



**TÉCNICO**  
LISBOA

# **Computer Control**

## **Identification and Computer Control of a Flexible Robot Arm Joint**

### **Group 3**

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# System Identification

- Previous knowledge:
  - System is causal
  - Non-minimum phase zero
  - Oscillatory behaviour
- Excitation signal:
  - Square wave with  $f = 0.1$  Hz
  - Amplitude is indifferent
- Sampling frequency:
  - $f_s = 10$  Hz
- Pre-processing
  1. Differentiation and LP filter
  2. Detrend
  3. Burning initial 20 seconds

# System Identification

- ARMAX model:
  - $na = 3, nb = 3, nc = 3, nk = 1$
  - Training accuracy (square wave): 98.398%
  - Test accuracy (PRBS wave): 96.150%
- State-Space Representation:

$$x(k+1) = \begin{bmatrix} 2.192 & -2.424 & 1.679 & -0.448 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k)$$
$$y(k) = [0.0057 \quad -0.098 \quad -0.137 \quad 0] x(k)$$

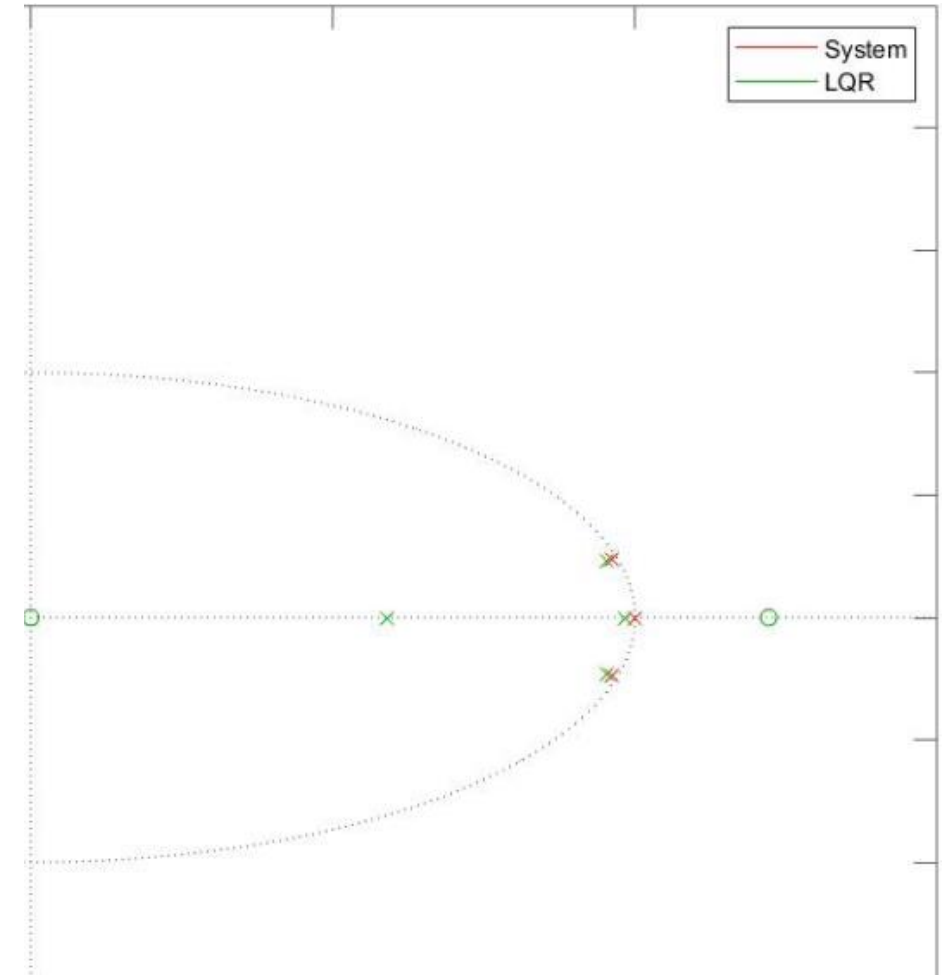
# System Identification

- After a few tests with the real system, it was concluded that the model previously identified was of too low order.
- New state space model:

$$x(k+1) = \begin{bmatrix} 3.293 & -4.851 & 5.170 & -4.579 & 2.538 & -0.571 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k)$$
$$y(k) = [0.0008 \quad 0.0029 \quad 0.0046 \quad -0.0114 \quad 0 \quad 0] x(k)$$

# LQR - Linear Quadratic Regulator

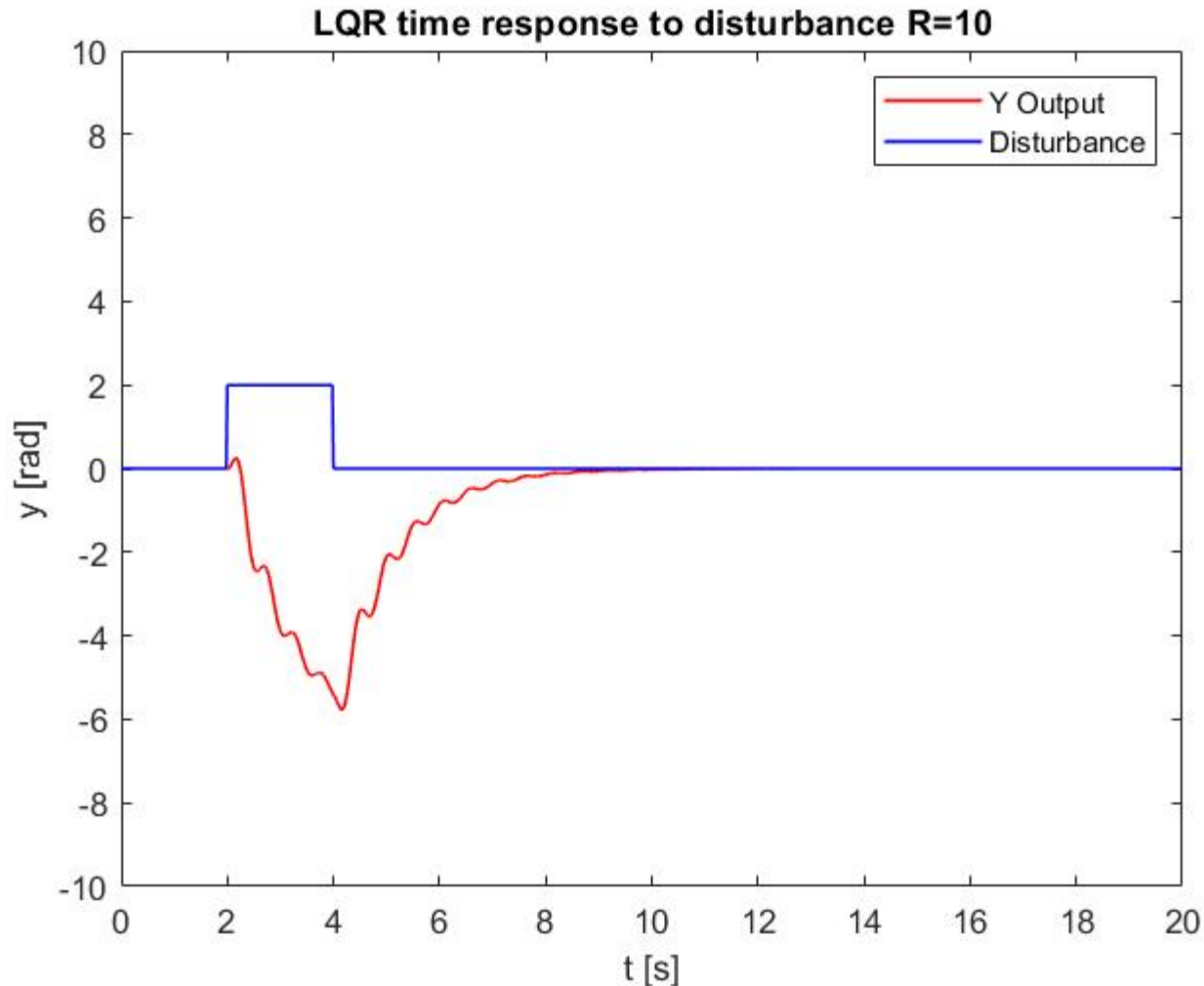
- The LQR allows optimal control with the guarantee of stability, achieved with the help of Symmetric Root-Locus
- For the case at hand, the control action is regulated by the “tuning knob”  $R$
- Trade-off in choosing  $R$ :
  - High  $R$ : **less sensitivity** to high-frequency noise but the system is **slower**
  - Low  $R$ : system is **faster** but control action might result in **damaged hardware**



Effect of the LQR with  $R=10$ . Dominant poles are pulled away from the unit circle resulting in a faster and less oscillating system

# LQR - Validation

A disturbance is given to the system. Time response and rate of decay are analysed.



The time response of the controlled system after the disturbance finishes, can be interpreted as the impulse response of the system.

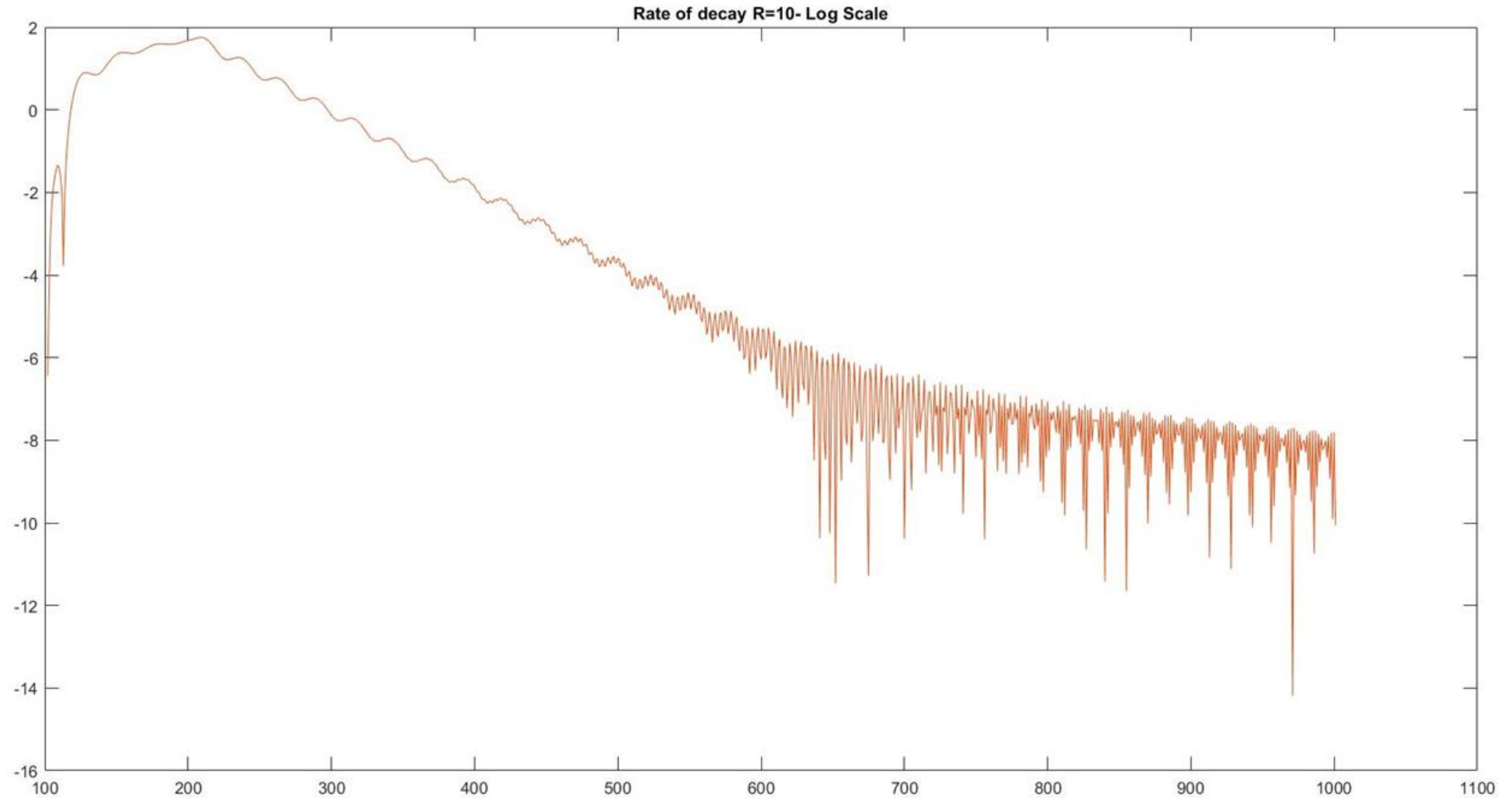
By making use of the properties of the logarithm, the slope of the rate of decay is calculated and compared with the dominant pole(s) of the controlled system.

$$\text{Log}(|\text{dominant pole}|) = -0.0188$$

$$\text{Slope of rate of decay} = -0.0173$$

$$\text{Difference} = 8\% < 10\%$$

# LQR - Validation

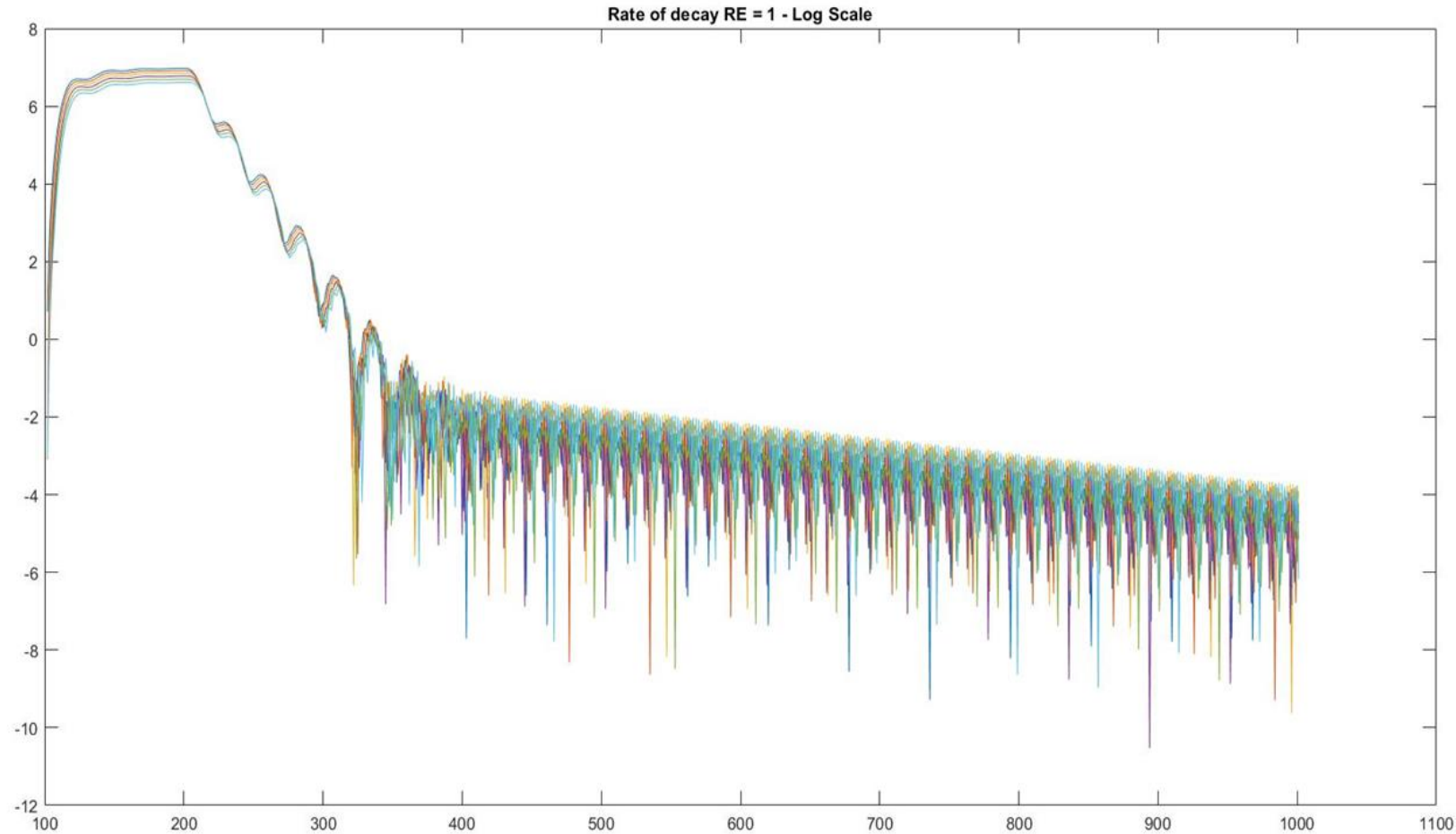


# LQE - Current Observer

- The LQE allows the estimation of state variables, even those that are not observable, with the Process and Sensor noise included in the system
  - QE : Covariance of Process Noise
  - RE : Covariance of Sensor Noise
- For the case at hand, similarly to the LQR, QE is fixed and RE is the “tuning knob” of the Observer. The poles of closed-loop will be the same symmetric Root-Locus as the poles of the LQR
- The LQE is validated in the same way as the LQR.
- Only constraint : LQE must be faster than the LQR, so that the estimated states can be used by the LQR



# LQE - Current Observer



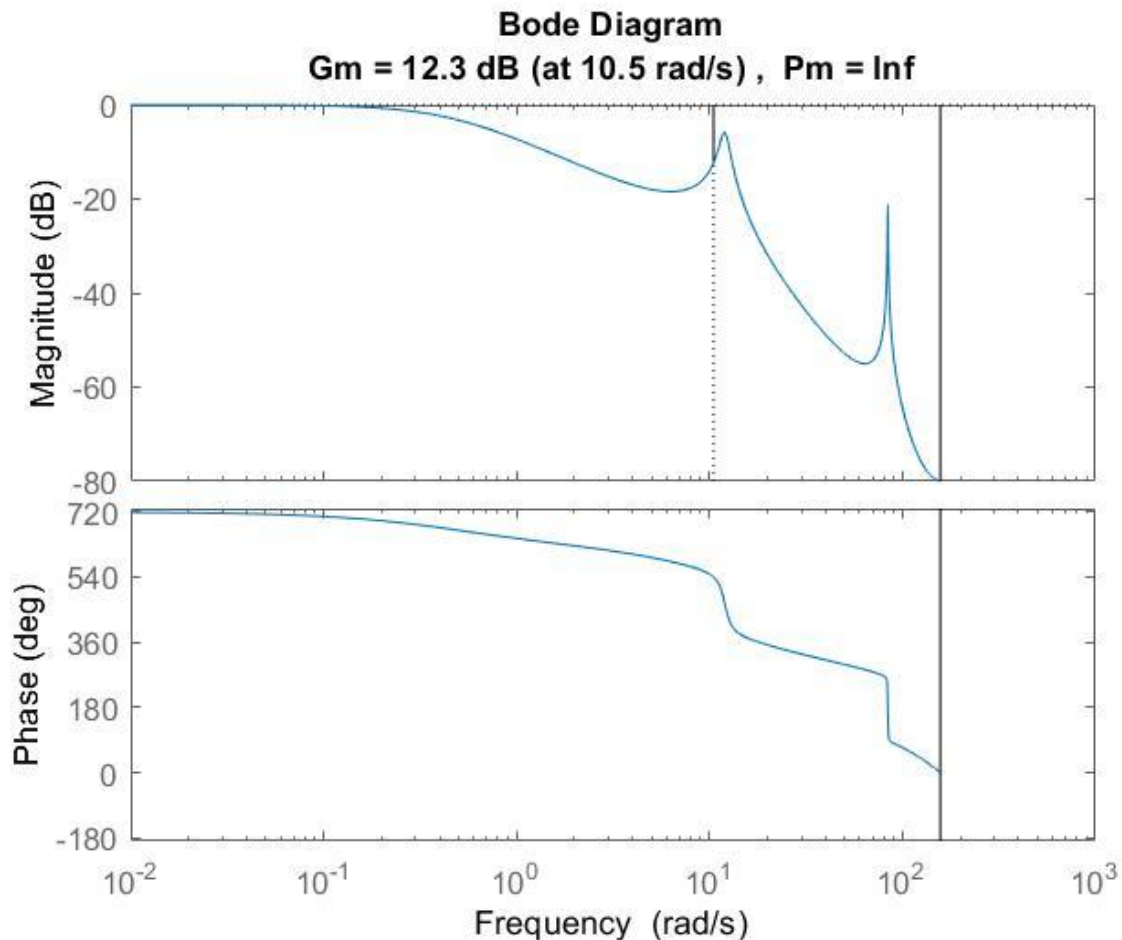
$\text{Log}(|\text{dominant pole}|) = -0.0550$

Slope of rate of decay = -0.0531

**Difference= 6% < 10%**

# LQG - Linear Quadratic Gaussian

The Linear Quadratic Gaussian is the joining of the LQR and LQE, according to the Separation Principle.



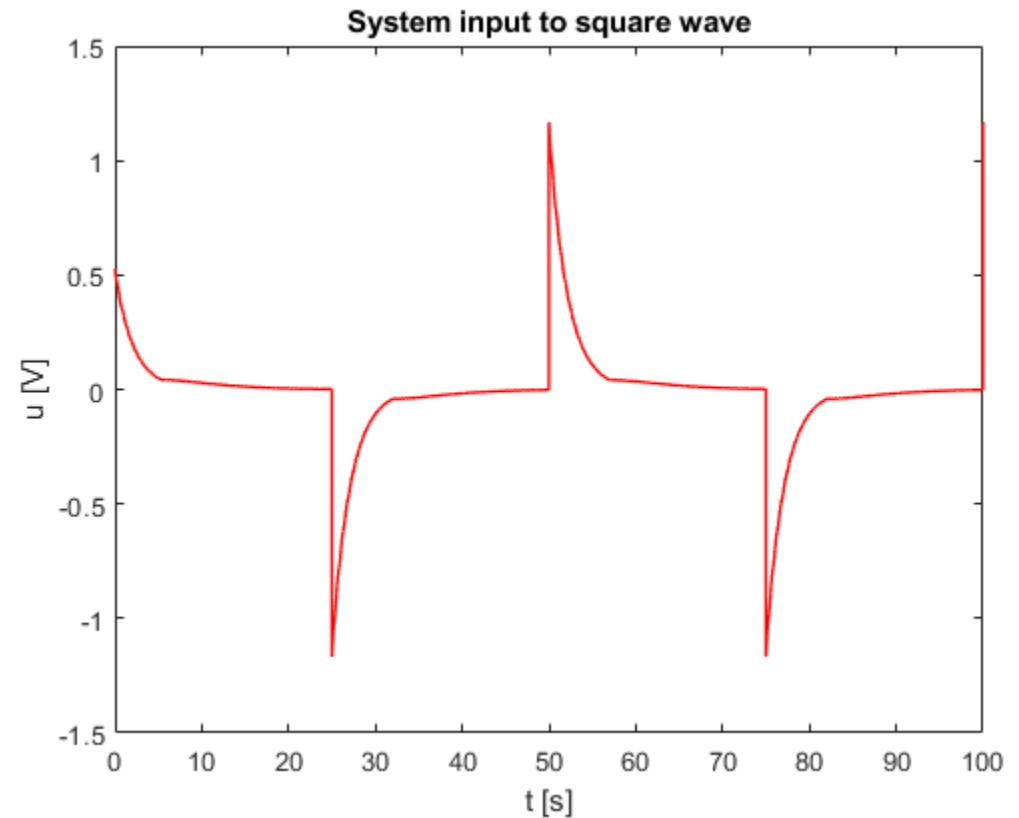
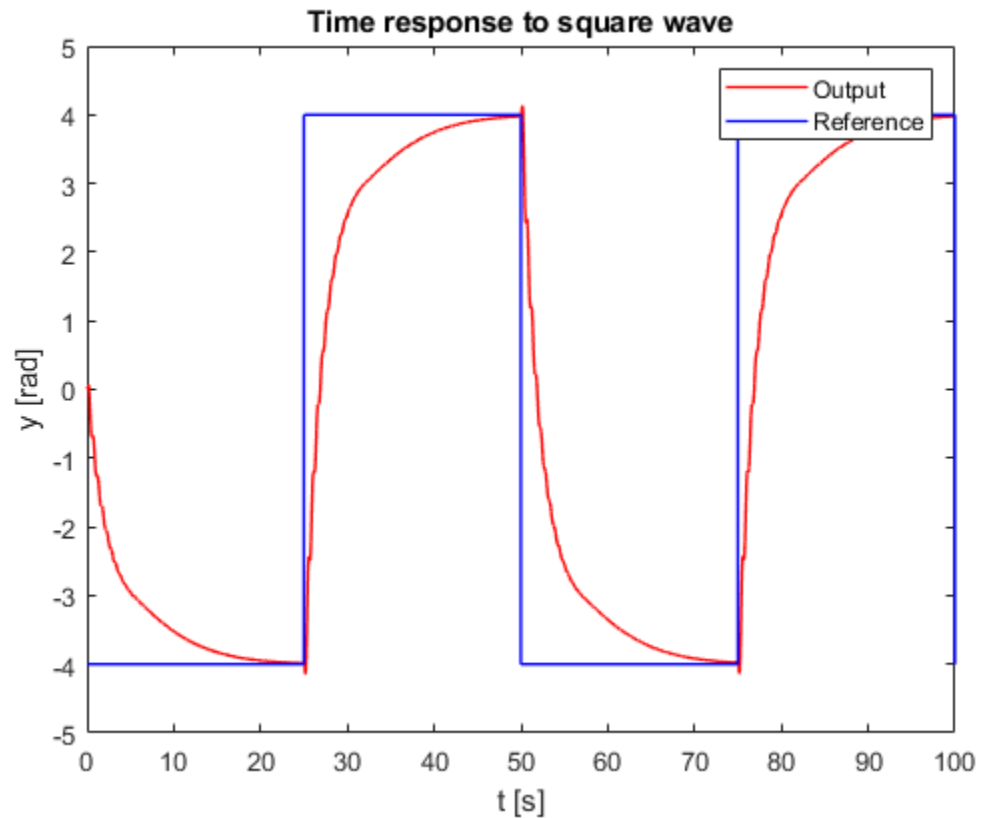
## Validation:

- Closed-loop gain of the LQG was computed and compared with the closed-loop gain of the LQR. Both gains are unitary.
- Analysis of the Time and Frequency responses.

# LQG - Linear Quadratic Gaussian

## Identified Model

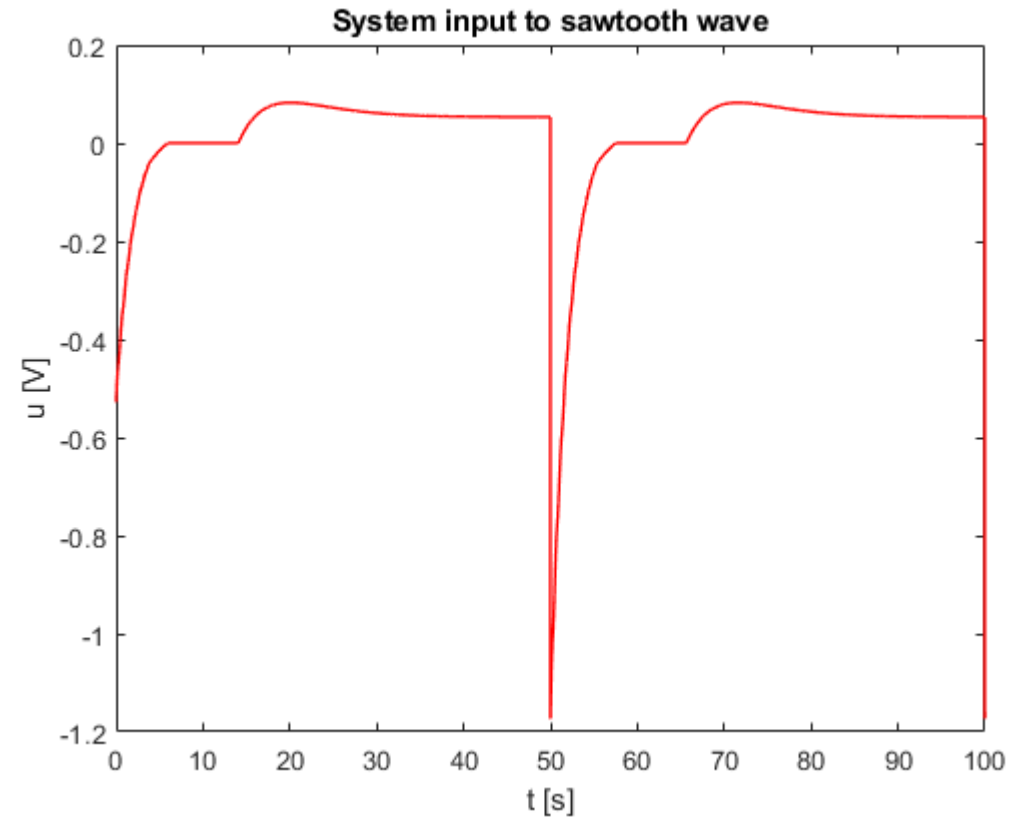
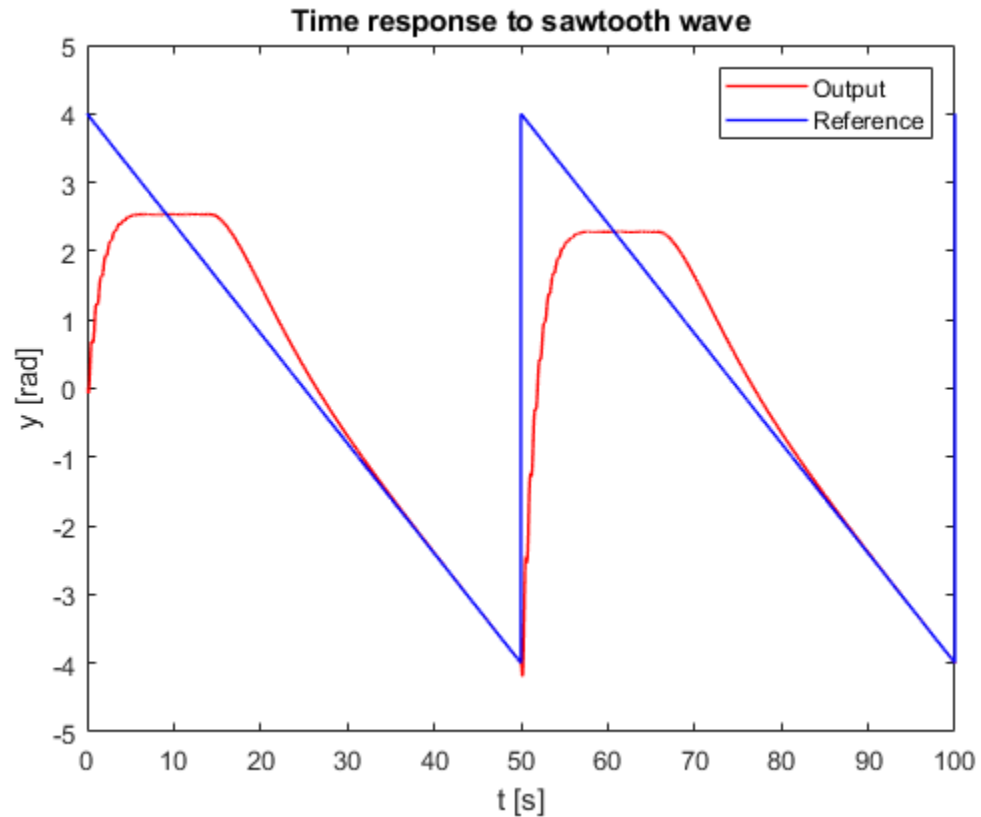
- $R=40$ ,  $R_E=1$



# LQG - Linear Quadratic Gaussian

## Identified Model

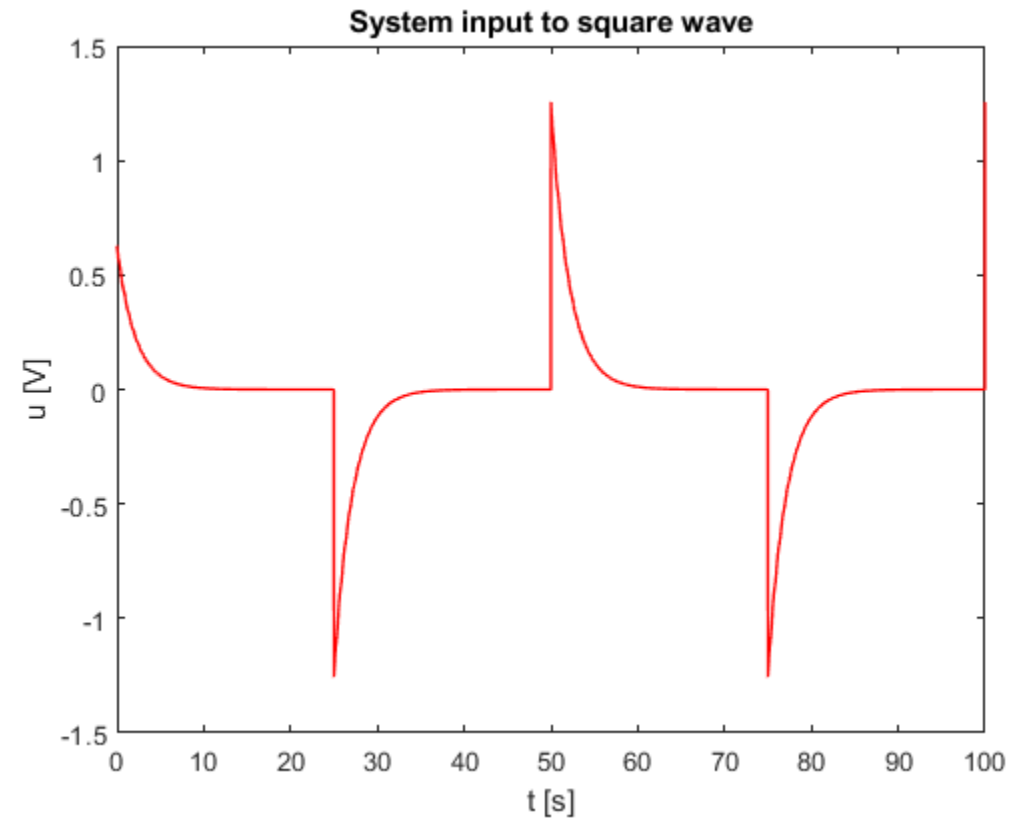
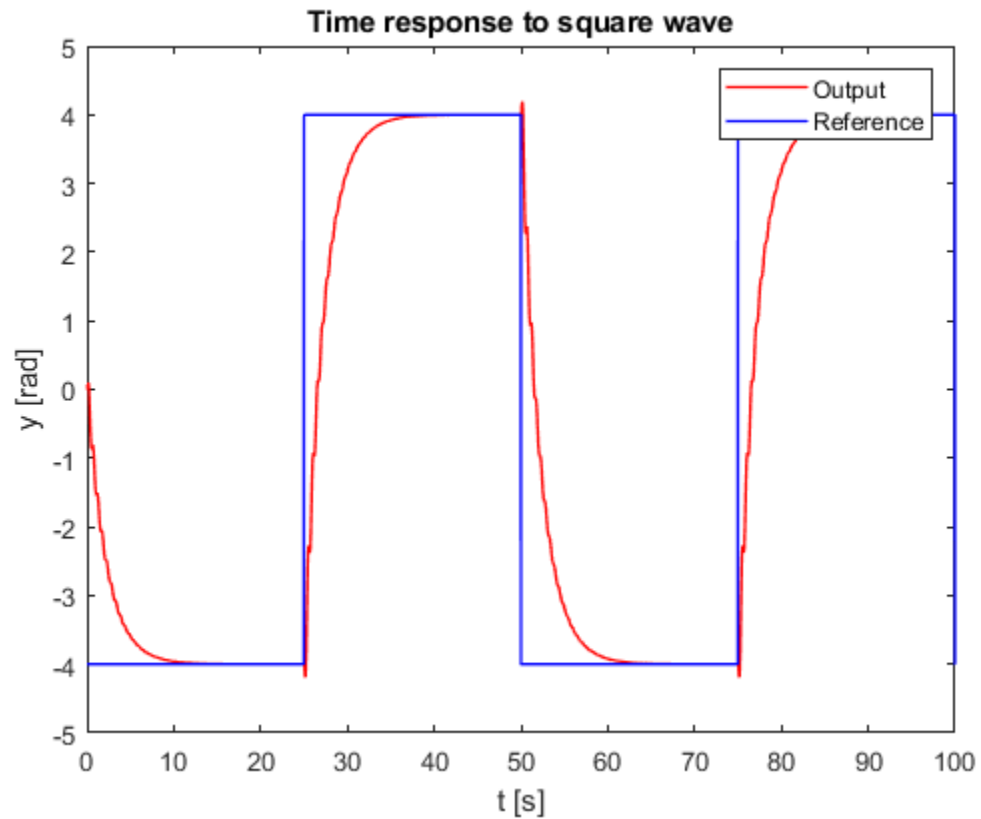
- $R=40$ ,  $R_E=1$



# LQG Test References

## Real System 1

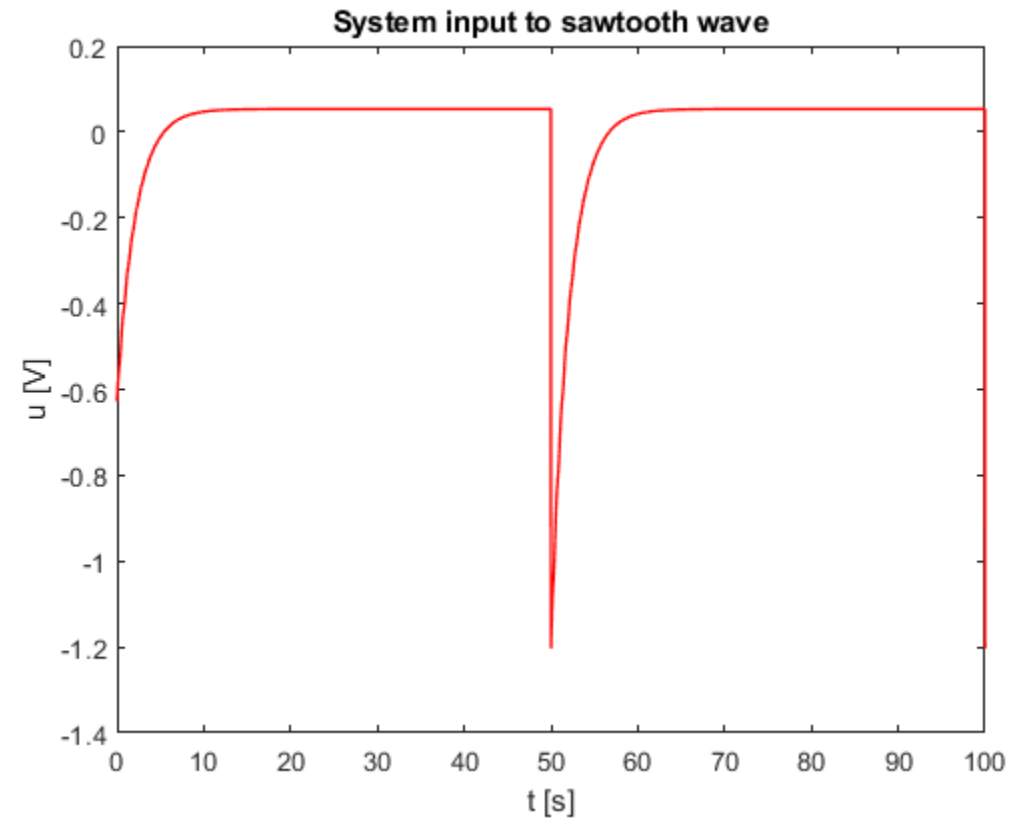
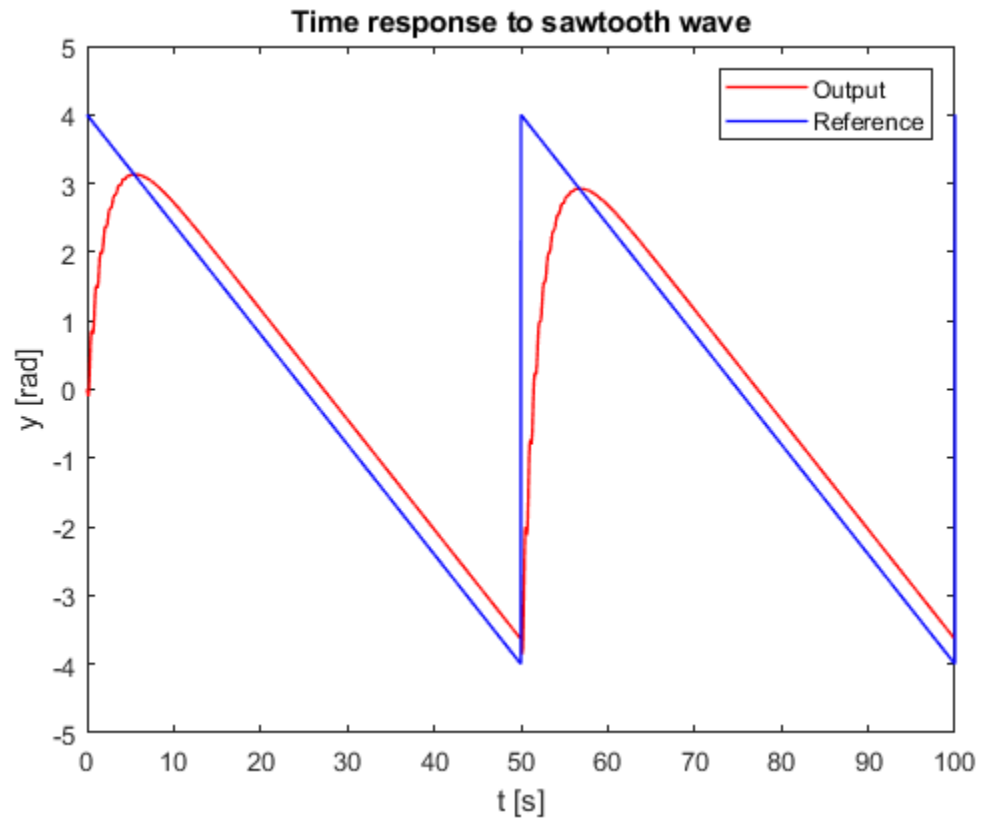
- $R=80$ ,  $RE=1$



# LQG Test References

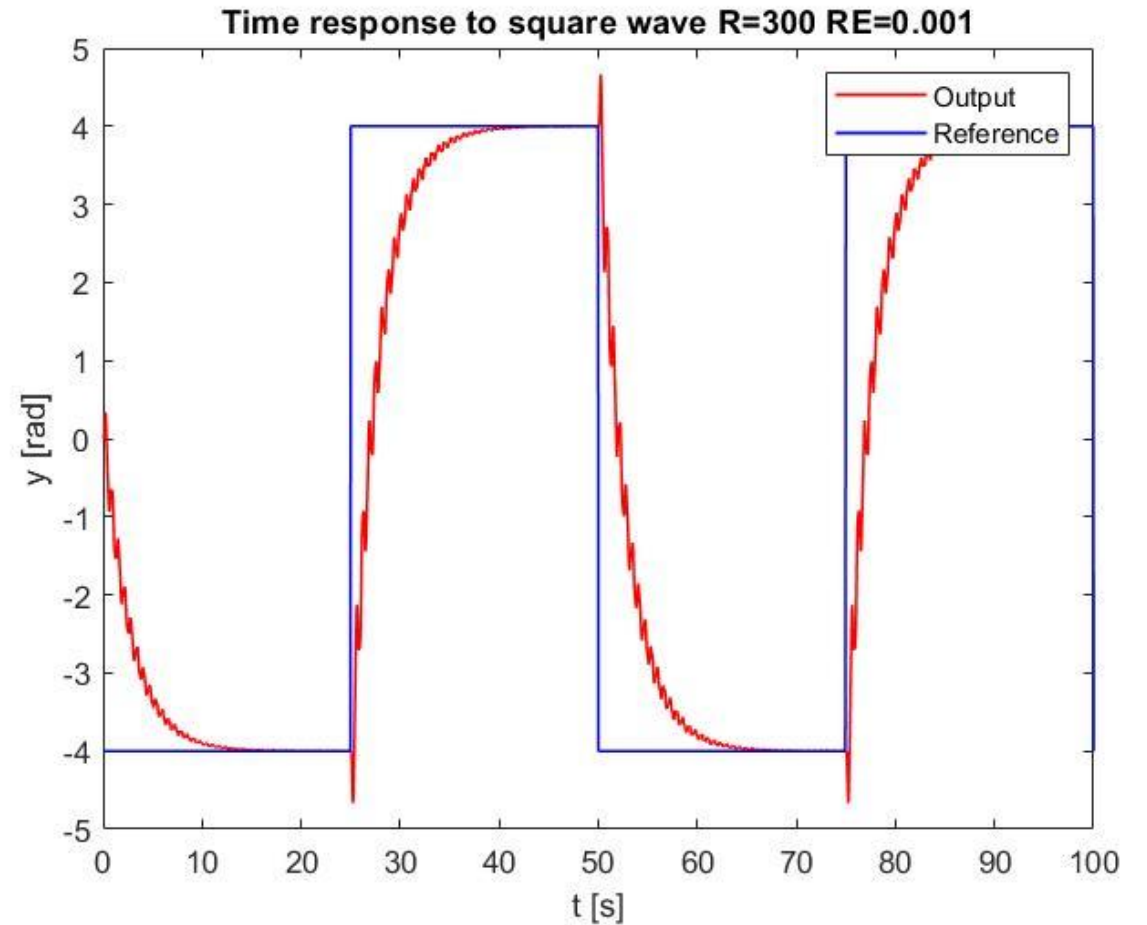
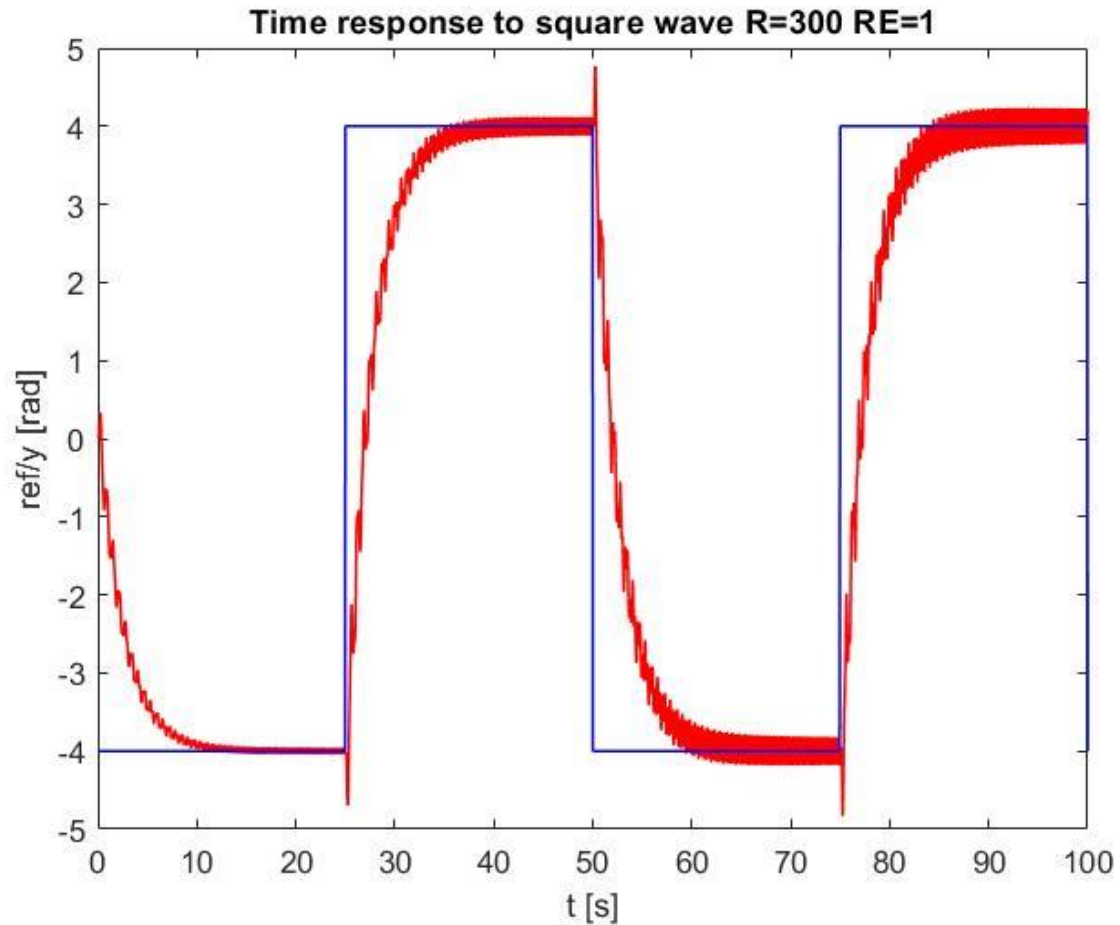
## Real System 1

- $R=80$ ,  $RE=1$



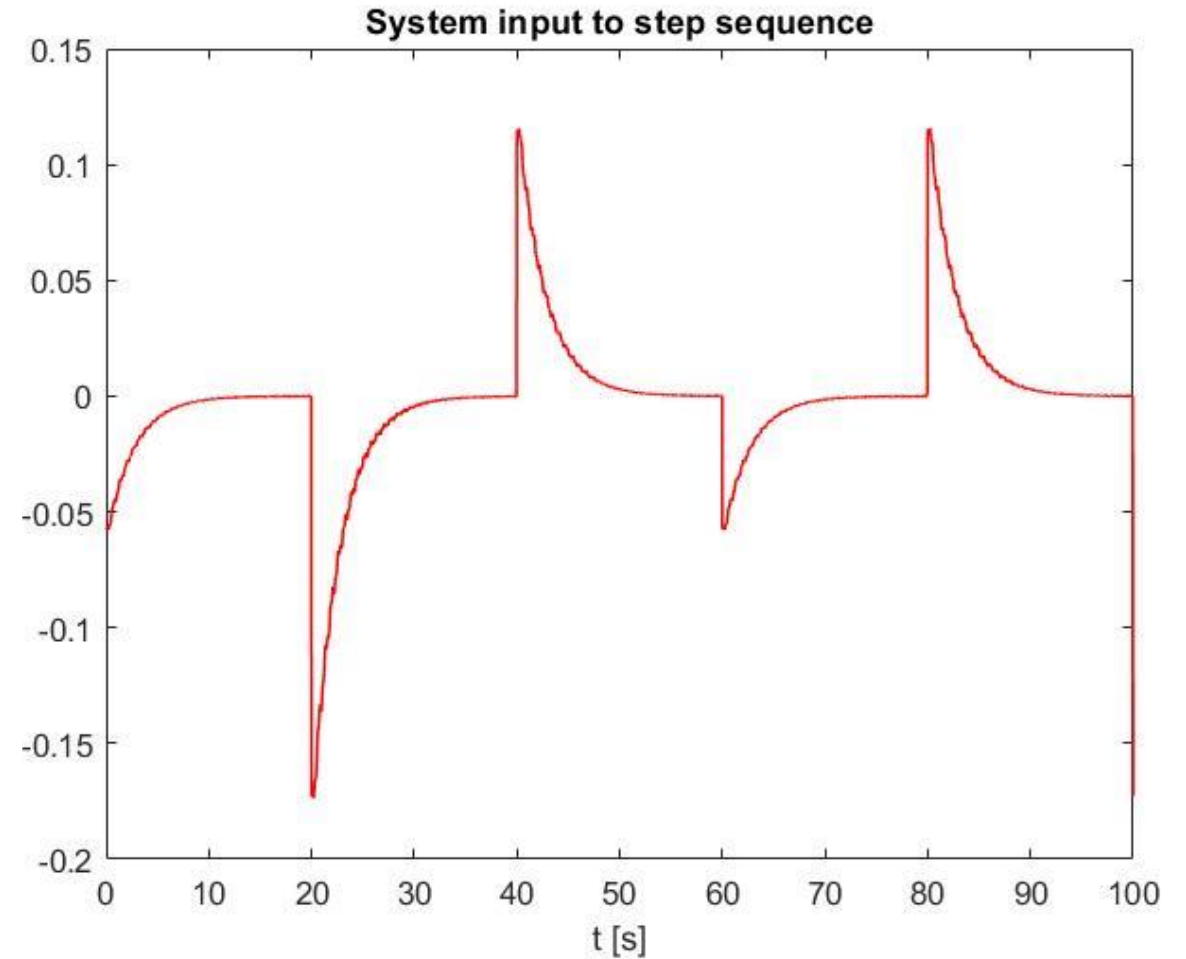
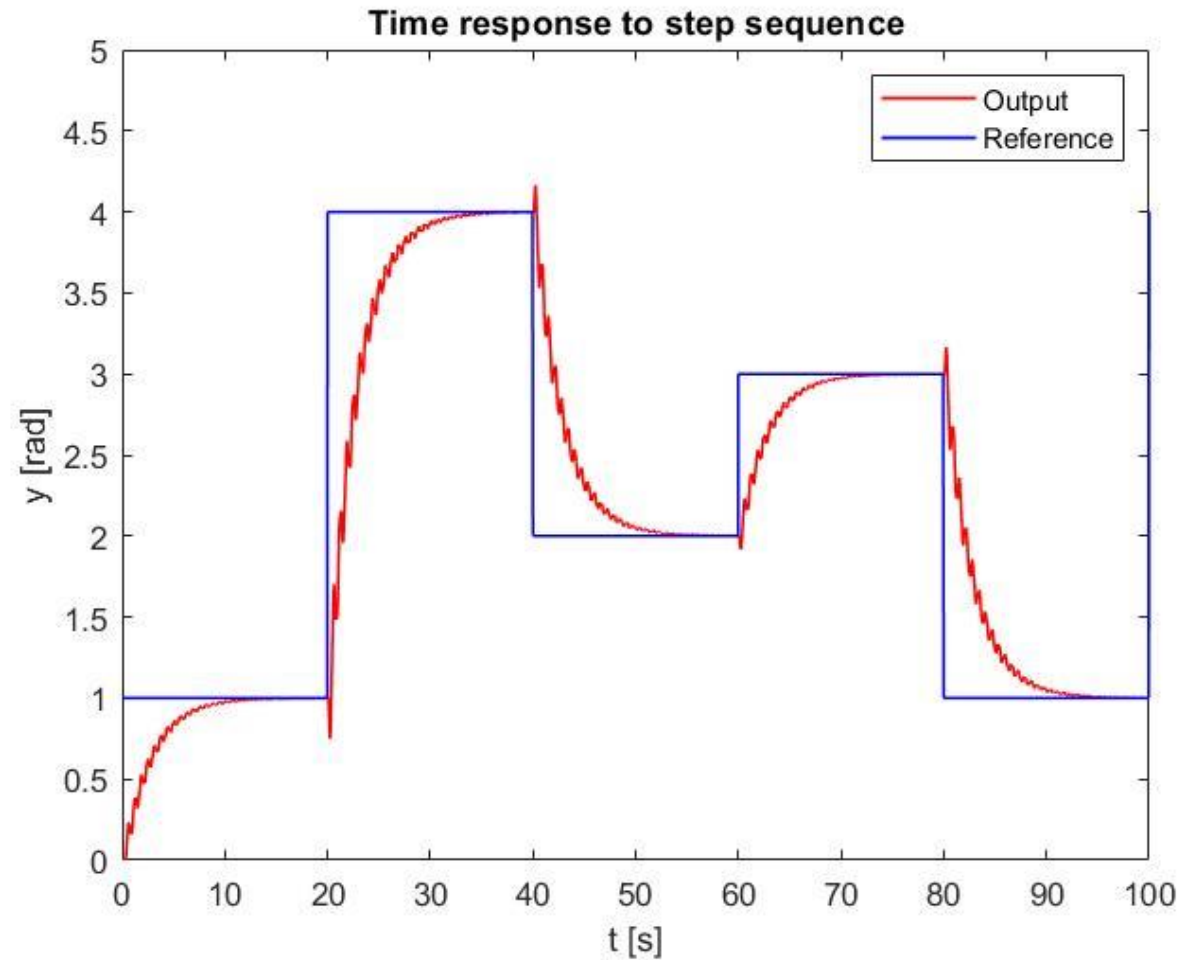
# LQG Test References

## Real System 2



# LQG Test References

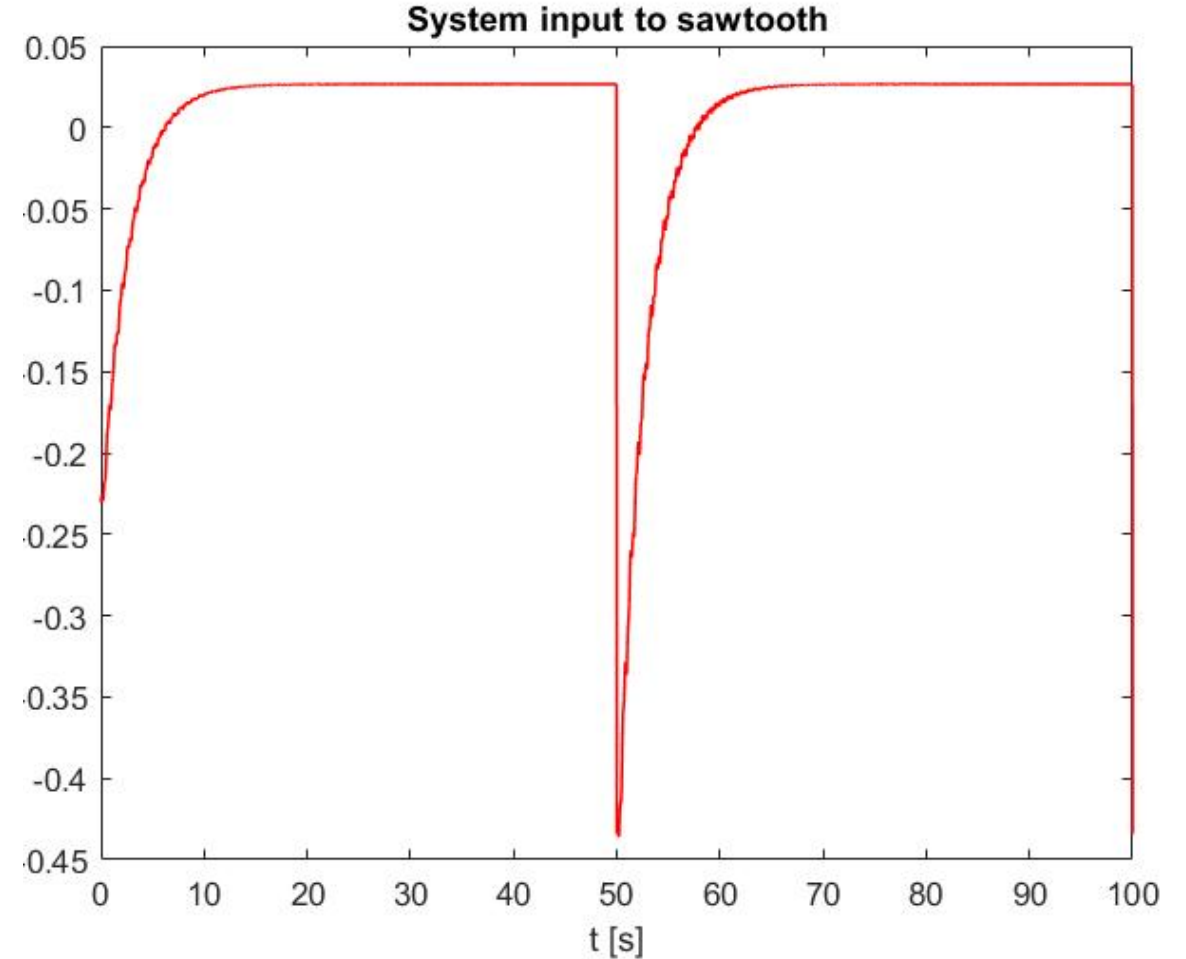
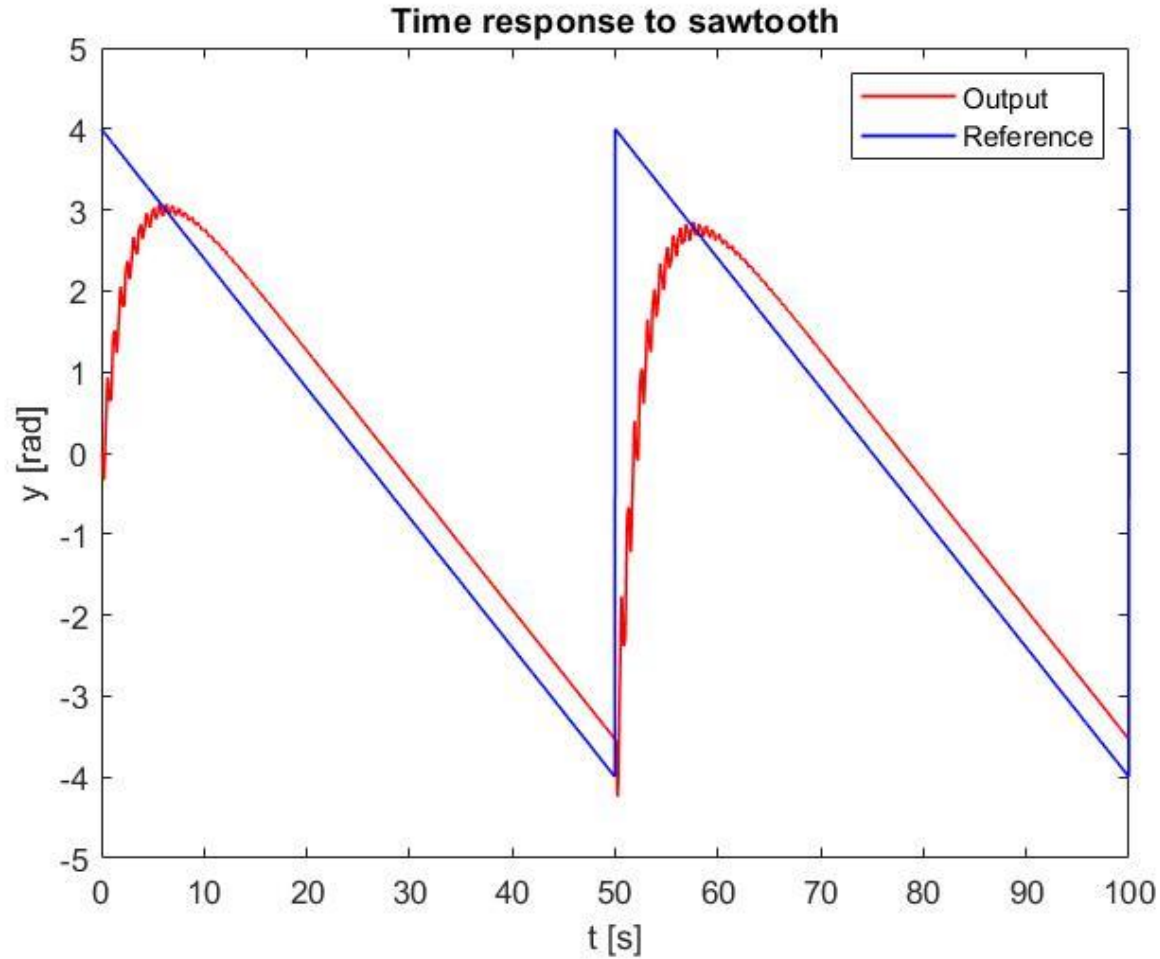
## Real System 2





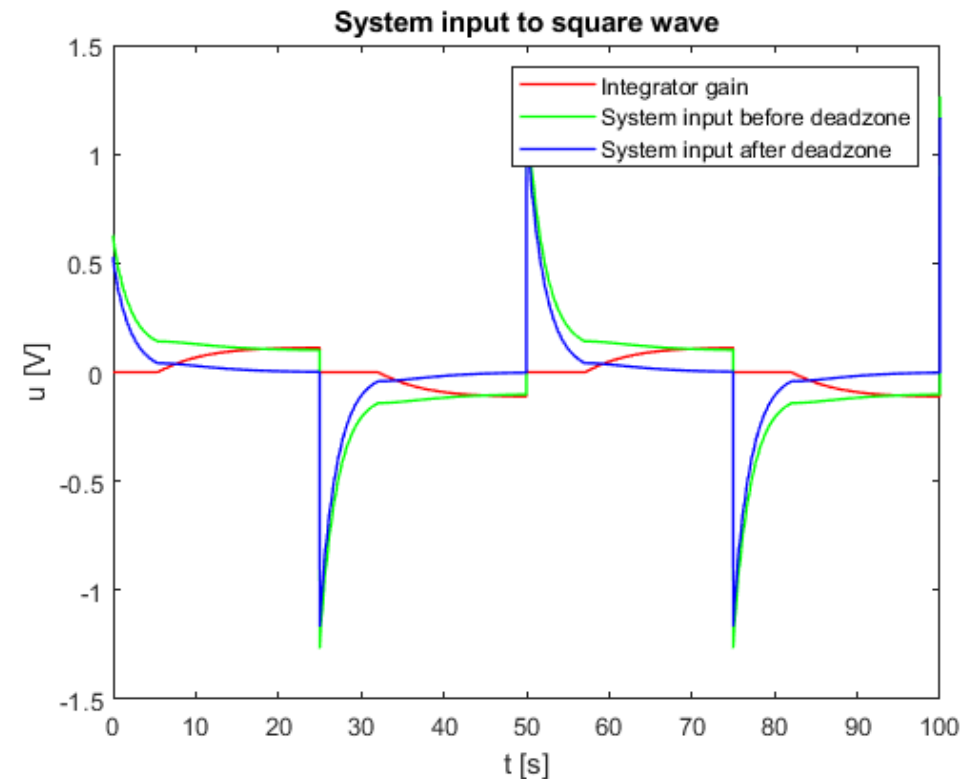
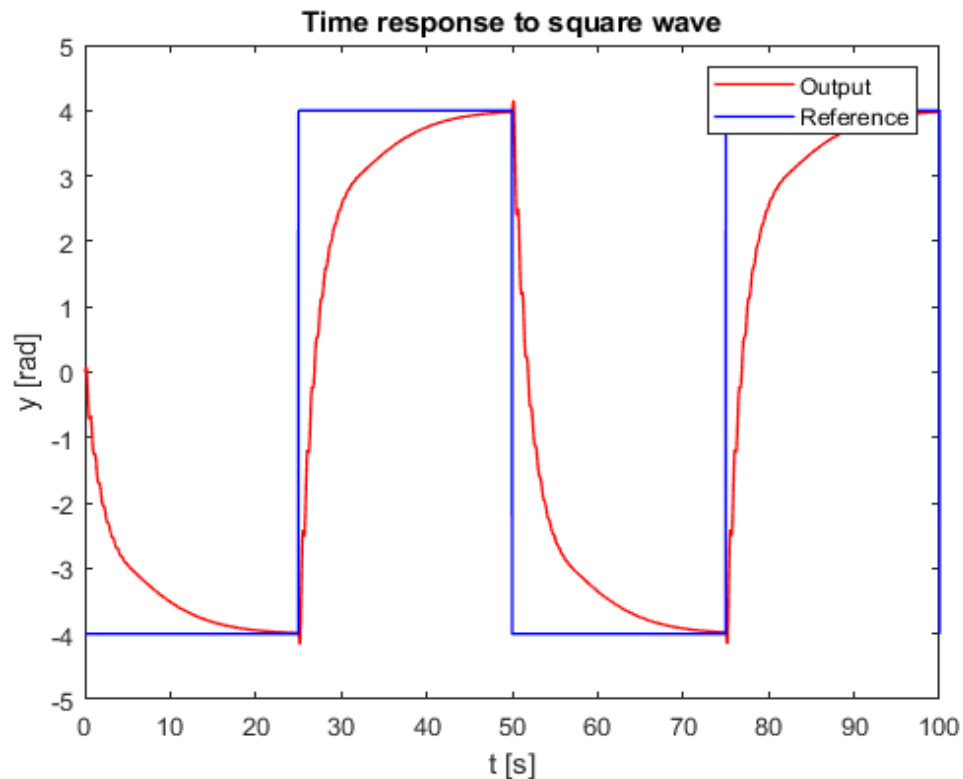
# LQG Test References

## Real System 2



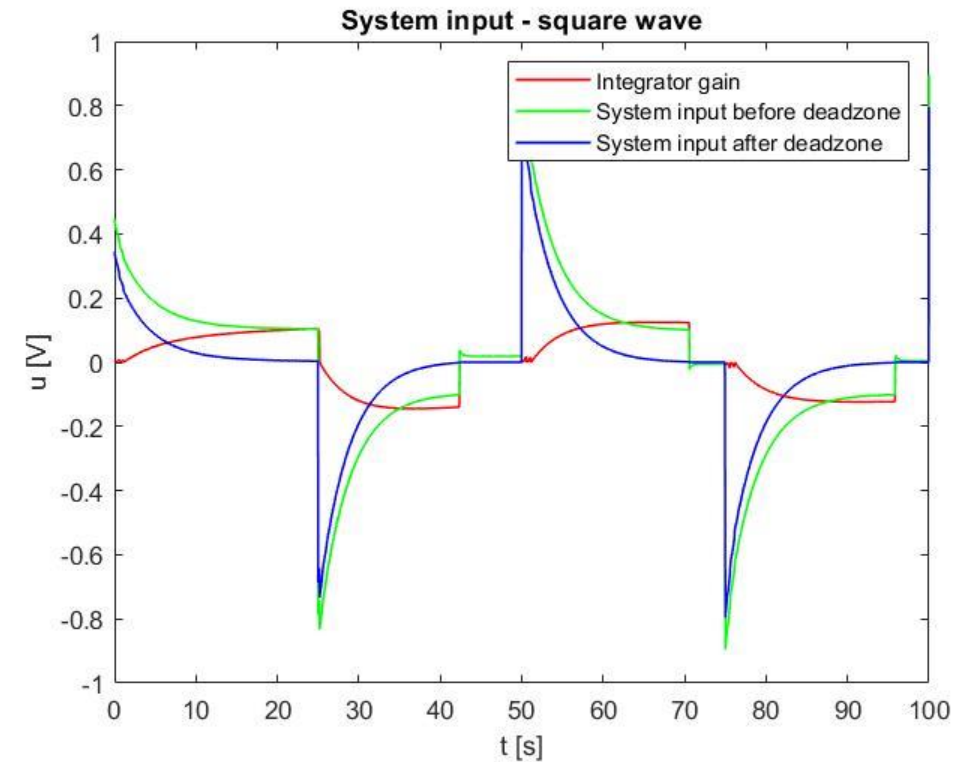
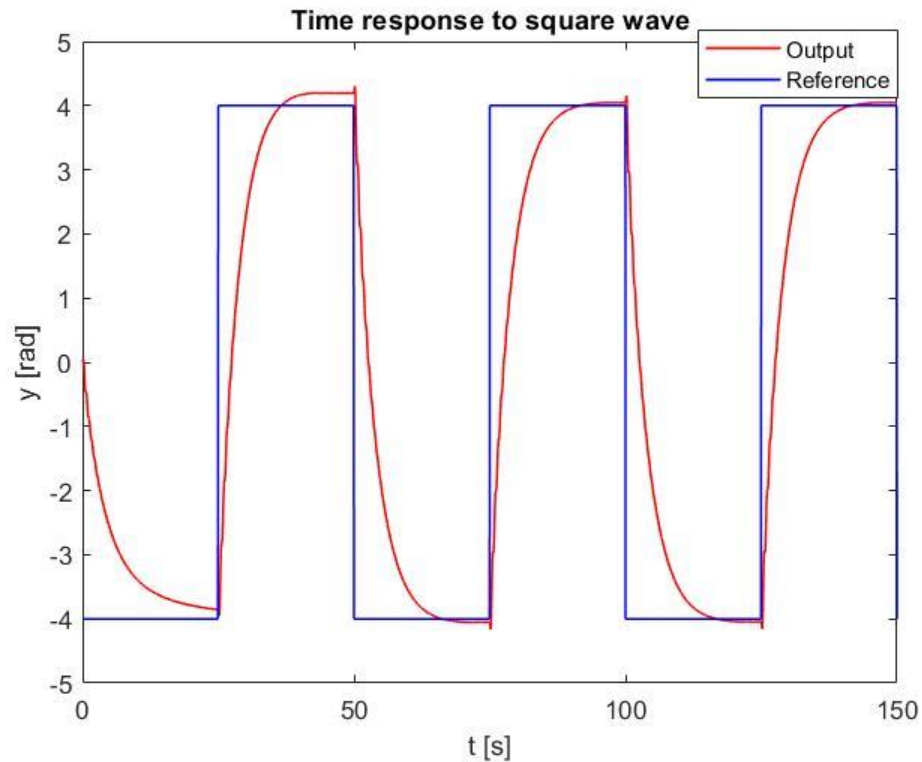
# The Dead-Zone Problem

- Real-world systems may have a deadzone (system inputs close to 0 do not make the motor move)
- **1st approach:** Activate integrator when the system output is close to the reference ( $\leq 1$  rad)



# The Dead-Zone Problem

- **2nd approach:** Activate integrator when the derivative of the error is far from zero. Dead-Zone compensation before the motor stops working



# Conclusions

- The identified model was able to mimic the behaviour of the real system, relatively well.
- The designed LQG performed well for both the simulated real system and the simulated disturbed real system.
- It was possible to tackle the Dead-Zone problem, exploring two different approaches that yielded good results.