Let A, B be nxm matrices, then:

1. C = A $\odot$ B is the elementwise (Hadamard) product, i.e $c_{ij} = a_{ij}b_{ij}$
2. Similarly, D = A $\oslash$ B is the elementwise division: $d_{ij} = a_{ij}/b_{ij}$
3. A $\star$ B is the discrete cross-correlation of A and B

Further let: **i** = (i, j) be a multi-index and **h** = (h_x, h_y) be a two-dimensional offset.

Starting from the sum:

$$T = \sum_{\mathrm{ij}} \left(z_{ij}^2 - z_{ij}\right)^2 = \sum_{h \in S_h} (z_i - z_{i+h})^2 = \sum_{h \in S_h} z_i^2 - 2z_i z_{i+h} + z_{i+h}^2$$
$$= \sum_{h \in S_h} z_i^2 - 2\sum_{h \in S_h} z_i z_{i+h} + \sum_{h \in S_h} z_{i+h}^2$$
$$= (\mathrm{Z} \odot \mathrm{Z}) \star \mathrm{J} - 2\,\mathrm{Z} \star \mathrm{Z} + \mathrm{J} \star (\mathrm{Z} \odot \mathrm{Z})$$

Where J is an all-ones matrix, with the same shape as Z

To obtain the normalizing factor (how many pairs from Z participate per lag) we can calculate:

$$\mathrm{D} = \mathrm{J} \star \mathrm{J}$$

Finally, the discrete 2D correlation function / variogram is given by:

$$\mathrm{C} = \mathrm{T} \oslash \mathrm{D} = [(\mathrm{Z} \odot \mathrm{Z}) \star \mathrm{J} - 2\,\mathrm{Z} \star \mathrm{Z} + \mathrm{J} \star (\mathrm{Z} \odot \mathrm{Z})] \oslash [\mathrm{J} \star \mathrm{J}]$$

Note: Computationally, flipping $\mathrm{J} \star (\mathrm{Z} \odot \mathrm{Z})$ along both axes is equivalent to computing $(\mathrm{Z} \odot \mathrm{Z}) \star \mathrm{J}$.

To obtain the distances (lags) for which C, we need to note that, if $Z\colon n\ x\ \mathrm{m}$, then

$\mathrm{Z} \star \mathrm{Z}\colon 2\mathrm{n} - 1 \times 2\mathrm{m} - 1$ matrix. The matrix of lags Q for which C was computed is then given:

$$\mathrm{Q}_{h_y, h_x} = \sqrt{(h_x \cdot \Delta \mathrm{x})^2 + \left(h_y \cdot \Delta \mathrm{y}\right)^2}$$

Where:

$h_y \in \{-(n_y - 1), -(n_y - 2), \dots, -1,0,1, \dots, n_y - 2, n_y - 1\}$ and

$h_x \in \{-(n_x - 1), -(n_x - 2), \dots, -1,0,1, \dots, n_x - 2, n_x - 1\}$

Finally:

1. We flatten **C** and **Q**
2. We bin the data with equal population bins based on C
3. We average the data in each bin
4. Plot the bin centers vs averaged C-values