Exponencial

$$p(x|\theta) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

$$L(\theta) = p(x_1, \cdots, x_N | \theta) = \prod_{i=1}^{N} p(x_i | \theta) = \prod \frac{1}{\mu} e^{-\frac{x}{\mu}}$$

$$L(\theta) = \left(\frac{1}{\mu}\right)^N e^{-\frac{1}{\mu}\sum_1^N x_i}$$

$$l(\theta) = +N \log\left(\frac{1}{\mu}\right) - \frac{1}{\mu}\sum_{i=1}^{N} x_i$$

$$\frac{\partial l}{\partial \theta} = 0 \iff N\mu - \sum_1^N x_i = 0 \iff \mu = \frac{\sum_1^N x_i}{N}$$

$$\boxed{\hat{\mu} = \bar{x}}$$

Normal

$$p(x|\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$L(\theta) = p(x_1, \dots, x_N|\theta) = \prod_{i=1}^{N} p(x_i|\theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$l(\theta) = \sum_{i=1}^{N} \left[\log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)\right] = -\frac{N}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{1}^{N}(x_i-\mu)^2$$

$$\frac{\partial l}{\partial \mu} = 0 \Rightarrow -\frac{1}{2\sigma^2}\cdot(-2)\sum_{i=1}^{N}(x_i-\mu) = 0 \Leftrightarrow \sum_{i=1}^{N} x_i = N\mu$$

$$\boxed{\hat{\mu} = \bar{x}}$$

$$\frac{\partial l}{\partial \sigma^2} = 0 \Rightarrow -\frac{N}{2}\frac{2\cancel{\pi}}{\cancel{\pi}\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{N}(x_i-\mu)^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow N\sigma^2 = \sum_{i=1}^{N}(x_i-\mu)^2$$

$$\boxed{\hat{\sigma}^2 = \frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}} \qquad \boxed{\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}}}$$

Rayleigh

$$p(x|\theta) = \frac{x}{\theta^2} e^{\left(\frac{-x^2}{2\theta^2}\right)}$$

$$L(\theta) = p(x_1, \dots, x_N|\theta) = \prod_{i=1}^{N} p(x_i|\theta) = \prod_{i=1}^{N} \frac{x_i}{\theta^2} e^{\left(\frac{-x_i^2}{2\theta^2}\right)}$$

$$l(\theta) = \sum_{i=1}^{N} \left[\log(x_i) - \log(\theta^2) - \frac{x_i^2}{2\theta^2} \right]$$

$$\frac{\partial l}{\partial \theta} = 0 \Rightarrow 0 - \frac{2N}{\sigma} + \frac{\cancel{2}}{\cancel{2}\sigma^3} \sum_{i=1}^{N} x_i^2 = 0 \Leftrightarrow$$

$$\Leftrightarrow \sigma^2 = \frac{1}{2N} \sum_{i=1}^{N} x_i^2$$

$$\boxed{\hat{\sigma} = \sqrt{\frac{1}{2N} \sum_{i=1}^{N} x_i^2}}$$