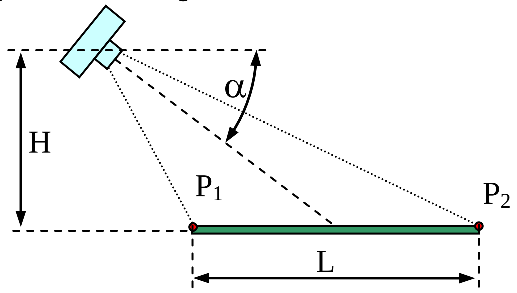
Universidade de Aveiro
Dep. de Engenharia Mecânica

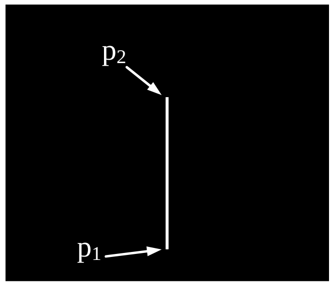
Sistemas de Visão e Percepção Industrial – English ver.

Exame de Época Normal - 18 de Junho de 2012

Mestrado Integrado em Engenharia Mecânica; Mestrado em Engenharia de Automação Industrial
Minor em Automação da Licenciatura em Matemática

1. A camera with 1315x1123 pixels, with 12 mm of focal distance and a *dot-pitch* of 90 pixels/mm, lays at a height H above an horizontal plane and is tilted downwards the α angle, as shown. The camera position is such that its optical axis intersects the middle point of a rod with length L and negligible diameter, which lays over the horizontal plane. The rod has P1 and P2 as its extreme points and is contained in the ZOY plane of the camera coordinate frame. The figure below on the right illustrates the projection of the rod into the camera image plane; pixel (1,1) is the bottom left pixel and pixels p1 and p2 are the images of P1 and P2.





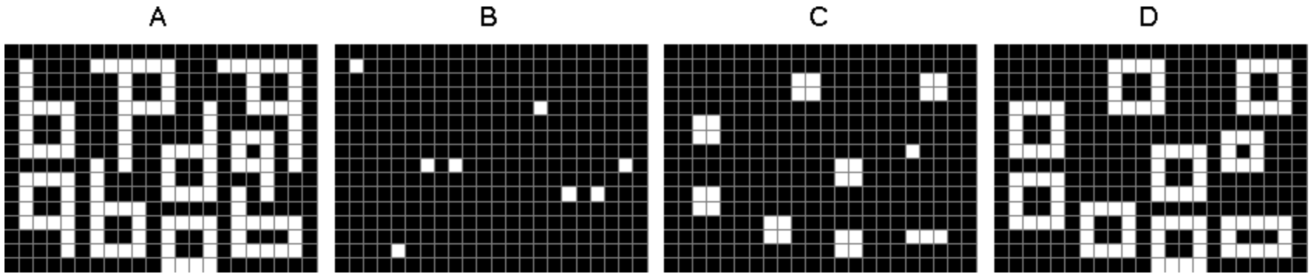
a) Indicate the numeric value of the camera intrinsic matrix.

b) Obtain the generic homogeneous coordinates of points P1 & P2 in the camera coordinate frame.

c) Write the homogeneous matrix expression to obtain the coordinates of pixels p1 & p2.

d) Knowing that L=1 meter and that the pixels y coordinates are y_{pix1}=60 and y_{pix2}=780, obtain the values of H and α.

2. Consider the following binary images:



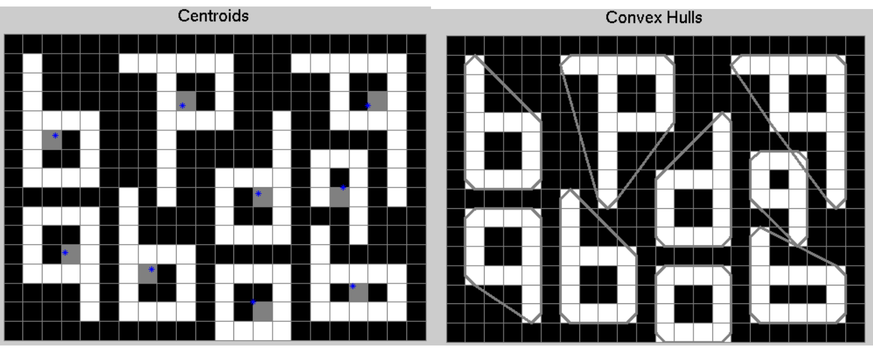
a) Indicate a convolution filter F with dimensions 3x3 and a function *g( )* such as: $\mathrm{B} = g\left(\mathrm{A} * \mathrm{F}\right)$. Justify your answer by illustrating the calculation for different points in A and their corresponding points in B.

b) Using exclusively morphological and set operations, indicate mathematically (using the notation from the formula table on the reverse side of this sheet) all the steps to obtain image C after image A or possible images derived from it. **NB**: It is NOT allowed to use Matlab functions such as `imfill()`, `imreconstruct()`, etc.

c) After image C, indicate an expression with morphological operations to obtain image D.

Regions

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	3	3	3	3	3	3	0	0	0	7	7	7	7	7	7	0
0	1	0	0	0	0	0	0	3	0	0	3	0	0	0	0	0	7	0	0	7	0
0	1	0	0	0	0	0	0	3	0	0	3	0	0	0	0	0	7	0	0	7	0
0	1	1	1	1	0	0	0	3	3	3	3	0	0	5	0	0	7	7	7	7	0
0	1	0	0	1	0	0	0	3	0	0	0	0	0	5	0	0	0	0	0	7	0
0	1	0	0	1	0	0	0	3	0	0	0	0	0	5	0	8	8	8	0	7	0
0	1	1	1	1	0	0	0	3	0	0	5	5	5	5	0	8	0	8	0	7	0
0	0	0	0	0	0	4	0	3	0	0	5	0	0	5	0	8	8	8	0	7	0
0	2	2	2	2	0	4	0	0	0	0	5	0	0	5	0	0	0	8	0	0	0
0	2	0	0	2	0	4	0	0	0	0	5	5	5	5	0	9	0	8	0	0	0
0	2	0	0	2	0	4	4	4	4	0	0	0	0	0	0	9	0	0	0	0	0
0	2	2	2	2	0	4	0	0	4	0	6	6	6	6	0	9	9	9	9	9	0
0	0	0	0	2	0	4	0	0	4	0	6	0	0	6	0	9	0	0	0	9	0
0	0	0	0	2	0	4	4	4	4	0	6	0	0	6	0	9	9	9	9	9	0
0	0	0	0	0	0	0	0	0	0	0	6	6	6	6	0	0	0	0	0	0	0

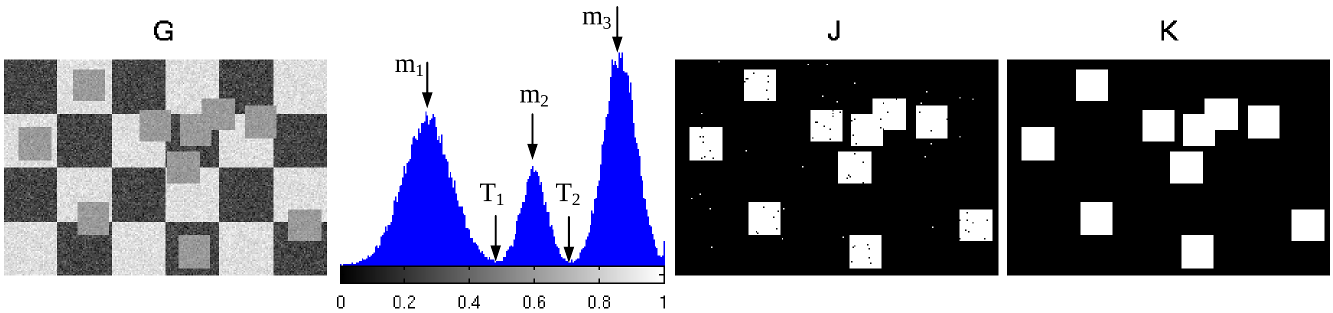


d) After the definition of image moments, calculate the coordinates of the pixel closest to the centroid of object 2 in image A (according to the region numbering form the previous figure).

e) Based on the Convex Hulls from above figure on the right, indicate the objects (from 1 to 9) with the largest and the smallest solidity. Justify with the indication of calculations. **NB**. A pixel is considered to be part of a geometric area if at least half of the pixel is "inside" that area.

f) Let X be a binary matrix {0; 1} defined equal to object 6 (4x4 pixels). Indicate the maximum value reached by the function $\left(\mathrm{A} \circledast \mathrm{X}\right)\left(r,s\right) = \sum_{(i,j)\in \mathrm{X}} \mathrm{A}\left(r+i, s+j\right) \cdot \mathrm{X}\left(i,j\right)$, and in how many points (r,s) that maximum value occurs.

3. **Consider a chess-like board where small square gray objects are placed. These objects have a gray level that, in average, lays between the gray levels of the chess board squares, as illustrated.**



a) Consider available the function `m=mmode(h,k)` that returns a vector `m` with the `k` modes of histogram `h`. For the histogram shown, and for `k=3`, this function returns the following: `m=[0.255 0.605 0.865]`. By using this function, indicate how can the thresholds T1 and T2 be obtained to segment the gray square objects?

b) Using function `B=im2bw(A,T)` which limits image `A` with threshold `T` (similar to Matlab), indicate an expression that allows to obtain the binary image J after G, T1 and T2.

c) Using the notation from the formula table below, indicate an expression with morphological operations to obtain image K after image J. Explain your answer. **NB**. Image J noise is more than only isolated points; in image K the squares are perfectly reconstructed without any deformation in the corners, for example.

4. **Consider a binary image (200 lines x 400 columns) with only 3 white pixels at the coordinates: $p1=[200\ 100]^T$, $p2=[300\ 150]^T$, $p3=[360\ 180]^T$, and where straight lines are sought.**

a) Which are the analytical expressions of the Hough transform (relation between ρ and θ) for p1, p2 e p3?

b) If the spatial resolution of the numerical Hough transform is made to be 2 pixels for the distances, and 1º for the angles (i.e., $\rho \in \{0, \pm 2, \pm 4, \ldots\}$ and $\theta \in \{0^o, \pm 1^o, \pm 2^o, \ldots\}$), determine if there is any accumulator [a pair (ρ,θ)] common to the Hough transforms of the 3 points and, in that case, indicate them. Explain your answer with calculations.

Grading: Question 1 – 5 Val. Question 2 – 9 Val. Question 4 – 4 Val. Question 3 – 2 Val.

Formula table:

Came intrinsic matrix: $\mathbf{K} = \begin{bmatrix} \alpha_x & 0 & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$

Image moments:

$$m_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}}, \quad m_{01} = \sum_x \sum_y y \cdot f(x,y),$$

$$m_{10} = \sum_x \sum_y x \cdot f(x,y)$$

Expressions for histograms:

$$\mu_n = \sum_{i=0}^{L-1} (i-\mu_0)^n h(i) \quad , \quad \mu_0 = \sum_{i=0}^{L-1} i h(i), \quad \text{with} \quad h(i)$$

normalized, i.e., $0 \le h(i) < 1, \forall i \in \{0,1,2,...,L-1\}$

Morphology:

$$\mathrm{A}_h = \{p \in \mathbb{Z}^2 : p = x+h, x \in \mathrm{A}\},$$

$$\mathrm{A}^C = \overline{\mathrm{A}} = \{p \in \mathbb{Z}^2 : p \notin \mathrm{A}\},$$

$$\mathrm{A} \backslash \mathrm{B} = \mathrm{A} - \mathrm{B} = \mathrm{A} \cap \mathrm{B}^C = \{p \in \mathbb{Z}^2 : (p \in \mathrm{A}) \wedge (p \notin \mathrm{B})\}$$

$$\mathrm{C} = \mathrm{A} \oplus \mathrm{B} = \{c \in \mathbb{Z}^2 : c = a+b, a \in \mathrm{A} \wedge b \in \mathrm{B}\} = \bigcup_{h \in B} \mathrm{A}_h,$$

$$\mathrm{C} = \mathrm{A} \ominus \mathrm{B} = \{c \in \mathbb{Z}^2 : \mathrm{B}_c \subseteq \mathrm{A}\} = \bigcap_{h \in B} \mathrm{A}_{-h}$$

$$\mathrm{D} = \mathrm{A} \otimes (\mathrm{B}, \mathrm{C}) = (\mathrm{A} \ominus \mathrm{B}) \cap (\mathrm{A}^C \ominus \mathrm{C})$$

$$\mathrm{A} \bullet \mathrm{B} = (\mathrm{A} \oplus \mathrm{B}) \ominus \mathrm{B}$$

$$\mathrm{A} \circ \mathrm{B} = (\mathrm{A} \ominus \mathrm{B}) \oplus \mathrm{B}$$

$$\bigcup_i \mathrm{A} \otimes (\mathrm{B}_i, \mathrm{C}_i) = \bigcup_i \left[(\mathrm{A} \ominus \mathrm{B}_i) \cap (\mathrm{A}^C \ominus \mathrm{C}_i) \right]$$

Propagation/reconstruction after marker A up to mask B using the structuring element C (recursive conditional dilations):

$$\mathrm{D} = \mathrm{A} \oplus|_B \mathrm{C} \quad \text{equivalent to:} \quad \begin{cases} \mathrm{X}_0 = \mathrm{A} \\ \mathrm{X}_i = (\mathrm{X}_{i-1} \oplus \mathrm{C}) \cap \mathrm{B} \\ \mathrm{D} = \mathrm{X}_i \Leftarrow \mathrm{X}_i = \mathrm{X}_{i-1} \end{cases}$$

Straight line polar equation: $x \cos\theta + y \sin\theta = \rho$

Trigonometry:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Solution of equation: $k_1 \cos\theta + k_2 \sin\theta = k_3$

$$\theta = 2\mathrm{atan2}\left(k_2 \pm \sqrt{k_1^2 + k_2^2 - k_3^2}, k_1 + k_3\right)$$