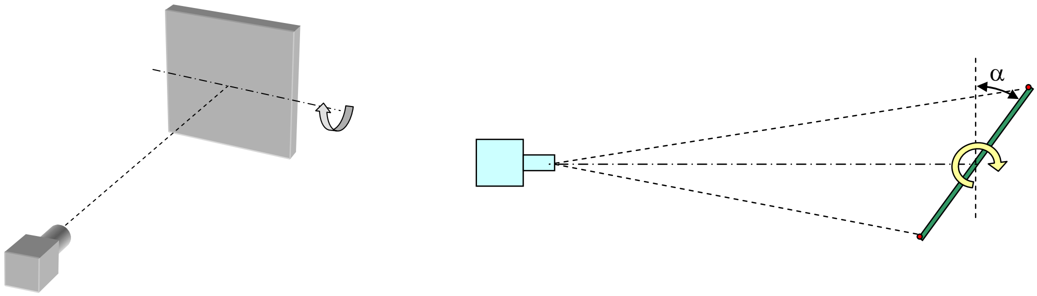
Universidade de Aveiro
Departamento de Engenharia Mecânica

Sistemas de Visão e Percepção Industrial

Exame de Época Normal - 27 de Junho de 2011 – *English Version*

Mestrado Integrado em Engenharia Mecânica; Mestrado em Engenharia de Automação Industrial
Minor em Automação da Licenciatura em Matemática

1. A camera with 1239x1023 *pixels* and a focal distance of 15 mm and a *dot-pitch* of 150 pixels/mm is placed initially at 3 m away from a square panel 80 cm wide. Initially the panel is parallel to the focal plane, but latter it can be re-oriented around its media axis.



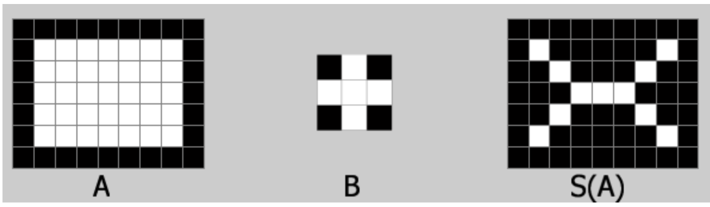
a) Indicate the numeric value of the camera intrinsic matrix for the initial conditions.

b) Given a generic point P=$[x_S\ y_S\ z_S]^T$ on the panel, write down the equation of the perspective transformation to obtain the corresponding point on the image in pixel coordinates p=(x_{pix}, y_{pix}). Consider that the geometric centre of the CCD coincides with the intersection of the optical axis with the focal plane, and that the pixel counting starts on the lower left corner of the image at pixel (1,1).

c) For a given orientation α, calculate what id the minimum distance at which the camera may lay to the panel axle of rotation in order that no point of the panel projects further than 500 pixels from the image centre.

2. A erosion operation repeated recursively *k* times over an image A using the structuring element B é represented by: $\mathrm{C} = \mathrm{A} \ominus k\,\mathrm{B} = ((((\mathrm{A} \ominus \underbrace{\mathrm{B}) \ominus \mathrm{B}) \ominus \mathrm{B}) \cdots \ominus \mathrm{B})}_{k\ vezes}$ **where, for *k*=0, we have C=A.**

We define a morphological skeleton S of A by the expression $S(\mathrm{A}) = \bigcup_{k=0}^{K} S_k(\mathrm{A})$**, where** $S_k(\mathrm{A}) = (\mathrm{A} \ominus k\,\mathrm{B}) \backslash ((\mathrm{A} \ominus k\,\mathrm{B}) \circ \mathrm{B})$**; for k=0 we have:** $S_0(\mathrm{A}) = \mathrm{A} \backslash (\mathrm{A} \circ \mathrm{B})$**. The intention is now to calculate the skeleton S(A) using the structuring B whose origin is its central point.**

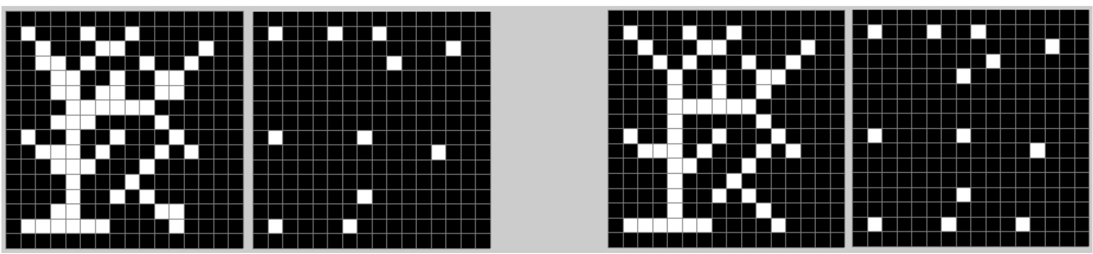


a) Determine and represent $(\mathrm{A} \ominus k\,\mathrm{B})$ for k=0, k=1, e k=2.

b) Determine and represent $(\mathrm{A} \ominus k\,\mathrm{B}) \circ \mathrm{B}$ for k=0, k=1, e k=2.

c) Using the definition given above, obtain and represent *S_k(A)* for k=0, 1 e 2, e demonstrate that the morphological skeleton is given by *S(A)* as represented in the image above..

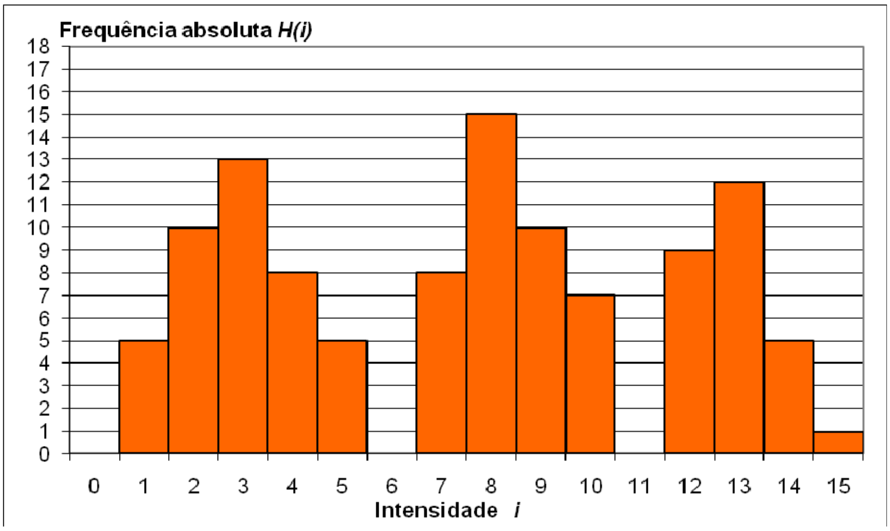
d) Based on the following images, indicate a convolution filter F, as well as the subsequent operation *g*() to obtain image T_i after the corresponding image X_i: in brief, indicate F and *g()* such as $\mathrm{T}_i = g(\mathrm{X}_i * \mathrm{F}),\ i = 1, 2$, where the operator "*" is "convolution", or filtering operation, also known as Filt2(F,X_i). In other words, the intention id to obtain all the pixels that are terminal points of a skeleton (even if in intermediate phases of calculation). Image (X_2) represents a final skeleton and T_2 contains its terminal pixels; e X_1 represents a skeleton in a intermediate phase and T_1 represents its terminal pixels in that phase.



X_1 T_1 X_2 T_2

3. Consider image analysis after its histogram

a) In a thresholding process, if the image histogram is trimodal, which thresholding technique is more appropriate? Justify your answer.



b) Knowing that the image associated to the shown histogram obeys a format 2:3, calculate its dimensions in pixels.

c) Calculate the mean image intensity regarding the show histogram.

d) If pixels of intensity 1 are changed into intensity 2, pixels of intensity 15 and 14 are changed into intensity 13, when performing a contrast expansion on the resulting histogram, calculate how many pixels of intensity 9 are there on the new image.

e) Calculate and compare with a critical point of view, the Uniformity coefficient (U) of the original histogram (illustrated) versus the Uniformity coefficient of the histogram after contrast expansion performed in the previous question.

Grading: Question 1 – 5 points. Question 2 – 8 points. Question 3 – 7 points.

Brief formulae set

Camera intrinsic matrix: $\mathbf{K} = \begin{bmatrix} \alpha_x & 0 & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$

Image moments:

$$m_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$

$$\bar{x} = \frac{m_{10}}{m_{00}} \;,\; \bar{y} = \frac{m_{01}}{m_{00}},\; m_{01} = \sum_x \sum_y y \cdot f(x,y),$$

$$m_{10} = \sum_x \sum_y x \cdot f(x,y)$$

Operations on histograms

$$\mu_n = \sum_{i=0}^{L-1} (i-\mu_0)^n h(i) \quad , \quad \mu_0 = \sum_{i=0}^{L-1} i h(i), \quad \text{with} \quad h(i)$$

normalised, i.e., $0 \le h(i) < 1, \forall i \in \{0,1,2,\dots,L-1\}$

Contrats expansion:

$$g(x,y) = (L-1)\frac{f(x,y) - \min[f(x,y)]}{\max[f(x,y)] - \min[f(x,y)]}$$

Histogram Uniformity Coefficient:

$$U = \sum_{i=0}^{L-1} h^2(i)$$

Morphology:

$$\mathrm{A}_h = \{p \in \mathbb{Z}^2 : p = x + h, x \in \mathrm{A}\},$$

$$\mathrm{A}^C = \overline{\mathrm{A}} = \{p \in \mathbb{Z}^2 : p \notin \mathrm{A}\},$$

$$\mathrm{A} \setminus \mathrm{B} = \mathrm{A} - \mathrm{B} = \mathrm{A} \cap \mathrm{B}^C = \{p \in \mathbb{Z}^2 : (p \in \mathrm{A}) \wedge (p \notin \mathrm{B})\}$$

$$\mathrm{C} = \mathrm{A} \oplus \mathrm{B} = \{c \in \mathbb{Z}^2 : c = a + b, a \in \mathrm{A} \wedge b \in \mathrm{B}\} = \bigcup_{h \in B} \mathrm{A}_h ,$$

$$\mathrm{C} = \mathrm{A} \ominus \mathrm{B} = \{c \in \mathbb{Z}^2 : c + b \in \mathrm{A}, \textit{para todos } b \in \mathrm{B}\} =$$
$$= \{c \in \mathbb{Z}^2 : \mathrm{B}_c \subseteq \mathrm{A}\} = \bigcap_{h \in B} \mathrm{A}_{-h}$$

$$\mathrm{D} = \mathrm{A} \otimes (\mathrm{B},\mathrm{C}) = (\mathrm{A} \ominus \mathrm{B}) \cap (\mathrm{A}^C \ominus \mathrm{C})$$

$$\mathrm{A} \bullet \mathrm{B} = (\mathrm{A} \oplus \mathrm{B}) \ominus \mathrm{B}$$

$$\mathrm{A} \circ \mathrm{B} = (\mathrm{A} \ominus \mathrm{B}) \oplus \mathrm{B}$$

$$\bigcup_i \mathrm{A} \otimes (\mathrm{B}_i, \mathrm{C}_i) = \bigcup_i \left[(\mathrm{A} \ominus \mathrm{B}_i) \cap (\mathrm{A}^C \ominus \mathrm{C}_i) \right]$$