Васил Христов, №62431, СИ, група 2.

Контролна работа №1.

2.3. Решете задачата на Коши.

$$\begin{cases} 4(\sqrt{\pi}x+1)y^3y' = 2\sqrt{\pi}y^4 + (\sqrt{\pi}x+1)\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right), \\ y(0) = -1 \end{cases}$$

~~Това е уравнение на~~

$$4(\sqrt{\pi}x+1)y^3y' = 2\sqrt{\pi}y^4 + (\sqrt{\pi}x+1)\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right), \quad x \neq -\frac{1}{\pi}$$

$$y' = \frac{(\sqrt{\pi}x+1)\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)}{4(\sqrt{\pi}x+1)y^3} + \frac{2\sqrt{\pi}y^4}{4(\sqrt{\pi}x+1)y^3}$$

$$y' = \frac{\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)}{4y^3} + \frac{\sqrt{\pi}y}{2(\sqrt{\pi}x+1)}$$

$$y' = \frac{\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)}{4} \cdot y^{-3} + \frac{\sqrt{\pi}}{2+2\sqrt{\pi}x} \cdot y$$

$$y' = \frac{\sqrt{\pi}}{2+2\sqrt{\pi}x} \cdot y + \frac{\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)}{4} \cdot y^{-3}$$

1) $y = 0$ е решение

2) $y \neq 0 \Rightarrow$ Делим на y^{-3}

$$\frac{y'}{y^{-3}} = \frac{\sqrt{\pi}}{2+2\sqrt{\pi}x} \cdot y^4 + \frac{\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)}{4}$$

Полагаме $z(x) = y^4$, $z' = 4y^3y' = \frac{4y'}{y^{-3}}$

$$\frac{1}{4}z' = \frac{\sqrt{\pi}}{2+2\sqrt{\pi}x} \cdot z + \frac{\sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)}{4} \Rightarrow z' = \frac{2\sqrt{\pi}}{\sqrt{\pi}x+1} \cdot z + \sin\left(\frac{\sqrt{\pi}}{\sqrt{\pi}x+1}\right)$$

$$a(x) = \frac{2\pi}{\pi x+1}, \quad b(x) = \sin\left(\frac{\pi}{\pi x+1}\right)$$

$$z' = e^{\int \frac{2\pi}{\pi x+1}dx}\left[C + \int \sin\left(\frac{\pi}{\pi x+1}\right)\cdot e^{-\int \frac{2\pi}{\pi x+1}dx}dx\right]$$

$$z' = e^{2\ln|\pi x+1|}\left[C + \int \sin\left(\frac{\pi}{\pi x+1}\right)\cdot e^{-2\ln|\pi x+1|}dx\right]$$

$$z' = e^{\ln(\pi x+1)^2}\left[C + \int \sin\left(\frac{\pi}{\pi x+1}\right)\cdot \frac{1}{(\pi x+1)^2}dx\right]$$

$$z' = (\pi x+1)^2\left[C + \frac{\cos\left(\frac{\pi}{\pi x+1}\right)}{\pi^2}\right]$$

$$z' = (\pi x+1)^2\left(C + \frac{\cos\left(\frac{\pi}{\pi x+1}\right)}{\pi^2}\right)$$

$$z = y^4 \Rightarrow y' = \sqrt[4]{(\pi x+1)^2\left(C + \frac{\cos\left(\frac{\pi}{\pi x+1}\right)}{\pi^2}\right)}$$

$$y(0) = -1 \Rightarrow$$

$$\Rightarrow -1 = \sqrt[4]{(\pi\cdot 0+1)^2\left(C + \frac{\cos\left(\frac{\pi}{\pi\cdot 0+1}\right)}{\pi^2}\right)}$$

$$-1 = \sqrt[4]{C + \frac{\cos\pi}{\pi^2}}$$

$$-1 = \sqrt[4]{C - \frac{1}{\pi^2}}$$

$$1 = C - \frac{1}{\pi^2} \Rightarrow C = 1 + \frac{1}{\pi^2} = \frac{\pi^2+1}{\pi^2}$$

$$\boxed{C = \frac{\pi^2+1}{\pi^2}}$$

$$\boxed{y^* = \sqrt[4]{(\pi x+1)^2\left(\frac{\pi^2+1}{\pi^2} + \frac{\cos\left(\frac{\pi}{\pi x+1}\right)}{\pi^2}\right)}}$$