesis test, at the 0.03 level of significance, population mean 500 ml and to help determine whether the filling process should continue operating or be stopped and corrected.

Given Values

- Sample
- Sample size = 30
- Sample mean = 505 ml

- Population
- Population mean = 500 ml
- Standard deviation = 10 ml
- Significance level 0.03

p –Value approach

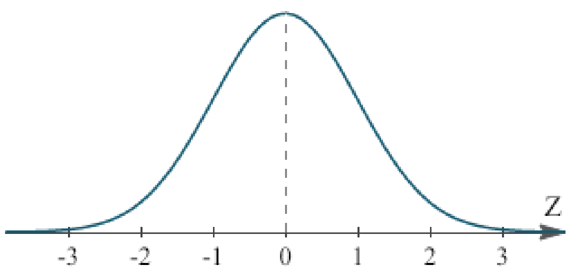
Two-Tailed Tests About a Population Mean: σ Known

1. Determine the hypotheses.
2. Specify the level of significance.
3. Compute the value of the test statistic.

H_0: $\mu = 500$
H_a: $\mu \neq 500$

$\alpha = .03$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{505 - 500}{10 / \sqrt{30}} = 2.74$$



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In [9]: 1-stats.norm.cdf(2.74)

Out[9]: 0.003071959218650444
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In [10]: (1-stats.norm.cdf(2.74))*2

Out[10]: 0.006143918437300888
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Two-Tailed Tests About a Population Mean: σ Known

***p* –Value Approach**

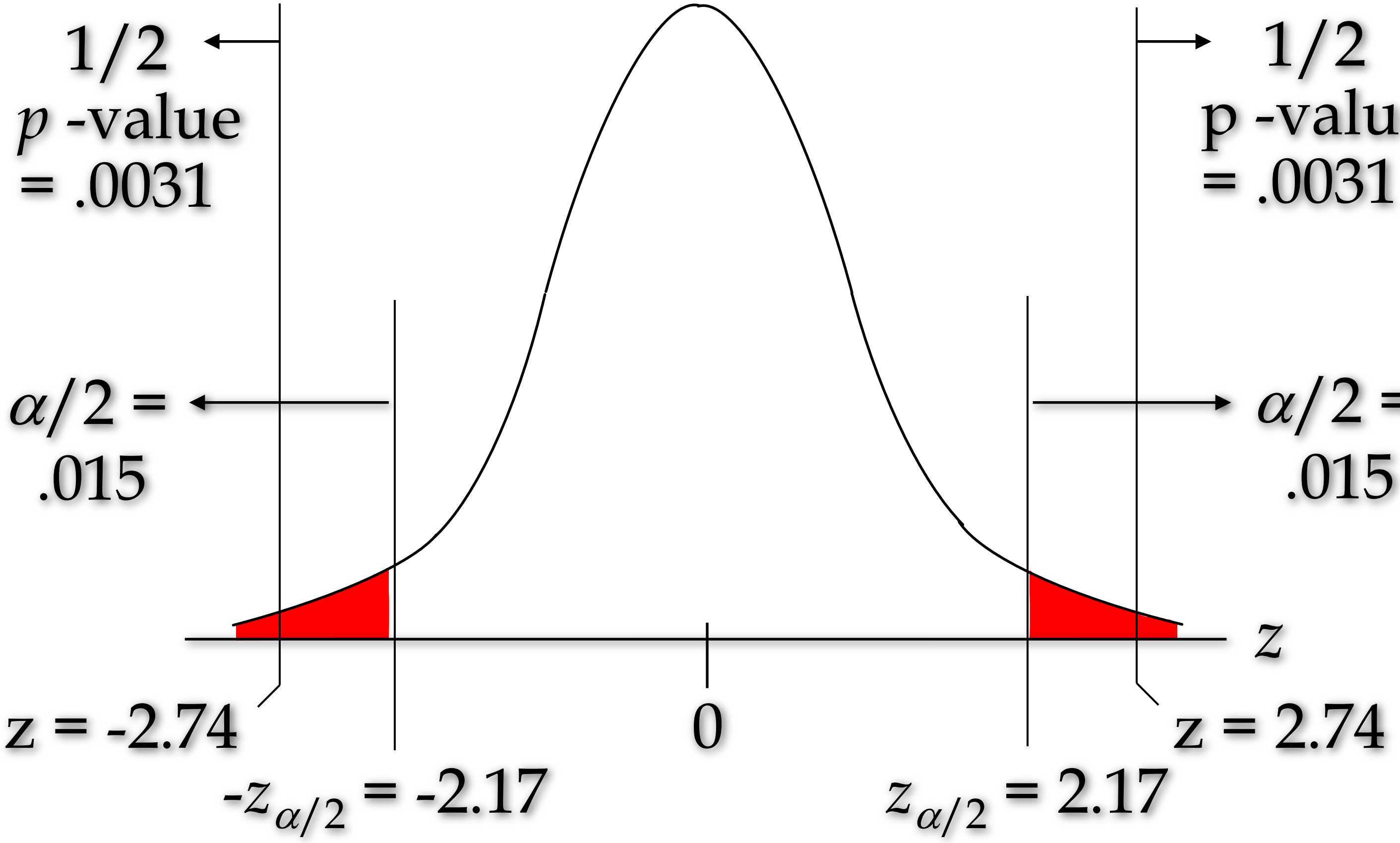
4. Compute the p –value.
   – For $z = 2.74$, p–value = 2(1 - .9969) = .0061

5. Determine whether to reject H0.
   – Because p–value = .0062 < α = .03, we reject H_0.

There is no sufficient statistical evidence to infer that the null hypothesis is true (i.e. the mean filling quantity is not 500 ml)

Two-Tailed Tests About a Population Mean: σ Known

p-Value Approach



Critical Value Approach

Two-Tailed Tests About a Population Mean : σ Known

- **Critical Value Approach**

4. Determine the critical value and rejection rule, for $\alpha/2$ = .03/2 = .015, z.015 = 2.17

Reject H_0 if z < -2.17 **or** z > 2.17

5. Determine whether to reject H_0.

Because 2.74 > 2.17, we reject H0.

There is sufficient statistical evidence to infer that the null hypothesis is not true

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In [12]: stats.norm.ppf(0.015)
Out[12]: -2.1700903775845606
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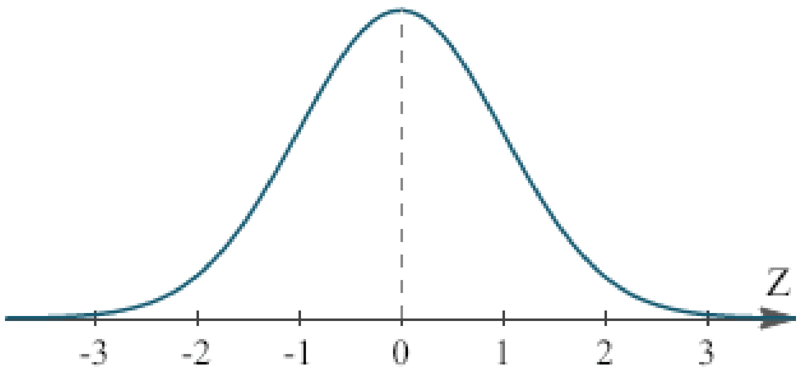
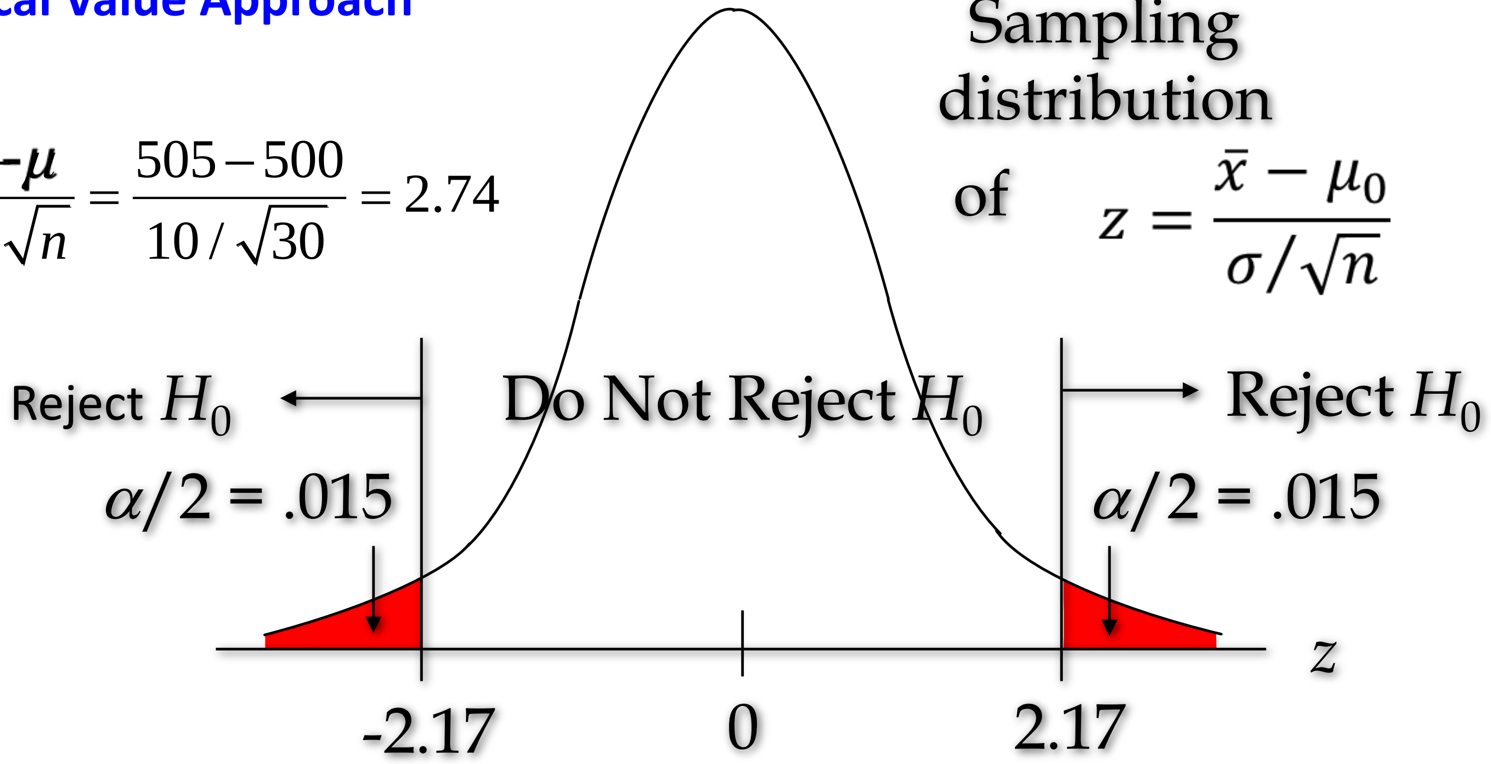


Two-Tailed Tests About a Population Mean : σ Known

Critical Value Approach

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{505 - 500}{10 / \sqrt{30}} = 2.74$$



Confidence Interval Approach

Confidence Interval Approach to Two-Tailed Tests About a Population Mean

- Select a simple random sample from the population and use the value of the sample mean to develop the confidence interval for the population mean μ.
- If the confidence interval contains the hypothesized value 500, do not reject H_0.
- Otherwise, reject H_0.
- Actually, H_0 should be rejected if μ_0 happens to be equal to one of the end points of the confidence interval.

Confidence Interval Approach to Two-Tailed Tests About a Population Mean

The 97% confidence interval for 500 is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 505 \pm 2.17\frac{10}{\sqrt{30}} = 505 \pm 3.9619$$

$$= \quad 501.03814\ ,508.96186$$

Because the hypothesized value for the population mean, $\mu_0 = 500\text{ml}$, is not in this interval, the hypothesis-testing conclusion is that the null hypothesis, H_0: $\mu = 500$, is rejected.

Thanks