MODELAÇÃO DE SISTEMAS COMPLEXOS

PROJECT 3

Two-dimensional random walks. First passage time and the survival probability. Lévy flights.

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Part 1: Two-dimensional random walk

Task 1.1

The goal of this task is to simulate N=10,000 repetitions of symmetric random walks on a square lattice (two-dimensional). Then, by using t=5000, 5001, as final times of all random walks, the second goal is to compute the averaged probabilities, $<P(x, y, t)>$, in order to compare with the theoretical distribution function,

$$P(x, y, t) = \frac{1}{\pi t} e^{-(x^2+y^2)/t} \tag{1}$$

The graphic results (figure 1) visually show us that the experimental cloud of points in (x,y) are qualitativelly similar to the theoretical probability distribution. The behavior of the experimental results of probabilities is quite similar to the predicted one.

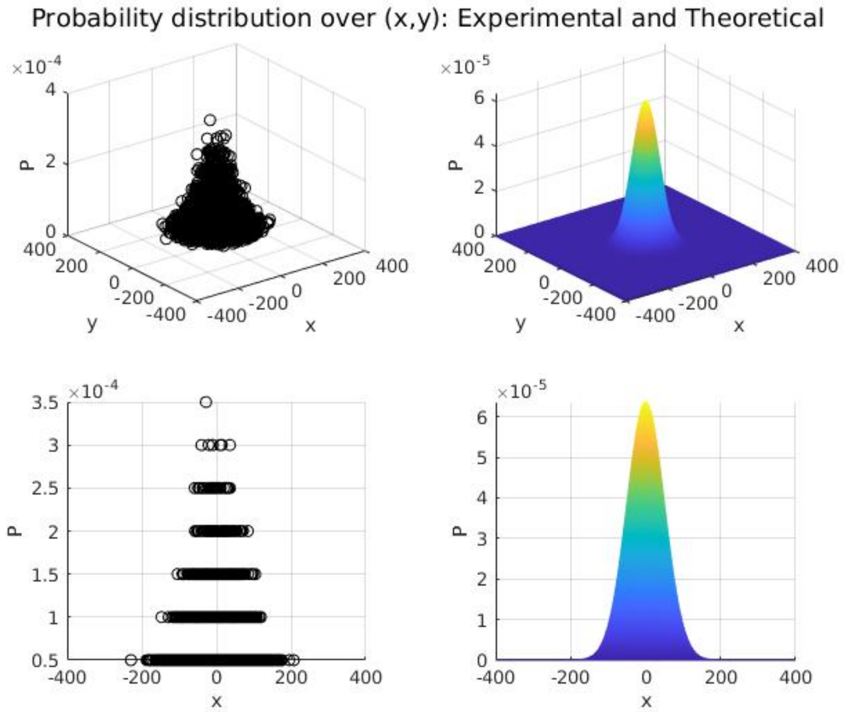


Figure 1: Probability distributions for the experimental points (in the left) and for the theoretical function (in the right), for the final positions of 10,000 2-D random walks. The two plots above represent P in function of both axis, and the bellow plots represent a projection in Y=0 plane.

—	$\sum_{x,y} < P(x,y,t) >$	$\sum_{x,y} < P(x,y,t) > x$	$\sum_{x,y} < P(x,y,t) > y$	$\sum_{x,y} < P(x,y,t) > (x^2+y^2)$
Results	1.0000	-0.7107	-0.7107	$4.9396 \cdot 10^3$

Table 1: Results of some summations to check the validity of the experimental probability distribution.

Although the experimental probability seems to have a similar behavior, spacially, to the theoretical probability distribution, the results are bad because they are roughly 10 times greater than what was supposed. So, in the implementation of the algorithm, something tecnical went wrong that is yet to know. Also some of the mathematical properties are not quite there, as shown in the summations table 1: the last three collumns don't show right values:

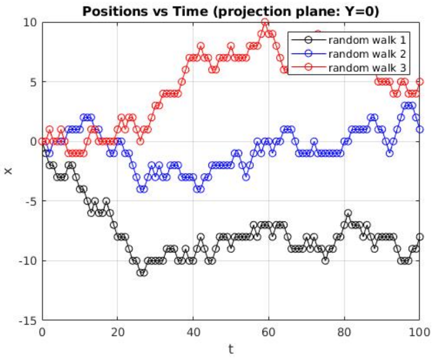
- for the second and third collumn, the resulting summations are equal (as expected) but not equal to zero;

- the final collumn of table 1 is not equal to t (t=5000).

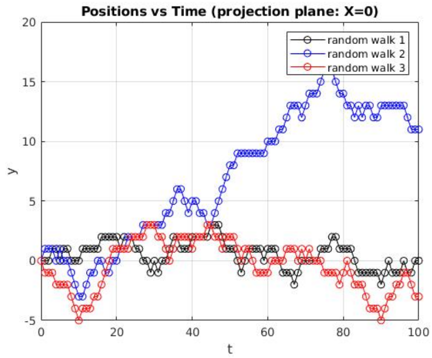
So, because of these mistakes, we don't show further statistical analysis such as correlation, to compare the experimental and theoretical distributions.

Task 1.2

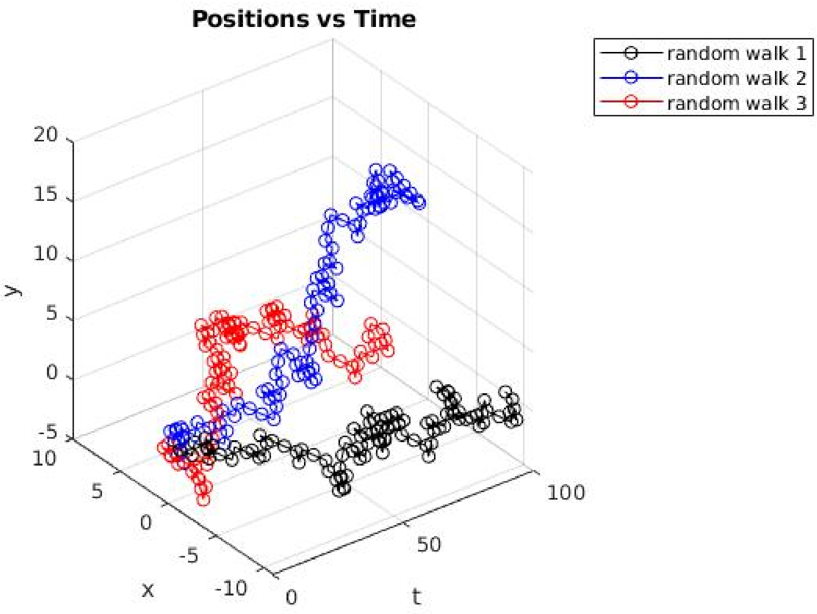
Here,three trajectories are show to depict how a 2-D random trajectory behaves in space-time, and how different they are from one another. The number of steps is t=100, and the the used algorithm is the same as the previous task.



(a) Positions of 2-D random walks projected in x-t plane



(b) Positions of 2-D random walks projected in y-t plane



(c) Positions of 2-D random walks

Figure 2: Representation in space-time of three trajectories of 2-D random walks after 100 jumps

Part 2: First passage time and the survival probability

There is an absorbing boundary along the vertical line at $x_c = -30$, i.e., particles walk in the semi-plane $x \leq 30$. The starting point is at $R = (0,0)$. If the particle hits this boundary, then the walk is stopped. To get good mathematical properties we have to repeat these random walks. The number of attempts $N = 50000$ times. (In other words, there are $N = 50000$ particles.)

Task 2.1

We compute the number of particles which hit the boundary for the first time during a time interval $[t, t + \Delta t]$, $\Delta t = 10$. The maximum number of jumps is 50,000 jumps. The graphical results of the first passage time probability, $F(t)$, are shown in figure 3. For the plot with the logarithmic scale the deviations of the results to the theory appear smaller, because the function is supposed to have a proportionality with $1/(t^{3/2})$, which has an exponential behavior. The correlation coefficient between the experimental results and the theoretic function (of $F(t)$) is 0.9895, which shows excellent similarity between the results and the predictions.

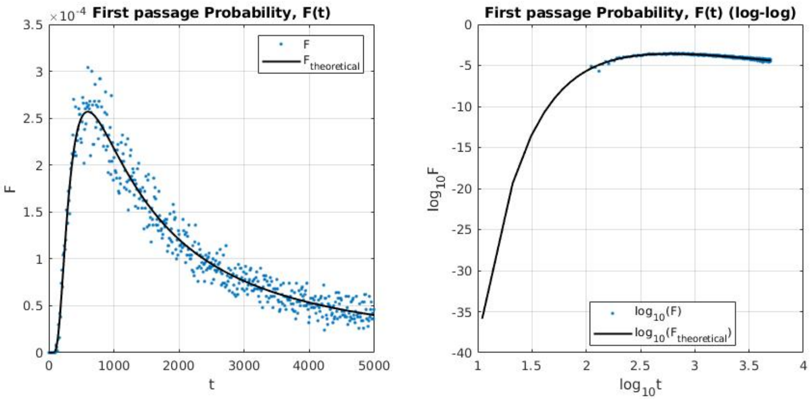


Figure 3: First passage probability with respect to time.

Also, we present (in figure 4) results on the survival probability , $S(t)$, i.e., the probability of a particle surviving the "hit" on the wall given by the plane $x_c = -30$. In each trajectory, we consider that after the particle hits the wall, it ceases to exist (in the code, it the increment of x and y stop, in the time cicle). Judging from the results in figure 4, they are very good. Also, there is a zoomed plot (in the same figure) to show that, in a close view, there is some little deviations, as one would expect from these random repetitions. Quantitativelly, the correlation between the experimental and theoretical distribution functions is even better than the first passage probability: It is 0.0.99995301. Why? Because there is less complexity in the mathematical function that survival probability portraits: for example, it's behaviour in time is monotone (always decreases), as in opposition to the latter.

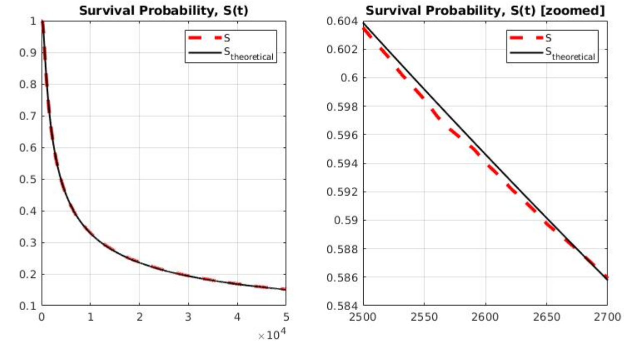


Figure 4: Survival probability with respect to time.

Part 3: Lévy flights

In this part, we study 2D-random walks with variable length of jumps. The probability $P(l)$ that a jump has length l is determined by the Lévy distribution,

$$P(l) = \frac{c}{l^{\mu}} \tag{2}$$

where c (normalization constant) is given by

$$c = \frac{\mu - 1}{1 - l_{max}^{1-\mu}} \tag{3}$$

where we consider $l_{min} = 1$ and $l_{max} = 1000$.

After a jump, the particle is at point $(x, y) = (l cos(\phi), l sin(\phi))$ with respect to the initial point.

Task 3.1

For the first task in this section, the exponent μ changes ($\mu = 1.6, 2, 2.6$) and, for each value of μ, N=1000 trajectories of random jumps (2D) are generated, following the algorithm given in the assignment. The results are given in figures 5 and 6. From visualizing the graphs, we see that, for variable lengths of jumps, for $\mu = 1.6$ and $\mu = 2.6$ the path is very unstable comparing to the variable lengths generated with $\mu = 2$. In other words, the path for $\mu = 2$ shows an approximate continuous function, and also doesn't go much far from the original position (0,0). For Lévy flights, according to the theory, optimal μ is 2, so this can explain why qualitativelly the results are better for this value. Furthermore, the random trajectory with constant jumps of l=1 is the one which presents less oscilations with respect to the original positions, as expected.

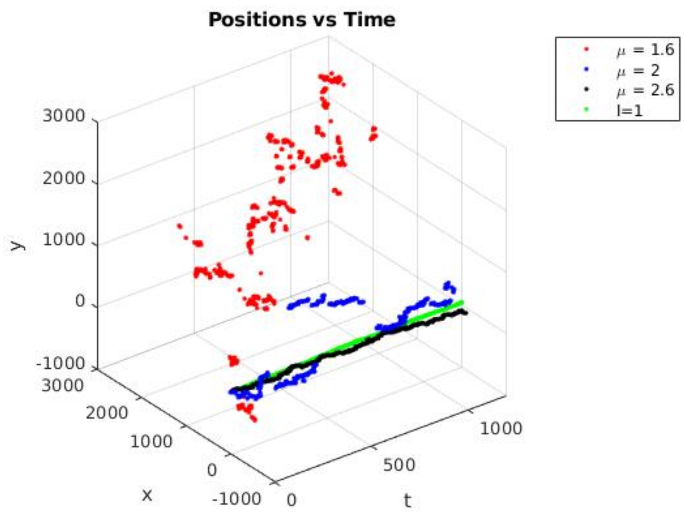


Figure 5: Trajectories of 3 Lévy flights

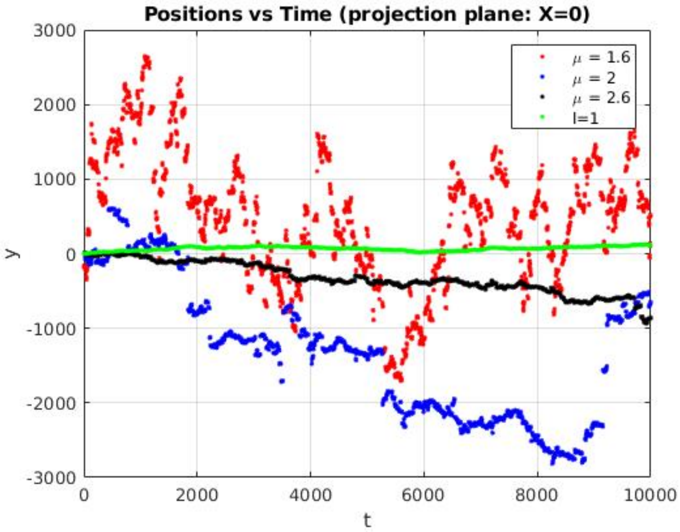


Figure 6: Trajectories of 3 Lévy flights, projected.

Task 3.2

Here, the purpose is to compare the probability distributions of N=50,000 repetitions of Lévy flights to the theoretic probability distribution:

$$P(l) = \frac{c}{l^{\mu}} \tag{4}$$

and, as shown in figure 7, the results of bar plots for all μ values are very similar to the Lévy distribution given in the equation above.

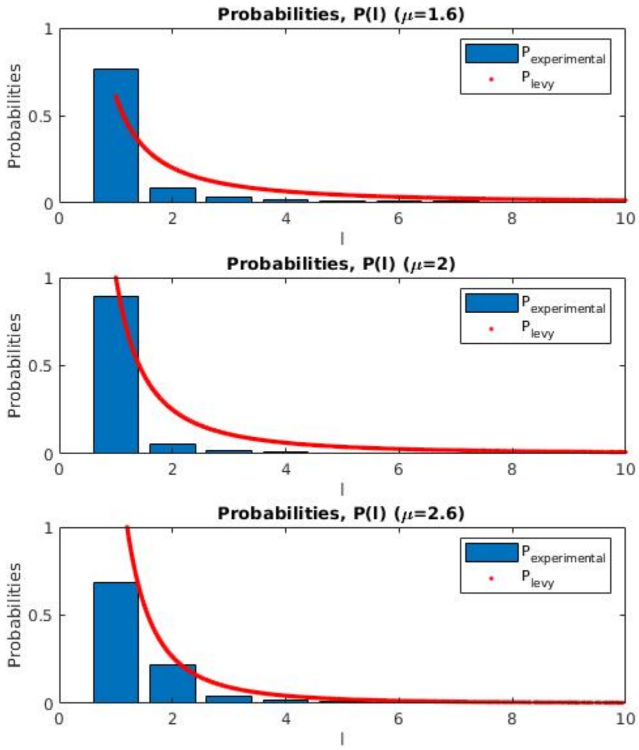


Figure 7: Probability of a jump having length l for different μ