Disordered and complex systems

Department of Physics, University of Aveiro, Aveiro, Portugal

Alexander V. Goltsev

Project 2.
One dimensional random walks.

1. One-dimensional random walks with symmetric jumps.

Simulate random walks of a particle on a one-dimensional chain. Probabilities to jump on the left and on the right are equal to 0.5.

1. A particle starts at t=0 at x=0 and makes t=50 steps. Plot three trajectories of random walks, i.e., plot the dependence of the particle position x(t) on time t:

$$x(t) = \sum_{i=1}^{t} S_i$$

   Here $S_i = \pm 1$ with the probability p=q=0.5.

2. Compute the probability P(t,x) to find the particle at time t (i.e., after t jumps) at site x. At t=0 the particle places at site x=0. Use (i) t=40, 41; (ii) t= 400, 401; (iii) t= 4000, 4001, then average over even and odd t and find the averaged probability $\overline{P}(x,t)$. Compare between your simulations and the theoretical distribution function:

$$P(x,t) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}$$

Algorithm:

1. For each time *t* generate *N*=50 000 trajectories (or more to get batter statistics). Find $P(x,t) = N(x)/N$ where *N*(x) is the number of trajectories which lead to point x. Average over even and odd times:

$$\overline{P}(x,t) = \frac{1}{2}(P(x,t) + P(x,t+1)).$$

Plot the averaged distribution function $\overline{P}(x,t)$. Compare with the theoretical distribution function $P(x,t)$.

2. Check that

$$\sum_x \overline{P}(x,t) = 1, \quad \sum_x \overline{P}(x,t)x = 0, \quad \sum_x \overline{P}(x,t)x^2 = t,$$

3. Plot on the same graph your results for (i)-(iii).

2. Random walks with a drift.

Simulate random walks of a particle on a one-dimensional chain with asymmetric probabilities. The probabilities to jump on the left and on the right equal to p=0.5-δ and q=0.5+ δ , respectively. Take δ =0.015. Compute the probability *P(t,x)* to find this particle after time *t* (i.e., after *t* jumps) at site *x*. At *t* =0 the particle places at site *x* =0. Use (i) *t* =40, 41; (ii) *t* = 400, 401; (iii) *t* = 4000, 4001, then average over even and odd *t* and find the averaged probabilities $\overline{P}(x,t)$. Compare between your simulations and a theoretical distribution function,

$$P(x,t) = \frac{1}{\sqrt{2\pi t}} e^{-(x-2t\delta)^2/2t} .$$

Algorithm:

1. For each time *t* generate *N*=50 000 trajectories (or more to get a better statistics). Find $P(x,t) = N(x)/N$ where *N*(x) is the number of trajectories which lead to point *x*. Then average over even and odd times :

$$\overline{P}(x,t) = \frac{1}{2}(P(x,t) + P(x,t+1)) .$$

Plot the averaged distribution function $\overline{P}(x,t)$. Compare with the theoretical distribution function $P(x,t)$.

2. Check that

$$\sum_x \overline{P}(x,t) = 1, \quad \sum_x \overline{P}(x,t)(x-2t\delta) = 0, \quad \sum_x \overline{P}(x,t)(x-2t\delta)^2 = t .$$

3. Plot on the same graph your results for (i)-(iii).