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An Investor's Guide to Floating-Rate Notes: Conventions, Mathematics and Relative Valuation

TABLE OF CONTENTS — PAGE

FIGURES

Virtually every sector of the fixed-income market contains bonds with floating-rate coupons. The popularity of these securities arises from the reluctance of many investors to accept a long-maturity fixed bond, particularly in periods when market participants fear rising interest rates. Consider a hypothetical bond that is issued at a price of 100, accrues a coupon of some easily observed index plus some prespecified spread (which resets to the new index level whenever the index level changes [instantaneous reset]), has no credit risk or embedded options, and the timing of the payment of principal is certain. In this case, the intuition that floating-rate notes have no price sensitivity and should always trade at or near 100 is valid.

Of course, such bonds do not exist in today's markets, and therefore, all floating-rate securities (FRN) do have some degree of price sensitivity. In reality, no floating-rate securities reset instantaneously, and there typically is a lag before the index rate used in coupon accrual resets to the market index rate. Corporate and emerging market floaters have exposure to changes in credit spreads. Floating-rate mortgage-backed securities and many products in the structured note market usually have embedded options such as caps and floors. Mortgage-backed FRNs have the additional feature of the prepayment option which affects the amortization of principal. In addition to the usual effect that faster or slower prepayments have on the performance of a mortgage bond, unexpected prepayment speeds will affect the value of the embedded caps and floors.

Investors who purchase FRNs are usually aware of these issues. However, it is not always easy to evaluate the implied risks and the appropriate compensation for taking on these risks. In fact, given the nature of floating-rate securities, it is not trivial to *measure* the compensation that the investor is receiving for taking risks. A framework by which to measure risk-return trade-offs is particularly important for investors that have the flexibility to invest in FRNs across several different sectors.

The purpose of this research is twofold. First, we provide a reference that describes in detail the various conventions and tools used in FRN markets. Next, we discuss the relative valuation of different types of FRNs and provide some frameworks by which to assess their risk/return trade-offs.

The text is structured as follows:

- In the first section, we explicitly define pricing relationships and commonly used terms such as *discount margin* and *reset margin*. One particularly useful formulation of the standard pricing equation allows us to decompose any FRN into a par floater and a fixed annuity. We provide examples that use the formulas and illustrate the sensitivities of these measures to pricing assumptions (pages 2-6).

- We then describe various sensitivity measures such as *effective duration*, *partial duration* and *spread duration* and compare these measures to those of fixed-coupon bonds (pages 7-9).

- Next, we discuss the relationship between FRNs, fixed corporate bonds and derivatives markets (pages 9-13).

- In the following section, we illustrate the various concepts discussed thus far with an example using Eurodollar perpetual floating-rate notes (pages 13-16).

• We then address the relationship between FRNs of different indices. Topics discussed are: (1) the reasons for buying FRNs of a particular index; (2) the use of derivatives markets to evaluate relative FRN values of different indices; (3) some statistical methods to assess the propensity of these indices to move together; and (4) an example that uses statistics to evaluate "conventional wisdom" of the relationship between prime and Treasury rates (pages 16-25).

• Finally, we discuss the valuation of embedded optionality in FRNs. We address this topic with respect to two types of securities: floored-corporate FRNs and floating-rate mortgage securities (pages 25-29).

FLOATING-RATE NOTE PRICING: EQUATIONS AND DEFINITIONS

In this section, we explicitly define the various pricing equations and market conventions used in floating-rate security markets. This section is designed to serve as a reference, although we will provide examples to make the exposition more lucid. Readers who are already familiar with these concepts should skip this section; however, we will refer to the equations shown here in the subsequent text.

Consider the following bullet corporate floating-rate note (FRN) which matures at the end of period T and pays a coupon which resets every period to a particular rate, I_t, plus a specified spread called the reset margin (RM). The note is noncallable and nonputable. The principal amount is \$100. For ease of explanation, we assume that the pricing date is the reset date.

If $I_1,\ldots, I_{T-1}$ were known at time 0, the pricing relationship of the floater would be as follows:

$$P(0,T) =$$

$$100 * \left[\frac{I_0 + RM}{(1 + d(1) + S)} + \frac{I_1 + RM}{(1 + d(2) + S)^2} + \ldots + \frac{I_{T-1} + RM + 1}{(1 + d(T) + S)^T}\right],$$

$$= 100 * \left[\sum_{i=1}^{T}\frac{I_0 + RM}{(1 + d(i) + S)^i} + \frac{1}{(1 + d(T) + S)^T}\right] \quad (1)$$

where P(t,T) is the price of a floater with T periods until maturity observed at period t,

RM	=	the reset margin. This margin is fixed at issuance,
d(t)	=	the discount rate of a risk-free \$1 paid at time t, and
S	=	the discount spread of the corporate issuer.

Floating-rate notes typically are "set in advance" which means that the index level that is applied to the coupon at time t is the observed index at time t-1 (I_{t-1}).

Because $I_1,\ldots, I_{T-1}$ are not observable at time 0, a projected coupon must be substituted. We denote the series of projected coupons as $X_1,\ldots, X_{T-1}$ (we will discuss the determinants of the X's later).

The pricing equation is then modified to:

$$P(0,T) = 100 * \left[\sum_{i=1}^{T} \frac{X_{i-1} + RM}{(1 + d(i) + S)^i} + \frac{1}{(1 + d(T) + S)^T} \right] \tag{2}$$

where $X_0 = I_0$ if pricing date is a reset date. For ease of explanation, we make this assumption going forward.

The trading convention is to quote FRN prices in terms of a **discount margin** (DM) which is analogous to the yield spreads on fixed corporate bonds. The discount margin is defined as the spread over the appropriate index that equates the price of the FRN to the present value of the projected cash flows. In other words, the DM is defined as follows:

$$(1 + d(i) + S)^i = (1 + I_0 + DM)(1 + X_1 + DM)...(1 + X_{i-1} + DM),$$

$$\text{for } i = 1, ..., T. \tag{3}$$

Therefore, Equation (2) can be rewritten in terms of the discount margin as follows:

$$P(0,T) = 100 * \left[\frac{I_0 + RM}{(1 + I_0 + DM)} + \frac{X_1 + RM}{(1 + I_0 + DM)(1 + X_1 + DM)} \right.$$
$$\left. +...+ \frac{1 + X_{T-1} + RM}{(1 + I_0 + DM)(1 + X_1 + DM)...(1 + X_{T-1} + DM)} \right] \tag{4}$$

Equation (4) is simply the standard pricing equation with the appropriate discount rate for the issuer (d(i) + S) restated in terms of the projected coupon indices $(I_0, X_1, ..., X_{T-1})$.

Example

Consider a new issue FRN that is indexed to three-month LIBOR (LIB3) + 20 basis points, has a maturity of two years, pays and resets quarterly, and is priced at par at time 0 (P(0,T)=100). Suppose that on the pricing date, LIB3=6.06% and for now, we make the assumption that $LIB3_1 = LIB3_2 = ... = LIB3_{T-1} = 6.06\%$ and T=8 quarters.[1] LIB3 FRNs typically pay on an actual/360 basis and, for simplicity of this example, we assume that each quarter has 91 days. Therefore, the quarterly coupon in our example is:

$$Coupon_t = 100 * (LIB3_{t-1} + 20 \text{ bp}) * (91/360)$$
$$= 100 * (0.0606 + 0.0020)(91/360) = 1.58$$

[1] We will discuss the use of alternative assumptions later in the report.

Note that the index and discount margin in the denominator are accrued on the same basis as the coupon. We can rewrite Equation (4) as:

$$100 = 100 * \left[\frac{(0.0606 + 0.0020) * (91/360)}{(1 + ((0.0606 + DM) * (91/360)))} + \frac{(0.0606 + 0.0020) * (91/360)}{(1 + ((0.0606 + DM)(1 + 0.0606 + DM)(91/360)(91/360)))} + \ldots + \frac{1 + ((0.0606 + 0.0020) * (91/360))}{(1 + ((0.0606 + DM) * (91/360)))(1 + ((0.0606 + DM) * (91/360)))\ldots(1 + ((0.0606 + DM) * (91/360)))}\right] \quad (5)$$

The discount margin is the value of DM that solves Equation (5). In this example, since the bond is priced at par, it is clear that the solution is *DM=0.0020=20 basis points.* In fact, whenever a floater is priced at par, RM=DM. However, *if the price were to drop instantaneously to 99.75*, then substituting by 99.75 into the left hand side of Equation (5), we obtain *DM=33 basis points.* The following rule applies to FRNs:

- If RM > DM, then Price > 100
- If RM < DM, then P < 100
- If RM = DM, then P = 100

The pricing formulas described above help to answer the following questions:

- We assumed that future levels of LIB3 are equal to today's level. This is the convention in U.S. corporate, mortgage and asset-backed markets. For a given DM, how does the calculated price change under different assumed coupon levels? What is the appropriate assumption to use?
- What is the sensitivity of the FRN's price to changes in the Treasury yield curve (effective and partial durations)? What is the price sensitivity of the FRN with respect to changes in DM (spread duration)?
- How are the reset margins of new issues or the discount margins of secondary-market issues determined? How should the RMs and DMs differ across indices (for example, prime versus LIB3) for the same issue?

In the following analysis, we provide some answers to these questions. In evaluating these issues, it is useful to rewrite Equation (4) by adding DM-DM=0 to each coupon:

$$P(0,T) = 100 * \left[\frac{I_0 + RM + (DM - DM)}{(1 + I_0 + DM)} + \frac{X_1 + RM + (DM - DM)}{(1 + I_0 + DM)(1 + X_1 + DM)} + \ldots + \frac{1 + X_{T-1} + RM + (DM - DM)}{(1 + I_0 + DM)(1 + X_1 + DM)\ldots(1 + X_{T-1} + DM)}\right] \quad (6)$$

Recombining terms in Equation (6) yields the following:

$$P(0,T) =$$

$$100 * \left\{ \left[\frac{I_0 + DM}{(1 + I_0 + DM)} + \frac{X_1 + DM}{(1 + I_0 + DM)(1 + X_1 + DM)} + ... + \frac{1 + X_{T-1} + DM}{(1 + I_0 + DM)(1 + X_1 + DM) ... (1 + X_{T-1} + DM)} \right] + \right.$$

$$\left. \left[\frac{RM - DM}{(1 + I_0 + DM)} + \frac{RM - DM}{(1 + I_0 + DM)(1 + X_1 + DM)} + ... + \frac{RM - DM}{(1 + I_0 + DM)(1 + X_1 + DM) ... (1 + X_{T-1} + DM)} \right] \right\} \quad (7)$$

Although at first glance, Equation (7) may appear to be an unnecessarily long restatement of the pricing relationship, it actually provides some valuable intuition in understanding FRNs. The first term in the brackets represents a FRN whose reset margin is equal to its discount margin. Therefore, the value of this quantity is par. The second term is the present value of an annuity that pays the difference between the reset margin and the discount margin. Therefore, Equation (7) simplifies into the following expression:

$$P(0,T) = 100 + \text{present value of an annuity that pays } (RM - DM) \quad (8)$$

The first payoff of viewing the pricing equation in this way is to confirm the rules of premiums and discounts that were described above. If RM = DM, the second term of Equation (8) is zero and the bond is priced at par. If RM is greater than DM, the second term is positive and the price of the floater is greater than par (premium). If RM is less than DM, the second term is negative and the price of the floater is less than par (discount).

Sensitivity of Discount Margin Calculations to Assumptions

Equations (7) and (8) also provide a framework in which to evaluate the questions posed earlier. Recall that the first question was: *For a given DM, how does the calculated price change under different assumed index levels?* We use an example to illustrate the answer to this question.

Example

Consider the following corporate FRN:

- Index — LIB3
- Reset Margin — 100 basis points
- Reset/Pay Frequency — Quarterly/Quarterly
- Maturity — 5 years
- For simplicity, assume that the settlement date is a coupon date.

Suppose we were to consider two sets of index projections:

(1) $LIB3_0 = LIB3_1 = LIB3_{T-1} = 6.31\%$; and

(2) $LIB3_t = LIB3_{t-1} + 25bps$.

(i.e., $LIB3_0 = 6.31\%$, $LIB3_1 = 6.56\%$,..., $LIB3_{t-1} = 10.81\%$).

Case (2) was chosen arbitrarily to illustrate the differences across assumptions, but these projections are similar to the forward LIB3 rates implied by the swap curve in a sharply upward-sloping yield curve environment.

We substitute the projections, parameters and various discount margins into Equation (7) and solve for price. The results are shown in Figure 1.

Figure 1. Prices Under Various Discount Margins and Assumed Index Levels

Discount Margin	Assumed Index Projection (1) All LIB3 remain at current level (6.31%)	Assumed Index Projection (2) LIB3 increase 25bps each quarter
200	95.87	96.02
100	100.00	100.00
0	104.34	104.18

The calculated price is:

- the same under either assumption when the discount margin is equal to the reset margin;
- *greater* under the rising index assumption when the discount margin exceeds the reset margin (discount); and
- *lower* under the rising index assumption when the reset margin exceeds the discount margin (premium).

The terms under the first set of brackets in Equation (7) will equal 100 **regardless of the assumed index level**. This is true because the discount rates in the denominator of each term will exactly offset any increase or decrease in assumed coupon. However, the values of the terms under the second bracket will change under various assumptions of the index level. Specifically, a higher assumed level of the index will increase the rate at which the fixed annuity is discounted. We can use Equation (8) to define the price premium as follows:

Premium = P(0,*T*) - 100 = Present Value of Annuity which pays (RM-DM)

$$= 100 * \left[\frac{RM - DM}{(1 + I_0 + DM)} + \frac{RM - DM}{(1 + I_0 + DM)(1 + X_1 + DM)} + ... + \frac{RM - DM}{(1 + I_0 + DM)(1 + X_1 + DM)...(1 + X_{t-1} + DM)}\right] \quad (9)$$

As the projected index levels increase, the value of the premium decreases. Because a discount can be viewed as a negative premium, increasing the projected indices results in a lower discount and a higher price. This example demonstrates that the price of the bond for a given DM is clearly sensitive to the projected index level except for the special case in which RM=DM. The longer the maturity of the bond and the greater the difference between RM and DM, the more pronounced the differential between coupon projection methods. This example illustrates that the discount margin, although a convenient tool by which to quote prices on FRNs, is a flawed measure of relative value.

Equations (7) and (8) also provide insight into our second question: *What is the sensitivity of the FRN's price to changes in the Treasury yield curve?*

Effective Duration

Effective duration is commonly defined as the sensitivity of a bond's price to a parallel shift of the Treasury yield curve, keeping yield spreads constant. If the spread between the index rate and Treasury rate is held constant, this definition is equivalent to saying that *effective duration measures the sensitivity of the price of the FRN to changes in the level of its index.* Henceforth, we use these two definitions interchangeably.

Consider two cases:

- **Case (1)**. Suppose P(0,T)=100. Then RM=DM and the annuity shown in Equation (7) is equal to zero. Now, suppose I_0 instantaneously increased by a small amount to $I_0 + \Delta I$. After the next reset, coupons are increased to reflect the higher level of I. Equation (7) will now be written as:

$$\text{New P} = 100 * \left[\frac{I_0 + DM}{(1 + I_0 + \Delta I + DM)} + \sum_{i=2}^{T}\left(\frac{X_1 + RM + \Delta I + DM}{(1 + I_0 + \Delta I + DM)\ldots(1 + X_{i-1} + DM)}\right.\right.$$
$$\left.\left. +\ldots+ \frac{1}{(1 + I_0 + \Delta I + DM)\ldots(1 + X_T + \Delta I + DM)}\right)\right] = 100 * \left[\frac{1 + I_0 + DM}{1 + I_0 + \Delta I + DM}\right]. \qquad (10)$$

The terms sum in this manner because the value at time 1 of all subsequent projected cash flows is equal to par. Therefore, *the effective duration of a noncallable, nonputable bullet FRN is the modified duration of a bond that matures at the next reset date*.

Example

Recall the example from the previous section:

- Index — LIB3
- Reset Margin — 100 basis points
- Reset/Pay Frequency — Quarterly/Quarterly
- Maturity — 5 years
- Settlement date — Reset date.

Suppose that this FRN is priced at par.

If the index level increases by ten basis points instantaneously **(**ΔI = 0.0010), the new price is given by Equation (10) as follows:

$$\text{New P} = 100 * \left[\frac{1 + ((0.0631 + 0.01)(91/360))}{1 + ((0.0631 + 0.001 + 0.01)(91/360))}\right] = 99.9752. \qquad (11)$$

Similarly, a ten-basis-point decrease in the index ($\Delta I = 0.0010$) gives us a new price of 100.0243. Therefore, the average price change for a ten-basis-point shift in the index level is equal to (100.0243 - 99.9752)/2 = 0.0248 = 0.0248% of purchase price. Since effective duration is defined to be the price change for a 100-basis-point change in yields,

Effective Duration = 0.0248% x 10 = 0.248%.

Note that this is the modified duration of a zero coupon bond which matures in 91 days and has a yield of 1.85% (0.0731)(91/360).

• **Case (2)**. RM ≠ DM (the FRN is trading at a discount or a premium). From Equation (7), we can view this bond as a "portfolio" of a long position in a par floater and a long position in an annuity paying RM-DM (or a short position in an annuity paying DM-RM if a discount). As shown in Case (1), the long floater will have an effective duration equal to the modified duration of a bond that matures at its next reset. The annuity position will have a longer duration. In the case of a discount, the annuity piece will reduce the effective duration of the "portfolio." If the annuity is a significant enough portion of the "portfolio," the effective duration of the discount floater actually may be negative. Unfortunately, in this case, the method used to project the coupons *will affect the effective duration* because the effective duration of the annuity piece will differ for different index levels.

Example

Suppose the LIB3 floater described above had a dollar price of 85. Using the portfolio approach, the floater can be decomposed into a long position in a FRN priced at 100 and a short position in an annuity worth 15. The present value of that annuity is given by the terms under the second set of brackets in equation (7). We assume that all subsequent levels of LIB3 are equal to the current level of 6.31%, and we can solve for the DM which sets that annuity value equal to -15.[2] In this case, the DM equals 492 basis points. We have shown earlier that the effective duration of the par floater piece is equal to 0.248 years. The effective duration of an annuity that pays DM-RM (392 basis points) for five years is equal to 2.367 years.[3] Using the portfolio approach and the fact that the effective duration of a portfolio is equal to the market-value-weighted average of the components of the portfolio, we can obtain the effective duration of the discount FRN as follows:

Effective Duration = (100/85)*(0.25) - (15/85)*(2.37) = -0.12 years.

Partial Durations

A careful, quantitative analysis of partial durations requires detail that is beyond the scope of this paper. However, several intuitive points can be made from our simple pricing equations:

• If the coupon of the FRN is indexed to a short rate and the bond is priced at par, then from Equation (10), the price will be sensitive only to movements in the index, and other partial durations will be zero.

• If the bond is at a premium or a discount, we can take the "portfolio" approach discussed above. Shifts of intermediate rates will affect the price of the bond to the extent that the annuity in the portfolio is affected.

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[2] A short position in a portfolio is indicated by a negative present value.

[3] Note that the duration of the annuity is dependent upon the discount rate and time to maturity but independent of the size of the annuity payment.

- Some FRNs are indexed to a longer maturity rate such as the two-year constant maturity Treasury (CMT2). Even if they are priced at par, such floaters are sensitive to shifts of various portions of the curve because their prices are sensitive to shifts in the longer forward rates.

Spread Duration

We define *spread duration* as the price sensitivity of a FRN with respect to changes in its DM. To derive some intuition of the spread duration, it is easiest to consider equation (4). Recall that Equation (4) is given by the following:

$$P\,(0,T) = 100 * \left[\frac{I_0 + RM}{(1 + I_0 + DM)} + \frac{X_1 + RM}{(1 + I_0 + DM)(1 + X_1 + DM)} + ... + \frac{1 + X_{T-1} + RM}{(1 + I_0 + DM)(1 + X_1 + DM)...(1 + X_{T-1} + DM)} \right] \tag{4}$$

It is clear from Equation (4) that any increase in DM affects the denominator of Equation (4) but does not increase the numerator. In fact, increasing the DM of a floater will have the same price effect as increasing the yield on a fixed bond that pays the projected coupons out to the maturity of the floater. Therefore, a longer-maturity FRN will have a spread duration equal to the modified duration of a fixed bond paying the projected coupons out to the maturity of the floater. The method used to project the coupons will also affect this measure.

Example

The five-year LIB3 floater used in our previous examples has a spread duration of 4.15 years when priced at par and the future levels of LIB3 are assumed to be the current level (6.31%). This is the modified duration of a bond that pays a quarterly coupon of 1.83% (7.31/4%) for five years.

RELATIONSHIP BETWEEN DISCOUNT MARGINS ON FLOATERS AND FIXED CORPORATE SPREADS

Figure 2 shows the discount margins of various types of corporate FRNs observed on March 10, 1995. One immediate observation from Figure 2 is that discount margins increase as the maturity of the FRN increases (term structure). In the previous section, we showed that for a given reset frequency, FRNs priced at par will have the same effective duration regardless of their maturity. However, longer-maturity floaters have longer spread durations. Therefore, at least a portion of the term structure of DMs for a given issuer likely represents the compensation for taking on spread duration risk. We now discuss how this term structure of DM is determined by focusing on FRNs indexed to LIB3.

Figure 2. Corporate Floating-Rate Note Indication Levels, as of 10 Mar 95 (Quality: A2/A)

	Discount Margin (bp)					
	Index					
Maturity	LIB3	TB3	Prime	Fed Funds	CMT2	CP1
One-Year	+0	+23	-255	+25	+10	+5
Two-Year	+10	+33	-250	+28	+18	+8
Five-Year	+25	+55	-240	+30	+25	+15

Index	Level
LIB3	6.31%
TB3	5.93
Prime	9.00
Fed Funds	5.93
CMT2	6.82
CP1	6.08

bp Basis points. LIB3 Three-month LIBOR. CMT2 Two-year Constant Maturity Treasury. CP1 One-month Commercial Paper. TB3 Three-month Treasury Bill Yield.
Note: All rates quoted in CD equivalent except CMT2, which is quoted in bond equivalent.
Source: Salomon Brothers Inc.

Three-Month LIBOR (LIB3)-Indexed Floating-Rate Notes

Consider the following transaction:

- Purchase a newly issued fixed corporate bond with a maturity of five years which is priced at 100 with a coupon of the five-year Treasury ($TSY5_0$) plus spread (CS).

- Enter into a five-year interest rate swap in which the investor pays a fixed-rate plus a swap spread (SS) and receives the three-month LIBOR observed at reset date ($LIB3_{t-1}$).[4] Figure 3 shows the market levels of swap spreads observed on March 10, 1995.[5] Figure 4 shows the net cash flows of this transaction.

Figure 3. Swap Spread Indication Levels — Fixed-rate versus Three-Month LIBOR, as of 10 Mar 95

	Spreads to Treasury (bps)	
Maturity	Receive Fixed	Pay Fixed
One Year	+30	+33
Two Year	+26	+29
Three Year	+26	+29
Four Year	+26	+29
Five Year	+25	+28

bp Basis points.
Source: Salomon Brothers Inc.

Figure 4. Net Cash Flow of Swapped Corporate Bond

Transaction	Cash Flow at Quarter t=1	Cash Flow at Quarter t=2
Buy Fixed bond		$(TSY5_0+CS)/2$
Interest Rate Swap		
Receive Float	$+LIB3_0*Act/360$	$+LIB3_1*(Act/360)$
Pay Fixed		$-(TSY5_0+SS)/2$
Net Cash Flow	$+LIB3_0*Act/360$	$(LIB3_1*Act/360)+(CS-SS)/2$

bp Basis points. LIB3 Three-month LIBOR. TSY5 Five-year Treasury yield.
Source: Salomon Brothers Inc.

[4] Swaps are conventionally "set in advance," which means that the time t floating-rate payment is LIB3 observed at time t-1.

[5] Actual swap rates may deviate from the indication levels shown in Figure 3 because of a differential in credit risk for the counterparties. See *Pricing of Interest Rate Swap Default Risk*, Eric H. Sorenson and Thierry F. Bollier, Salomon Brothers Inc., October 1993 for a discussion on the determinants of this adjustment.

The net cash flow approximates the cash flow on a LIBOR floater where

Reset margin (RM)=spread on fixed corporate bond-swap spread = CS-SS.

The swapped fixed bond differs from a FRN only by slight differences in timing of cash flows because the synthetic will receive its "reset margin" semiannually and the FRN will receive the reset margin quarterly.

Therefore, the fixed corporate bond and swap markets provide a good "rule of thumb" as to where LIB3 floaters should trade. For corporate issuers with similar credit quality to that implied by the swap market, this rule of thumb should hold fairly closely. However, there likely will be some deviation when the credit quality of the corporate issuer differs substantially from the credit quality of the swap counterparty.[6] Some theoretical models of credit spreads provide analytic solutions for the price of a FRN, thereby quantifying the difference between swapped fixed bonds and corporate FRNs. However, a thorough discussion of these models is beyond the scope of this analysis.[7]

Does this relationship hold in practice?

On March 10, 1995, the following sets of prices were observed in the market:

	Yield Spread (bps)
(a) GMAC (Baa1/BBB+) 6.35% of 6/28/98=	+68/Actual
Swap spreads: pay fixed-three year =	+29
four year=	+29

Corporate spread-swap spread= 68bps - 29bps = 39bps.

A GMAC FRN with a coupon of LIB3 plus 25 basis points and maturing on 2/2/98 was trading at a discount margin of LIB3 plus 37.5 basis points.

	Yield Spread (bps)
(b) Nations Bank (A2/A) 7.50% of 2/15/97=	+40/Actual
Swap spreads: pay fixed two year=	+29

Corporate spread - swap spread = 40bps - 29bps = 11 bps.

Nations Bank FRN with a coupon of LIB3 plus six basis points and maturing in 11/96 was trading at a discount margin of 12 basis points.

In these two cases, the relationship between swapped fixed corporate bonds and floaters held quite closely.

An Alternative Specification

Investors that are familiar with the swap market know that a close relationship exists between the fixed swap rate and the yield on a strip of Eurodollar futures contracts. In this analysis we will not provide a detailed description of hedging swaps with Eurodollar futures contracts but will briefly discuss the relationship between corporate floaters and Eurodollar futures.[8]

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[6] For an extreme example, see *Brady Bond Fixed-Floating Spreads — Forget History*, Vincent J. Palermo, et al., Salomon Brothers Inc, December 15, 1993.

[7] See "A Simple Approach to Valuing Risky Fixed and Floating-Rate Debt," Francis A. Longstaff and Eduardo S. Schwartz, *The Journal of Finance*, July 1995, and "A Probabilistic Approach to the Valuation of General Floating-Rate Notes with an Application to Interest Rate Swaps," Nicole El-Karoui and Helyette Geman, *Advances in Futures and Options Research*, volume 7, JAI Press Inc., 1994.

[8] For a detailed discussion of hedging swaps with Eurodollar futures, see *Eurodollar Futures and Options*, Galen Burghardt, et al., Probus Publishing Company, 1991. For a discussion on the effect of convexity on the relative pricing between swaps and Eurodollar futures, see "A Question of Bias," Galen Burghardt and Bill Hoskins, *Risk*, March 1995.

Suppose we entered into the following transactions:

- Purchase a newly issued floating-rate note with a maturity of five years that is priced at 100 with a quarterly coupon of LIB3 plus a reset margin (RM).
- Buy the series of Eurodollar futures contracts maturing each quarter until the maturity of the FRN. The notional amount of the contracts equal the par amount of the bond.

If a Eurodollar futures contract that matures at time t is held to maturity, the payoff at time t is:

$$F_{t,t} - F_{0,t} = 100 - LIB3_t - [100 - FLIB3_{0,t}].$$

where $F_{t,T}$ = the time t price of a Eurodollar futures contract that matures at time T.

$FLIB3_{t,T}$ equals the yield on a T-period Eurodollar futures contract observed at time t. This rate will be closely related to the forward $LIB3_t$ but will differ slightly because of the "mark-to-market" feature of futures contracts. Henceforth, we will use the terms "futures yield" and "forward rate" interchangeably.

The net cash flows of these transactions can be approximated as shown in Figure 5.

Figure 5. Net Cash Flow of Buying a Floater and a Series of Eurodollar Futures Contracts

Transaction	Cash Flow at Any Quarter t
Buy Floater	$(LIB3_{t-1} + RM)*(Days/360)$
Buy Futures Contract	$(\text{Forward } LIB3_{t-1} - LIB3_{t-1})/4$
Net Cash flow	$\text{Forward } LIB3_{t-1} + RM + \varepsilon$

ε The difference in interest as a result of day count conventions.

The description of the cash flows in Figure 5 is slightly imprecise because of the following: (1) the maturity date on the contracts will not exactly match the coupon dates of the bond; (2) the bond coupon is set in advance to $LIB3_{t-1}$ but the cash flow of the futures contract takes place at t-1; and (3) the mark-to-market feature of futures contracts.

We essentially have created a synthetic fixed bond that pays the forward rates plus a reset margin using prices that are observed at time 0. Therefore, the synthetic should trade at a similar yield spread as actual fixed bonds of the same issuer. Note that the partial durations are not matched to those of a fixed-coupon-paying bond of the same maturity. If the Eurodollar curve is upward sloping, the synthetic will have lower coupons in the early quarters and higher coupons in later quarters.

Does this relationship hold in practice?

Figure 6. Eurodollar Futures Contract Prices, 10 Mar 95

Maturity	Price	Yield
6/19/95	$93.43	6.57%
9/18/95	93.20	6.80
12/18/95	92.98	7.02
3/18/96	92.91	7.09
6/17/96	92.84	7.16
9/16/96	92.80	7.20
12/16/96	92.72	7.28
3/17/97	92.76	7.24
6/16/97	92.72	7.28
9/15/97	92.70	7.30
12/15/97	92.63	7.37
3/16/98	92.64	7.36

Source: Salomon Brothers Inc.

Recall from the earlier example that a secondary-market GMAC FRN with approximately three years to maturity traded at a discount margin of 37.5 basis points. At the same time, three-year fixed GMAC bonds were trading at a yield spread of 68 basis points.

Suppose GMAC issued a new floating-rate note with a dated date of 3/13/95 priced at 100 with a reset margin of 37.5 basis points. We test how closely the theoretical relationship described above holds by substituting the futures rate shown in Figure 6 for the appropriate coupon each quarter and allowing the interest to accrue on an Actual/360 basis. The calculated yield on this synthetic fixed bond was 7.57%, equal to the three-year Treasury + 64 basis points.

Given the relationship between FRNs, swap markets and futures contracts, one may argue that the appropriate method of projecting the coupons on FRN is to the forward rates because an investor can effectively fix his coupons to the forwards. Nonetheless, the convention in U.S. corporate, mortgage and asset-backed markets has always been to use current LIBOR levels as the projected coupon in calculating DM. It is likely that this convention has persisted because of its simplicity and because of the difficulty in practice of creating arbitrage opportunities from the mispricings. If the market prices FRNs using a fixed index, then through transactions like the previous two examples, investors occasionally can synthetically create a "cheap" fixed corporate bond. However, it is extremely unlikely that a fixed corporate bond of the same issuer, coupon and maturity exists which an investor can costlessly short sell, and thus collect arbitrage profits. Furthermore, as we saw in Figure 1, in most cases, the differences in methodology will be small.

A CASE STUDY: EURODOLLAR PERPETUAL FLOATERS

One application that nicely demonstrates the points made in the previous two sections is the case of *perpetual* FRNs, which are common in the Eurobond market. A typical perpetual FRN will represent a promise to pay the level of an index plus a prespecified spread (RM) forever. These securities usually reset and pay frequently (for example, every three months). In the mid-1980s, banks issued many of these secirities in the Eurobond market at par. A 1984 article that discussed the popularity of FRNs states, "Since an FRN coupon is reset to market levels every three or six months, Eurodollar FRN investments are similar to rolling over funds in the certificate of deposits market."[9] A portfolio manager quoted in the same article considered a floater portfolio as "sort of an insurance policy against rising rates."[10]

One typical A2/A-rated perpetual issue has a coupon of six-month LIBOR (LIB6) + 12.5 basis points, reset and paid quarterly. Most of the perpetual issues are callable and some have floors, but for the purpose of this example, we assume that these securities do not contain embedded options. Like most bonds in this market, this issue currently trades at a substantial discount. In this case, the bond has an offer price of 80.5.[11] If floaters are indeed an "insurance policy" or a similar investment to rolling CDs, what caused such drastic price depreciation?

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9 "The FRN Dilemma," *Institutional Investor — International Edition*, April 1984, pp. 215-218.

10 "The FRN Dilemma."

11 As of July 7, 1995.

For the purpose of this example, suppose that this security was issued at par on August 1, 1985. LIB6 on this date was 8.25%. We can rewrite the pricing Equation (4) for a perpetual as follows:

$$P = 100 * \left[\sum_{i=1}^{\infty} \frac{X_{i-1} + RM}{(1 + I_0 + DM)...(1 + X_{t-1} + DM)}\right] \qquad (13)$$

Suppose we project the index to some constant level (X_i=I for all i). Then, using the solution for an infinite geometric series, we obtain the following analytic solution for Equation (13):

$$\text{Price of perpetual} = 100 * \frac{I + RM}{I + DM} \qquad (14)$$

Equation (14) is consistent with our previous pricing expressions in the sense that if RM=DM, then the price will equal 100 irrespective of the projected index.

Despite the extremely long maturity (infinite), Equation (10) is still appropriate to evaluate the effective duration of a perpetual FRN priced at par. Therefore, the effective duration on August 1, 1985, was slightly less than 0.25 years. In this sense, the security is a similar investment to rolling certificates of deposit.

Yet, now consider the spread duration of these bonds. By differentiating Equation (14) with respect to DM and dividing by the price, we obtain the following expression:

$$\text{Spread duration of perpetual FRN} = \frac{1}{I + DM} \qquad (15)$$

Using the August 1, 1985, LIB6 rate of 8.563% as the projected coupon, Equation (15) becomes:

$$\text{Spread duration of perpetual FRN} = \frac{1}{0.08563 + 0.00125} = 11.51 \text{ years.} \qquad (16)$$

The long spread duration is the major risk of the perpetual FRN relative to traditional short-term strategies, such as rolling CDs or buying short-dated floaters. If LIB6 had increased by 100 basis points on August 1, 1985, the perpetual would have experienced a price depreciation of only approximately $0.25 since its effective duration is roughly 0.25 years. However, an increase in the market discount margin of 100 basis points because of either deteriorating credit or technical factors would have resulted in a price depreciation of $11.51. In the case of perpetual floaters, spreads widened and to the surprise of some investors, the price was quite sensitive to it.

The perpetual also provides a nice example of how an investor can use the swap market in assessing relative value in the FRN market. Figure 7 shows the Treasury and swap rates that were observed on the date that the perpetual floater in our example was priced.

Figure 7. Treasury and Swap Rates, 7 Jul 95

Maturity	Treasury Yield	Swap Spreads	Fixed Swap Rate
10 year	6.08%	42bp	6.50%
30 year	6.55	45	7.00

bp Basis points.
Source: Salomon Brothers Inc.

Suppose an investor purchased the perpetual floater at 80.50 on August 1, 1995, and entered into a thirty-year swap in which he pays LIB6 and receives the fixed swap rate.[12] The net cash flows for 30 years is given in Figure 8.

Figure 8. Net Cash Flow of Swapped Perpetual Floater

Transaction	Cash Flow at Time t
Buy Perpetual Floater	$+(LIB6_{t-1}+0.125) * (Act/360)$
Interest Rate Swap	
Pay Float	$-LIB6_{t-1} * (Act/360)$
Receive Fixed	$(TSY30_0+SS)/2$
Net Cash Flow	$(TSY_0+SS)/2+(0.125*(Act/360))$

Act Actual number of days. LIB6 Six-month LIBOR. SS Swap spread. TSY30 30-year Treasury yield.

Therefore, the investor has locked in a fixed semiannual coupon of approximately 7.00% + 0.125% = 7.125% for 30 years at a dollar price of 80.50. The synthetic differs from a fixed corporate bond in the sense that, after 30 years, the bondholder does not receive principal but rather still owns a perpetual stream of floating-rate cash flows. The following methodology can be used to calculate the present value of this stream:

(1) Assume a LIB6 (LIB6') for the coupons from year 30 onward;

(2) Use the perpetual pricing formula in Equation (14) to obtain a value of these cash flows at the end of year 30;

(3) Discount this value back to today.

This methodology allows us to obtain the yield on the synthetic by solving the following equation:

$$\text{Price} = \sum_{i=1}^{60} \frac{(TSY30_0 + \text{Swap Spread} + RM_{\text{perpetual FRN}})/2}{(1 + \text{yield}/2)^i} + PV\ (\text{perpetual which pays } LIB6' + RM)$$

where PV(.) = Present Value (17)

[12] Assume July 7, 1995 Treasury and swap rates are prevalent on that date. The floating rate on plain-vanilla swaps is usually LIB3. The difference on the fixed rate between a swap which receives either LIB3 and a swap which receives LIB6 is typically not more than a few basis points. For simplicity, we assume that these rates are the same.

Of course, this calculation depends on our assumption of LIB6′, but the first cash flow for which the assumption is important is 30 years hence. Many investors are reluctant to enter into a 30-year swap because of credit and liquidity issues. We can also use Equation (17) to calculate the yield of a synthetic that is swapped for ten years by substituting a term of 20 semi-annual periods rather than 60. In this case, the yield on the synthetic is more sensitive to the assumptions of LIB6′ because the assumed level enters the pricing relationship only ten years hence.

Figure 9 shows the spread over the "old" 30-year Treasury of the perpetual floater swapped for ten and thirty years under various assumptions for LIB6 in the "tail."[13]

Figure 9. Yield Spread of Swapped Perpetual over 30-Year Treasury

Assumed LIB6 After Maturity of Swap	Yield Spread of Perpetual Swapped for Thirty Years	Yield Spread of Perpetual Swapped for Ten Years
10.00%	252bp	330bp
7.00	226	191
6.50	221	164
5.75	214	121
3.00	184	-82

bp Basis points. LIB6 Six-month LIBOR.
Source: Salomon Brothers Inc.

The historical average over the past ten years of LIB6 was 6.55%.

An investor can use Figure 9 to compare the spreads of the swapped perpetual to the spreads of long corporate fixed bonds. For a perpetual that is swapped for 30 years, even after imposing extremely conservative assumptions to the unswapped cash flows, the spread does not change much. Of course, because of lesser liquidity, an investor should expect the synthetic to trade at wider spreads than a fixed issue. Figure 9 enables the investor to assess how much he is being compensated for accepting lower liquidity.

[13] Corporate bonds are conventionally not spread off the newest issued 30-year Treasury but the previous issue. In this case, we use the yield of the 7.50% of 11/24.

Recall Figure 2 which shows the indication discount margins of generic A-rated FRNs across various indices as of March 10, 1995. For the reader's convenience, we reproduce this table below. These levels also approximate where the reset margin on a generic new issue FRN of A-rated credit quality would be set.

Figure 2. Corporate Floating-Rate Note Indication Levels, as of 10 Mar 95 (Quality: A2/A)

	Discount Margin (bp)					
	Index					
Maturity	LIB3	TB3	Prime	Fed Funds	CMT2	CP1
One-Year	+0	+23	-255	+25	+10	+5
Two-Year	+10	+33	-250	+28	+18	+8
Five-Year	+25	+55	-240	+30	+25	+15

Index	Level
LIB3	6.31%
TB3	5.93
Prime	9.00
Fed Funds	5.93
CMT2	6.82
CP1	6.08

bp Basis points. CMT2 Two-year Constant Maturity Treasury. CP1 One-month Commercial Paper. LIB3 Three-month LIBOR. TB3 Three-month Treasury Bill yield.
Note: All rates quoted in CD equivalent except CMT2, which is quoted in bond equivalent.
Source: Salomon Brothers Inc.

A quick addition of the various index levels to the appropriate new issue reset margins show that, for the same credit quality, the current coupon on FRNs across indices and maturities are not the same. Figure 10 shows a comparison of FRNs of the various indices to a LIB3 floater. The previous sections illustrate that the coupons typically increase across maturities to compensate the investor for taking on additional *spread duration risk*. It is important to understand the reasons that these FRNs may trade at different current coupons for the *same maturity* FRN.

Figure 10. Current Coupon Comparison of Corporate Floating-Rate Notes, as of 10 Mar 95 (Quality: A2/A)

	Current Coupon versus LIB3 Floater (bp)				
	Index				
Maturity	TB3[a]	Prime	Fed Funds	CMT2[a]	CP1[b]
One-Year	-23	+14	-13	+52	-15
Two-Year	-24	+9	-20	+50	-23
Five-Year	-17	+4	-33	+42	-34

[a] Adjusted to reflect the difference in day count convention. [b] Adjusted to reflect the difference in pay frequency.
CMT2 Two-year Constant Maturity Treasury Yield. CP1 One-month commercial paper. LIB3 Three-month LIBOR.
TB3 Three-month Treasury bill yield.
Source: Salomon Brothers Inc.

Four reasons that an investor may purchase a floater with a lower initial coupon are:

(1) **To avoid risk of deviating from benchmark.** Many short-term investors, such as money market funds and securities lending accounts, buy short maturity floaters and hold them until maturity. As long as the bond does not default, these funds earn a return of the index plus the reset margin. The objective of the managers of such funds is to earn a modest excess return over some benchmark (often some rate such as LIB3) while maintaining a very low tolerance for substantive underperformance over

any period of time. Given their extreme risk aversion, the natural choice for such fund managers is to purchase floaters which match their benchmarks unless the current coupon advantage is sufficiently enticing.

(2) **Institutional restrictions.** Some investors are prohibited from buying floaters of certain indices.

(3) **More frequent reset.** In the previous section, we showed that the effective duration of a FRN without options priced at par is approximated by the time to the next reset. Therefore, a short-term investor who has an outlook of immediate Fed tightening may prefer a daily reset Fed funds FRN or a weekly reset TB3 FRN rather than one indexed to LIB3 that resets quarterly, even if the current coupon is lower.

(4) **Speculation on relative movements between rates.** Many investors may accept a lower coupon on an FRN to benefit from an expected change in the relationship between two rates (basis). For the various FRN shown in Figures 9 and 10, this basis arises for a number of reasons:

• **LIB3-TB3.** This is the spread between two indices with the same maturity, but LIB3 will always be the higher rate because it reflects the rate on Eurodollar deposits that are subject to credit risk and have less liquidity than Treasury bills. This spread, which is variable through time, is often known as the "TED spread."[14]

• **CMT2-LIB3, LIB3-Fed Funds**. These spreads have two components: (1) the difference between a longer-maturity rate and a shorter-maturity rate; and (2) the difference between a credit-risky rate and a Treasury rate. Thus, in addition, to the TED spread, the CMT2-LIB3 spread reflects the term structure of Treasury yields from the two-year to three-month part of the curve. We must emphasize that the decision between choosing a CMT2 quarterly reset floater over a LIB3 quarterly reset floater *does not* constitute a decision to extend effective duration because the effective duration is driven by the reset frequency. In fact, as we shall discuss in more detail later, the decision about which of these indices to invest in reflects a view on the *slope* of the Treasury curve rather than the level. Similarly, the LIB3-Fed funds basis reflects the slope of the Treasury curve on the very short end (overnight to three months) as well as the TED spread.

• **CP1-LIB3, Prime-LIB3**. These bases are similar to CMT2-LIB3 spreads in that they comprise both the term structure of Treasury yields and some credit spread (although not necessarily the same credit component as that which drives the TED spread). These spreads differ from the Treasury-LIBOR spreads discussed above because their statistical properties differ and there is no futures market in which to hedge them.

Earlier, we showed that a strong relationship exists between the fixed corporate bond, swap and FRN markets. There is also an active basis swap market which links the LIBOR-indexed FRN market to FRNs of the same issuer but indexed to alternative rates.

Consider the following transactions:

• Purchase a newly issued LIB3 floater with two years to maturity that is priced at 100 and pays a quarterly coupon of LIB3 plus a reset margin (RM).

[14] The common usage of the term "TED spread" refers to the spread between prices on the Treasury bill futures contract and the Eurodollar futures contract. This distinction is irrelevent for the purpose of this discussion, and henceforth we will "misuse" the term as the difference between the spot rates.

• Enter into a two-year basis swap in which the investor pays LIB3 and receives the average daily Federal funds rate plus a basis swap spread (BSS).

Figure 11. Net Cash Flows of a LIB3 FRN and a LIB3-Fed Funds Basis Swap

Transaction	Cash Flow at Quarter t
Buy LIB3 Floater	$+(LIB3_{t-1}+RM^{LIB3})$ * Act/360
Basis Swap	
Receive Federal Funds+Basis Swap Spreads	+(Avg. Daily Fed Funds+BSS) * Act/360
Pay LIB3	$-LIB3_{t-1}$ * Act/360
Net Cash Flow	+(Avg. Daily Fed Funds+RM^{LIB3}+BSS) * Act/360

Source: Salomon Brothers Inc.

• This net cash flow approximates the cash flow on a Fed Funds floater where

$$\text{Reset margin } (RM^{FF}) = RM^{LIB3} + BSS. \quad (18)$$

Understanding the Current Coupon Differential Between CMT2 and LIB3

Because an actively traded and liquid market exists for Eurodollar futures, we can look to those markets to help determine the current coupon differential between a FRN indexed to a longer Treasury rate and one indexed to LIB3. Consider a FRN that promises to pay CMT2 plus a reset margin (RM^{CMT2}) for five years. In theory, the issuer could hedge the exposure by selling the two-year Treasury rate forward each quarter over the five-year life of the bond. However, since there is not an active market in futures on the two-year Treasury yield, the hedge would involve a complicated position of long and short Treasuries.

A more popular alternative is to hedge the exposure in the liquid Eurodollar futures market. Although CMT2 is a two-year par rate and LIB3 is a three-month spot rate, the two-year forward rate can be expressed as a function of a series of three-month forward rates. The number of each contract needed to hedge this position can be determined by calculating the partial duration of the CMT2 floater with respect to each of the three-month forward rates. Although the determination of the number of contracts needed to hedge the CMT2 exposure is beyond the scope of this discussion, the following observations are important in understanding the determinants of relative coupon differentials between CMT2 FRNs and LIB3 FRNs.

• The typical hedge amounts for a five-year maturity CMT2 floater consists of a short position in Eurodollar futures contract from quarter 0 to quarter 7 and a long position in contracts from quarter 8 to quarter 27. The long positions become much larger from quarter 20 to quarter 27. Intuitively, the last promised cash flow is a two-year rate 4.75 years forward. Therefore, this bond will be sensitive to forward rates all the way out to the 6.75 year portion of the curve.

• Although the Eurodollar futures curve comprises the Treasury forward rates and a forward TED spread, the issuer is exposed to changes in the TED spread since the hedge is designed assuming constant spreads.

• Because the CMT2 floater can be hedged by futures contracts, the current coupon differential between CMT2 and LIB3 floaters will reflect differences in the **forward** CMT2-LIB3 spread. To be more specific, ignoring the effect of convexity on the optimal hedge ratios and differential option-adjusted spreads due to liquidity, the reset margin of a CMT2 floater is priced so that its present value equals that of a LIB3 floater of the same maturity *if the forward rates are realized.*[15]

Figure 12 shows the spreads between forward CMT2 and forward LIB3 observed on March 10, 1995. This table is a good starting point for an investor to assess the yield curve view that is embedded in the relative coupons of these floaters. If the investor believes that future yield curves will be steeper than those implied by the forwards shown in Figure 12, then he should consider buying a CMT2 floater.[16] Of course, other factors such as liquidity also should enter into the decision.

Because the reset frequencies are equal and a parallel shift of the yield curve will have approximately the same effect on all of the forwards, the FRNs have equal sensitivity to small, parallel shifts of the yield curve in either direction. However, the bonds have different sensitivities to reshapings of the curve (partial durations). On March 10, 1995, substantial flattening from the two-year to three-month part of the curve was priced into the CMT2 floater, and this is why it receives a higher coupon at the pricing date.

Figure 12. Comparison of Forward CMT2 to Forward LIB3, 10 Mar 95

Years Forward	Forward CMT2	Forward LIB3	Difference
0.00	6.82%	6.31%	0.51%
0.25	6.95	6.48	0.47
0.50	7.06	6.77	0.29
0.75	7.13	6.99	0.14
1.00	7.17	7.08	0.09
1.25	7.23	7.15	0.08
1.50	7.24	7.19	0.05
1.75	7.26	7.27	-0.01
2.00	7.26	7.24	0.02
2.25	7.29	7.27	0.02
2.50	7.28	7.29	-0.01
2.75	7.29	7.35	-0.06
3.00	7.27	7.34	-0.07
3.25	7.28	7.37	-0.09
3.50	7.27	7.40	-0.13
3.75	7.28	7.43	-0.15
4.00	7.28	7.44	-0.16
4.25	7.31	7.45	-0.14
4.50	7.33	7.46	-0.13
4.75	7.38	7.48	-0.10

CMT2 Two-year Constant Maturity Treasury. LIB3 Three-month LIBOR.
Note: CMT2 forwards are calculated from the Salomon Brothers Treasury Model Curve. LIB3 forwards are calculated from the swap curve.

The Use of Historical Data in Assessing Basis Risk

In comparing indices such as Fed funds, TB3 and Prime to LIB3, the term structure of interest rates are not as important because the maturities embedded in these rates are near the maturity of LIB3. Furthermore, for Fed Funds and Prime, there is no liquid futures market with long maturity contracts in which an issuer can hedge its financing rates by buying the indices forward.[17] Therefore, there are not "arbitrage" relationships to dictate how these FRNs should trade relative to each other.

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[15] The pricing coupons of the CMT2 floater will differ from the forwards because of convexity. This error will increase with the term of the index. We can account for the liquidity differentials by obtaining the present value after discounting at the appropriate option-adjusted spread.

[16] In assessing the relative coupons through time, one must account for the fact that LIB3 FRNs typically pay on an actual/360 basis and the CMT2 pay on a 30/360 basis. On March 10, 1995, this differential was worth ten basis points per annum.

[17] Fed Funds futures contracts are traded on the Chicago Board of Trade but the open interest is small and the contracts do not extend longer than one year in maturity.

For example, consider an investor that is benchmarked to the Fed funds rate and fears an imminent Fed tightening. He makes the decision to purchase a two-year floater on March 10, 1995 and compares a daily reset Fed funds floater to one of the same credit quality indexed to LIB3. We define the relative coupon at any quarterly payment date t as:

$$\text{Relative Coupon}_t = ([\text{LIB3}_{t-1} + \text{RM}^{\text{LIB3}}] - [\text{Average Daily Fed Funds observed over period t} + \text{RM}^{\text{FF}}] * (\text{days}/360) = ([\text{LIB3}_{t-1} - \text{Avg.Daily Fed Funds observed over period t}] + [\text{RM}^{\text{LIB3}} - \text{RM}^{\text{FF}}])* (\text{days}/360). \quad (19)$$

If the Fed Funds rate does not change over the first quarter,

$$\text{Relative Coupon}_t = [(6.31\% - 5.93\%) + (0.10\% - 0.28\%)] * (92/360) \approx 5 \text{ basis points.} \quad (20)$$

Should the investor accept the five-basis-point advantage and take on the basis risk and the longer time to reset, given his views on Fed tightening? If the Fed tightens, how long does it take for LIB3 to adjust?

Historical data can help the investor to assess such risk-reward trade-offs. For example, he can compare current spreads between the indices to historical average spreads to determine whether the basis is at historically wide or tight levels. Furthermore, the data provide a measure of the propensity of these rates to move together. Estimates of historical volatilities and correlations between the indices allow the derivation of an expected change in one index *conditional* on a specified change in the other index.

A sufficient amount of data is available on these rates to provide good estimates of the average levels of these spreads and their propensity to move together. However, in practice, many statistical issues and problems exist that must be addressed. They include:

- What frequency of data should be used to estimate the parameters?
- Over what time period should the estimate be calculated? In other words, over what time period is the assumption of stationary parameters a good one?
- Does the data warrant a more complicated time-varying parameter model such as Generalized Autoregressive Conditional Heteroscedasticity (GARCH)?[18]
- In comparing changes in rates, are only contemporaneous correlations important or should we account for various lags?

Figure 13 shows a comparison of the average spreads between various indices. These tables illustrate that the estimates can differ significantly across measurement frequencies and across subperiods.

[18] See *How to Model Volatility: Grappling over GARCH*, Joseph J. Mezrich, Robert F. Engle, Ashihan Salih, Salomon Brothers Inc, September 1995.

Figure 13. Comparison of Average Spreads of Various Indices

	Prime	CP1	Fed Funds	LIB3	LIB1	TSY2	TB3
Average Rates Mar 82-Jan 95							
Quarterly	9.35%	7.40%	7.59%	7.65%	7.52%	7.98%	6.80%
Monthly	9.29	7.16	7.34	7.54	7.42	7.91	6.74
Weekly	9.32	7.13	7.14	7.56	7.36	7.92	6.76
Daily	9.32	7.16	7.19	7.56	7.37	7.93	6.78
10 Mar 95	9.00	6.08	5.93	6.31	6.13	6.82	5.93
Average Spreads to 3-Month T-Bill							
Quarterly	2.55%	0.60%	0.79%	0.85%	0.72%	1.18%	x
Monthly	2.55	0.42	0.60	0.80	0.68	1.17	x
Weekly	2.56	0.37	0.38	0.80	0.60	1.16	x
Daily	2.54	0.38	0.41	0.78	0.59	1.15	x
10 Mar 95	3.07	0.15	0.00	0.38	0.20	0.89	x

CP1 One-month commercial paper. LIB1 One-month LIBOR. LIB3 Three-month LIBOR. TB3 Three-month Treasury bill yield. TSY2 Two-year Treasury yield.
Source: Salomon Brothers Inc.

Figure 14. Average Daily Spreads to Three-Month T-Bill for Various Time Periods

	Prime	CP1	Fed Funds	LIB3	LIB1	TSY2
Full Sample	2.54%	0.38%	0.41%	0.78%	0.59%	1.15%
3/82-6/86	2.50	0.29	0.55	1.07	0.83	1.49
6/86-9/90	2.30	0.62	0.65	0.90	0.72	0.83
9/90-1/95	2.85	0.23	0.06	0.39	0.24	1.14
10 Mar 95	3.07	0.15	0.00	0.38	0.20	0.89

CP1 One-month commercial paper. LIB1 One-month LIBOR. LIB3 Three-month LIBOR. TSY2 Two-year Treasury yield.
Source: Salomon Brothers Inc.

Spreads of LIB1, LIB3 and CP1 over TB3 clearly have grown tighter through time. Regardless of the subperiod or frequency of measurement, one might have concluded on March 10, 1995, that prime spreads were historically wide and TSY2 spreads were historically tight. Other spreads are near their averages, at least over the previous five years.

If spreads between indices tend to be mean reverting, then one may conclude that prime floaters are "rich" relative to LIB3 given that Figure 10 shows that the two securities were essentially receiving the same coupon. As we discussed in detail earlier, the CMT2 floaters receive a high current coupon because some flattening of the yield curve is priced into the bond. However, historical spreads indicate that the CMT2-LIB3 spreads are tighter than average historical levels, which may suggest that CMT2 floaters offer value.[19]

Using historical average spreads between indices does not address the issue of how we incorporate the view of imminent Fed tightening into the decision process. Correlations provide a measure of the propensity of two random variables to move together. Figure 15 shows estimated standard deviations and correlations between one period changes in various indices for the full sample and the most recent one third of the sample. We will show how these estimates can be used to incorporate an investor's views on Fed funds into the choice of FRNs.

[19] Of course, even if an investor believes historical spreads are applicable today, he may prefer to hold prime floaters because of the frequent reset or to not hold CMT2 floaters for reasons such as liquidity.

Figure 15. Correlations Between One Period Changes in Various Indices

Mar 82-Jan 95	Prime	CP1	Fed Funds	LIB3	LIB1	TSY2	TB3
Monthly Frequency							
Prime	1.00						
CP1	0.59	1.00					
Fed Funds	0.34	0.50	1.00				
LIB3	0.64	0.79	0.51	1.00			
LIB1	0.60	0.76	0.54	0.93	1.00		
TSY2	0.57	0.62	0.35	0.86	0.72	1.00	
TB3	0.47	0.70	0.40	0.80	0.75	0.74	1.00
Standard Deviation	**0.36**	**0.56**	**0.81**	**0.50**	**0.53**	**0.45**	**0.66**
Weekly Frequency							
Prime	1.00						
CP1	0.10	1.00					
Fed Funds	0.09	0.17	1.00				
LIB3	0.27	0.27	0.33	1.00			
LIB1	0.23	0.28	0.35	0.84	1.00		
TSY2	0.12	0.19	0.26	0.66	0.52	1.00	
TB3	0.09	0.19	0.32	0.61	0.49	0.71	1.00
Standard Deviation	**0.16**	**0.70**	**0.39**	**0.23**	**0.24**	**0.18**	**0.22**
Daily Frequency							
Prime	1.00						
CP1	0.06	1.00					
Fed Funds	0.02	0.26	1.00				
LIB3	0.04	0.52	0.16	1.00			
LIB1	0.04	0.47	0.13	0.56	1.00		
TSY2	0.03	0.18	0.07	0.27	0.20	1.00	
TB3	0.04	0.19	0.08	0.24	0.17	0.60	1.00
Standard Deviation	**0.07**	**0.09**	**0.30**	**0.11**	**0.12**	**0.08**	**0.10**

Sep 90-Jan 95	Prime	CP1	Fed Funds	LIB3	LIB1	TSY2	TB3
Monthly Frequency							
Prime	1.00						
CP1	0.55	1.00					
Fed Funds	0.20	-0.19	1.00				
LIB3	0.57	0.55	0.31	1.00			
LIB1	0.49	0.39	0.33	0.88	1.00		
TSY2	0.52	0.40	0.20	0.79	0.58	1.00	
TB3	0.64	0.48	0.30	0.86	0.75	0.79	1.00%
Standard Deviation	**0.28**	**0.32**	**0.69**	**0.31**	**0.42**	**0.32**	**0.25**
Weekly Frequency							
Prime	1.00						
CP1	0.23	1.00					
Fed Funds	0.15	0.23	1.00				
LIB3	0.24	0.37	0.01	1.00			
LIB1	0.12	0.40	0.04	0.74	1.00		
TSY2	0.09	0.29	0.11	0.53	0.38	1.00	
TB3	0.15	0.28	0.18	0.46	0.40	0.68	1.00%
Standard Deviation	**0.14**	**0.19**	**0.34**	**0.13**	**0.18**	**0.13**	**0.08**
Daily Frequency							
Prime	1.00						
CP1	0.14	1.00					
Fed Funds	-0.02	0.08	1.00				
LIB3	0.08	0.37	0.00	1.00			
LIB1	0.04	0.25	0.04	0.48	1.00		
TSY2	0.02	0.19	0.06	0.21	0.12	1.00	
TB3	0.07	0.21	0.10	0.21	0.11	0.59	1.00%
Standard Deviation	**0.06**	**0.07**	**0.31**	**0.06**	**0.10**	**0.06**	**0.04**

CP1 One-month commercial paper. LIB1 One-month LIBOR. LIB3 Three-month LIBOR. TB3 Three-month Treasury bill yield. TSY2 Two-year Treasury yield.
Source: Salomon Brothers Inc.

We can make several observations from Figures 15 and 16:

- Across nearly all of the frequencies and subperiods, Fed funds is the most volatile (highest standard deviation) index.

- In general, the correlations are smaller the more frequently the data are sampled.

- Some of the estimated parameters change substantially over time. One of the most difficult issues to resolve in the use of statistical measures to assess financial risk is determining which set of data is appropriate to use to estimate the parameters. On one hand, one would prefer the longest time series of data possible in order to obtain more *precise*[20] estimates. On the other hand, if the true relationship between the variables has changed over time, then some of the earlier observations will not be relevant in estimating the new parameter.

Figures 13-15 show that the estimated distributions of LIB1 and LIB3 have changed through time with the average spreads and volatilities decreasing substantially since the late 1980s. Many market participants would argue that a fundamental shift in the relationship between LIBOR rates and Treasury rates has occurred. If this is true, then the estimated parameters derived from the most recent subperiod would be the most applicable in measuring the current basis risk.

Figure 15 also presents an example whereby it may be more appropriate to use the full sample of data. For the last subperiod (September 1990 - January 1995) and monthly frequency, the estimated correlation between CP1 and Fed funds is -0.19. However, this estimate was driven by a large negative correlation in late 1990 and 1991. This period had a large impact on the subperiod estimates because of the small number of observations. The estimated correlation between CP1 and Fed funds over the entire period (March 1982 - January 1995) was 0.50. The 1991 period makes a much smaller contribution to the estimate for the full sample. Many market participants would argue that the full sample estimate appears more reasonable in this case.

Prime-Treasury Spreads

One "conventional wisdom" is that banks raise their prime lending rate quickly as Treasury rates increase but react much more slowly when interest rates drop. This phenomenon would have positive implications for prime floaters because the coupon would reset quickly to higher rates but slowly to lower rates, thereby improving their performance relative to other FRNs, keeping all else equal. Our simple statistics shown in Figure 15 do not tell whether this conventional wisdom is substantiated because our estimates represent an "average" of rising and falling interest rate environments. Figure 16 gives average changes in rates for Prime and Fed funds and correlations in different environments. The environments are determined by whether the Fed funds rate rises ($\Delta FF > 0$) or falls ($\Delta FF <= 0$) for the given period.

[20] Precision refers to the degree of error in estimating the parameters.

Figure 16. Comparison of Prime Rate Changes to Fed Funds Changes in Different Interest Rate Environments

	ΔFF>0			ΔFF<=0		
	Correlation (ΔFF, ΔPrime)	Average ΔPrime	Average ΔFF	Correlation (ΔFF, ΔPrime)	Average ΔPrime	Average ΔFF
Mar 82-Jan 95						
Monthly	-0.06	0.05	0.48	0.30	-0.15	-1.10
Weekly	-0.08	0.01	0.21	0.37	-0.03	-0.62
Daily	-0.04	0.00	0.21	0.15	0.00	-0.23
Mar 82-Jun 86						
Monthly	-0.12	-0.01	0.66	0.39	-0.29	-0.92
Weekly	-0.19	-0.03	0.34	0.13	-0.04	-0.36
Daily	-0.01	0.00	0.25	0.05	-0.01	-0.20
Jun 86-Sep 90						
Monthly	0.27	0.09	0.38	0.06	-0.04	-0.44
Weekly	0.33	0.03	0.13	0.09	-0.02	-0.12
Daily	0.02	0.00	0.16	0.01	0.00	-0.12
Sep 90-Jan 95						
Monthly	-0.22	0.06	0.40	0.22	-0.12	-0.46
Weekly	0.04	0.01	0.18	0.19	-0.02	-0.20
Daily	-0.08	0.00	0.22	0.07	0.00	-0.15

ΔFF One-period change in Fed funds rate. ΔPrime One-period change in prime rate.
Source: Salomon Brothers Inc.

The evidence in Figure 16 does not support the conventional wisdom. If Prime rates moved with Treasury rates in rising rate environments but remained "sticky" in falling rate environments, then we should see higher correlations and substantially positive changes in the Prime rate in rising rate environments. However, the data does not support the conjecture except possibly in the June 1986 - September 1990 subperiod. In fact, Figure 16 provides more support to the conjecture that the Prime rate falls with decreases in Treasury rates but is "sticky" in rising rate environments.

Using the Statistics to Assess the Basis Risk

The average one-period change of each of these indices was close to zero for all frequencies and periods. We can say that our *unconditional* expectation of the change in any particular index over any frequency equals zero. Given that the appropriate frequency and period has been chosen, the estimated correlations allow an estimate of an expectation *conditional* on a particular view. For example, if the investor believes that the Fed will raise the Fed funds rate (FF) by 50 basis points within the next week, what increase in LIB3 should he expect?

- The link between conditional expectations and correlations can be seen in the standard regression equation:

$$y = a + \beta x. \tag{21}$$

$$\beta \text{ is defined as } \frac{\text{covariance }(x,y)}{\text{std}(x)^2} \tag{22}$$

$$\text{correlation }(x,y) = \frac{\text{covariance }(x,y)}{\text{std }(x)\text{ std }(y)}\ ,$$

where std (x) = standard deviation of x. (23)

Also, taking expectations of the standard regression equation shows that:

$$a = \overline{y} + \beta\overline{x},$$

where $\overline{x}$ = the expected value of x and $\overline{y}$ = the expected value of y. (24)

Substituting these equations into the standard regression equation yields the following relationship:

$$E(\text{y conditional on x}) = \overline{y} + \left(\frac{\text{correlation }(x,y) * \text{std}(y)}{\text{std}(x)}\right)(x - \overline{x}),$$

where E(.) indicates expected value. (25)

We can substitute the estimated values from the table for weekly frequency into Equation (25), to obtain the following relationship:

$$E[\text{change in LIB3}] = 0 + \left(\frac{(0.328)(0.226)}{0.390}\right)(\text{change in FF} - 0)$$

$$= 0.19 * \text{change in FF}. \qquad (26)$$

Suppose, as stated earlier, we want to investigate the case in which the change in FF equals 50 basis points over the next week. Then, to get the updated or "conditional" expectation, we substitute 0.50% for change in FF in Equation (26) yielding the following result:

E[change LIB3 conditional on change in FF=0.5%]

= 0.19 * 0.50% = 0.095% = 9.5 basis points.

The statistic R^2 provides a measure of the "goodness of fit" of the regression model. For a univariate regression (one variable), R^2 equals correlation$(x,y)^2$. Therefore, in our case, $R^2=(0.328)^2=10.8\%$, implying that the change in the Fed funds rate only "explains" 10.8% of the total variation in LIB3.

In this example, we only considered contemporaneous correlations. To better quantify basis risk, we also should consider lagged changes of indices. A *vector autoregression model*[21] estimates the effects of shocks to a particular index and on its expected future levels and those of other indices. An investor could use such a model to estimate relative cash flows between two FRNs over time conditional on a particular change of one of the indices.

[21] For a technical definition and discussion of vector autoregression models, see *Time Series Analysis*, James D. Hamilton, Princeton University Press, 1994.

Thus far, we have only discussed FRNs that do not contain embedded options. However, some corporate and asset-backed floaters are subject to maximum or minimum coupons or have call provisions. Many structured notes comprise fairly complicated option positions. Floating-rate mortgage securities such as adjustable-rate mortgages (ARMs) or floating-rate CMOs have maximum and minimum coupons and are subject to prepayment options. In comparing these securities to noncall securities of the same issuer or to each other, an investor must: (1) identify the option position embedded in the bond; and (2) value the option position.

For some FRNs with embedded options, it is relatively straightforward to decompose the security into a portfolio of a long position in a noncall FRN and a position in some commonly traded option.

Example — Floored Corporate FRNs

Consider a FRN with the following characteristics:

- Rating — A3/A-,
- Maturity — 7/15/03,
- Coupon — LIB3+12.5 basis points,
- Minimum coupon — 4.35%,
- Reset/pay frequency — quarterly/quarterly.

This security can be decomposed into a portfolio of a long position in an unfloored FRN with a coupon of LIB3 plus 12.5 basis points and a long position in a LIBOR floor maturing on the same date with a strike equal to 4.225%. Figure 17 illustrates that the portfolio replicates the cash flows of the floored floater.

Figure 17. Cash Flow of Portfolio Consisting of an Unfloored FRN and a Long Floor

Portfolio	Cash Flow at Time t
Long Unfloored Floater	$LIB3_{t-1}$ + 0.125%
Long Floor With a 4.225% Strike	Max[4.225% - $LIB3_{t-1}$, 0]
Net Cash Flow	Max[4.35%, $LIB3_{t-1}$ + 0.125%]

FRN Floating-rate note. LIB3 Three-month LIBOR.
Source: Salomon Brothers Inc.

This approach is extremely useful in relative valuation. Suppose that, on July 15, 1995 (exactly eight years from maturity), the market price of the floored floater was 98. On this date, a fixed bond of the same issuer maturing on February 1, 2003 was trading at a yield spread of 92 basis points over the seven year Treasury. For simplicity, assume that this is also the fair yield of a fixed bond which matures on July 15, 2003. On July 15, 1995, the fixed-rate on an eight-year swap was 6.45%. Therefore, the floored floater could be swapped into a fixed coupon of 6.575% (6.45% plus a 0.125% reset margin). Using a yield spread of 92 basis points, the present value of the swapped floater (ignoring the value of the floor) is 97.85. Therefore, the implied price of the embedded floor is equal to 0.15 (98.00-97.85). Using an arbitrage-free term structure model, for an eight-year floor with a strike equal to 4.225% and a dollar price of 0.15, we obtain an implied volatility of 12.5%. Since the implied volatility of offered floors of similar maturity and strikes on 7/15/95 was 23%, the floored floater appears "cheap".

Several caveats apply to this type of analysis. One practical issue is that if the floored FRN is less liquid than the portfolio of an unfloored floaters and an over-the-counter (OTC) floor, then we should expect it to be slightly cheaper to reflect higher transaction costs of unwinding the position. Another issue is whether an investor can extract the "cheapness" of the floored floater. Suppose an investor agreed with our analysis but wants to purchase a FRN because he has a view that interest rates will rise in the near term. He asks, "Even though the embedded floor appears cheap at a premium of $0.15, why should I pay that premium when I have such strong views that rates will rise?"

One answer that is often given is that the investor can extract the cheapness today by writing an OTC 4.225% strike floor with the same coupon dates and maturity as the FRN. Assuming the bid volatility of the OTC floor was 21%, such a floor would provide the investor with an up-front premium of $0.96. Therefore, he now created a synthetic unfloored FRN at approximately 26/32 of a point "cheap" ($0.96-$0.15). This is a completely valid argument if the investor's intention is to keep the long and short position until maturity and does not have to mark this position to market. If the floors were to come into the money at any time over the life of the bond, the difference between the floored floater coupon and that of an unfloored floater would exactly offset any cash flow that the investor would have to pay on the short floor.

However, suppose the investor had an investment horizon shorter than the eight-year maturity of the FRN and either had to mark the combination to market or trade out of the position at the short horizon. Suppose also that the implied volatility on the OTC floors increased to 30%. There is no mechanism to ensure that the embedded option in the FRN would increase to reflect the higher market volatility. Therefore, the investor may have a marked-to-market loss on the short floor position and no marked-to-market gain on the FRN. Alternatively, suppose LIB3 decreased by 50 basis points from its July 15 level of 5.81%. If the implied volatilities on the OTC floors remained constant, then they would appreciate because the option would be closer to the money. Once again, the FRNs may not necessarily increase in price accordingly; they may just trade at even lower implied volatilities.

These caveats do not invalidate the analysis as a measure of richness or cheapness. Purchasing this FRN under the market conditions given in the example still appears to be a cheap way of adding convexity because the FRN should eventually experience price appreciation due to the changing option value if rates decrease. This example simply illustrates that the mispricing does not represent "arbitrage" opportunities.

Other types of FRNs contain much more complicated embedded option positions. Many structured notes are bonds that pay a coupon that is some function of LIB3. Although the component parts may not be as obvious as in the previous example, these securities can often be decomposed into a replicating portfolio in a similar manner.[22] However, mortgage-backed FRNs do not lend themselves to such a clean decomposition because of the existence of the prepayment option.

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[22] For some examples, see *Floating-Rate Securities: Current Markets and Risk/Return Trade-Offs in Rising Interest Rate Environments*, Raymond J. Iwanowski, Salomon Brothers Inc. April 6, 1994.

Mortgage FRNs

A complete discussion of all of the relevant factors that influence the ARM and floating-rate CMO markets would require much more detail and analysis than we intend to devote in this piece. However, we will broadly outline some of the main issues that an investor should consider in comparing a mortgage-backed FRN to those of other asset classes, particularly in the context of our discussion of optionality.

Like all mortgage-backed securities (MBS), floating-rate CMOs and ARMs differ from corporate and government bonds in that the securities amortize through time. Furthermore, although the securities are priced with respect to some scheduled amortization, this schedule is based on projected prepayments. Actual prepayments that deviate from scheduled prepayments will cause the amortization schedule to change over time.

Actual prepayment speeds that are faster (slower) than pricing prepayment speeds positively affect the total return performance of fixed mortgage-backed securities when the bond is purchased at a discount (premium) and negatively affect performance when the bond is purchased at a premium (discount). This intuition also holds for floating-rate MBS. However, suppose that the floating-rate mortgage is priced at or near 100. Why should prepayment variability affect this bond? As long as spreads remain the same, shouldn't the investor be able to reinvest fast prepayments back into another floater with the same spread to current LIB3?

The main reason that prepayment variability affects the price of a floating-rate MBS is the effect that prepayments have on the embedded options. Specifically, both ARMs and floating-rate CMOs have maximum and minimum coupons.[23] ARMs also have *periodic caps*, which represent the maximum amount the coupon could change at any particular reset.

Some investors attempt to "uncap" floating-rate CMOs by decomposing the floating-rate MBS into an uncapped floater and a short position in LIBOR caps with maturities equal to the average life of the CMO. A more sophisticated approach to the same idea is to value a short position in an *amortizing* cap where the amortization schedule is set to the amortization schedule of the MBS at the pricing date. However, this "replicating portfolio" idea misses the effect of prepayment variability. Although actual prepayments are difficult to predict and the best forecasting models always will have some error, it is quite predictable that, all other factors constant, prepayments speed up when interest rates fall and slow when interest rates rise. This phenomenon increases the value of the caps because the "maturity" of the caps increases as they become nearer the money. This effect decreases the value of the bond because the bond has an embedded *short* position in the caps.

For example, suppose a floating-rate CMO with a reset margin of 40 basis points and a 9.40% cap is priced on July 15, 1995 and had a weighted-average life of five years. The market implied volatility for five-year 9% OTC caps of 24.5% which results in a "value" for the cap, ignoring extension risk, of $1.06.[24] Now, suppose the LIBOR yield curve experiences an instantaneous parallel increase in rates of 50 basis points. Because the cap is 50 basis points closer to becoming "in-the-money," a

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[23] The floors are typically struck at very low levels, in some cases at 0%. For the ensuing discussion and without loss of generality, we assume that the securities have caps but no floors.

[24] Floating-rate CMOs typically are indexed to one-month LIBOR and pay monthly.

five-year cap would now be worth \$1.45. However, suppose that the CMO extends such that at the new yield curve levels, the market would be pricing the CMO to a seven-year average life. Now, the price of a seven year cap under the same volatility and the new yield curve is \$2.87.[25] This example illustrates that "uncapping" the floating-rate MBS in this manner overvalues the security because the method ignores the extension risk.

One method of comparing floating-rate MBSs to FRNs of other asset classes is through the use of the option-adjusted spread (OAS) methodology. Standard OAS methodology takes a term structure of interest rates and a volatility assumption and values the security over a distribution of possible interest rate paths. The parameters of the distribution are calibrated such that the current term structure is correctly priced and arbitrage opportunities are precluded. For mortgages, a prepayment function is applied at each path, and therefore, the OAS calculation will incorporate the effect of extension on the value of the caps. However, it must be emphasized that the calculated OAS is a function of the assumed term structure and prepayment models. Even the most accurate models will capture the true distributions with some error. Therefore, a comparison of the OAS of a complex optionable security such as a floating-rate MBS to a FRN without options should incorporate some additional spread because of "model risk."

Alternatively, the canonical decomposition methodology[26] extends the replicating portfolio idea to account for the negative convexity implied by the extension risk of the caps. Specifically, this approach uses an "arbitrage-free" term structure model to determine a portfolio of Treasury bonds, caps and floors that replicate the cash flows of the floating-rate MBS across all possible paths. The decomposition allows an investor to evaluate the risks embedded in a complex security in the context of a portfolio of very liquid instruments. In addition, this portfolio does not require continuous rebalancing. Of course, like the OAS methodology, the accuracy by which the cash flows of the portfolio replicate the cash flows of the FRN along all paths is subject to the accuracy of the term structure and prepayment model.

The mortgage example illustrates another important intuition in the valuation of embedded options in FRNs. The spot LIB1 observed on July 15, 1995 was 5.88%. Given the 9.00% strike on the embedded caps, these options are currently over 300 basis points "out of the money". An investor that is familiar with option pricing theory may be surprised that options so far "out of the money" would be valued so highly. In reality, caps comprise a set of options on *forward rates*. Therefore, one should look at the implied forward rates over the life of the cap to determine "in-the-moneyness." In an upward sloping yield curve environment, the options may be much nearer to the money than implied by the difference between the strike and the spot LIB1.

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[25] This value is slightly overstated because longer caps typically get priced at lower volatility. At 22% volatility, the value of the cap is \$2.36.

[26] See "Arbitrage-Free Bond Canonical Decomposition," Thomas S.Y. Ho, Global Advanced Technology and this volume, April 1995.

CONCLUSION

Floating-rate securities have been, and will continue to be, popular in practically all asset classes of fixed-income markets, particularly for investors that are bearish on interest rates. Many of the features of these markets make comparisons of the risks, and even the compensations for taking risks, difficult to quantify. In this report, we present many of the measures and conventions that are currently used in the various FRN markets. We hope that these discussions will be useful as a reference that makes these commonly used measures more concrete. We then describe in the detail the risks that the various instruments are subject to and offer frameworks in which an investor can assess these risks.