Хипергеометрично уравнение

$$ {}_2F_1(a;b|c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \quad ; \quad |z|<1 $$

$$ (q)_n = \begin{cases} 1 & , \quad n=0 \\ q(q+1)\cdots(q+n-1) & , \quad n>0 \end{cases} $$

Хипергеометрична ф-я

$$ z(1-z)\frac{d^2w}{dz^2} + (c-(a+b+1)z)\frac{dw}{dz} - abw = 0 $$

регулярни сингулярни точки $1, 0, \infty$

локални решения за $z=0$

$$ {}_2F_1(a;b|c;z) $$

$$ z^{1-c}\,{}_2F_1(1+a-c, 1+b-c; 2-c; z) $$

$$ \frac{d^2y}{dx^2} = xy \quad - \text{ Airy or Stokes eq.} $$

$$ Ai(x) = \frac{1}{\pi}\int_0^{\infty} \cos\left(\frac{t^3}{3}+xt\right)dt $$

$$ Bi(x) = \frac{1}{\pi}\int_0^{\infty}\left[\exp\left(-\frac{t^3}{3}+xt\right) + \sin\left(\frac{t^3}{3}+xt\right)\right]dt $$

Уравнение на Хойн

$$ \frac{d^2w}{dz^2} + \left[\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\varepsilon}{z-a}\right]\frac{dw}{dz} + \frac{\alpha\beta z - q}{z(z-1)(z-a)}w = 0 $$

$$ \varepsilon = \alpha+\beta-\gamma-\delta-1 $$

локални решения за $\underline{z=0}$

$H(a;q;\alpha;\beta;\gamma;\delta;z)$ – ф-я на Хойн!

$$ z^{1-\gamma}H(a;(a\delta+\varepsilon)(1-\gamma)+q;\alpha+1-\gamma,\beta+1-\gamma,2-\gamma;\delta;z) $$

Уравнение на Лъжандър

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + l(l+1)y = 0$$

$$y_1(x) = 1 + \sum_{n=1}^{\infty}(-1)^n \frac{[(l-2n+2)\dots(l-2)l][(l+1)(l+3)\dots(l+2n-1)]}{(2n)!}x^{2n}$$

$$y_2(x) = x + \sum_{n=1}^{\infty}(-1)^n \frac{[(l-2n+1)\dots(l-3)(l-1)][(l+2)(l+4)\dots(l+2n)]}{(2n+1)!}x^{2n+1}$$

Уравнение на Матьо

$$\frac{d^2y}{dx^2} + (a - 2q\cos 2x)y = 0$$

$C(a,q,x)$ - четна

$S(a,q,x)$ - нечетна ф-я на Матьо!

$$C(a,0,z) = \cos(\sqrt{a}\,z)$$

$$S(a,0,z) = \sin(\sqrt{a}\,z)$$

У. е на Ламе (Хойн)

$$\frac{d^2y}{dx^2} + \frac{1}{2}\left(\frac{1}{x-e_1} + \frac{1}{x-e_2} + \frac{1}{x-e_3}\right)\frac{dy}{dx} \; \frac{Bx+A}{4(x-e_1)(x-e_2)(x-e_3)}\,y = 0$$

или

$$\frac{d^2y}{dx^2} + (A + B\wp(x))y = 0$$

$\wp(x)$ - P-ф-ция Вейерщрас

$\wp(x)$ е реш на ДУ

$$\wp'^2 = 4\wp^3 - g_2\wp - g_3$$