

### Client-Level Fairness in Federated Learning

- Problem) Non-uniform performance distribution of a global model,  $\theta$

$F_1(\theta) = 0.1,$

$F_2(\theta) = 2.3,$

$F_3(\theta) = 9.5,$

$F_4(\theta) = 0.6,$

$F_5(\theta) = 1.1$

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!! VIOLATION of the CLIENT-LEVEL FAIRNESS !!

- Simple Solution) Use of an adaptive mixing coefficient,  $\mathbf{p}$

$$\min_{\theta \in \Theta \subseteq \mathbb{R}^d} F(\theta) := \sum_{i=1}^K \mathbf{w}_i F_i(\theta), \mathbf{w}_i = \frac{n_i}{\sum_{j=1}^K n_j} \quad \rightarrow \quad \sum_{i=1}^K \mathbf{p}_i F_i(\theta), \mathbf{p} \in \Delta_{K-1}$$

Imposing **LARGER** coefficients to local updates with **LARGER** losses

- Sample Deficiency) Only a single single response vector (e.g., local losses  $[F_1(\theta), \dots, F_K(\theta)]^\top$ ) is given for deciding a single mixing coefficient vector,  $\mathbf{p} = [p_1, \dots, p_K]^\top \dots$

### Fair FL Algorithms are an Online Convex Optimization Algorithms

- Exponentiated Gradient
- For all  $t = 1, \dots, T$ , suppose we want to minimize a decision loss  $\ell^{(t)}(\mathbf{p}) = -\langle \mathbf{p}, \mathbf{r}^{(t)} \rangle$  sequentially, for a response vector  $\mathbf{r}^{(t)} \in \mathbb{R}^K$  and a decision variable  $\mathbf{p} \in \Delta_{K-1}$ :

$$\mathbf{p}^{(t+1)} = \operatorname{argmin}_{\mathbf{p} \in \Delta_{K-1}} \ell^{(t)}(\mathbf{p}) + \eta R(\mathbf{p})$$

• When the regularizer  $R(\mathbf{p}) = \sum_{i=1}^K p_i \log p_i$ , a negative entropy, with a step size  $\eta \in \mathbb{R}_{\geq 0}$ , the closed form update is given as follows.

$$p_i^{(t+1)} = \frac{p_i^{(t)} \exp(r_i^{(t)} / \eta)}{\sum_{j=1}^K p_j^{(t)} \exp(r_j^{(t)} / \eta)}$$

Method	Response, $r_i^{(t)}$	Last Decision, $p_i^{(t)}$	Step Size, $\eta$	New Decision, $p_i^{(t+1)}$
FedAvg	0	$w_i$	1	$\propto w_i$
q-FedAvg (AFL if $q \rightarrow \infty$ )	$q \log F_i(\theta^{(t)})$	$w_i$	1	$\propto w_i F_i^q(\theta^{(t)})$
TERM	$F_i(\theta^{(t)})$	$w_i$	$1/\lambda$	$\propto w_i \exp(\lambda F_i(\theta^{(t)}))$
PropFair	$-\log(M - F_i(\theta^{(t)}))$	$w_i$	1	$\propto \frac{w_i}{M - F_i(\theta^{(t)})}$

### Follow-The-Regularized-Leader (FTRL)

- OCO is certainly a suitable choice for making decision under the sample deficiency.
- Existing fair FL algorithms were NOT devised for online learning.
- Better decision-making is possible with the following modified objective.

$$\mathbf{p}^{(t+1)} = \operatorname{argmin}_{\mathbf{p} \in \Delta_{K-1}} \sum_{\tau=1}^t \ell^{(\tau)}(\mathbf{p}) + \eta^{(t+1)} R(\mathbf{p}) \quad (\text{or } R^{(t+1)}(\mathbf{p}))$$

### AAgFF: Addaptive Aggregation for Fair Federated Learning

- AAgFF-S: Cross-Silo FL Setting
- Number of clients ( $K$ ) < Total communication rounds ( $T$ )

• Full synchronization of clients
- $$\mathbf{p}^{(t+1)} = \operatorname{argmin}_{\mathbf{p} \in \Delta_{K-1}} \sum_{\tau=1}^t \tilde{\ell}^{(\tau)}(\mathbf{p}) + \frac{\alpha}{2} \|\mathbf{p}\|_2^2 + \frac{\beta}{2} \sum_{\tau=1}^t (\langle \mathbf{g}^{(\tau)}, \mathbf{p} - \mathbf{p}^{(\tau)} \rangle)^2$$
- Runtime:  $\mathcal{O}(K^2 + K^3)$  (cubic complexity due to weighted simplex projection)
- AAgFF-D: Cross-Device FL Setting
- Number of clients ( $K$ )  $\gg$  Total communication rounds ( $T$ )

• Partial synchronization of clients (i.e., client sampling is required)

$$\mathbf{p}^{(t+1)} = \operatorname{argmin}_{\mathbf{p} \in \Delta_{K-1}} \sum_{\tau=1}^t \tilde{\ell}^{(\tau)}(\mathbf{p}) + \frac{L_\infty \sqrt{t+1}}{\sqrt{\log K}} \sum_{i=1}^K p_i \log p_i$$

- Runtime:  $\mathcal{O}(K)$  (closed-form update exists)
- Doubly-Robust Estimator for Partially Observed Response (in the Cross-Device Setting)
- Denote  $\mathcal{C} = P(i \in S^{(t)})$  as a client sampling probability,

$S^{(t)}$  is an index set of selected clients, and  $\bar{\mathbf{r}}^{(t)} = \frac{1}{|S^{(t)}|} \sum_{i \in S^{(t)}} r_i^{(t)}$ . Then,

$$\check{r}_i^{(t)} = \left(1 - \frac{\mathbb{I}(i \in S^{(t)})}{\mathcal{C}}\right) \bar{\mathbf{r}}^{(t)} + \frac{\mathbb{I}(i \in S^{(t)})}{\mathcal{C}} r_i^{(t)},$$

which satisfies  $\mathbb{E}[\check{\mathbf{r}}^{(t)}] = \mathbf{r}^{(t)}$ .

### Vanishing Regret Guarantees

- ☺ **Decision Objective** (Regret Minimization):  $\text{Regret}^{(T)}(\mathbf{p}^\star) \triangleq \sum_{t=1}^T \ell^{(t)}(\mathbf{p}^{(t)}) - \sum_{t=1}^T \ell^{(t)}(\mathbf{p}^\star)$
- ☹ **Decision Loss** (Negative Logarithmic Growth):  $\ell^{(t)}(\mathbf{p}) \triangleq -\log(1 + \langle \mathbf{p}, \mathbf{r}^{(t)} \rangle)$
- ☺ **Theorem 1** (Regret of **AAgFF-S**):  $\text{Regret}^{(T)}(\mathbf{p}^\star) \leq 2L_\infty K \left(1 + \log\left(1 + \frac{T}{16K}\right)\right)$
- ☺ **Theorem 2** (Regret of **AAgFF-D** w/o sampling):  $\text{Regret}^{(T)}(\mathbf{p}^\star) \leq 2L_\infty \sqrt{T \log K}$
- ☺ **Corollary 3** (Regret of **AAgFF-D** w/ sampling):  $\mathbb{E}[\text{Regret}^{(T)}(\mathbf{p}^\star)] \leq 2L_\infty \sqrt{T \log K}$

### Improved Uniformity in Performance Distribution

(Number of Clients:  $K$  / Number of Rounds:  $T$ )

Dataset	Berka (AUROC)				MQP (AUROC)				ISIC (Acc. 5)			
	Avg. (↑)	Worst (↑)	Best (↑)	Gini (↓)	Avg. (↑)	Worst (↑)	Best (↑)	Gini (↓)	Avg. (↑)	Worst (↑)	Best (↑)	Gini (↓)
FedAvg	80.09 (2.45)	48.06 (25.15)	<b>99.03</b> (1.37)	10.87 (4.11)	56.06 (0.06)	41.03 (4.33)	76.31 (8.42)	8.63 (0.91)	87.42 (2.11)	69.92 (6.78)	92.57 (2.56)	4.84 (1.17)
AFL	79.70 (4.14)	49.02 (25.89)	<b>98.55</b> (2.05)	10.58 (5.03)	56.01 (0.30)	41.28 (3.92)	75.54 (6.77)	<b>8.56</b> (1.24)	87.39 (2.31)	68.17 (10.09)	93.33 (2.18)	4.80 (1.74)
q-FedAvg	79.98 (3.89)	<b>49.44</b> (26.15)	98.07 (2.73)	10.62 (5.22)	<b>56.89</b> (0.42)	40.22 (3.06)	<b>79.38</b> (9.09)	8.68 (0.57)	41.59 (16.22)	20.38 (23.24)	58.08 (28.52)	22.25 (10.02)
TERM	<b>80.11</b> (3.08)	48.96 (25.79)	<b>99.03</b> (1.37)	10.86 (4.73)	56.47 (0.19)	40.73 (4.36)	76.80 (8.30)	8.67 (1.43)	<b>87.89</b> (1.69)	<b>77.32</b> (5.84)	<b>96.00</b> (3.27)	<b>3.77</b> (0.94)
FedMGDA	79.24 (2.96)	46.38 (24.11)	<b>99.03</b> (1.37)	11.64 (4.84)	53.02 (1.67)	34.91 (2.22)	69.65 (3.89)	10.33 (0.44)	42.36 (14.94)	21.44 (21.30)	59.21 (28.52)	22.25 (10.02)
PropFair	79.61 (4.49)	<b>49.44</b> (26.15)	98.07 (2.73)	<b>10.47</b> (5.04)	56.60 (0.39)	<b>41.71</b> (3.80)	<b>79.09</b> (7.40)	8.74 (0.87)	83.88 (2.50)	58.36 (11.63)	91.35 (2.48)	7.91 (2.10)
AAgFF-S (Proposed)	<b>80.93</b> (2.96)	<b>52.08</b> (23.59)	<b>99.03</b> (1.37)	<b>10.16</b> (3.80)	<b>56.63</b> (0.54)	<b>41.79</b> (4.43)	<b>75.56</b> (6.53)	<b>8.38</b> (0.77)	<b>89.76</b> (1.03)	<b>85.17</b> (3.87)	<b>98.22</b> (1.66)	<b>2.52</b> (0.38)

◀ Cross-Silo  
Setting ( $K < T$ )

Dataset	CelebA (Acc. 1)				Reddit (Acc. 1)				SpeechCommands (Acc. 5)			
	Avg. (↑)	Worst 10% (↑)	Best 10%(↑)	Gini (↓)	Avg. (↑)	Worst 10%(↑)	Best 10%(↑)	Gini (↓)	Avg. (↑)	Worst 10%(↑)	Best 10%(↑)	Gini (↓)
FedAvg	90.79 (0.53)	<b>55.76</b> (0.84)	<b>100.00</b> (0.00)	7.86 (0.30)	10.76 (1.45)	2.50 (0.21)	20.86 (3.64)	25.66 (0.49)	<b>75.51</b> (1.08)	7.93 (2.87)	<b>100.00</b> (0.00)	24.58 (1.34)
q-FedAvg	<b>90.88</b> (0.19)	55.73 (0.85)	<b>100.00</b> (0.00)	<b>7.82</b> (0.21)	<b>12.76</b> (0.32)	<b>3.38</b> (0.20)	<b>21.81</b> (0.19)	<b>23.34</b> (0.34)	<b>73.34</b> (0.47)	<b>11.19</b> (0.47)	<b>100.00</b> (0.00)	<b>23.16</b> (0.13)
TERM	90.71 (0.65)	55.66 (0.93)	<b>100.00</b> (0.00)	7.90 (0.38)	12.02 (0.16)	2.85 (0.41)	20.74 (0.65)	24.15 (1.05)	70.90 (2.96)	5.98 (1.10)	<b>100.00</b> (0.00)	26.37 (1.32)
FedMGDA	88.33 (0.63)	48.60 (25.85)	<b>100.00</b> (0.00)	9.75 (0.59)	10.58 (0.18)	2.35 (0.20)	19.09 (0.62)	25.20 (0.22)	72.45 (1.88)	9.65 (2.90)	<b>100.00</b> (0.00)	23.68 (1.27)
PropFair	87.25 (5.01)	48.11 (10.03)	<b>100.00</b> (0.00)	10.39 (3.43)	11.26 (0.71)	1.95 (0.32)	21.33 (0.92)	25.97 (1.02)	73.64 (3.31)	7.30 (1.02)	<b>100.00</b> (0.00)	24.97 (1.09)
AAgFF-D (Proposed)	<b>91.27</b> (0.07)	<b>56.71</b> (0.08)	<b>100.00</b> (0.00)	<b>7.54</b> (0.04)	<b>12.95</b> (0.39)	<b>4.75</b> (0.76)	<b>22.81</b> (1.36)	<b>22.59</b> (0.28)	<b>76.68</b> (0.80)	<b>14.54</b> (2.58)	<b>100.00</b> (0.00)	<b>21.42</b> (0.81)

◀ Cross-Device  
Setting ( $K \gg T$ )

