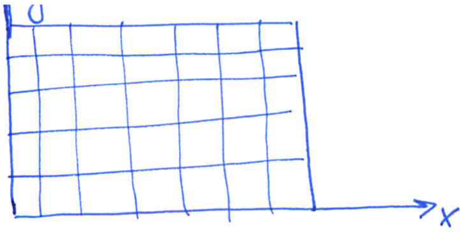

Уравнения акустики

$$\begin{cases}\rho \dfrac{\partial \vec{v}}{\partial t} + \nabla p = \vec{f}, \longrightarrow \dfrac{\partial v}{\partial t} + \dfrac{1}{\rho}\nabla p = \dfrac{f}{\rho}, \\ \dfrac{\partial p}{\partial t} + \rho c^2 (\nabla \cdot \vec{v}) = 0\end{cases}$$

2D случай:

$$\vec{q} = \begin{pmatrix} V_x \\ V_y \\ p \end{pmatrix} \qquad \begin{cases} \dfrac{\partial V_x}{\partial t} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = \dfrac{f_x}{\rho}, \\ \dfrac{\partial V_y}{\partial t} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial y} = \dfrac{f_y}{\rho}, \\ \dfrac{\partial p}{\partial t} + \rho c^2 \dfrac{\partial V_x}{\partial x} + \rho c^2 \dfrac{\partial V_y}{\partial y} = 0 \end{cases}$$

$$\frac{\partial}{\partial t}\underbrace{\begin{pmatrix} V_x \\ V_y \\ p \end{pmatrix}}_{q} + \underbrace{\begin{pmatrix} 0 & 0 & \frac{1}{\rho} \\ 0 & 0 & 0 \\ \rho c^2 & 0 & 0 \end{pmatrix}}_{A_x} \frac{\partial}{\partial x}\begin{pmatrix} V_x \\ V_y \\ p \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\rho} \\ 0 & \rho c^2 & 0 \end{pmatrix}}_{A_y} \frac{\partial}{\partial y}\begin{pmatrix} V_x \\ V_y \\ p \end{pmatrix} = \begin{pmatrix} \frac{f_x}{\rho} \\ \frac{f_y}{\rho} \\ 0 \end{pmatrix}$$

$$\frac{\partial \vec{q}}{\partial t} + A_x \frac{\partial \vec{q}}{\partial x} + A_y \frac{\partial \vec{q}}{\partial y} = \vec{F}$$

Рассмотрим случай $\vec{F} = 0$, $\dfrac{\partial q}{\partial y} = 0$

$$\frac{\partial \bar{q}}{\partial t} + A_x \frac{\partial \bar{q}}{\partial x} = 0 \Rightarrow \frac{\partial \bar{q}}{\partial t} + \Omega^{-1}\Lambda\Omega \frac{\partial \bar{q}}{\partial x} = 0 \Rightarrow$$

$$\Rightarrow \Omega \frac{\partial \bar{q}}{\partial t} + \Omega\Omega^{-1}\Lambda\Omega \frac{\partial \bar{q}}{\partial x} = 0 \Rightarrow \frac{\partial \Omega \bar{q}}{\partial t} + \Lambda \frac{\partial \Omega \bar{q}}{\partial x} = 0$$

$$\Omega = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{\rho c}{2} & 0 & \frac{1}{2} \\ \frac{\rho c}{2} & 0 & \frac{1}{2} \end{pmatrix}; \quad \Omega^{-1} = \begin{pmatrix} 0 & -\frac{1}{\rho c} & \frac{1}{\rho c} \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -c & 0 \\ 0 & 0 & c \end{pmatrix}$$

Решаем уголком $\dfrac{\partial W_i}{\partial t} + \lambda_i \dfrac{\partial W_i}{\partial x} = 0$ и $\underline{\bar{q}^{n+1} = \Omega^{-1}\bar{W}^{n+1}}$

Для A_y получим

Λ без изменений, $\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{\rho c}{2} & \frac{1}{2} \\ 0 & \frac{\rho c}{2} & \frac{1}{2} \end{pmatrix}$

$$\Omega^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\rho c} & \frac{1}{\rho c} \\ 0 & 1 & 1 \end{pmatrix}$$