PROJECT MMCE

Choose appropriate numerical method and implement it to solve the problem:

$$\frac{\partial u_1}{\partial t} = \varkappa_1 \frac{\partial^2 u_1}{\partial x^2}, \quad -\infty < x < 0, \qquad 0 < t \le T, \tag{1}$$

$$\frac{\partial u_2}{\partial t} = \varkappa_2 \frac{\partial^2 u_2}{\partial x^2}, \qquad 0 < x < \infty, \qquad 0 < t \le T; \tag{2}$$

initial conditions:

$$u_1(x,0) = 0, \quad -\infty < x < \infty, \tag{3}$$

$$u_2(x,0) = u_0, \quad 0 < x < \infty; \tag{4}$$

boundary conditions:

$$u_1(-\infty,t) = 0, \quad t > 0, \tag{5}$$

$$u_2(+\infty,t) = u_0, \quad t > 0, \tag{6}$$

ideal contact conditions:

$$u_1(0,t) = u_2(0,t), \quad t > 0, \tag{7}$$

$$k_1 \frac{\partial u_1}{\partial x}(0,t) = k_2 \frac{\partial u_2}{\partial x}(0,t), \quad t > 0. \tag{8}$$

Make computational experiments:

A) for bodies of the same thermo-physical characteristics;

B) for bodies of different thermo-physical characteristics;

Determine "the actual infinity" depending on the time T so, that the boundary conditions are fulfilled with sufficient accuracy.

Compare the numerical solution with the exact one:

$$u_1(x,t) = \frac{\beta u_0}{1+\beta}\left[1 + erf\left(\frac{x}{2\sqrt{\varkappa_1 t}}\right)\right],$$

$$u_2(x,t) = \frac{u_0}{1+\beta}\left[\beta + erf\left(\frac{x}{2\sqrt{\varkappa_2 t}}\right)\right],$$

where $erf(z)$ is the special function "error function":

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt, \qquad erf(-z) = -erf(z), \qquad \frac{\sqrt{k_2\rho_2 c_2}}{\sqrt{k_1\rho_1 c_1}} = \beta.$$