I. Wavenumber-frequency space

The *wavenumber vector* (or wave vector) of a plane wave $\vec{k} = \{k_x, k_y, k_z\}$ is the propagation vector giving both the magnitude and direction of arrival of the incident plane wave. Assuming a plane wave with wavelength λ, the magnitude of the wavenumber vector is the *wavenumber* of the wave $|\vec{k}| = k$ measured in units of radians per meter.

$$|\vec{k}| = k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda} \tag{1}$$

where f is the frequency of the incident plane wave (ranging from 20 Hz to 20 kHz for audible sound) and c is the speed of sound waves in air approximately 340 m/s.

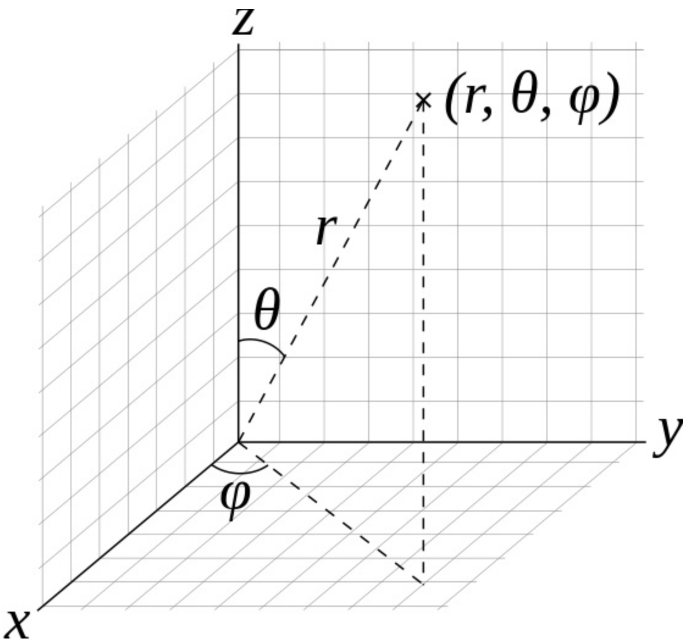

Fig. 1. Spherical coordinate system (image courtesy of Wikipedia)

In most situations, a three-dimensional Cartesian grid represents space, with time being the fourth dimension. Other coordinate systems may be defined as well, and for certain problems, it's more convenient to use spherical coordinates. Here a point is represented by its distance r from the origin, its azimuth ϕ within the equatiorial plane, and its angle θ down from the vertical axis. In Fig. 1 a right handed orthogonal coordinate system is depicted along with a spherical coordinate system. The angle θ is known as the elevation and is the normal incidence angle, and ϕ is denoted the azimuth which is the angle in the XY plane. For a wave propagating in spherical coordinates, the wave vector is related to the Cartesian coordinates by simple trigonometric formulas

$$\begin{aligned} k_x &= k \sin\theta \cos\phi \\ k_y &= k \sin\theta \sin\phi \\ k_z &= k \cos\theta \end{aligned} \tag{2}$$

where the x-component of the wave vector, k_x, determines the rate of change of the phase of a propagating plane wave in the x-direction. The same definitions apply for the y- and z-directions.

II. Beampattern

When describing the response of an array which is not discrete, but can be sampled at all points within an area, the term *aperture smoothing function* is used. The aperture smoothing function of an array is given as

$$W(\vec{k}) = \int_{-\infty}^{\infty} w(\vec{x}) e^{j\vec{k}\vec{x}} \tag{3}$$

where $\vec{x}$ describes the extent of the array. The discrete version of the aperture smoothing function is named the *array factor* (AF), and describes the spatial response of an M element array. The total response of an array is called the *array pattern*, or *beampattern* (BP), and will not only be a function of the array geometry, but also of the radiation pattern of the individual elements. The spatial response of the element is called the *element pattern* (EP). The beampattern which gives the complete pattern representation of the array can be found by multiplying the array factor and the element pattern. This assumes that the element pattern is identical for each element. The beampattern of an array can then be stated as

$$BP = EP \cdot AF \tag{4}$$

For an array with isotropic elements, the beampattern of the array is the same as the array factor. The positions of the M array sensors, or array elements, are given as

$$\vec{x} = \{x_m, y_m, z_m\} \tag{5}$$

where m ranges from 0 to $M-1$ and each sensor has a weight w_m. The total beampattern of an array with isotropic elements is the weighted sum of the individual elements and may be calculated as

$$\begin{aligned} W(\vec{k}) &= \sum_{m=0}^{M-1} w_m e^{j\vec{k}\vec{x}} \\ W(k_x, k_y, k_z) &= \sum_{m=0}^{M-1} w_m e^{j(k_x x_m + k_y y_m + k_z z_m)} \\ W(k_x, k_y, k_z) &= \sum_{m=0}^{M-1} w_m e^{jk(\sin\theta\cos\phi x_m + \sin\theta\sin\phi y_m + \cos\theta z_m)} \\ W(k_x, k_y, k_z) &= \sum_{m=0}^{M-1} w_m e^{j\frac{2\pi f}{c}(\sin\theta\cos\phi x_m + \sin\theta\sin\phi y_m + \cos\theta z_m)} \end{aligned} \tag{6}$$