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Понижаване на реда на ДУ

1) $F(x, y^{(k)}, y^{(k+1)}, \dots, y^{(n)}) = 0$

пол $z = y^{(k)}$

Пример 1) $x^2 y'' - y'^2 = 0$

пол $y' = z \to x^2 z' - z = 0$

$x^2 z' = z \qquad z = 0$ ерем. $\to y' = 0 \to y = const$

$\frac{z'}{z} = \frac{1}{x^2} \to \int \frac{z' dx}{z} = \int x^{-2} dx + c$

$\int \frac{dz}{z} = -\frac{1}{x} + c \to \ln z = -\frac{1}{x} + c$

$\Rightarrow z = \underbrace{e^c}_{k} e^{-1/x} \to y' = k e^{-1/x}$

$y(x) = k \int e^{-1/x} dx + k_1$

Дом.

2) $y''' = -\frac{1}{2} y''^2 \qquad$ 3) $x y'' = y' + x \sin\left(\frac{y'}{y}\right)$

2) $F(y, y', \dots y^{(n)}) = 0$ (Автономно у-е)

пол $y' = p(y) \to y'' = p' \cdot y' = p' p$ и т.н.

Пр $y^3 y'' + 1 = 0$ пол $y' = p(y) \to y'' = p'p$

$y^3 p'p + 1 = 0 \to p'p = -\frac{1}{y^3}$

$\int p'p\, dy = -\int y^{-3} dy + c \to \int p\, dp = -\frac{y^{-2}}{-2} + c$

$\frac{p^2}{2} = \frac{1}{2y^2} + c \to p^2 = \frac{1}{y^2} + 2c$

$y' = \pm\sqrt{\frac{1}{y^2} + 2c} \to y' = \pm \frac{\sqrt{2cy^2 + 1}}{y}$

$\int \frac{y}{\sqrt{2cy^2+1}} dy = \int dx + c_1 \to \frac{1}{c2} \int \frac{d(cy^2+1)}{(2cy^2+1)^{1/2}} = x + c_1$

1ви $c \neq 0$

$$\rightarrow \frac{1}{c^2} \frac{(2cy^2+1)^{1/2}}{1/2} = x + C_1$$

2ри $c = 0 \rightarrow \int y\,dy = \int dx + C_1$

$$\frac{y^2}{2} = x + C_1$$

<u>Дом</u>

1) $xyy'' - xy'^2 = yy'$

2) $yy'' = y'^2 + 15y^2\sqrt{x}$

3) $F(x, y, y', \dots, y^{(n)}) = 0$

$$F(x, y, y', \dots, y^{(n)}) = \frac{d}{dx} G(x, y, \dots y^{(n-1)}) = 0$$

Пълни производни (първи интеграли)

<u>Пример</u> $yy'' = y'^2 \rightarrow y'^2 - yy'' = 0 \quad : y'^2$

$$\left(\frac{y}{y'}\right)' = \frac{y'y' - yy''}{y'^2} = 0$$

$$\frac{d}{dx}\left(\frac{y}{y'}\right) = 0 \rightarrow \frac{y}{y'} = C \rightarrow \frac{y'}{y} = \frac{1}{C}$$

$$\int \frac{dy}{y} = \frac{1}{C}\int dx + C_1$$

$$\ln y = \frac{1}{C}x + C_1 \rightarrow y = e^{\frac{1}{C}x + C_1}$$

<u>Дом</u>

1) $yy''' + 3y'y'' = 0$

2) $yy''' = 2y''^2$