

# MATHEMATICS

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Paper 9709/11

Paper 11

## General comments

Whilst there were some very pleasing scripts scoring high marks Examiners were disappointed to see quite a significant proportion of scripts scoring very low marks. Scripts in this category unfortunately showed little understanding of the topics examined. One other feature of this paper was the particularly poor performance on **Question 1**. Further comment on this question appears below.

## Comments on specific questions

### **Question 1**

Most candidates gained little or no credit on this question. It was expected that many candidates would consider a right-angled triangle having one angle of  $x$  radians and with sides of  $k$ , 1 and  $\sqrt{1+k^2}$ . Candidates appeared to be confused with the fact that  $\tan x = k$ . Furthermore, they often proceeded in the first two parts to write  $\tan(\pi - x)$  and  $\tan\left(\frac{\pi}{2-x}\right)$  as  $\tan \pi - \tan x$  and  $\tan\left(\frac{\pi}{2}\right) - \tan x$  respectively. The error

in part (i) often led to the right answer but in these cases no credit was given. Some candidates were able legitimately to reach the right answer to part (i), and thereby to gain credit, by using the identity for  $\tan(A - B)$ , which is not a syllabus item for Paper 1. Very few candidates were able to make progress with part (iii). Those that did, either used the right-angled triangle approach or happened to know that  $1 + \tan^2 A = \sec^2 A$ , which is also not a syllabus item for Paper 1, and used the route:

$$\sin x = \tan x \cos x = \frac{k}{\sec x} = \frac{k}{\sqrt{1+\tan^2 x}} = \frac{k}{\sqrt{1+k^2}}.$$

Answers: (i)  $-k$ ; (ii)  $\frac{1}{k}$ ; (iii)  $\frac{k}{\sqrt{1+k^2}}$ .

### **Question 2**

Many candidates did quite well on this question, and in particular on part (i). The most common error was the failure to apply the power to the coefficient as well the term in  $x$ . Hence terms such as  $2x^5$  instead of  $32x^5$  and  $240x^3$  instead of  $-240x^3$  were often seen. A few examples of ascending powers of  $x$  were also seen. Part (ii) was answered rather less successfully than part (i) as some candidates were not so adept at distinguishing which terms would combine to give the required term in  $x$ .

Answers: (i)  $32x^5 - 240x^3 + 720x$ ; (ii) 240.

### **Question 3**

Candidates enjoyed reasonable success with this question. On the whole candidates showed a good understanding of the appropriate formulae, although some candidates confused the formula for the sum to  $n$  terms with the formula for the  $n$ th term. Solving the simultaneous equations presented few difficulties.

Answers: (i) 10, 1.5; (ii) 25.



#### Question 4

This question was reasonably well done. Some candidates found the intersections with the  $x$ -axis (0 and 6) and found themselves having to calculate the additional areas under the curve between 0 and 1 and between 5 and 6. Some took these to be areas of triangular regions. Most candidates performed the integration correctly but arithmetic mistakes were quite prevalent after applying limits to the integrated function. A significant number found the area under the curve correctly between  $x = 1$  and 5 but then failed to subtract the area of the rectangle.

Answer:  $10\frac{2}{3}$ .

#### Question 5

The first part of the question was usually attempted well with most candidates using the identity  $\sin^2 x + \cos^2 x = 1$  correctly. Part (ii), however, was not well done with many candidates not realising that  $-5 < -5\cos^2 x < 0$ . Significant numbers of candidates seemed to get confused with the presence of the minus sign and some even got confused with 'greatest' and 'least' - frequently writing the answers reversed. In part (iii) most candidates reached  $\cos^2 x = 0.6$  but very few realised that a second solution is given by the equation  $\cos x = -\sqrt{0.6}$ . In addition, most candidates did not appreciate that the range given in the question,  $0 < x < \pi$ , required answers to be given in radians. The few who gave answers in radians often did not give answers correct to 3 significant figures.

Answers: (i) 2, -5; (ii) 2, -3; (iii) 0.685, 2.46.

#### Question 6

Part (i) was quite well done for the most part with the majority of candidates attempting to integrate, although, having been given  $\frac{dy}{dx}$  some candidates were unsure what to do, with a significant number opting to substitute  $x = 9$ ,  $y = 2$  into the given equation, obtaining a gradient of 3, and then using the standard equation of a straight line having this gradient and passing through (9, 2). In part (ii) many candidates found the  $x$ -coordinate of the stationary point correctly but often made mistakes subsequently, either in finding the second derivative or in substituting the wrong value into it. Candidates needed to show clearly why they thought the stationary point was a minimum, rather than merely stating 'minimum'.

Answers: (i)  $y = 2x^{\frac{3}{2}} - 6x + 2$ ; (ii) 4, Minimum.

#### Question 7

In part (i) fully correct answers were not seen as frequently as might be expected. Mistakes in differentiation and in finding the coordinates of A were commonplace. Candidates were more successful with part (ii). It was, however, somewhat surprising to see so many candidates applying a standard procedure (Pythagoras) for finding the distance between two points when this particular situation merely required only  $6.75 + 4$ .

Answer: (ii)  $10\frac{3}{4}$ .



### Question 8

In part (i) most candidates found the gradient of  $AB$  correctly and deduced that  $m = 1$ . Some candidates confused themselves by denoting the gradient of  $AB$  to be  $m$  when the question defines the gradient of  $BC$  to be  $m$ . Mistakes in forming the equations of  $AC$  and  $BC$  in part (ii) were unfortunately very common and hence the coordinates of  $C$  were often either not found or found incorrectly. In part (iii) mistakes in finding the mid-point of  $AB$  were common with candidates often subtracting the coordinates instead of adding. Some candidates did not appear to understand the term ‘perpendicular bisector’ and attempted to find the equation of a perpendicular through  $A$  or  $B$ . As in part (ii) mistakes were often made in forming the equation of the perpendicular bisector.

Answers: (i) 1; (ii)  $(-1, 6)$ ; (iii)  $(5, 12)$ .

### Question 9

This question was often a good source of marks for candidates and even weaker candidates often scored quite well. The first part was usually well done with the first two coefficients usually being correct. A common wrong answer for  $c$  was  $-2$ . In part (ii) a reasonable proportion of candidates answered that the range of  $f$  greater than or equal to *their*  $c$ . In part (iii) most candidates found the end-points correctly although often the inequality signs were not dealt with correctly. Candidates might be better advised in future to first solve the corresponding *equality* before turning their attention to the inequality. Part (iv) was particularly well done. Almost all candidates formed the composition of correctly and a large proportion went on to find the correct value of  $k$ .

Answers: (i)  $2(x - 3)^2 - 11$ ; (ii)  $f(x) > -11$ ; (iii)  $-1 < x < 7$ ; (iv) 22.

### Question 10

In part (i) most candidates found  $\vec{OB}$  correctly, although a significant number subtracted  $\vec{OC}$  from  $\vec{OA}$ . There were very few attempts at finding the unit vector and this appears to be a part of the syllabus that was not familiar to a large proportion of the candidates. In part (ii) most candidates simply found the angle between the two given vectors  $\vec{OA}$  and  $\vec{OC}$  (which they did accurately for the most part) rather than the angle between vectors  $\vec{OB}$  and  $\vec{AC}$ . Part (iii) was well done on the whole.

Answers: (i)  $\frac{1}{6}(4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$ ; (ii)  $74.2^\circ$ ; (iii) 15.4.



# MATHEMATICS

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Paper 9709/12

Paper 12

## General comments

There were many excellent scripts, but also some low-scoring scripts. Candidates often struggled to provide a reasonable depth of thought on their offerings to the last question and the required handling of radian measure was disappointing. The standard of presentation was good from the majority of candidates, though it must be mentioned again that dividing the paper into two halves and working down both sides is a practice not to be encouraged.

## Comments on specific questions

### Question 1

This proved to be a good starting question. Virtually all candidates realised the need to expand the brackets, collect terms in  $\sin x$  and  $\cos x$  and then to use the relationship  $\frac{\sin x}{\cos x} = \tan x$ . A small minority preferred to divide the original equation by  $\cos x$  and to obtain an equation in  $\tan x$  directly. The resulting equation  $\tan x = -\frac{3}{4}$  presented some difficulty, with many candidates failing to realise that the solutions were in the 2nd and 4th quadrants. Several others failed to give solutions correct to one decimal place as required by the rubric and it was also common to see  $36.9^\circ$  left as one of the solutions.

Answer:  $143.1^\circ$ ,  $323.1^\circ$ .

### Question 2

The majority of candidates attempted to use the formula  $V = \pi \int y^2 dx$ , though occasionally the  $\pi$  was omitted. A small minority used the formula for area instead of volume. The most common errors were either to square  $\frac{a}{x}$  as  $\frac{a}{x^2}$  or to integrate  $\frac{a^2}{x^2}$  as  $\frac{a^3}{3} \times \frac{x^{-1}}{-1}$ . Use of limits was accurate and there were a large number of perfectly correct solutions.

Answer: 6.

### Question 3

- (i) This was poorly answered with only a small minority of candidates realising the need to find the value of  $f(x)$  at the stationary point. Most attempts at this were, surprisingly, by attempting to complete the square, but only a minority of these were correct. Those who obtained the stationary point at  $(1, 2)$  or who correctly completed the square as  $-2(x - 1)^2 + 2$ , often incorrectly expressed the range as  $f(x) > 2$ , rather than  $f(x) < 2$ .
- (ii) This was well answered with most candidates obtaining a correct expression for  $gf(x)$  and realising the need to set the resulting equation to  $k$  before using ' $b^2 - 4ac$ ' on the resulting quadratic. Common errors were to equate to  $k$  and then to ignore the  $k$  or to make sign errors in  $a$ ,  $b$  or  $c$ .

Answers: (i)  $f(x) < 2$ ; (ii) 13.



#### Question 4

This proved to be a source of high marks with candidates confidently finding the gradients and then the equations of both  $L_1$  and  $L_2$  before solving the simultaneous equations to find the coordinates of  $C$ . The most common error was to assume that  $L_1$  was the same as  $AB$  and to then find that the point of intersection was in fact the point  $B$ .

Answer: (5, 5).

#### Question 5

- (i) This was well answered with the majority of candidates equating the scalar product of vectors  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  to 0 and then solving to find  $p$ . A few candidates incorrectly assumed that the scalar product equalled  $-1$ , whilst others went badly astray with poor algebra.
- (ii) This was reasonably well answered with most candidates using vector  $\overrightarrow{AB}$  correctly as  $\mathbf{b} - \mathbf{a}$ . Obtaining the modulus as  $\sqrt{40 + (p-1)^2}$  was usually correct, though many errors occurred in equating  $40 + (p-1)^2$  to either 7 or even  $\sqrt{7}$ . The expansion of  $(p-1)^2$  was often incorrect, and roughly half of those working directly with  $(p-1)^2 = 9$  took  $(p-1)$  as 3 instead of  $\pm 3$ .

Answers: (i) 5; (ii) 4,  $-2$ .

#### Question 6

- (i) A sizeable minority of all candidates assumed that the term in  $x^2$  was  $10ax^2$  instead of  $10a^2x^2$ . A very small minority failed to simplify the binomial coefficients, but this was not penalised if used correctly in parts (ii) and (iii).
- (ii) The vast majority of candidates realised that there were two terms in  $x$  in the expansion of  $(1-2x)(1+ax)^5$  and there were a large number of correct answers.
- (iii) Errors in part (i) affected the answer to part (iii), though most candidates realised that the coefficient of  $x^2$  came from considering exactly two terms in the expansion of  $(1-2x)(1+ax)^5$ .

Answers: (i)  $1+5ax+10a^2x^2$ ; (ii) 0.4; (iii)  $-2.4$ .

#### Question 7

- (a) This presented many candidates with difficulty. Several candidates confused ‘inclusive’ with ‘exclusive’ and a few weaker candidates assumed that the progression was geometric. Finding that there were 41 terms in the progression proved to be too difficult for most candidates and consequently there were few correct solutions.
- (b) This was accurately answered by most candidates. Use of  $S_3 = a + ar + ar^2$  and  $S_3 = \frac{a(1-r^3)}{(1-r)}$  were equally common. The accuracy mark for part (ii) was ‘follow-through’ and the 2 marks for this part were almost always gained.

Answers: (a) 8200; (b)(i) 45, (ii) 27.

#### Question 8

- (i) This caused a lot of difficulty with candidates often failing to recognise the need to eliminate  $h$  from the equations  $4xh + 2x^2 = 96$  and  $V = x^2h$ .
- (ii) This was very well done with candidates confidently setting  $\frac{dV}{dx} = 0$ . Surprisingly at least a half of all candidates obtained  $x = 4$ , but failed to find the stationary value of  $V$ .

- (iii) This was generally well answered with the majority of candidates considering the sign of the second differential.

Answers: (ii) 64; (iii) Maximum.

### Question 9

This was a source of high marks with most candidates correctly reducing the two equations to a quadratic equation in  $x$ . Virtually all candidates used the correct formula for the area under a curve and the standard of integration was generally good. Most preferred to find the area under the curve separately from the area under the line. Those attempting to reduce the question to a single integration by using the formula  $A = \int (y_1 - y_2)dx$  often made sign errors in subtracting prior to integration. Use of limits was generally accurate. A surprising number of candidates thought it necessary to split the area under the curve into 3 separate integrals, working from  $-1$  to  $0$ , from  $0$  to  $2$  and from  $2$  to  $3$ .

Answer:  $10\frac{2}{3}$ .

### Question 10

- (i) This was well answered with a pleasing majority of candidates correctly dealing with the composite function and including the ' $\times 2$ '. Common errors were to ignore the ' $\times 2$ ', to divide by 2, to ignore the differential of  $-4x$  or to multiply the whole expression, including the ' $-4$ ', by 2.
- (ii) This presented difficulty with many candidates failing to realise the need to find the value of  $y$  when  $x = 0$  and at least a third of all others obtaining an incorrect value. Many other candidates offered the equation of the normal instead of the tangent.
- (iii) This was badly answered with many candidates not realising that 'increasing function' implies  $\frac{dy}{dx} > 0$ . Of those correctly using  $(2x - 3)^2 - 4 > 0$  many obtained the answer  $(2x - 3) > 2$  as the required answer, whilst many others offered the answer  $\frac{1}{2} < x < 2\frac{1}{2}$ . Many ignored the inequality completely.

Answers: (i)  $(2x - 3)^2 - 4$ ; (ii)  $2y = 10x - 9$ ; (iii)  $x > 2\frac{1}{2}$ ,  $x < \frac{1}{2}$ .

### Question 11

The question as a whole was poorly answered.

- (i) Reducing the equation  $4 - 3 \sin x = 2$  to  $\sin x = \frac{2}{3}$ , proved too difficult for many candidates. Most solutions were left in degrees rather than radians, and even when candidates used radians, finding the second solution,  $\pi - 0.730$ , proved to be too difficult.
- (ii) The sketches were generally good, though many candidates wasted a lot of time by drawing accurate graphs. The main loss of marks came from sketches that used lines rather than curves. Several sketches were also seen that were reflections of the correct curve in the line  $y = 4$ .
- (iii) There were very few correct solutions. Weaker candidates often attempted to relate 'no solution' automatically with ' $b^2 - 4ac$ '. It was very rare for candidates to link the answer to this part with the sketch in part (ii).
- (iv) Most candidates failed to see that this part of the question referred to the part of the graph to the right of  $x = \pi$ , and that for  $g$  to have an inverse, it needed to be one-one, that is to the 'right' of  $\frac{3\pi}{2}$ .

- (v) There were very few correct solutions. In only about a half of all attempts did candidates realise the need to consider the solution of the equation  $\sin x = \frac{1}{3}$ . Of these, only a third worked in radians, and only a handful of these recognised that, because of the domain of g, the required value of x was the obtuse, rather than the acute, value.

Answers: (i) 0.730, 2.41; (iii)  $k < 1$ ,  $k > 7$ ; (iv)  $\frac{3\pi}{2}$ ; (v) 2.80.



# MATHEMATICS

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Paper 9709/13

Paper 13

## General comments

Candidates generally found the paper to be accessible. There were few poor scripts and no indications that candidates were short of time.

Most candidates clearly made reference to the list of formulae, although in some cases formulae/identities were quoted and not used. Some candidates quoted an incorrect formula.

## Comments on specific questions

### Question 1

- (i) Most candidates found  $r = -\frac{1}{2}$ , though some merely quoted a false value such as  $+\frac{1}{2}$  or  $-2$ . A few candidates wrongly tried to find the sum of the first 10 terms but almost all did correctly find the 10th term. Some candidates never showed a fractional answer and went straight to decimals, with some failing to gain the accuracy mark by giving an answer of  $-0.023$  (not correct to 3 significant figures, as is required by the rubric.)
- (ii) Those candidates who had tried to treat part (i) as an arithmetic progression came to a halt here (or started again). Most efforts showed working so that a slip in dealing with  $-\left(-\frac{1}{2}\right)$  still earned the method mark, although this was not awarded for use of a common ratio of magnitude greater than 1.

Answers: (i)  $-\frac{3}{128}$ ; (ii) 8.

### Question 2

- (i) This question was, on the whole, well done, although a significant minority of candidates did not interpret 'in descending powers' correctly. Most candidates handled the '2' correctly but some made mistakes with the minus sign – all three terms being positive in a large number of cases.
- (ii) Whilst there were a variety of approaches (some candidates having a full expansion and product of 14 terms to deal with), most of the better candidates quickly realised that just two products were necessary to obtain the answer. A few candidates confused the issue by treating  $(1 + x^2)$  as  $(1 + x)^2$ .

Answers: (i)  $x^6 - 12x^4 + 60x^2$ ; (ii) 48.



### Question 3

- (i)  $a + b = 10$  was stated by most candidates, but the second equation  $a - \frac{1}{2}b = 1$  was often incorrect with varying wrong values for  $\cos\left(\frac{2\pi}{3}\right)$  used.
- (ii) Some candidates had only one end to the range, while others failed to answer this part.
- (iii) Many candidates failed to appreciate the meaning of exact value here and went straight from the equation with their  $a$  and  $b$  values and gave their answers in decimals. Of those proceeding correctly by using surd form and bringing in  $\cos\left(\frac{\pi}{6}\right)$ , the majority went on to give the correct answer with just a few making sign errors.

Answers: (i) 4, 6; (ii)  $-2 \leq f(x) \leq 10$ ; (iii)  $4 - 3\sqrt{3}$ .

### Question 4

- (i) This was generally well answered using  $\tan x = \frac{\sin x}{\cos x}$  and then  $\sin^2 x = 1 - \cos^2 x$ . Some candidates had minor difficulties with the algebraic manipulation required, and some failed to show the necessary steps in their working.
- (ii) The solution of the quadratic equation was handled well with  $\cos x = \frac{1}{2}$  and 2 being seen in almost all cases. Some candidates then confused themselves with  $60^\circ$  and  $300^\circ$  often being seen, sometimes as the two solutions, sometimes with one of these and one of the correct solutions, and sometimes as excess solutions.

Answer: (ii)  $120^\circ, 240^\circ$ .

### Question 5

A large minority of candidates attempted to integrate in part (i) in order to find the normal (even though a line equation is not obtained) and then attempted to find a line equation in part (ii).

- (i) Starting from  $\frac{dy}{dx}$  seemed to confuse some candidates. Some started to differentiate again and others to integrate, yet all that was necessary was the substitution of  $x = 2$  to find the gradient of the curve and the negative reciprocal of this to find the gradient of the normal, followed by substitution of all necessary information into the equation of a straight line.
- (ii) For those candidates tackling this part correctly, the most common error was having no constant of integration, or writing '+c' but making no attempt to find this. The actual integration was generally well handled; the original power of  $-\frac{1}{2}$  being well dealt with in most cases as was the need to divide by 3 because of the  $3x$  in the bracket.

Answers: (i)  $3y + x = 35$ ; (ii)  $y = 4\sqrt{3x - 2} + 3$ .



### Question 6

In part (i) the use of, for example,  $\mathbf{p} \cdot \mathbf{q} = p_1 q_1 + p_2 q_2 + p_3 q_3 = |\mathbf{p}||\mathbf{q}| \cos \theta$  was common to almost all answers but only about half of the candidates actually realised that vectors  $\overrightarrow{AB}$  with  $\overrightarrow{CB}$  (or  $\overrightarrow{BA}$  with  $\overrightarrow{BC}$ ) were necessary to obtain the required angle. The most common mistake was the use of  $\overrightarrow{AB}$  with  $\overrightarrow{BC}$  giving  $116.4^\circ$ , but other candidates used, for example,  $\overrightarrow{OA}$  with  $\overrightarrow{OB}$ , while others still attempted to evaluate a 'triple scalar product'.

In part (ii) candidates working with triangle  $ABC$  scored well, although some used long-winded geometrical methods to find length  $AC$ . Some candidates, however, seemed to think that what was needed was  $OA + OB + OC$ .

Answers: (i)  $63.6^\circ$ ; (ii) 18.32.

### Question 7

- (i) Many candidates correctly used trigonometry to find half of the angle required and wrote this down as either 0.643 or 0.644 which, when doubled, does not give 1.287. It is likely that they actually had a more accurate answer on their calculator display and doubled that, but greater care should be taken when the question says 'Show that...' rather than 'Find...'. Another method employed was to use the cosine rule to find the required angle. Some candidates used degrees and then converted to radians.
- (ii) Finding an arc length was straightforward but some candidates were unsure which sections to add together. One common wrong answer was  $2 \times 10 \times 1.287 + 24$ , whereas others who did realise that 1.287 gave the wrong arc and correctly used  $2 \times 10 \times 1.855$  forgot to add the lengths of the two straight pieces. Occasionally candidates went to great lengths to find all the arc lengths and added them together – not realising that they had found the total circumference.
- (iii) In a similar way, some candidates added four sectors to find the total area of the circle, offering this as their answer. Many candidates showed no clear overall plan. It was expected that candidates would use either 'full circle area minus the two shaded segments' or 'two sector areas based on 1.855 plus two triangle areas based on 1.287'. In practice candidates made attempts at various triangles, various sectors and occasionally the shaded segments, but many were unable to draw their work together coherently.

Answers: (ii) 61.1 cm; (iii) 281 or 282 cm<sup>2</sup>.

### Question 8

- (i) Large numbers of candidates seemed not to comprehend the words 'perpendicular bisector' and either omitted this part or found the equation of the line  $AC$ . Some did obtain the correct gradient for a perpendicular but did not give coordinates for the mid-point of  $AC$  and took their line through either  $A$  or  $C$ .
- (ii) Of the candidates who failed to understand what was needed in part (i), several did find the equation of  $BD$  here and thus were able to find the coordinates of points  $B$  and  $D$  usually as the intercept on the  $y$ -axis and then from vector moves, although various geometrical methods were also used with varying degrees of success.



- (iii) Candidates tended to use various triangles but confusion arose over halving, doubling etc. with 20 and 80 as fairly common wrong answers. A small, but significant, minority seemed to think that the area of a rhombus is the product of two of its sides.

Other methods seen included taking an upright rectangle drawn through points  $A$ ,  $B$ ,  $C$  and  $D$  and taking the area of various triangles from this. Lack of clear layout sometimes made it difficult to award even method marks if an incorrect answer was produced.

The matrix method was used by several candidates but some did not seem to fully understand it. Some never had, or forgot, the ' $\frac{1}{2}$ ', some only had 3 products each way although using it for the rhombus and some failed to realise that the second set of products were to be taken from the first set.

**Answers:** (i)  $y + 3x = 9$ ; (ii) (0,9), (4,-3); (iii) 40.

### Question 9

Some candidates worked with the function  $x^2 + 4$  or with the function  $x^2 + 5x + 4$ .

- (i) Forming and solving a quadratic equation gave  $x$ -values for points  $A$  and  $B$  and was well handled by the majority. Not all realised the need for differentiation and 'gradient = 0' in order to find point  $M$ , some assuming that it was midway between  $A$  and  $B$  in terms of its  $x$ -value, while others treated the curve as a quadratic. Several candidates achieved purely fortuitous answers for one or more of the three points from wrong working.
- (ii) For the better performing candidates the required volume proved simple and swift to determine correctly. For the slightly less accomplished candidates several problems were apparent: some performed integration on  $y$  not  $y^2$ , some integrated  $y^2$  for the given function but failed to consider the relevance of the line  $y = 5$ , some found the squaring of  $y$  difficult often losing the middle term, some used incorrect limits or used correct limits incorrectly.

Examiners were sometimes unable to award a mark for use of limits when they could see no evidence of how limits were used.

**Answers:** (i) (1, 5), (4, 5), (2, 4); (ii) 56.5 or  $18\pi$ .

### Question 10

- (i) Candidates generally seemed to realise that using the two given equations to remove one of the variables and then using  $b^2 - 4ac$  would produce the required result. Others differentiated the curve function but this was only of use if then equated to  $-k$  from the line equation and further algebra was necessary.
- (ii) Some candidates answered in precisely the form required, others listed  $a$ ,  $b$  and  $c$  but a few gave  $2((x - 2)^2 + 3)$  as an answer. This then led to further errors in some cases.
- (iii) Most candidates offered the range correctly or correct following through from their 'c'.
- (iv) Again most candidates realised the answer to be when  $x + b = 0$ .
- (v) This was very well done by the better candidates, with just some indecision by some over the + sign. Others struggled, starting from the original function and making algebraic errors along with interchanging  $x$  and  $y$  at will in the hope of reaching the answer.

Some candidates successfully started from the original function as  $y = \dots$  and then, using  $y$  as part of the constant term, solved as a quadratic in  $x$ .

**Answers:** (i) 4 or 12; (ii)  $2(x - 2)^2 + 6$ ; (iii)  $f(x) \geq 6$ ; (iv) 2; (v)  $\sqrt{\frac{(x-6)}{2}} + 2$ .



# MATHEMATICS

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Paper 9709/21

Paper 21

## General comments

The standard of presentation was usually satisfactory. There was a wide range of marks with many weaker candidates displaying a lack of knowledge of the basic techniques of differentiation and integration. All parts of the paper seemed accessible, but weaker candidates scored very poorly on the final question. Many candidates in **Questions 3, 4 and 5** were able to earn marks for part (ii) even after failing to score in part (i). There were no signs of lack of time being a problem for the candidates.

## Comment on specific questions

### Question 1

Methods of solution involving linear or quadratic inequalities/equations proved equally popular. Graphical approaches were very few and rarely successful. Often, good work was spoilt by having inequality signs the wrong way round in the final answer. Some candidates used set notation or open/closed interval notation.

*Answer:*  $x < -1$ ,  $x > 4$ .

### Question 2

There were no problems here for stronger candidates. Some candidates used  $\log(x + 2) = \log x + \log 2$  or  $\log(x + 2) = (\log x) + 2$  or  $\ln 8 - \ln 2 = \frac{\ln 8}{\ln 2}$ .

### Question 3

- (i) This was well done by most candidates, although wrong expansions of  $\tan(\theta + 45^\circ)$  were sometimes seen.
- (ii) Poorer candidates who failed in part (i) usually found that  $\tan x = \frac{1}{2}$  or  $\frac{1}{3}$  but failed to then find the inverse tangents. Where extra solutions were given, these were usually (but not always) outside the given range. Use of radians was rarely seen.

*Answer:* (ii)  $18.4^\circ$ ,  $26.6^\circ$ .

### Question 4

- (i) The standard long division approach was predominant, but a few solutions by inspection were seen. Often candidates lost the last mark for failing to identify their remainder or quotient; equally many followed through correctly.
- (ii) This part was generally well done; candidates who failed to score in part (i) often made progress here. Synthetic division was often seen and others factorized  $f(x)$  as  $(x + 1)(x^2 + 2x + 2)$ .

*Answer:* (i)  $x + 2$ ,  $3x + 4$ .



**Question 5**

- (i) There were some good solutions but many poor attempts which failed to recognize that  $2^{-x} = y^{-1}$ .
- (ii) Some candidates who failed to score in part (i) made progress here. Weaker candidates solved the quadratic equation successfully, but failed to find  $x (= \frac{\ln y}{\ln 2})$ . Some poor rounding was seen.

*Answer:* (ii) 0, 1.58.

**Question 6**

- (i) Most solutions were fully correct, but some candidates at no point attempted to differentiate. Some use made of differentials.
- (ii) Again some candidates who failed to score in part (i) made progress here. A few candidates falsely sought the equation of the normal. Quite a number incorrectly calculated  $\frac{dy}{dx}$  at (1, 2).

*Answer:* (ii)  $2x - 5y + 8 = 0$ .

**Question 7**

- (i) A significant number of candidates failed to score. A number of  $y = 2 - x$  lines were drawn with a positive gradient and a few  $y = e^{-x}$  and  $y = \ln x$  graphs were drawn in place of a  $y = e^{2x}$  graph. Some candidates' attempts at graphs of the latter function were drawn through the origin.
- (ii) Better candidates did well here, though some candidates failed to evaluate their expression, merely stating that it was greater than 0 or less than 0.
- (iii) Often this part was not attempted. Others in effect attempted part (iv) by attempting a numerical approach.
- (iv) This was generally well done, but some candidates gave answers to too few or too many decimal places.

*Answer:* (iv) 0.27.

**Question 8**

- (i) There were many good complete solutions, with a few using the product rule. Weaker candidates merely quoted the results or went down routes involving  $\tan \theta$  or  $\cot \theta$ .
- (ii) There were not many successful attempts. As well as from sign errors, many solutions never involved  $\int (\cosec^2 x - 1) dx$ .
- (iii) Often the last mark was lost as candidates could not integrate  $\cosec^2 x$ . Other solutions featured sign errors.

*Answer:* (iii)  $-\frac{1}{2} \cot x + c$ .

# MATHEMATICS

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Paper 9709/22

Paper 22

## General comments:

No question proved difficult for the majority of candidates, though weaker candidates struggled with **Questions 2, 4 (b) and 8(ii)**. Often good work was spoilt by the inappropriate use of approximate values; this was noticeable in **Questions 4(b) and 5(ii)**. No question proved inaccessible to candidates. There were no signs that a lack of time hampered candidates.

## Comments on specific questions:

### **Question 1**

Almost all candidates successfully took logs of each side, but a surprising number ended with the reciprocal of the correct answer.

*Answer:* 2.49.

### **Question 2**

- (i) Many candidates attempted, without success, to integrate exactly. Others had 2 or 4 ordinates instead of 3 and/or used an incorrect version of the trapezium rule.
- (ii) Few candidates made comparison between the areas under the 2 trapeziums and the exact area. Reference was often made to the shape of the curve, to its concavity or convexity, but rarely to the trapeziums.

*Answer:* (i) 0.21.

### **Question 3**

Solutions involving linear inequalities/equations were less frequent than those involving squaring each side to obtain quadratic expressions. Often the former method led only to the single critical value  $x = 5$ . The latter technique invariably produced the 2 critical values of  $x$ , but sometimes with an inequality sign the wrong way around.

*Answer:*  $-1 < x < 5$ .

### **Question 4**

- (a) A surprising number of solutions involved  $k \sin 2x$ , where  $k = \pm 1, \pm 2$  or  $-\frac{1}{2}$ , instead of  $\frac{1}{2} \sin 2x$ .
- (b) Many candidates failed to set  $\tan^2 x = \sec^2 x - 1$  and attempted to write down an (incorrect) exact indefinite integral. Others failed to use the exact values  $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$  and  $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ . A sizeable minority, however, produced completely correct solutions.

*Answer:* (b)  $2\sqrt{3} - \frac{\pi}{2}$ .



### Question 5

- (i) Good solutions were produced, except for the minority of candidates who believed that  $\frac{dy}{dx}$  consisted of a single term.
- (ii) Otherwise excellent work was often spoiled by the use of approximate values for  $y(1)$  and  $y'(1)$ .

Answer: (ii)  $y = \frac{2}{e}x - \frac{1}{e}$ .

### Question 6

- (i) Graphs were often poor, with straight lines for  $y = 2 - x^2$  and a variety of incorrect shapes for  $y = \ln x$ .
- (ii) Many candidates lost a mark by incorrect calculation of  $y = \ln x + x^2 - 2$  at  $x = 1.3$  or  $1.4$ , or by merely starting that these values were less than 0 and greater than 0 respectively, without any calculations being shown.
- (iii) This was sometimes not attempted, but attempts usually produced a correct conclusion.
- (iv) This was well answered by the majority of candidates, but otherwise excellent solutions were sometimes spoilt by the answer being given to 3 or 4 decimal places.

Answer: (iv) 1.31.

### Question 7

- (i) A few candidates evaluated  $p(-3)$  and/or  $p(+1)$ , and in rare cases the 30 and 18 were replaced with zeros.
- (ii) This was inaccessible to those who could not solve part (i) correctly, but those who did succeed in part (i) almost always produced good solutions to part (ii).

Answers: (i) 1, -13; (ii)  $(x - 2)(x + 3)(2x - 1)$ .

### Question 8

- (i) This was usually well attempted, but a minority of candidates were reluctant to assign values to sine and cosine of  $30^\circ$  and  $60^\circ$ .
- (ii) Many more candidates than expected could not reduce the equation to the form  $\sin 2x = \frac{1}{\sqrt{3}}$ ; instead a value of  $\tan x$  (usually  $2\sqrt{3}$  or  $(2\sqrt{3})^{-1}$ ) was obtained, with  $\sec x$  not set equal to  $(\cos x)^{-1}$ .

Answer: (ii)  $17.6^\circ, 72.4^\circ, 197.6^\circ, 252.4^\circ$ .



# MATHEMATICS

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Paper 9709/23

Paper 23

## General comments

No question proved difficult for the majority of candidates, though weaker candidates struggled with **Questions 2, 4 (b) and 8(ii)**. Often good work was spoilt by the inappropriate use of approximate values; this was noticeable in **Questions 4(b) and 5(ii)**. No question proved inaccessible to candidates. There were no signs that a lack of time hampered candidates.

## Comments on specific questions

### Question 1

Almost all candidates successfully took logs of each side, but a surprising number ended with the reciprocal of the correct answer.

Answer: 2.49.

### Question 2

- (i) Many candidates attempted, without success, to integrate exactly. Others had 2 or 4 ordinates instead of 3 and/or used an incorrect version of the trapezium rule.
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Answer: (i) 0.21.

### Question 3

Solutions involving linear inequalities/equations were less frequent than those involving squaring each side to obtain quadratic expressions. Often the former method led only to the single critical value  $x = 5$ . The latter technique invariably produced the 2 critical values of  $x$ , but sometimes with an inequality sign the wrong way around.

Answer:  $-1 < x < 5$ .

### Question 4

- (a) A surprising number of solutions involved  $k \sin 2x$ , where  $k = \pm 1, \pm 2$  or  $-\frac{1}{2}$ , instead of  $\frac{1}{2} \sin 2x$ .
- (b) Many candidates failed to set  $\tan^2 x = \sec^2 x - 1$  and attempted to write down an (incorrect) exact indefinite integral. Others failed to use the exact values  $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$  and  $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ . A sizeable minority, however, produced completely correct solutions.

Answer: (b)  $2\sqrt{3} - \frac{\pi}{2}$ .



**Question 5**

- (i) Good solutions were produced, except for the minority of candidates who believed that  $\frac{dy}{dx}$  consisted of a single term.
- (ii) Otherwise excellent work was often spoiled by the use of approximate values for  $y(1)$  and  $y'(1)$ .

*Answer:* (ii)  $y = \frac{2}{e}x - \frac{1}{e}$ .

**Question 6**

- (i) Graphs were often poor, with straight lines for  $y = 2 - x^2$  and a variety of incorrect shapes for  $y = \ln x$ .
- (ii) Many candidates lost a mark by incorrect calculation of  $y = \ln x + x^2 - 2$  at  $x = 1.3$  or  $1.4$ , or by merely starting that these values were less than 0 and greater than 0 respectively, without any calculations being shown.
- (iii) This was sometimes not attempted, but attempts usually produced a correct conclusion.
- (iv) This was well answered by the majority of candidates, but otherwise excellent solutions were sometimes spoilt by the answer being given to 3 or 4 decimal places.

*Answer:* (iv) 1.31.

**Question 7**

- (i) A few candidates evaluated  $p(-3)$  and/or  $p(+1)$ , and in rare cases the 30 and 18 were replaced with zeros.
- (ii) This was inaccessible to those who could not solve part (i) correctly, but those who did succeed in part (i) almost always produced good solutions to part (ii).

*Answers:* (i) 1, -13; (ii)  $(x - 2)(x + 3)(2x - 1)$ .

**Question 8**

- (i) This was usually well attempted, but a minority of candidates were reluctant to assign values to sine and cosine of  $30^\circ$  and  $60^\circ$ .
- (ii) Many more candidates than expected could not reduce the equation to the form  $\sin 2x = \frac{1}{\sqrt{3}}$ ; instead a value of  $\tan x$  (usually  $2\sqrt{3}$  or  $(2\sqrt{3})^{-1}$ ) was obtained, with  $\sec x$  not set equal to  $(\cos x)^{-1}$ .

*Answer:* (ii)  $17.6^\circ, 72.4^\circ, 197.6^\circ, 252.4^\circ$ .



# MATHEMATICS

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Paper 9709/31

Paper 31

## General comments

Some Centres had a number of candidates who appeared not to be well prepared for the examination as a whole, although there were odd questions for which they could produce a full solution. The questions or parts of questions that were generally done well were **Question 2** (solving a trigonometry equation), **Question 4** (differential equation), **Question 6 (iii)** (iteration) **Question 8** (partial fractions) and **Question 10** (vector geometry). Those that were done least well were **Question 1** (inequalities), **Question 3** (solving equations using logarithms) and **Question 7** (Argand diagram).

There are still candidates who present their work in double column format and this practice should be discouraged.

**Questions 4 and 8** required candidates to find a specific exact answer given in the question. Some candidates appeared to either not know or be unsure of the what this meant since they either used their calculator to undertake all the working and then checked on their calculator that the exact value produced this same value, or having secured the exact given answer they then proceeded to give an approximate answer from their calculator. Whilst the latter was treated by ISW (ignore subsequent working), the former approach was not worthy of credit.

## Comments on specific questions

### **Question 1**

The fact that this question contained the parameter  $a$  caused difficulties on two fronts. Candidates often made a slip and lost an  $a$ , hence they were unable to simplify their answer, having a multiple of  $a$  term to try to combine with a constant term. The other problem saw them forgetting that they were solving for  $x$  and as a result saw them trying to express  $a$  in terms of  $x$ . The usual approach was to try to square both sides but failure to square the coefficient of the modular right-hand side was common amongst weaker candidates, and even if this was forthcoming, poor squaring of the linear terms, often both of them, produced a quadratic equation that they were unable to solve by factorisation. Such candidates might have done better to create two linear equations and to solve each, since the squaring approach contains many potential pitfalls. Only a few candidates who reached  $x = \frac{a}{3}$  and  $x = 7a$  were able to combine them into the correct final solution.

Answer:  $\frac{a}{3} < x < 7a$ .



### Question 2

Some candidates scored well. However, either incorrect substitution for  $\cos 2\theta$ , or poor algebra, saw too many candidates start with an incorrect quadratic equation, something that should not have happened since the work to get to this stage was minimal. Several candidates who had the correct quadratic equation then produced incorrect factors, seeming only to care to match up the coefficients of  $x^2$  and the constant term, completely ignoring the coefficient of the linear term. Whilst a method mark was available for those who could show that they could factorise or apply the formula correctly to their quadratic equation, attempts just producing incorrect numbers from the calculator could not score. This is an important point for candidates to realise, namely, that marks are gained at various stages throughout the solution and failure to show full details can prevent these marks being scored.

*Answer:*  $48.6^\circ$ ,  $131.4^\circ$ ,  $270^\circ$ .

### Question 3

Although most candidates knew what was required, many appeared not to know the laws of logarithms, so often logarithm terms were multiplied instead of being added and the index was deemed to apply to both  $x$  and  $y$ . With calculators, candidates are expected to obtain the answers correctly to 3 significant figures. This is another instance where candidates could gain method marks provided they clearly showed their correct logarithm work and then their substitution, but again scored nothing if incorrect numbers simply appeared from a calculator.

*Answers:* (i) 1.50, 6.00.

### Question 4

Most candidates could state the required expansions, but too many then did not realise that this meant that they had the  $\cos 2x$  and  $\cos 4x$  required for the next part. Again few saw the continuation of this new additional information in order to undertake the integration part of the question, and commenced into pages of trying to integrate by parts. Even some who took the hint then thought that the integral of  $\cos 2x$  was

$-\frac{1}{2} \sin 2x$ , and similarly with a negative sign for the  $\sin 4x$  term. In order to gain the method mark for substituting the limits, candidates had to be showing clearly the substitution of their limits into integrals containing  $\sin 2x$  and  $\sin 4x$  and the exact evaluation of these expressions. Any calculator work, or failure to show the actual evaluation of the individual sine terms by just jumping to the given exact answer, failed to collect this method mark. It should be noted that exact answers are not possible if candidates use their calculator, since all calculators are only accurate to a particular number of decimal places.

### Question 5

Separating the variables was too often incorrect, as was the coefficient of the  $\ln(y^2 + 4)$  term following candidate's integration with respect to  $y$ . At this level candidates should have been able to spot that the integral with respect to  $y$  was  $\frac{1}{2} \ln(y^2 + 4)$ . Poor separation saw negligible marks being available, whilst a few good candidates came to grief when they had to apply exponentiation in order to obtain an expression for  $y^2$ .

*Answer:*  $y^2 = 4(x^2 - 1)$ .



### Question 6

There were only a few successful attempts at part (i) as the idea of constructing an equation seemed unfamiliar to most. Furthermore, relating a  $\theta$  in their sector formula to something usually involving  $(x - \pi)$ , which was the approach that was usually adopted, proved difficult. In addition, there were a number of candidates who appeared to not know the formula for the area of a triangle or, when they did, failed to realise that  $\sin(x - \pi) = \sin x$ . Fortunately the next two parts met with more success, and there were many complete solutions for both sections. Where candidates did have a problem was in part (ii) when they

thought that simply looking at the value of  $\frac{3}{4}\pi - \sin x$  at  $x = 1.3$  and at  $x = 1.5$  would suffice. The standard

approach is the investigation of the change of sign in  $x - (\frac{3}{4}\pi - \sin x)$ . In part (iii) some candidates did not work to 4 decimal places as required in the question.

Answer: (iii) 1.38.

### Question 7

Part (i) was usually correctly answered, but the answer of zero was often seen for the modulus due to the inclusion of the complex number  $i$ . Part (ii) usually had the correct points shown, but these should always be clearly labelled and not left as a very faint dot. Unfortunately, neither the perpendicular bisector, nor the centre of the circle, tended to match up with the points 1 and  $i$  and the point  $u$  respectively, hence resulting in the final part of (ii) being impossible to complete successfully. Part (iii) of this question was something that candidates found difficult. Most attempts seemed to be based on misunderstandings of what was required, e.g. some candidates sought the least value of  $\text{mod } z$ .

Answers: (i)  $\sqrt{8}$ ,  $45^\circ$ ; (iii)  $\sqrt{7}$ .

### Question 8

This was usually attempted with a considerable amount of success, but candidates should have been able to obtain the partial fractions in a few lines of working rather than take at least a page. The second part of this

question highlights the linking of parts of the question. With an answer given, the expression  $\frac{2}{(x+1)(x+3)}$  had to be clearly identified on the right-hand side of the expansion before going to the given answer in order to gain the method mark and hence the final accuracy mark. The integration in part (iii) was usually correct apart from several candidates having sign errors or logarithms for all four integrals. Again, the given answer demanded full details of the candidate's working, namely the clear substitution of both limits into all the terms and the actual evaluation of each term prior to any attempt at their combination, other than the two logarithm terms.

Answer: (i)  $\frac{1}{x+1} - \frac{1}{x+3}$ .

### Question 9

Some candidates chose to ignore the hint in the question and went straight ahead with the chain rule. However, whichever approach candidates used, they usually had some success before algebraic/differentiation errors prevented them establishing the given answer. In the final part candidates should have realised that whatever happened in the early part, they were now given the correct expression in order to continue the question and find its maximum value. The usual problem here was that once candidates had differentiated they were unable to clear the denominator successfully from their expression by either multiplying throughout or cross-multiplying if they already had non-zero terms on both sides of the equation.

Answer: (ii)  $\frac{1}{2}$ .



**Question 10**

Candidates' knowledge of vectors appears to be improving and there were many correct solutions to part (i), including the checking that all three component equations were satisfied. In part (ii) too many candidates failed to choose the direction vectors in order to obtain the angle. Full details of the candidate's working were essential here for them to score the method marks if they made any errors, or if they started with incorrect vectors. Part (iii) again saw many candidates fail to choose the correct vectors from which to find relevant simultaneous equations or the vector product. Those who use the vector product should show full details of their calculations as otherwise, when errors are made, it is impossible to know whether the vector product is being correctly attempted. For example, it was quite common to see  $a_i b_j + a_j b_i$  applied instead of  $a_i b_j - a_j b_i$ .

Answers: (ii)  $74.2^\circ$ ; (iii)  $5x - 3y - 4z = -2$ .



# MATHEMATICS

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Paper 9709/32

Paper 32

## General comments

The standard of work varied considerably and there was a wide range of marks obtained. However there were a considerable number of excellent responses to this paper. Candidates appeared to have sufficient time, and though nearly all questions discriminated well, no question or part of a question seemed to be of undue difficulty. The questions that were generally done well were **Question 4** (iteration) and **Question 9** (vector geometry). Those that were done least well were **Question 3** (trigonometry) and **Question 7** (differential equation).

The majority of scripts were clear and easy to follow. Previous reports have requested Centres to discourage candidates from presenting their answers in a double column format as such work is sometimes difficult to follow and mark. Examiners were pleased to find fewer instances of scripts in which this practice was evident. Earlier reports have also stressed that candidates should provide full and clear working in support of their answers, and in particular when providing the working that leads up to an answer that is already given in the question paper, e.g. as in **Question 2** of this paper. But each session a sizeable minority of candidates lose marks because they fail to provide sufficient evidence to convince Examiners that correct methods have been correctly applied. Clear working is necessary to demonstrate that a correct method is being used before any marks assigned to or dependent upon the application of that method can be given.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on specific questions

### Question 1

Candidates with a sound grasp of the laws of indices and logarithms answered this well, though occasionally they did not give the final answer to the appropriate degree of accuracy. Some multiplied the numerator and denominator of the given fraction by  $(2^x + 1)$ , obtained a quadratic in  $2^x$ , found the correct root  $\frac{3}{2}$  and the spurious root  $-1$  (which they usually rejected), and went on to complete the question correctly.

A considerable number of candidates however made all manner of mistakes of principle. For example,  $5(2^x)$  was taken to be  $10^x$ , and  $(10^x - 2^x)$  was thought to be  $8^x$ . The errors of taking  $\ln(a + b)$  to equal  $\ln a + \ln b$  and  $\ln(a - b)$  to equal  $\ln a - \ln b$  also appeared regularly.

Answer: 0.585

### Question 2

This question required two applications of the method of integration by parts. Most candidates made a good attempt at the first of these applications, but there were often errors of signs and only the better candidates carried out the second application accurately. A method mark for the correct substitution of limits was available to candidates who had integrated twice. Some of those who had made errors of sign or coefficient in their three-term indefinite integral were so determined to conclude with the given answer that they were prepared to alter the integral or invalidate the substitution to achieve that end, and thus forfeited the method mark. Even more surprisingly, some with the correct indefinite integral, whose attempt at substitution and evaluation failed to give the required answer, altered the correct integral.



### Question 3

Denied the use of a calculator, a considerable number of candidates were unable to find an exact value for  $\sin a$  and made no progress. The majority used a 3, 4, 5 triangle or a trigonometric identity to show that  $\sin a = \frac{4}{5}$ , though even at that stage errors were made. In part (i) the correct expansion was usually seen, though some said that  $\sin(a - 30^\circ)$  was equal to  $\sin a - \sin 30^\circ$ . The relevant trigonometric ratios for  $30^\circ$  seemed well known and this part was answered quite well.

The first task in part (ii) was usually done well,  $\tan 2a$  being found by means of the formulae for  $\tan 2A$  or for  $\sin 2A$  and  $\cos 2A$ . The error of taking  $\tan 2a$  to be  $2 \tan a$  was seen occasionally.

The final task of finding  $\tan 3a$  was successfully completed by using either the  $\tan(A + B)$  formula with  $A = a$  and  $B = 2a$  or the formulae for  $\sin 3A$  and  $\cos 3A$ . However many candidates used incorrect equivalents for  $\tan 3a$  such as  $3\tan a$ ,  $\tan a + \tan 2a$ ,  $\frac{3 \tan a}{1 - \tan^3 a}$ , and  $\tan(2 \tan\left(\frac{3}{2}a\right))$  where  $\tan\left(\frac{3}{2}a\right) = \frac{3}{2} \tan a$ .

Answers: (i)  $\frac{1}{10}(4\sqrt{3} - 3)$ ; (ii)  $-\frac{24}{7}, -\frac{44}{117}$ .

### Question 4

This was very well answered on the whole. In part (i) the main source of error was failure to use the quotient rule accurately, either omitting  $v^2$  from the denominator or using  $uv' - vu'$  as numerator. Those who used the product rule rarely lost the method mark for its application.

Most candidates understand that using an iterative formula  $x_{n+1} = f(x_n)$  with initial value  $x_1$  involves the generation of a sequence  $x_1, x_2 = f(x_1), x_3 = f(x_2), \dots$ , and usually part (ii) was answered well. However there appear to be candidates who believe they should substitute successive values of  $n$  and thus generate the sequence  $f(1), f(2), f(3), \dots$ . Apart from this, the main errors were (a) failure to set the calculator to radian mode, (b) giving  $\pi$  the value 180, (c) failure to give iterates to 4 decimal places, and (d) giving both 4.49 and 4.50 as final answer.

Answer: (ii) 4.49.

### Question 5

In part (i), candidates who used  $p(-\frac{1}{2}) = 0$  and  $p(-2) = 9$  to obtain simultaneous equations in  $a$  and  $b$  completed the section quickly and accurately, provided they avoided sign errors in their simplifications. Those who used long division tended to fail because of slips in the working or not continuing the division to a remainder not involving  $x$ . Candidates forming identities involving unknown quadratics and using inspection usually started with correct comparisons of coefficients but did not continue the work far enough to reach equations in  $a$  and  $b$ .

In part (ii) many who divided by  $(2x + 1)$  found the correct quadratic factor, but failed to state the complete factorisation. Synthetic division with multiplier  $-\frac{1}{2}$  produced the factor  $2x^2 + 4x - 6$ , but some candidates wishing to associate it with  $(2x + 1)$  failed to halve the coefficients. The most common false start was to attempt division by  $(x + 2)$ . Some seemed to ignore the information about  $(2x + 1)$ , used the factor theorem on the cubic, found the factor  $(x - 1)$  and went on to complete the part correctly.

Answers: (i) -4, -3; (ii)  $(2x + 1)(x + 3)(x - 1)$ .



### Question 6

In part (i) those who applied implicit differentiation to the given equation often arrived at a correct expression for  $\frac{dy}{dx}$  but could not see how to obtain the given result. Those who rearranged the equation to make  $\ln y$  or  $y$  the subject tended to be more successful. In part (ii) most candidates obtained the correct  $x$ -coordinate of the point and used a correct method for finding the equation of the tangent, but errors in calculating the gradient were common and some candidates even went on to replace the found gradient with the gradient of the normal.

Answer: (ii)  $y + 4x + 1 = 0$ .

### Question 7

This question was poorly answered. Quite a few candidates began with an incorrect separation of variables and made no progress at all. The majority did separate correctly and made reasonable attempts at integrating  $\frac{1}{e^{2t}}$ , but only a minority converted the other integral to that of  $\sec^2 x$ . Those who completed the integrations correctly were fairly successful in determining the constant of integration, though some made sign errors when substituting the constant into their solution. The question asked for  $x$  in terms of  $t$ , but some only gave  $\tan x$  in terms of  $t$ .

By part (ii) only a few were working with a correct solution to the differential equation. Some showed a good understanding of this question and expressed their answer satisfactorily. Others, perhaps as a result of calculations with large values of  $t$ , believed that  $x$  became constant. Some asserted that it equalled 26.6. In part (iii) many incorrectly said that  $x$  was directly proportional to  $t$ . Some tried to compose explanations based on their correct solutions to part (i) and the behaviour of the exponential and inverse tangent function. Only a few went back to the original equation and used the fact that the time derivative of  $x$  is positive.

Answers: (i)  $x = \tan^{-1}(\frac{1}{2} - \frac{1}{2}e^{-2t})$ ; (ii)  $x$  tends to  $\tan^{-1}(\frac{1}{2})$ .

### Question 8

Most candidates treated the modulus and argument problems separately in part (i). The trigonometric work was often very protracted and the presentation of the work was sometimes imprecise, for example some wrote down an expression equal to the tangent of the argument but called it the 'argument'. A pleasing number of candidates began by reducing  $z$  to polar form. Some then read off the modulus and argument, but others did not seem to realise the potential of what they had found and so embarked on trigonometric manipulations to find the modulus and argument. In part (ii) the strategy of multiplying numerator and denominator by the conjugate seemed to be widely known. Lengthy solutions were again the norm and candidates did not always clearly indicate the real part of  $\frac{1}{z}$ .

### Question 9

This was a straightforward test of vector geometry and candidates responded very well. Nearly all had a sound approach to part (i), though a few equated the scalar product to 4 or 13 instead of 0. The most common error was to follow  $3a + 6 = 0$  with  $a = -3$ . Again in part (ii), most candidates made progress to a final answer. If marks were lost, they were usually lost for numerical slips rather than for errors of principle.

Answers: (i) -2; (ii) 3.



**Question 10**

There were a good number of correct solutions to part (i), but also many badly flawed attempts. Apart from occasional numerical and algebraic slips in work with a sound foundation, serious errors were quite common in the setting up of an identity from which the constants were to be determined. For example, there were situations where Examiners found  $2x^3 - 1$  was proposed to be identical to a quartic in  $x$ , rather than a cubic. Similarly when the work began with a long division to determine  $A$ , errors were made in setting up an identity to determine the remaining constants.

In part (ii) the integrations were generally done well. The main errors were in the integration of  $\frac{C}{x^2}$  and  $\frac{D}{2x-1}$ . Most candidates with a correct indefinite integral provided sufficient evidence and manipulation of logarithms to justify the given answer, but some omitted essential working.

*Answers:* (i) 1, 2, 1, -3.



# MATHEMATICS

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Paper 9709/33

Paper 33

## General comments

Many candidates' work was of a high standard, but there were also a significant number of candidates scoring low marks.

The questions, or parts of questions, that were generally done well were **Question 3** (solving a trigonometry equation), **Question 5** (calculus), **Question 6** (iteration) **Question 9** (partial fractions) and **Question 10** (vector geometry), the two latter questions being particular well done by a large number of candidates. Those that were done least well were **Question 2** (using logarithms), **Question 4** (differential equation) and **Question 7** (trigonometry and integration) and **Question 8** (Argand diagram).

Examiners were pleased to find that candidates mainly did not present their work in double column format.

**Questions 5 and 7** required candidates to find an exact answer. Some candidates appeared to not know what this meant since they used their calculator to undertake all their working.

## Comments on specific questions

### **Question 1**

The usual approach was to try to square both sides but failure to square the coefficient of the modular right-hand side was common, and even if this was forthcoming, poor squaring of the linear terms saw a quadratic equation that candidates were unable to solve by factorisation. Such candidates might have done better if they had created two linear equations and solved each, since the squaring approach contains many potential pitfalls. Providing working was clearly shown for the solution of the quadratic equation, via factorisation or formula, a method mark was gained, even if applied to an incorrect equation. However for candidates who resorted to the calculator to solve the incorrect quadratic equation it was very difficult for the Examiner to know what the candidate had done during the solution process, especially if further errors had occurred, so the gaining of this method mark was very unlikely if a calculator approach was adopted.

Answer:  $-5 < x < \frac{1}{3}$ .

### **Question 2**

This question caused problems for those candidates who seemed unable to take logarithms of the term  $Ae^{2x}$ . Further problems occurred because once candidates saw  $2x$  they deemed the gradient to be 2, forgetting about the coefficient of  $\ln y$ . Even those candidates who managed to obtain the correct gradient often then ran into trouble in finding a correct value of  $A$ , since in dividing through by the factor 3 they had failed to divide the constant  $\ln A$ , which meant that when setting  $x = 0$ , an incorrect equation resulted.

Answers: (i)  $\frac{2}{3}$ , (ii) 4.48.



### Question 3

Candidates usually gave a correct expansion for  $\tan(45^\circ - x)$ , but then too often poor algebra/arithmetic meant that an incorrect quadratic equation was produced. However, again if a clear method was shown then a method mark was available.

Answer:  $15.7^\circ, 119.3^\circ$ .

### Question 4

Candidates would have been well advised to immediately convert the right-hand side to the single fraction  $\frac{4-x^2}{4x}$ . Those that delayed tended to produce things far removed from anything for which marks could realistically be given. It was quite common to see each individual term inverted. Candidates would have been well advised to apply a substitution in order to perform the integration of the term in  $x$ , even though they should have been capable of doing it mentally. Many appeared to just guess, so  $2\ln(4-x^2)$  and  $\frac{1}{2}\ln(4-x^2)$

were common. Candidates had difficulty in exponentiation, particularly when more than a single term was involved on one particular side of the equation. Whilst candidates should learn how to perform this operation, it is something that can often be avoided if the constant is determined prior to integration and then the logarithm terms combined into a single term before proceeding to exponentiate.

Answer:  $x^2 = 4 - 3\exp(-\frac{1}{2}t)$ .

### Question 5

Usually the differentiation in part (i) and integration in part (ii) were correct, but occasionally sign errors or the position of the 2 was wayward. In part (i) the exact value of  $p$  was required, so  $p = 0.693$  scored zero. It should be stressed that candidates need to obtain non-zero terms on both sides of an equation before they attempt to take logarithms. In part (ii), with the given answer, all the details of substituting  $p = \ln 2$ , together with the simplification of each term, needed to be clearly seen in order to gain the method mark.

Answer: (i)  $\ln 2$ .

### Question 6

There were many fully correct attempts at part (i), although occasionally a candidate used minus the correct quotient formula or omitted  $v^2$  from the quotient formula. Whilst such errors are self-cancelling when the derivative is equated to zero, marks are not awarded for such errors. Surprisingly it took many candidates much working to acquire the given answer, whilst others found the final operation a step too far, despite the simple algebra involved. Where candidates did have a problem was in part (ii) when they thought that simply looking at the value of  $\frac{x+1}{\ln x}$  at  $x = 3$  and at  $x = 4$  would suffice. The standard approach is the investigation of the change of sign in  $x - \frac{x+1}{\ln x}$ . The main error in part (iii) was the failure to work to 4 decimal places as requested.

Answer: (ii) 3.59.



### Question 7

Part (i) was usually established correctly by all those candidates starting from the left-hand side with the expansion of  $\cos(2\theta + \theta)$ . Those starting from the right-hand side usually were unable to make much progress. Again with the given answer it was essential to show each correct trigonometric substitution together with all appropriate algebra to acquire the marks. The reason for the given answer was to guide candidates to use that as their substitution in part (ii); unfortunately many ignored this and went into lengthy integration by parts or read the question that was being asked as the integral of  $\cos 3\theta$ , so simply substituted the expression just derived. This had the same effect as integration by parts and candidates were unable to progress. Candidates were asked for an exact answer so they should have shown the given limits clearly substituted into  $\sin 3\theta$  and  $\sin \theta$ , followed by the exact evaluation of each of these terms, as candidates were being tested to see whether they knew the various values of  $\sin \theta$  and not what their calculator produced.

Answer: (ii)  $\frac{2}{3} - \frac{3}{8}\sqrt{3}$ .

### Question 8

Obviously there were many different approaches seen to this question, but again Examiners were looking for detailed working. So, for the method of multiplying the terms out the expansion of  $(1 + i\sqrt{3})^2$  was required followed by the detailed evaluation of this expansion multiplied by  $(1 + i\sqrt{3})$ . Most candidates knew the other complex root. The Argand diagram proved a real problem to many candidates since they seemed unable to

link the point  $(1 + i\sqrt{3})$  to the circle and then to  $\arg z \leq \frac{\pi}{3}$ , with circles in different quadrants from the point

and from the line of  $\arg z = \frac{\pi}{3}$ . In fact this line was rarely labelled and usually came from the centre of the circle instead of from the origin through the centre of the circle and beyond, not just to the centre of the circle. Occasionally everything was correct but candidates opted to shade the wrong semicircle.

Answer: (a)  $1 - i\sqrt{3}$ .

### Question 9

This question was usually undertaken with a considerable amount of success. Some candidates failed to take care when multiplying out and so had an extra factor of  $(2 + x)$  in all their terms. Most candidates knew what was required in part (ii) but obtaining the correct expressions  $2^{-1}(1 + \frac{x}{2})^{-1}$  and  $2^{-2}(1 + \frac{x}{2})^{-2}$  in order to commence the expansions proved too difficult for many.

Answers: (i)  $\frac{1}{1-2x} + \frac{1}{2+x} - \frac{2}{(2+x)^2}$ ; (ii)  $1 + \frac{9}{4}x + \frac{15}{4}x^2$ .

### Question 10

Candidates appear to be improving in their knowledge of vectors and there were many correct solutions to part (i). In part (ii) too many candidates failed to choose the direction vector and the vector normal to the plane in order to obtain the angle. Full details of candidates' working were essential here for them to score the method marks if they made any error, or if they had the incorrect vectors to start with. In part (iii) the candidates had many methods available to choose from, but the essential part of any of these methods is to commence with the correct vectors. In the vector product method many candidates failed to choose these. Here it is again necessary to stress that full details of working are essential since otherwise, when errors are made, it is impossible to know whether the vector product is actually being applied correctly. It was quite common to see  $a_i b_j + a_j b_i$  applied instead of  $a_i b_i - a_j b_j$ .

Answers: (i)  $4i + 3j$ ; (ii)  $26.5^\circ$ ; (iii)  $6x + 4y - 7z = 36$ .



# MATHEMATICS

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Paper 9709/41

Paper 41

## General comments

The work of candidates was generally well presented, which helps candidates in developing a solution to a problem. It continues to be the practice among a significant minority of candidates to limit the scope for presenting the work well by drawing a mid-line down the page and working in two columns, thus reducing the width of the working space to half the width of the page. Candidates should be discouraged from adopting this practice.

## Comments on specific questions

### Question 1

Most candidates realised the need to apply Newton's second law to the motion of the car, and the need to use  $F = \frac{P}{V}$  to find the driving force. However, although the driving force and the resistance to motion were both taken into account by almost all candidates, many candidates omitted the component of the weight of the car.

Answer:  $0.845 \text{ ms}^{-2}$ .

### Question 2

Parts (i) and (ii) were very well attempted. However in part (iii) many candidates inappropriately used kinematic equations. The value  $a = 0.09 \text{ ms}^{-2}$  was often used although it has no relevance to this part.

Also in part (iii) some candidates implicitly assumed that the maximum speed occurs when  $t = 9.5$  (so that  $2 \times \frac{1}{2} \times 1.5V = 1.08$ ) or when  $t = 11$  (so that  $\frac{1}{2}(0+V) \times 3 = 1.08$ ). Candidates who obtained the answer  $0.72 \text{ ms}^{-1}$  by one of these methods scored one mark; both marks would have been scored if the working had been accompanied by an explanation that the answer obtained is the same irrespective of the value of  $t$  at which the maximum speed occurs.

Answers: (i)  $0.09 \text{ ms}^{-2}$ ; (ii)  $1.08 \text{ m}$ ; (iii)  $0.72 \text{ ms}^{-1}$ .

### Question 3

- (i) This part was very well attempted.
- (ii) This part was much less well attempted. Despite obtaining the correct answer in part (i) many candidates used the value of the weight of the ring as the normal reaction. A significant minority of candidates used the relevant formula effectively as  $\mu = \frac{R}{F}$ , instead of the correct  $\mu = \frac{F}{R}$ .

Answer: (ii) 1.62.



#### Question 4

This question was very well attempted. Almost all candidates used the method whose first stage is to find the components  $X_N$  and  $Y_N$  of the resultant. Errors made by just a few candidates were of the types  $Y = 370 + 250 \times 0.28 = 440$  and  $X = 160 + 250\cos 0.96 = 409.96$ . Almost all candidates applied the correct formula for  $R$ , but some candidates used the incorrect trigonometric function in finding the required angle.

*Answers:* 500 N,  $36.9^\circ$ .

#### Question 5

A few candidates seemed to have a problem with interpretation of the scenario, leading them to believe the motion to be downwards, or that the speed required in part (i) is the final speed rather than the initial speed. For some candidates the notions of energy gain and energy loss were confused in both parts of the question. Candidates should be encouraged to read questions carefully.

Notwithstanding the foregoing, part (i) was generally very well attempted.

In part (ii) the gain in potential energy is a specific requirement of the question. However, some candidates, although having the correct equation  $0.3gh = \frac{1}{2} \times 0.3 \times 3^2 - 0.39$ , did not state the answer for potential energy gain. A fairly frequently occurring error was the wrong sign attached to the 0.39, leading to incorrect answers of 1.74 J and 0.58 m.

*Answer:* (i)  $3 \text{ m}^{-1}$ ; (ii) 0.96 J, 0.32 m.

#### Question 6

- (i) This part of the question was well attempted by a large proportion of candidates, even though the characteristics of horizontal and vertical motion for the different particles and the presence of a frictional force make the question more than usually challenging for a connected particles problem.

Nevertheless a significant number of candidates had the weight of  $A$  (2 N), in the Newton's second law equation, instead of the frictional force (0.6 N) and some others had both the weight and the frictional force in the equation.

In finding the required velocity of  $B$ , there were various different values seen for the distance travelled by  $B$ .

- (ii) Almost all candidates realised the relevance of the formula  $v^2 = u^2 + 2as$  in this part of the question. However a very large proportion of candidates used  $a = 6$ , its value in part (i), and many others used  $a = g$ , notwithstanding the horizontal motion. Similarly various different values for  $s$  were used by candidates.

*Answers:* (i)  $6 \text{ ms}^{-2}$ ; (ii)  $3.29 \text{ ms}^{-1}$ .

#### Question 7

- (i) Many candidates obtained  $A = 4$  satisfactorily, but the next step proved to be more difficult. It was fairly common to see erroneous methods used that involve the given information that  $s(15) = 225$ .

One such erroneous method is to write  $v = \frac{s}{t} \Rightarrow \frac{B}{15} = \frac{225}{15} \Rightarrow B = 3375$  and another is to write

$$s = \int Bt^{-2} dt \Rightarrow s = -\frac{B}{t} \Rightarrow 225 = (-)\frac{B}{15} \Rightarrow B = (-)3375.$$

- (ii) Almost all candidates obtained  $s = -\frac{B}{t} + C$  satisfactorily, but very many candidates just used  $C = 225$  or  $C = 0$  instead of  $s(15) = 225$  (and hence  $C = 450$ ). A significant number of other candidates wrote  $225 = -\frac{3375}{15} + C$  but wrongly deduced from this equation that  $C = 0$ .

- (iii) Candidates who had an incorrect expression for  $s(t)$  from part (ii) could not score full marks in this part, although many such candidates did score the two method marks.

Of these method marks the second was more often scored than the first. This arose because some candidates did not use  $s(t)$  as found in part (ii) to equate with 315, and some did use  $s(t)$  as found in part (ii) but equated it with 90 ( $= 315 - 225$ ). Most candidates who did find a value of  $t$ , correct or otherwise, substituted into  $\frac{3375}{t^2}$  to obtain an answer for the required speed.

Answers: (i) 4; (ii)  $(450 - \frac{3375}{t})$  m; (iii)  $5.4 \text{ ms}^{-1}$ .



# MATHEMATICS

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Paper 9709/42

Paper 42

## General comments

The work of candidates was generally well presented, which helps candidates in developing a solution to a problem. It continues to be the practice among a significant minority of candidates to limit the scope for presenting the work well by drawing a mid-line down the page and working in two columns, thus reducing the width of the working space to half the width of the page. Candidates should be discouraged from adopting this practice.

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Answers: 500 N,  $36.9^\circ$ .

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Answer: (i)  $3 \text{ m}^{-1}$ ; (ii) 0.96 J, 0.32 m.

#### Question 6

- (i) This part of the question was well attempted by a large proportion of candidates, even though the characteristics of horizontal and vertical motion for the different particles and the presence of a frictional force make the question more than usually challenging for a connected particles problem.

Nevertheless a significant number of candidates had the weight of  $A$  (2 N), in the Newton's second law equation, instead of the frictional force (0.6 N) and some others had both the weight and the frictional force in the equation.

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Answers: (i)  $6 \text{ ms}^{-2}$ ; (ii)  $3.29 \text{ ms}^{-1}$ .

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- (i) Many candidates obtained  $A = 4$  satisfactorily, but the next step proved to be more difficult. It was fairly common to see erroneous methods used that involve the given information that  $s(15) = 225$ . One such erroneous method is to write  $v = \frac{s}{t} \Rightarrow \frac{B}{15} = \frac{225}{15} \Rightarrow B = 3375$  and another is to write  $s = \int Bt^{-2} dt \Rightarrow s = -\frac{B}{t} \Rightarrow 225 = (-)\frac{B}{15} \Rightarrow B = (-)3375$ .

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- (iii) Candidates who had an incorrect expression for  $s(t)$  from part (ii) could not score full marks in this part, although many such candidates did score the two method marks.

Of these method marks the second was more often scored than the first. This arose because some candidates did not use  $s(t)$  as found in part (ii) to equate with 315, and some did use  $s(t)$  as found in part (ii) but equated it with 90 ( $= 315 - 225$ ). Most candidates who did find a value of  $t$ , correct or otherwise, substituted into  $\frac{3375}{t^2}$  to obtain an answer for the required speed.

Answers: (i) 4; (ii)  $(450 - \frac{3375}{t})$  m; (iii)  $5.4 \text{ ms}^{-1}$ .



# MATHEMATICS

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Paper 9709/43

Paper 43

## General comments

The paper allowed candidates to show what they knew. Many excellent scripts were seen but there were also a significant number of candidates who were not prepared for tackling the paper. The paper clearly differentiated between candidates. **Questions 3(ii), 5(ii), 6(ii) and 7(ii), (iii)** were found to be more difficult while weaker candidates were able to access at least the first part of most questions.

Presentation was often of a high standard with clear working and formulae to support solutions, although in some cases minimal working and a lack of diagrams made solutions more difficult to follow. Some candidates presented work with two columns of work on each page which is confusing in the examination situation. The majority of candidates followed the rubric giving answers to the correct degree of accuracy and using  $10 \text{ ms}^{-2}$  for the acceleration due to gravity.

## Comments on specific questions

### Question 1

This question did not prove to be straightforward for candidates, many of whom overlooked the fact that the resultant was in the direction of the 6.8 N force. Some candidates equated the two horizontal components and obtained  $\alpha = 21.3^\circ$ . Others who treated the situation as one of equilibrium and resolved in both directions, obtained  $\alpha = 39^\circ$ , and then attempted to use Pythagoras' Theorem to work out the magnitude of the resultant, not realising that the vertical component of the resultant should be zero.

Answers: 48.9, 2N.

### Question 2

The majority of candidates recognised that calculus was required and many obtained full marks. Occasionally candidates used  $a = \frac{dv}{dt}$  to find the time taken correctly but then used constant acceleration to calculate the displacement of the particle. Alternatively, a few candidates obtained the time erroneously using  $v = 1.2t - 0.12t^2 = u + at$  before applying either  $s = \int v dt$  or another constant acceleration formula.

Answer: 3.13 m.

### Question 3

- (i) Most candidates calculated the work done correctly, although a few worked as if the load was moving on an inclined plane rather than pulled by an inclined string and thus calculated work done as  $25 \times 40$ . Other errors seen were 80 used instead of 40 and  $\sin 30^\circ$  used in place of  $\cos 30^\circ$ .



- (ii) Candidates were asked to find the gain in kinetic energy and it was expected that this be shown explicitly. Some found the kinetic energy at *B* rather than the kinetic energy gain whilst others used a work/energy equation but without identifying the gain in kinetic energy. The question was worded ‘hence find the speed’ and so candidates using Newton’s second law rather than an energy approach were penalised. Some candidates successfully obtained the speed first from a work/energy equation and then used it to find the gain in kinetic energy. One commonly seen value for speed was  $7.03 \text{ ms}^{-1}$  obtained from assuming zero speed at *M*. Another common erroneous value  $7.13 \text{ ms}^{-1}$  was obtained from premature approximation of  $25\cos 30^\circ$  to 21.7.

**Answers:** (i) 866 J; (ii) 866 J,  $7.14 \text{ ms}^{-1}$ .

#### Question 4

- (i) The answer was given in the question and candidates frequently obtained the correct value for *T* with suitable correct working. However, candidates sometimes attempted to reach the given value by trial and error finding a variety of inappropriate calculations leading to approximately 3.
- (ii) This situation of connected particles with two inclined planes was unfamiliar and, although many candidates found the coefficient of friction correctly, there were also those who did not include all components in their equation of motion for particle *B* or who made sign errors in doing so. Common erroneous answers were  $\mu = 0.84$  (using  $a = 0.25 \text{ ms}^{-2}$  up the plane) and  $\mu = 0.79$  (omitting  $ma$ ). Errors were sometimes due to the application of  $F = \mu R$  with  $R = 0.24g$  or  $0.24g\sin 60^\circ$  instead of  $0.24g\cos 60^\circ$  or due to a final step  $\mu = \frac{R}{F}$ .

**Answer:** (ii) 0.74.

#### Question 5

- (i) Most candidates applied appropriate constant acceleration formulae but some either calculated the acceleration without then stating a value for *d* or worked with  $v = u + dt$  and obtained  $d = -0.25 \text{ ms}^{-2}$ .
- (ii) Many candidates believed that the rebound speed was unchanged at  $1.1 \text{ ms}^{-1}$  even though they were asked to ‘find the speed’. This led to a common wrong value of 3.64 seconds for the time taken.
- (iii) Candidates often appeared to sketch a speed-time graph rather than a velocity-time graph with both stages of motion shown above the horizontal axis. Some candidates forgot to add 1.2 seconds to their time from part (ii) when labelling their graph.

**Answers:** (i) 1.5 m, 0.25; (ii)  $1 \text{ ms}^{-1}$ , 4 s.

#### Question 6

- (i) The acceleration was usually calculated correctly. The calculation of the angle  $\alpha$  was most straightforward for those who used  $a = g\sin \alpha$ . Rounding of the angle to  $10^\circ$  was sometimes seen whilst some candidates did not attempt to find the angle.
- (ii) The value of *t* was found most effectively by those who realised that particle *P* took *t* + 2 seconds and then used  $\frac{1}{2}a(t+2)^2 - \frac{1}{2}at^2 = 4.9$ . Many candidates did not appear to realise that there were two stages to the motion and frequently omitted the distance travelled by *P* from *O* to *A*. A common incorrect answer was thus  $t = 1.4$  from  $s_P - s_Q = 4.9 = 3.5t + \frac{1}{2}(1.75t^2) - \frac{1}{2}(1.75t^2)$ . A few candidates used  $s_P + s_Q$  in place of  $s_P - s_Q$ . Others used *t* – 2 instead of *t* + 2.

**Answers:** (i)  $1.75 \text{ ms}^{-2}$ ,  $10.1^\circ$ ; (ii) 0.4.

**Question 7**

Candidates often attempted this question with no apparent force diagram and found difficulty with the unfamiliar situation of two frictional interfaces. Many of the errors seen were due to confusion about which box or boxes to use. A number of candidates failed to attempt part (ii) or part (iii).

- (i) This part of the question often gained full marks but occasionally candidates used  $R = 2500$  instead of  $R = 4500$  leading to  $\mu = 1.26$ .
- (ii) Many candidates did not realise that it was necessary to consider box A only. The value 2 given in the question sometimes confused candidates and  $900 = 450a$  was regularly seen.
- (iii) This proved challenging to many candidates. Some considered boxes A and B combined whilst others considered box B only but extra or missing terms were common.

*Answers:* (i) 0.7; (iii) 4050.



# MATHEMATICS

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Paper 9709/51

Paper 51

## General comments

Most candidates attempted all the questions.

It was encouraging to see that good clear diagrams were drawn by many candidates to help them with their solutions, and this enabled their work and reasoning to be more easily followed.

Only a few candidates used premature approximation and rounded answers to less than 3 significant figures. It should be noted that on this paper  $g = 10$  should be used and not  $g = 9.8$  or  $9.81$ , and that the formulae list can be used by candidates to check that they are using the correct formula.

**Questions 2, 3, 4 and 6(i)** proved to be a good source of marks for many candidates, while **Question 7** proved to be the most difficult question on the paper and helped to discriminate between the average and the above average candidates.

## Comment on specific questions

### Question 1

Many candidates used the incorrect formula for the centre of mass of the semicircular wire, even though the correct formula is given in the formulae list. Attempts were made at a moment equation but often the quoted weights were ignored and other values were used.

*Answer:* 8.78 cm.

### Question 2

- (i) Most candidates recognised that the weight would act vertically through the lowest point of the cone. Quite a number of candidates failed to obtain the correct distance of the centre of mass of the cone from its base, even though the formula is given in the formula list.
- (ii) This part of the question was generally well done.

*Answer:* (i) 5.25.

### Question 3

- (i) This part of the question was generally well done. On occasions candidates used  $T \cos \theta = mg$  and then used  $\tan \theta = 0.75$  without establishing this.
- (ii) Usually this part of the question was well done. Some candidates had a dimensionally wrong equation using  $a = \frac{mv^2}{r}$  instead of  $a = \frac{v^2}{r}$ .

*Answer:* (ii)  $3 \text{ ms}^{-1}$ .



#### Question 4

This question was generally well done. Some candidates were unable to find the centre of mass of a triangular lamina and other candidates made errors in the moment equation. When taking moments, occasionally the force  $T\sin 30^\circ$  was used as a moment.

Answer: 13 N.

#### Question 5

- (i) This part of the question was usually well done. When the equation of the trajectory was used from the formula list then candidates often scored full marks.
- (ii) Candidates who used the formula for the range generally scored full marks. Some candidates found half the distance since they only calculated the time to the highest point of the flight.
- (iii) Not many of the sketches resulted in full marks. A significant number of candidates drew separate sketches for the two trajectories.

Answer: (ii) 38.4 m, 17.8 m.

#### Question 6

- (i) In this part of the question candidates often scored full marks by resolving vertically to find the tension and then by using  $T = \frac{\lambda x}{l}$  to find  $\lambda$ .
- (ii) Often in this part of the question elastic energy =  $\frac{\lambda x^2}{2l}$  was used but when the energy equation was set up either sign errors occurred or the elastic energy at M was omitted.

Answers: (i) 6.25 N; (ii) 4.90 ms<sup>-1</sup>.

#### Question 7

- (i) Newton's second law was used but too often  $0.25a = 5 - x$  was seen instead of  $0.25a = -(5 - x)$ . Candidates knew how to separate the variables and integrate to solve the differential equation but often no constant of integration was seen.
- (ii) The integration of  $\frac{1}{10-2x}$  often resulted in either  $-2\ln(10 - 2x)$  or  $-\ln(10 - 2x)$ . Again when the integration was carried out the constant of integration was sometimes omitted. Very few candidates were able to carry out the algebra required to obtain  $x = 5(1 - e^{-2t})$ , and of those who did only a few were able to explain why  $x < 5$  for all values of  $t$ .

Answers: (i)  $0.25v \frac{dv}{dx} = -(5 - x)$ ; (ii)  $x = 5(1 - e^{-2t})$ .



# MATHEMATICS

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Paper 9709/52

Paper 52

## General comments

Most candidates attempted all the questions.

It was encouraging to see that good clear diagrams were drawn by many candidates to help them with their solutions, and this enabled their work and reasoning to be more easily followed.

Only a few candidates used premature approximation and rounded answers to less than 3 significant figures. It should be noted that on this paper  $g = 10$  should be used and not  $g = 9.8$  or  $9.81$ , and that the formulae list can be used by candidates to check that they are using the correct formula.

**Questions 2, 3, 4 and 6(i)** proved to be a good source of marks for many candidates, while **Question 7** proved to be the most difficult question on the paper and helped to discriminate between the average and the above average candidates.

## Comment on specific questions

### Question 1

Many candidates used the incorrect formula for the centre of mass of the semicircular wire, even though the correct formula is given in the formulae list. Attempts were made at a moment equation but often the quoted weights were ignored and other values were used.

*Answer:* 8.78 cm.

### Question 2

- (i) Most candidates recognised that the weight would act vertically through the lowest point of the cone. Quite a number of candidates failed to obtain the correct distance of the centre of mass of the cone from its base, even though the formula is given in the formula list.
- (ii) This part of the question was generally well done.

*Answer:* (i) 5.25.

### Question 3

- (i) This part of the question was generally well done. On occasions candidates used  $T \cos \theta = mg$  and then used  $\tan \theta = 0.75$  without establishing this.
- (ii) Usually this part of the question was well done. Some candidates had a dimensionally wrong equation using  $a = \frac{mv^2}{r}$  instead of  $a = \frac{v^2}{r}$ .

*Answer:* (ii)  $3 \text{ ms}^{-1}$ .



#### Question 4

This question was generally well done. Some candidates were unable to find the centre of mass of a triangular lamina and other candidates made errors in the moment equation. When taking moments, occasionally the force  $T\sin 30^\circ$  was used as a moment.

Answer: 13 N.

#### Question 5

- (i) This part of the question was usually well done. When the equation of the trajectory was used from the formula list then candidates often scored full marks.
- (ii) Candidates who used the formula for the range generally scored full marks. Some candidates found half the distance since they only calculated the time to the highest point of the flight.
- (iii) Not many of the sketches resulted in full marks. A significant number of candidates drew separate sketches for the two trajectories.

Answer: (ii) 38.4 m, 17.8 m.

#### Question 6

- (i) In this part of the question candidates often scored full marks by resolving vertically to find the tension and then by using  $T = \frac{\lambda x}{l}$  to find  $\lambda$ .
- (ii) Often in this part of the question elastic energy =  $\frac{\lambda x^2}{2l}$  was used but when the energy equation was set up either sign errors occurred or the elastic energy at M was omitted.

Answers: (i) 6.25 N; (ii)  $4.90 \text{ ms}^{-1}$ .

#### Question 7

- (i) Newton's second law was used but too often  $0.25a = 5 - x$  was seen instead of  $0.25a = -(5 - x)$ . Candidates knew how to separate the variables and integrate to solve the differential equation but often no constant of integration was seen.
- (ii) The integration of  $\frac{1}{10-2x}$  often resulted in either  $-2\ln(10 - 2x)$  or  $-\ln(10 - 2x)$ . Again when the integration was carried out the constant of integration was sometimes omitted. Very few candidates were able to carry out the algebra required to obtain  $x = 5(1 - e^{-2t})$ , and of those who did only a few were able to explain why  $x < 5$  for all values of  $t$ .

Answers: (i)  $0.25v \frac{dv}{dx} = -(5 - x)$ ; (ii)  $x = 5(1 - e^{-2t})$ .

# MATHEMATICS

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Paper 9709/53

Paper 53

## General comments

Most candidates attempted all the questions.

It was encouraging to see that good clear diagrams were drawn by many candidates to help them with their solutions, and this enabled their work and reasoning to be more easily followed.

Only a few candidates used premature approximation and rounded answers to less than 3 significant figures. It should be noted that on this paper  $g = 10$  should be used and not  $g = 9.8$  or  $9.81$ , and that the formulae list can be used by candidates to check that they are using the correct formula.

**Questions 1, 2 and 4** proved to be a good source of marks for many candidates, while **Questions 6 and 7** proved to be the two questions which helped to discriminate between the average and the above average candidates.

## Comments on specific questions

### **Question 1**

This initial question proved to be accessible to candidates.

A few candidates made the error of finding the horizontal and vertical distances from the start and then used  $\tan \alpha = \frac{y}{x}$  instead of  $\tan \alpha = \frac{v_y}{v_x}$ .

*Answer:*  $59.0^\circ$  below the horizontal.

### **Question 2**

Again this question proved to be accessible to candidates.

A handful of candidates used the wrong formula for the distance of the centre of mass from the base, even though the formula appears on the formulae list. Many candidates recognised that they needed to take moments.

*Answer:* 0.48 kg.

### **Question 3**

- (i) Some candidates did not realise that the equation of trajectory is given in the formulae list.
- (ii) Again in this part of the question the formula for the range is given in the formula list and all that a candidate needed to do was to solve the given quadratic equation in  $\tan \theta$  to find both values of  $\theta$  and then substitute into the given formula. Some candidates who did not use the range formula made errors when trying to use more long-winded methods.

*Answer:* (ii) 91.1 m.



#### Question 4

- (i) Unfortunately a number of candidates used the wrong formula for the centre of mass. A few candidates tried to take moments about a point other than A but forgot that there were forces acting at A.
- (ii) This part of the question was generally well done. Candidates knew to resolve horizontally and vertically and were then able to find the resultant force and its direction.

Answer: (ii) 4.47 N at 26.6° with the vertical.

#### Question 5

- (i) Most candidates knew to resolve vertically in order to find the tensions and then to use Newton's second law in a horizontal direction using  $a = \frac{v^2}{r}$  for the acceleration. This part of the question was generally well done.
- (ii) The idea that  $T_{PB} = 0$  was missed on a few occasions which then created problems. When  $T_{PB} = 0$  was used it was then simply a matter of resolving vertically to find the tension in QB and then to use Newton's second law horizontally to find the speed of the ball.

Answers: (i) 3.12 m; (ii) 0 N,  $4.24 \text{ ms}^{-1}$

#### Question 6

While this question proved to be difficult for many candidates, more able candidates did score well.

- (i)  $0.5v \frac{dv}{dx} = -3v^{\frac{1}{2}}$  was often seen, but a few candidates omitted the minus sign and this then caused problems. Candidates usually knew to separate the variables and integrate. Occasionally the constant of integration was omitted.
- (ii) The use of  $v = \frac{dx}{dt}$  was often seen and again variables were usually separated and an integration was attempted. Unfortunately some candidates could not integrate  $(27 - 9x)^{\frac{2}{3}}$ . Again the constant of integration was occasionally omitted.

Answer: (ii) 2.63 m.

#### Question 7

- (i) Most candidates attempted to use the principle of conservation of energy but made sign errors in their equations. Elastic energy =  $\frac{\lambda x^2}{2l}$  was used by most candidates.
- (ii) Finding the greatest speed was often attempted either by a differentiation method or by a completing the square approach.
- (iii) Some candidates did not realise that the greatest tension would occur when  $v = 0$  and when the value of  $x$  is a maximum. Once this value of  $x$  was found it could be substituted into  $T = \frac{\lambda x}{l}$ .

Answers: (ii)  $8.31 \text{ ms}^{-1}$ ; (iii) 18.9 N.



# MATHEMATICS

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Paper 9709/61

Paper 61

## General comments

Many candidates found the paper challenging, with the binomial and normal distributions proving particularly difficult. A majority of candidates attempted most of the questions in order, and there was no evidence of lack of time being a problem. Clear, legible working is more likely to lead to the correct solution. Care should be taken to show preliminary working and maintain the required level of accuracy if marks are to be maximised. In questions inviting comments, it is not sufficient to produce definitions; the comments must be relevant and in the context of the situation.

## Comments on specific questions

### **Question 1**

Most candidates were able to obtain one equation involving the mean, but the equation using probability featured less often. Solving the two simultaneous equations was generally successful.

Answers: 0.2, 0.25.

### **Question 2**

Whilst this question was a main source of credit for many candidates, many marks were lost through carelessness.

- (i) Stems should be 0, 1, 2, 3 in a column, and the leaves ordered in columns. Candidates should be recommended to replace the diagram if and when they recognise an error has occurred. The key should be given with the appropriate unit, in this instance ‘people’.
- (ii) The quartiles presented problems for many candidates, who were unable to handle the concept of the 5.5th and 16.5th values. The interquartile range was surprisingly either unknown or overlooked by a lot of candidates.
- (iii) Credit was awarded here for comments indicating that the mode was of little or no use in this context.

Answers: (ii) 19, 10, 24, 14.

### **Question 3**

There was a disappointing response to this question. Those candidates aware of normal standardisation techniques rarely obtained full marks. Errors included: non-use of the tables; 1.45 was cited, or truncated to 1.5, instead of 1.045; 0.313 or 0.31 was used instead of -0.313; premature approximations whilst solving the simultaneous equations, and rounding errors finally for  $\mu$  and  $\sigma$ . A simple sketch with the given information on it would help to eliminate some of these errors.

Answers: 23.0, 6.70.



#### Question 4

- (i) Some candidates attempted unsuccessful arithmetical solutions to justify the statement.
- (ii) Candidates failed to recognise the need to find the cost of a revolving drum ride. Those that did then often divided by 3 instead of 10. There was a marked inability to substitute correctly in the variance formula, with  $\bar{x}^2$ ,  $\bar{x}x$  or  $(fx)^2$  often used. Another frequent error was the omission of the subtraction of the square of the mean.

Answer: (ii) 1.03.

#### Question 5

Very few candidates demonstrated familiarity with the binomial distribution.

- (i) Worthwhile attempts were usually successful, but were often marred by rounding errors in the final answer.
- (ii) Either  $np < 5$  or  $nq < 5$  are the tests for the ineligibility of using the normal approximation for the binomial distribution. Candidates often quoted  $npq$ , the size of  $n$ ,  $p = \frac{1}{2}$  and  $p \neq \frac{1}{2}$  as reasons.
- (iii) Some candidates presented correct answers, having used binomial expressions for the requisite number of terms. Many candidates failed to apply a continuity correction or did not realise that 'at least 13' implies the use of 12.5 in the standardisation procedure. There were many instances of applying the wrong tail, or not using tables to convert from a z-value to a probability. Again, a simple sketch would help in the former case.

Answers: (i) 0.311; (iii) 0.181.

#### Question 6

Very few correct solutions for any parts of this question were submitted.

- (i) Some candidates realised that there were five different situations and submitted 5 as the answer. Those who recognised that this was a question on combinations invariably evaluated 5 products of 2 combinations, not realising that an odd number for Lucy meant an odd number for Monica.
- (ii) Few candidates gained credit other than for writing 6!. Most attempts were by considering all possibilities and subtracting those with 3 china mugs together and those with 2 together; unfortunately the latter was often overlooked.
- (iii) Again, 12! was often the only source of credit, with 3! frequently appearing in the denominator.

Answers: (i) 512; (ii) 151 200; (iii) 3960.

#### Question 7

- (i) Most candidates successfully attained the given answer.
- (ii) The few candidates who drew a tree diagram identified the correct outcomes. Most candidates submitted a solution based on  $P(1\text{st correct}) \times P(2\text{nd correct})$ , overlooking  $0.7 \times 0.2 \times 0.95$ .
- (iii) Very few candidates submitted reasonable solutions, with the concept of conditional probability not materialising beyond some quoted formula.

Answers: (ii) 0.781; (iii) 0.372.



# MATHEMATICS

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Paper 9709/62

Paper 62

## General comments

The paper contained a variety of questions including some parts that required some knowledge of the meaning of terms commonly used in the syllabus, for example the mean, mode, median, exclusive and independent events. These parts were found difficult by many candidates who just quoted standard answers not in context and did not apply this knowledge to the particular situation in the question.

One important failure by candidates was the inability to work with 4 (or more) significant figures in order to give an answer correct to 3 significant figures. This caused many answers to be inaccurate and thus marks were unnecessarily lost.

Another failure was an inability to manipulate fractions, even with the use of a calculator. This was noticeable particularly in **Question 4**.

## Comments on specific questions

### **Question 1**

- (i) This question on finding the mean and standard deviation of 7 data values could be done by calculator in SD mode, but many candidates used the standard formulae to evaluate both mean and standard deviation. Those candidates who used 3 significant figures in working out the standard deviation obtained a value of 12.2 instead of 12.3 for the standard deviation and lost a mark.
- (ii) The majority of candidates answered this part correctly.
- (iii) Candidates from some Centres were well prepared and gave good reasons for their choice in part (ii). However there were at least an equal number who gave largely fictional answers, reflecting a lack of understanding of the usefulness of mean, mode or median, or did not know, or quoted general standard answers. Many candidates scored poorly on this question.

**Answers:** (i) 18.9, 12.3; (ii) median; (iii) mode, 10, inappropriate because it is the smallest number; mean inappropriate because it is affected by the outlier, 48

### **Question 2**

This question was standard: finding a probability using the normal distribution, followed by a binomial situation which involved using the probability found in part (i). It was pleasing that almost all candidates managed to standardise successfully with only a few candidates using a continuity correction (for which they obtained no marks). However, a very large proportion of candidates found the wrong probability. A diagram would have helped, showing that the probability had to be greater than 0.5. In part (ii) one mark was awarded for recognising a binomial situation regardless of which probability was used, and another mark was given for the correct unsimplified version of the answer, allowing follow-through from their previous part. Again, premature approximation from part (i) often led to a loss of an accuracy mark in part (ii) with 0.214 being seen.

**Answers:** (i) 0.854; (ii) 0.215.



### Question 3

In many Centres this question was well done. Candidates knew they had to find the median for central tendency and the interquartile range for spread. Finding these correctly from the graph gave 4 marks. Interpreting these in relation to the context of the question gave a further 2 marks. However, there were some Centres where candidates did not appear to know what to do. Some found the mean and standard deviation, which gained 4 marks but took a long time.

Answers: median  $A = 2.0$ , median  $B = 3.8$ , interquartile range  $A = 0.9$ , interquartile range  $B = 2.3$ , country  $B$  has heavier babies and greater spread than country  $A$ .

### Question 4

Almost all candidates standardised successfully in both parts, but many were unable to eliminate one variable and thus could not proceed further. Of those who did, dividing by  $\frac{3}{5}$  also proved an obstacle, even with the aid of a calculator. Part (ii) involved looking up a probability backwards in the normal tables to gain a z-value. There was only one answer, 1.047, but answers were seen of 1.045, 1.046, 1.048 and so on. These candidates had not used their normal tables carefully enough and so lost a mark. The question asked candidates to express  $\mu$  in terms of  $\sigma$ . Many candidates could not cope with this, expressing  $\sigma$  in terms of  $\mu$  or ignoring the minus sign.

Answers: (i) 0.952, (ii)  $-1.57\sigma$ .

### Question 5

Most candidates correctly identified 6 outcomes which satisfied the requirements of Q. Some then divided by 6<sup>2</sup> having not realised that this was a 12-sided die. The last two parts on exclusive and independent events were answered perfectly by candidates from a few Centres but a majority of candidates had trouble remembering which was which, getting the requirements for each correct, and then applying these conditions to the relevant question.

Answers: (i)  $\frac{1}{24}$ ; (ii)  $\frac{1}{9}$ ; (iii) Since  $P(Q \cap R) = 0$ , yes, they are mutually exclusive; (iv) Since  $P(Q) \times P(R) \neq 0$ , they are not independent.

### Question 6

This routine question was well attempted by the majority of candidates who gained full marks. Those who did not gain full marks lost them mainly because they failed to recognise that this was not a binomial situation but one where the probabilities change each time a goose is chosen. Candidates appeared to forget that the sum of probabilities in a probability distribution table should add up to 1. Almost everybody managed to get 0, 1, 2 for the values taken by the random variable and were given credit for knowing how to find the variance. Part (ii) was the best attempted part of the whole paper. It was pleasing to see a recognition of conditional probability.

Answers: (i) 0,  $\frac{1}{7}$ ; 1,  $\frac{4}{7}$ ; 2,  $\frac{2}{7}$ ; (ii)  $\frac{20}{49}$ ; (iii)  $\frac{5}{32}$ .

### Question 7

This question, surprisingly, was one of the best attempted overall on the whole paper. Everybody managed to answer part (i) correctly, and a very large proportion managed most of parts (ii), (iii) and (iv). Candidates should realise that a brief word of what they are finding would help gain method marks, whereas a wrong answer with no working invariably gets no marks at all.

Answers: (i) 362 880; (ii) 282 240; (iii) 504; (iv) 168; (v) 476.



# MATHEMATICS

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Paper 9709/63

Paper 63

## General comments

There was no evidence that candidates had any problems with lack of time. Candidates usually gained most of the marks available on **Questions 3** and **5**, whilst **Questions 1** and **4** proved the most challenging. A considerable number of candidates lost marks because they did not work to an appropriate degree of accuracy. Candidates at this level should appreciate that an integer larger than 1 is inappropriate when the question asks for a probability and that a standard deviation must be positive.

## Comments on specific questions

### **Question 1**

The most frequent answer was  ${}^7C_3 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^4$ , which incorrectly assumed that the probabilities of a red sweet and a non-red sweet were constant. This and nearly all other attempts involving probabilities gained the method mark for multiplying a probability by  ${}^7C_3$ . Those candidates who used combinations were usually more successful. Sometimes the number of red sweets (3) was subtracted from the total number of sweets (52), resulting in work which involved 49 rather than 39.

*Answer:* 0.176.

### **Question 2**

Attempts at this question produced a wide spread of marks. An appreciable number of candidates made a genuine attempt at only one of the two parts, often gaining both of the marks for that part. A few candidates used 81 instead of 82.

- (i) Several solutions reached -3.5 but did not continue to add this to 130.
- (ii) This part was more often answered well. The majority of incorrect answers arose from either using 126.5 instead of -3.5 or 6.9 instead of 6.9<sup>2</sup>. Some candidates did use a correct expression involving 126.5 in order to calculate the value of  $\sum x^2$  (= 1316088.52), but very few of these candidates continued to obtain the correct answer, usually failing to multiply 130<sup>2</sup> by 82 when expanding  $\sum(x - 130)^2$ .

*Answers:* (i) 127 cm; (ii) 4910.

### **Question 3**

- (i) This part was usually answered well. Nearly all mistakes were from either omitting  ${}^7C_6$  in an otherwise correct binomial expression, or calculating the probability of going to the park on 5 or more days.
- (ii) This part was nearly always answered correctly. Occasionally only one 2-factor product was considered. Tree diagrams were not used as often as usual.
- (iii) A few candidates used a probability other than 0.6 in this part. Some gave the mean or the standard deviation instead of the variance.

*Answers:* (i) 0.159; (ii) 0.51; (iii) 7.2.



#### Question 4

The majority of candidates gave a series of numbers, usually involving factorials, with little or no explanation. A wrong answer with no working is awarded no marks, so an answer of 96 only in part (ii) would be awarded no marks, whereas an answer  $5! - 4! = 96$  would be awarded both of the method marks.

- (i) Most candidates appreciated that the number of arrangements with cola at the ends was different from the number of arrangements when green tea or orange juice are at the ends. The number of candidates who then continued to calculate the number of permutations of 5 objects when 2 (or 3) are alike of the first kind and 2 are alike of a second kind was disappointing.
- (ii) It was expected that candidates would apply the ideas from part (i), subtracting the number of ways with the cans of cola together and the cans of green tea together from the number of ways with the cans of cola together and no other restrictions. Attempts involving the placement of the cans of green tea amongst the three arrangements of three cans of cola with the two cans of orange juice were more popular, though generally less successful. A small proportion of the candidates tried to list all of the possible arrangements, sometimes grouping their arrangements according to the position of the 3 cans of cola. This approach was the least successful.

Answers: (i) 50; (ii) 18.

#### Question 5

A few candidates did not understand the definition of the random variable  $X$ . Often this resulted in considering the set  $B$  only in part (i) and probabilities involving either sevenths or seventeenth in part (ii). The majority of candidates answered the first three parts correctly.

- (i) Adding the probabilities of a 0 from set  $A$  with a 2 from set  $B$  and a 2 from set  $A$  with a 0 from set  $B$  was slightly more popular than finding a possibility space.
- (ii) This part was usually answered correctly. Some tables included the values 1, 3 and 5, usually with the probability 0. Occasionally the value of 6 was overlooked.
- (iii) The most frequent error was failing to subtract  $(E(X))^2$  when calculating  $\text{Var}(X)$ . Candidates working to 3 significant figures obtained the answer 2.77 instead of 2.78, thereby losing the accuracy mark.
- (iv) This part proved quite challenging. Often candidates did not realise that conditional probability was appropriate. Several attempts involved  $\frac{3}{7}$  in the numerator as well as the denominator of a fraction.

Answers: (ii)  $0, \frac{24}{70}; 2, \frac{30}{70}; 4, \frac{13}{70}; 6, \frac{3}{70}$ ; (iii) 1.86, 2.78; (iv) 0.4.

#### Question 6

Some candidates counted from the wrong end of the leaves, so, for example, the median length of the insects in country  $X$  was given as 0.826 cm. Several candidates worked with 71 and 83 instead of 72 and 84 respectively.

- (i) A few candidates listed the quartiles but did not calculate the interquartile range.
- (ii) Whilst many candidates found this part rather difficult, a few realised that since  $q$  and  $r$  had to be involved they were able to correct any misconceptions in part (i). A frequent mistake was to leave the answers as 0.824 and 0.852, rather than 4 and 2 respectively.
- (iii) The most popular scale was 2 cm to represent 0.01 cm, which produced good results. Some candidates attempted to use the full width (or height) of their graph paper to cover 0.802 cm to 0.869 cm. This made drawing the box-and-whisker plots extremely difficult, and rarely correct. Only one number line should have been drawn, with the units written next to it. A few candidates superimposed their box-and-whisker plots.



- (iv) Those candidates familiar with the use of box-and-whisker plots to compare two sets of data realised that for 2 marks statements involving the lengths and the spreads were required.

Answers: (i) 0.825 cm, 0.019 cm; (ii) 4, 2; (iv) insects from country Y are longer on average; the lengths of insects from country Y have a larger range.

### Question 7

- (i) Nearly all attempts involved using the normal tables in the appropriate manner to find two z-values. The majority of incorrect z-values arose from using probabilities of 0.6, 0.66, 0.666, 0.667, 0.67 or 0.7 instead of 0.6667. Problems choosing the appropriate signs might have been eased with the aid of a sketch. Several candidates made mistakes solving their simultaneous equations, sometimes leading to a negative standard deviation and/or a mean quite different from 127 or 135. Only occasionally did one of these candidates go back and correct their working.
- (ii) Most candidates were able to use their mean and standard deviation in order to calculate the probability that the height jumped by a randomly chosen child was less than 145 cm. A large proportion of candidates lost the final accuracy mark through using 132 instead of 132.2.
- (iii) A few candidates realised that this part of the question could be answered independently of the other two parts. Sometimes a probability other than  $\frac{1}{3}$  was used, their answer to part (ii) being quite popular. Several candidates used the normal approximation to the binomial distribution, which was not appropriate.

Answers: (i) 132 cm, 6.29 cm; (ii) 0.978; (iii) 0.805.



# MATHEMATICS

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Paper 9709/71

Paper 71

## General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. Whilst there were many good scripts, there were also candidates who appeared completely unprepared for the paper. In general, candidates gave good answers to **Questions 2(i), 3(i), 4 and 6(i)**, whilst **Question 7** proved more demanding. **Question 1** also proved difficult for many candidates, who either omitted the question completely, or attempted it after completion of the rest of the paper. **Question 5(ii)** required both graphical knowledge and interpretation, and this was also poorly attempted.

Accuracy caused loss of marks for some candidates, though this was not as significant a problem as has been noted in the past. On the whole, presentation was good and an adequate amount of working was shown by candidates, though there were some cases where Examiners had to withhold marks due to lack of essential working. This was particularly noted in **Question 7** (see further comments below). Lack of time did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

## Comments on specific questions

### **Question 1**

Few candidates were able get both parts of this question correct. Part (i) was better attempted than part (ii). Common incorrect answers seen to part (i) included answers of  $\frac{1}{144}$ ,  $\frac{1}{48}$  and  $\frac{1}{6}$ . In part (ii) many candidates made statements referring to the 'lateness' of trains. Very few candidates gave a fully correct statement about arrival times.

**Answers:** (i)  $\frac{1}{12}$ ; (ii) Trains arrive every 12 minutes.

### **Question 2**

Part (i) of this question was well attempted. In part (ii) many candidates correctly equated half the width (0.0321) to  $zx\sqrt{\frac{pq}{n}}$ , and found the correct z-value of 2.233. However, not many candidates then went onto use a correct method to find the width of the confidence interval. All too often candidates, having worked correctly up to this point, thought the value of  $\alpha$  was 98.7.

**Answers:** (i) 0.145, 600; (ii) 97.4.

### **Question 3**

Part (i) was well attempted. In general, candidates used a correct method for part (ii); the main errors seen were the use of an incorrect z-value (1.282 rather than the correct value of 1.645), or the identification of an incorrect rejection region. Incorrect statements such as  $2.71 < m < 2.53$  were noted by Examiners.

**Answers:** (i) 0.941; (ii)  $m < 2.53$ ,  $m > 2.71$ .



#### Question 4

In general, this was a reasonably well attempted question. In part (i) candidates mostly used the correct distribution of ‘Mr – 5 × Mrs’, though an incorrect variance was sometimes seen. Some candidates incorrectly considered ‘5 × Mr – Mrs’. Errors in part (ii) included multiplying the variance by  $\frac{5}{8}$  rather than  $\frac{25}{64}$  to convert to miles, and weaker candidates used a conversion factor of  $\frac{8}{5}$ . Some candidates gave separate answers of 320 and 55.6 for Mr and Mrs Parry.

Answers: (i) 0.823; (ii) 376, 39.0.

#### Question 5

Many candidates used a correct method to find  $k$ , and some were successful in integrating and using the correct limits to reach the given answer for  $k$ . Errors noted were multiplication of 0.2 rather than division in the integrand, and weaker candidates were unable to deal with the integration of an exponential function at all. Part (ii) was not well attempted; whilst many candidates were able to sketch the correct shaped curve between 0 and 5, only a very few candidates considered less than 0 and greater than 5. In part (iii), some candidates were able to reach the correct final answer, but a particularly common error was to use the correct integral from 0 to  $T$ , but equate to 0.8 rather than 0.2. Candidates found this question more demanding than similar questions in the past.

Answer: (iii) 1.48 seconds.

#### Question 6

This question was well attempted with part (i), in particular, being a good source of marks for many candidates. Most candidates used a correct value of  $\lambda$  (4.4) and used a correct Poisson expression to find the probability of more than 2. Common errors on part (i) noted by Examiners included using separate means of 1.21 and 3.19 to find two separate probabilities, along with a possible subsequently combination. Part (ii) was reasonably well answered, though reaching the correct answer of  $n > 57.6$  did not always result in the correct value for  $n$  (58) being chosen, a value of 57 was often offered. Some candidates found the value of  $\lambda$  from a correct equation but did not realise that  $\lambda$  was equal to  $0.08n$ .

Answers: (i) 0.815; (ii) 58.

#### Question 7

Once again, candidates had difficulty in answering part (i) ‘in the context of the question’ as required. As this test was a one-tail test it was also important to reflect this in the contextual explanation of the Type I error. Part (i) also caused a loss of marks to many candidates who did not show all relevant working. A correct answer of 0.0342 only scored if the relevant method (comparison of the sums  $P(0) + P(1)$  and  $P(0) + P(1) + P(2)$  with 10%) had been clearly shown. Few candidates considered the total of 0.1087 and compared it to 0.1, merely showing the total and comparison of  $P(0) + P(1)$ . Some candidates thought that the probability of a Type I error had to be 0.1. More candidates showed relevant working in part (ii), though for  $H_0$  or  $H_1$ ,  $\mu$  was often used instead of  $\lambda$ , and some only considered  $P(2)$  instead of a sum. Some candidates correctly found the probability of a Type II error in part (iii), others found the wrong area, but gained some credit for a Poisson expression with a correct mean of 4.1.

Answers: (i) The number of white blood cells has decreased when it has not, 0.0342; (iii) 0.915.



# MATHEMATICS

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Paper 9709/72

Paper 72

## General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. Whilst there were many good scripts, there were also candidates who appeared completely unprepared for the paper. In general, candidates gave good answers to **Questions 2(i), 3(i), 4 and 6(i)**, whilst **Question 7** proved more demanding. **Question 1** also proved difficult for many candidates, who either omitted the question completely, or attempted it after completion of the rest of the paper. **Question 5(ii)** required both graphical knowledge and interpretation, and this was also poorly attempted.

Accuracy caused loss of marks for some candidates, though this was not as significant a problem as has been noted in the past. On the whole, presentation was good and an adequate amount of working was shown by candidates, though there were some cases where Examiners had to withhold marks due to lack of essential working. This was particularly noted in **Question 7** (see further comments below). Lack of time did not appear to be a problem for candidates.

Whilst the comments below indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

## Comments on specific questions

### **Question 1**

Few candidates were able get both parts of this question correct. Part (i) was better attempted than part (ii). Common incorrect answers seen to part (i) included answers of  $\frac{1}{144}$ ,  $\frac{1}{48}$  and  $\frac{1}{6}$ . In part (ii) many candidates made statements referring to the 'lateness' of trains. Very few candidates gave a fully correct statement about arrival times.

Answers: (i)  $\frac{1}{12}$ ; (ii) Trains arrive every 12 minutes.

### **Question 2**

Part (i) of this question was well attempted. In part (ii) many candidates correctly equated half the width (0.0321) to  $zx\sqrt{\frac{pq}{n}}$ , and found the correct z-value of 2.233. However, not many candidates then went onto use a correct method to find the width of the confidence interval. All too often candidates, having worked correctly up to this point, thought the value of  $\alpha$  was 98.7.

Answers: (i) 0.145,  $n = 600$ ; (ii) 97.4.

### **Question 3**

Part (i) was well attempted. In general, candidates used a correct method for part (ii); the main errors seen were the use of an incorrect z-value (1.282 rather than the correct value of 1.645), or the identification of an incorrect rejection region. Incorrect statements such as  $2.71 < m < 2.53$  were noted by Examiners.

Answers: (i) 0.941; (ii)  $m < 2.53$ ,  $m > 2.71$ .



#### Question 4

In general, this was a reasonably well attempted question. In part (i) candidates mostly used the correct distribution of ‘Mr – 5 × Mrs’, though an incorrect variance was sometimes seen. Some candidates incorrectly considered ‘5 × Mr – Mrs’. Errors in part (ii) included multiplying the variance by  $\frac{5}{8}$  rather than  $\frac{25}{64}$  to convert to miles, and weaker candidates used a conversion factor of  $\frac{8}{5}$ . Some candidates gave separate answers of 320 and 55.6 for Mr and Mrs Parry.

Answers: (i) 0.823; (ii) 376, 39.0.

#### Question 5

Many candidates used a correct method to find  $k$ , and some were successful in integrating and using the correct limits to reach the given answer for  $k$ . Errors noted were multiplication of 0.2 rather than division in the integrand, and weaker candidates were unable to deal with the integration of an exponential function at all. Part (ii) was not well attempted; whilst many candidates were able to sketch the correct shaped curve between 0 and 5, only a very few candidates considered less than 0 and greater than 5. In part (iii), some candidates were able to reach the correct final answer, but a particularly common error was to use the correct integral from 0 to  $T$ , but equate to 0.8 rather than 0.2. Candidates found this question more demanding than similar questions in the past.

Answer: (iii) 1.48 seconds.

#### Question 6

This question was well attempted with part (i), in particular, being a good source of marks for many candidates. Most candidates used a correct value of  $\lambda$  (4.4) and used a correct Poisson expression to find the probability of more than 2. Common errors on part (i) noted by Examiners included using separate means of 1.21 and 3.19 to find two separate probabilities, along with a possible subsequent combination. Part (ii) was reasonably well answered, though reaching the correct answer of  $n > 57.6$  did not always result in the correct value for  $n$  (58) being chosen, a value of 57 was often offered. Some candidates found the value of  $\lambda$  from a correct equation but did not realise that  $\lambda$  was equal to  $0.08n$ .

Answers: (i) 0.815; (ii) 58.

#### Question 7

Once again, candidates had difficulty in answering part (i) ‘in the context of the question’ as required. As this test was a one-tail test it was also important to reflect this in the contextual explanation of the Type I error. Part (i) also caused a loss of marks to many candidates who did not show all relevant working. A correct answer of 0.0342 only scored if the relevant method (comparison of the sums  $P(0) + P(1)$  and  $P(0) + P(1) + P(2)$  with 10%) had been clearly shown. Few candidates considered the total of 0.1087 and compared it to 0.1, merely showing the total and comparison of  $P(0) + P(1)$ . Some candidates thought that the probability of a Type I error had to be 0.1. More candidates showed relevant working in part (ii), though for  $H_0$  or  $H_1$ ,  $\mu$  was often used instead of  $\lambda$ , and some only considered  $P(2)$  instead of a sum. Some candidates correctly found the probability of a Type II error in part (iii), others found the wrong area, but gained some credit for a Poisson expression with a correct mean of 4.1.

Answers: (i) The number of white blood cells has decreased when it has not, 0.0342; (iii) 0.915.



# MATHEMATICS

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Paper 9709/73

Paper 73

## General comments

The paper proved accessible to the majority of candidates. Many scored highly, in particular on **Questions 3, 5, 6 and 7**. Candidates performed particularly well on applications of the normal and Poisson distributions. A significant minority found coping with unusual limits in the continuous random variable question challenging, and there is scope for improvement on significance testing and applying statistical principles in context. Solutions were generally very well presented, to the required accuracy, and lack of time did not appear to be a problem for candidates.

## Comments on specific questions

### Question 1

Most candidates wrote the two correct hypotheses although a number wrote  $\mu$  rather than  $p$  as the parameter. Few actually defined  $p$ .

The question required the calculation of the test statistic  $P(X < 3)$  using a binomial distribution with  $n = 20$  and  $p = \frac{1}{3}$ . A number of candidates only calculated  $P(X = 3)$ , and some

failed to secure the method mark for a correct comparison, merely talking about the result not being significant; a comparison with 0.025 was essential to perform the test. Candidates who did this then nearly always wrote a correct conclusion based on their test statistic. Use of a normal approximation to the binomial was seen on many occasions.

*Answer:* No evidence of a reduction in support for the Citizens Party.

### Question 2

Whilst nearly all candidates recognised that the test was two-tailed, many gave an explanation that lacked the required detail. A specific reference to the alternative hypothesis was needed, or a convincing explanation of why there were two tails. In part (ii) many good answers were seen, but a large number of candidates did not show enough to demonstrate that  $-1.75$  was compared to  $-1.645$ . This can be done on a sketch of the normal distribution provided both numbers are shown, or as an inequality. As in **Question 1** there were many answers that just stated 'reject  $H_0$ '. A number of candidates compared with  $-1.282$ . Most candidates who scored the method mark gave a correct conclusion. Part (iii) required the use of tables to find  $P(z < -1.75)$ , then doubling (as the test is two-tailed) and rounding to the nearest whole number. The most common errors were the answers 4%, 92% or 8.02%. A few candidates put 5%.

*Answers:* (i) two-tailed test; (iii) 8.

### Question 3

This question was very well answered. The vast majority of candidates knew that the sample mean was distributed normally and gave the correct parameters. A few candidates gave an entirely written explanation that often lacked the key points. In the second part some who had failed to score in part (i) then used the correct distribution to find the probability that the sample mean was greater than 64. Very few omitted the  $\sqrt{50}$ , and nearly all used normal tables correctly.

*Answers:* (i)  $N(62, \frac{8.2^2}{50})$ ; (ii) 0.0423.



#### Question 4

This question, which focused on Type I and Type II errors, was better answered than those on previous papers. The requirement to interpret a Type I error in context was met by most, although some candidates just wrote ‘reject  $H_0$  when true’. A small number of candidates described a Type II error, but some gave a contradictory statement such as ‘the mean has decreased but not decreased’. In part (ii) many candidates correctly calculated the required probability  $P(X < 1 | \lambda = 4.8)$  but others simply wrote 5% or attempted a normal calculation. In part (iii) many correct solutions were seen; the most common error was to calculate  $P(X < 1 | Po(0.9))$ , and a number of candidates calculated  $P(X > 1)$  rather than  $P(X < 1)$ .

Answers: (i) Conclude there is a reduced number of breakdowns when no reduction has occurred;  
(ii) 0.0477; (iii) 0.228.

#### Question 5

The major obstacle the candidates faced on this question was the definition of the range of the probability density function as  $x > 1$ . Whilst many realised this meant integration limits of 1 and  $\infty$ , many did not, using limits of 0 and 1 or 1 and 2 amongst others. This issue apart, candidates were generally able to perform all the necessary stages to verify that  $k = 3$  and to find  $E(X)$  and  $Var(X)$ , although some only found  $E(X^2)$ . A few candidates failed to combine  $xf(x)$  before integrating, and a small number made errors in integrating. Only the method marks could be scored without the correct limits.

Answers: (ii) 1.5, 0.75.

#### Question 6

This question was a good source of marks for the majority of candidates. They realised in part (i) that they should work with a normal distribution with  $\mu = 5 \times (22.4 + 20.3)$  and  $\sigma^2 = 5 \times (4.8^2 + 5.2^2)$ . Just a few candidates worked with a variance of  $25 \times (4.8^2 + 5.2^2)$ . Most candidates standardised and used tables correctly. In part (ii) most candidates correctly subtracted the means and added the variances and found the required probability. A few subtracted the variances. There were very few standardisation errors.

Answers: (i) 0.983; (ii) 0.383.

#### Question 7

The need to explain the use of a Poisson distribution in context caused difficulty. Some candidates quoted a proportion or all of the four main requirements but out of context, e.g. ‘events occur randomly’ rather than ‘patients arrive randomly’. Some, however, talked about features of the distribution such as ‘mean equals variance’, or conditions around approximating by or to a Poisson. Parts (ii)(a) and (b) were well answered. Most candidates realised that  $P(X > 1)$  was calculated by  $1 - P(X = 0)$  and nearly all could adjust the Poisson mean and find  $P(X < 4)$ , although some included  $P(X = 4)$  in this. In part (iii) most realised that the required approximation was  $N(336, 336)$ , but a large number failed to apply the continuity correction. To have insufficient vaccine, the required calculation was  $P(X > 370.5)$ . A good number of candidates calculated  $P(X < 369.5)$ , showing that they failed to understand what was required.

Answers: (i) Patients arrive at a constant mean rate, patients arrive at random; (ii)(a) 0.985; (b) 0.692;  
(iii) 0.0300.

