IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Lecture 4: Central Tendency and Dispersion

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Lecture objectives

- Central tendency
- Measures of Dispersion

Measures of Central Tendency

- Measures of central tendency yield information about "particular places or locations in a group of numbers."
- A single number to describe the characteristics of a set of data

Summary statistics

- Central tendency or measures of location
  - Arithmetic mean
  - Weighted mean
  - Median
  - Percentile
- Dispersion
  - Skewness
  - Kurtosis
  - Range
  - Interquartile range
  - Variance
  - Standard score
  - Coefficient of variation

Arithmetic Mean

- Commonly called ‘the mean’
- It is the average of a group of numbers
- Applicable for interval and ratio data
- Not applicable for nominal or ordinal data
- Affected by each value in the data set, including extreme values
- Computed by summing all values in the data set and dividing the sum by the number of values in the data set

Population Mean

$$\mu = \frac{\sum X}{N} = \frac{X_1 + X_2 + X_3 + \ldots + X_N}{N}$$

$$= \frac{24+13+19+26+11}{5}$$

$$= \frac{93}{5}$$

$$= 18.6$$

Sample Mean

$$\bar{X} = \frac{\sum X}{n} = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$$

$$= \frac{57 + 86 + 42 + 38 + 90 + 66}{6}$$

$$= \frac{379}{6}$$

$$= 63.167$$

Mean of Grouped Data

- Weighted average of class midpoints
- Class frequencies are the weights

$$\mu = \frac{\sum fM}{\sum f}$$

$$= \frac{\sum fM}{N}$$

$$= \frac{f_1 M_1 + f_2 M_2 + f_3 M_3 + \cdots + f_i M_i}{f_1 + f_2 + f_3 + \cdots + f_i}$$

Calculation of Grouped Mean

Class Interval	Frequency(f)	Class Midpoint(M)	fM
20-under 30	6	25	150
30-under 40	18	35	630
40-under 50	11	45	495
50-under 60	11	55	605
60-under 70	3	65	195
70-under 80	1	75	75
	50		2150

$$\mu = \frac{\sum fM}{\sum f} = \frac{2150}{50} = 43.0$$

Weighted Average

- Sometimes we wish to average numbers, but we want to assign more importance, or weight, to some of the numbers.

- The average you need is the ***weighted average.***

Formula for Weighted Average

$$Weighted\ \ Average = \frac{\sum xw}{\sum w}$$

where x is a data value and w is the weight assigned to that data value. The sum is taken over all data values.

Example

Suppose your midterm test score is 83 and your final exam score is 95. Using weights of 40% for the midterm and 60% for the final exam, compute the weighted average of your scores. If the minimum average for an A is 90, will you earn an A?

$$\boxed{\begin{aligned} Weighted\ Average &= \frac{(83)(0.40)+(95)(0.60)}{0.40+0.60} \\ &= \frac{32+57}{1} = 90.2 \end{aligned}}$$

You will earn an A!

Median

- Middle value in an ordered array of numbers
- Applicable for ordinal, interval, and ratio data
- Not applicable for nominal data
- Unaffected by extremely large and extremely small values

Median: Computational Procedure

- First Procedure
  - Arrange the observations in an ordered array
  - If there is an odd number of terms, the median is the middle term of the ordered array
  - If there is an even number of terms, the median is the average of the middle two terms
- Second Procedure
  - The median's position in an ordered array is given by (n+1)/2.

Median: Example with an Odd Number of Terms

Ordered Array

3 4 5 7 8 9 11 14 15 16 16 17 19 19 20 21 22

- There are 17 terms in the ordered array.
- Position of median = (n+1)/2 = (17+1)/2 = 9
- The median is the 9th term, 15.
- If the 22 is replaced by 100, the median is 15.
- If the 3 is replaced by -103, the median is 15.

Median: Example with an Even Number of Terms

Ordered Array

3 4 5 7 8 9 11 14 15 16 16 17 19 19 20 21

- There are 16 terms in the ordered array
- Position of median = (n+1)/2 = (16+1)/2 = 8.5
- The median is between the 8th and 9th terms, 14.5
- If the 21 is replaced by 100, the median is 14.5
- If the 3 is replaced by -88, the median is 14.5

Median of Grouped Data

$$Median = L + \frac{\frac{N}{2} - cf_p}{f_{med}}(W)$$

$Where:$

L = the lower limit of the median class

cf_p = cumulative frequency of class preceding the median class

f_{med} = frequency of the median class

W = width of the median class

N = total of frequencies

Median of Grouped Data -- Example

Class Interval	Frequency	Cumulative Frequency
20-under 30	6	6
30-under 40	18	24
40-under 50	11	35
50-under 60	11	46
60-under 70	3	49
70-under 80	1	50
	N = 50	

$$Md = L + \frac{\frac{N}{2} - cf_p}{f_{med}}(W)$$

$$= 40 + \frac{\frac{50}{2} - 24}{11}(10)$$

$$= 40.909$$

Mode

- The most frequently occurring value in a data set
- Applicable to all levels of data measurement (nominal, ordinal, interval, and ratio)
- Bimodal -- Data sets that have two modes
- Multimodal -- Data sets that contain more than two modes

Mode -- Example

- The mode is 44
- There are more 44s than any other value

35	41	44	45
37	41	44	46
37	43	44	46
39	43	44	46
40	43	44	46
40	43	45	48

Mode of Grouped Data

- Midpoint of the modal class
- Modal class has the greatest frequency

Class Interval	Frequency
20-under 30	6
30-under 40	18
40-under 50	11
50-under 60	11
60-under 70	3
70-under 80	1

$$Mode = L_{Mo} + \left(\frac{d_1}{d_1 + d_2}\right)w =$$

$$30 + \left(\frac{12}{12+7}\right)10 = 36.31$$

Percentiles

- Measures of central tendency that divide a group of data into 100 parts
- Example: 90th percentile indicates that at most 90% of the data lie below it, and at least 10% of the data lie above it
- The median and the 50th percentile have the same value
- Applicable for ordinal, interval, and ratio data
- Not applicable for nominal data

Percentiles: Computational Procedure

- Organize the data into an ascending ordered array
- Calculate the p th percentile location:

$$i = \frac{P}{100}(n)$$

- Determine the percentile's location and its value.

- If i is a whole number, the percentile is the average of the values at the i and (i+1) positions

- If i is not a whole number, the percentile is at the (i+1) position in the ordered array

Percentiles: Example

- Raw Data: 14, 12, 19, 23, 5, 13, 28, 17
- Ordered Array: 5, 12, 13, 14, 17, 19, 23, 28
- Location of 30th percentile:

$$i = \frac{30}{100}(8) = 2.4$$

- The location index, i, is not a whole number; i+1 = 2.4+1=3.4; the whole number portion is 3; the 30th percentile is at the 3rd location of the array; the 30th percentile is 13.

Dispersion

- Measures of variability describe the spread or the dispersion of a set of data
- Reliability of measure of central tendency
- To compare dispersion of various samples

Variability

Measures of Variability or dispersion

Common Measures of Variability

- Range
- Inter-quartile range
- Mean Absolute Deviation
- Variance
- Standard Deviation
- Z scores
- Coefficient of Variation

Range – ungrouped data

- The difference between the largest and the smallest values in a set of data
- Simple to compute
- Ignores all data points except the two extremes
- Example:

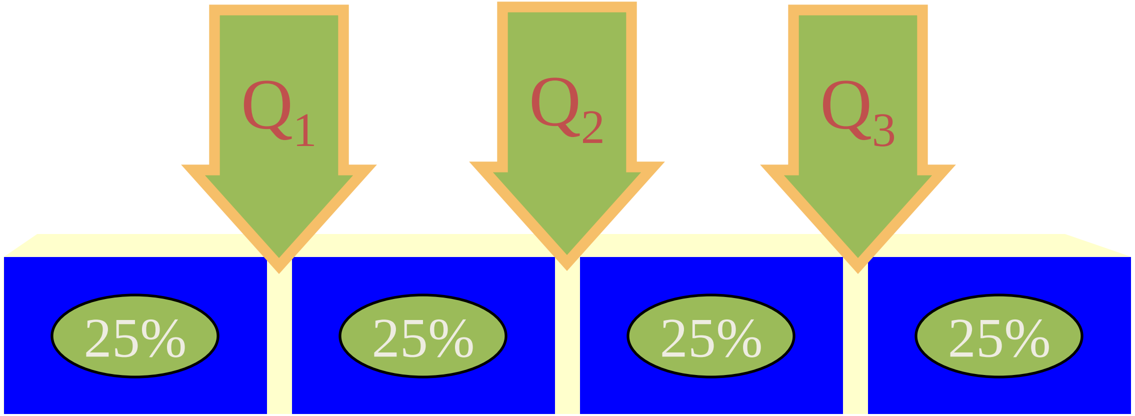
Range = Largest – Smallest = 48 - 35 = 13

35	41	44	45
37	41	44	46
37	43	44	46
39	43	44	46
40	43	44	46
40	43	45	48

Quartiles

- Measures of central tendency that divide a group of data into four subgroups
- Q_1: 25% of the data set is below the first quartile
- Q_2: 50% of the data set is below the second quartile
- Q_3: 75% of the data set is below the third quartile
- Q_1 is equal to the 25th percentile
- Q_2 is located at 50th percentile and equals the median
- Q_3 is equal to the 75th percentile
- Quartile values are not necessarily members of the data set

Quartiles



Quartiles: Example

- Ordered array: 106, 109, 114, 116, 121, 122, 125, 129
- Q_1

$$i = \frac{25}{100}(8) = 2 \qquad Q_1 = \frac{109+114}{2} = 111.5$$

- Q_2:

$$i = \frac{50}{100}(8) = 4 \qquad Q_2 = \frac{116+121}{2} = 118.5$$

- Q_3:

$$i = \frac{75}{100}(8) = 6 \qquad Q_3 = \frac{122+125}{2} = 123.5$$

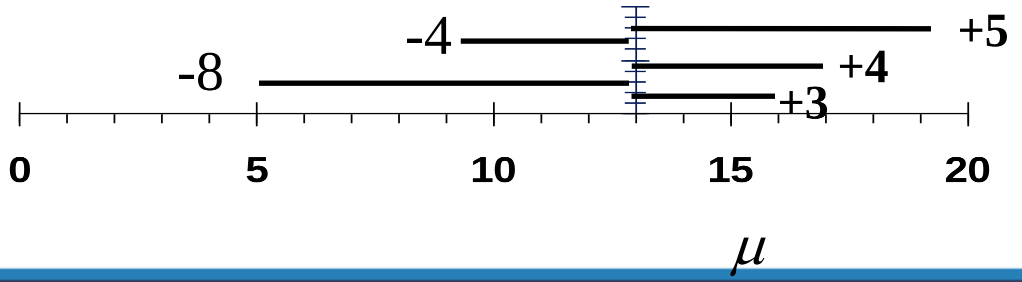
Interquartile Range

- Range of values between the first and third quartiles
- Range of the “middle half”
- Less influenced by extremes

$$Interquartile\ Range = Q_3 - Q_1$$

Deviation from the Mean

- Data set: 5, 9, 16, 17, 18
- Mean: $$\mu = \frac{\sum X}{N} = \frac{65}{5} = 13$$
- Deviations from the mean: -8, -4, 3, 4, 5



Mean Absolute Deviation

- Average of the absolute deviations from the mean

X	$X-\mu$	$\lvert X-\mu \rvert$
5	-8	+8
9	-4	+4
16	+3	+3
17	+4	+4
18	+5	+5
	0	24

$$M.A.D. = \frac{\sum \lvert X-\mu \rvert}{N} = \frac{24}{5} = 4.8$$

Population Variance

- Average of the squared deviations from the arithmetic mean

X	$X-\mu$	$(X-\mu)^2$
5	-8	64
9	-4	16
16	+3	9
17	+4	16
18	+5	25
	0	130

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$$

$$= \frac{130}{5}$$

$$= 26.0$$

Population Standard Deviation

- Square root of the variance

X	$X-\mu$	$(X-\mu)^2$
5	-8	64
9	-4	16
16	+3	9
17	+4	16
18	+5	25
	0	130

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$$

$$= \frac{130}{5}$$

$$= 26.0$$

$$\sigma = \sqrt{\sigma^2}$$

$$= \sqrt{26.0}$$

$$= 5.1$$

Sample Variance

- Average of the squared deviations from the arithmetic mean

X	$X - \bar{X}$	$(X - \bar{X})^2$
2,398	625	390,625
1,844	71	5,041
1,539	-234	54,756
1,311	-462	213,444
7,092	0	663,866

$$S^2 = \frac{\sum (X - \bar{X})^2}{n-1}$$

$$= \frac{663{,}866}{3}$$

$$= 221{,}288.67$$

Sample Standard Deviation

- Square root of the sample variance

X	$X - \overline{X}$	$(X - \overline{X})^2$
2,398	625	390,625
1,844	71	5,041
1,539	-234	54,756
1,311	-462	213,444
7,092	0	663,866

$$S^2 = \frac{\sum (X - \overline{X})^2}{n-1} = \frac{663{,}866}{3} = 221{,}288.67$$

$$S = \sqrt{S^2} = \sqrt{221{,}288.67} = 470.41$$

Uses of Standard Deviation

- Indicator of financial risk
- Quality Control
  - construction of quality control charts
  - process capability studies
- Comparing populations
  - household incomes in two cities
  - employee absenteeism at two plants

Standard Deviation as an Indicator of Financial Risk

	Annualized Rate of Return	
Financial Security	μ	σ
A	15%	3%
B	15%	7%