Supplementary File for Incentivizing High Quality Crowdsourcing Information using Bayesian Inference and Reinforcement Learning

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Lemma 1. *If* $x \sim \mathrm{Bin}(n, p)$, $\mathbb{E}t^x = (1 - p + tp)^n$ *holds for any* $t > 0$, *where* $\mathrm{Bin}(\cdot)$ *is the binomial distribution.*

Proof.

$$t^x = e^{x \log t} = m_x(\log t) = \left(1 - p + pe^{\log t}\right)^n \quad (1)$$

where $m_x(\cdot)$ denotes the moment generating function. □

Lemma 2. *For given* $n, m \geq 0$, *if* $0 \leq p \leq 1$, *we can have*

$$\begin{aligned}\sum\nolimits_{x=0}^{n}\sum\nolimits_{w=0}^{m} C_n^x C_m^w p^{x+w}(1-p)^{y+z} \times \\ B(x+z+1+t, y+w+1) = \\ \int_0^1 [(2p-1)x + 1 - p]^n [(1-2p)x + p]^m x^t \mathrm{d}x\end{aligned}$$

Proof. By the definition of the beta function (Olver, 2010),

$$B(x, y) = \int_0^{+\infty} u^{x-1}(1+u)^{-(x+y)} \mathrm{d}u \quad (2)$$

we can have

$$\begin{aligned}&\sum_{x,w} C_n^x C_m^w p^{x+w}(1-p)^{y+z} B(x+z+1+t, y+w+1) \\ &= \int_0^{+\infty} \mathbb{E}u^x \cdot \mathbb{E}u^z \cdot u^t \cdot (1+u)^{-(n+m+2+t)} \mathrm{d}u \quad (3)\end{aligned}$$

where we regard $x \sim \mathrm{Bin}(n, p)$ and $z \sim \mathrm{Bin}(m, 1-p)$. Thus, according to Lemma 1, we can obtain

$$\begin{aligned}&\int_0^{+\infty} \mathbb{E}u^x \cdot \mathbb{E}u^z \cdot u^t \cdot (1+u)^{-(n+m+3)} \mathrm{d}u \\ &= \int_0^{+\infty} \frac{[1-p+up]^n \cdot [p+(1-p)u]^m \cdot u^t}{(1+u)^{n+m+2+t}} \mathrm{d}u.\end{aligned} \quad (4)$$

For the integral operation, substituting u with $v - 1$ at first and then v with $(1 - x)^{-1}$, we can conclude Lemma 2. □

References

Frank W. J. Olver. *NIST Handbook of Mathematical Functions*. Cambridge University Press, 2010.