The role of replication in Bayesian data analysis

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The main points of this talk

1. Frequentist and Bayesian methods have different foci
2. Frequentist methods (NHST) only work well when power is high
3. Bayesian methods are useful when data are sparse
4. The replication crisis is partly driven by a "precision crisis" (low powered studies)
5. I will show how I use Bayesian methods to deal with the lack-of-precision problem in my field (psycholinguistics).

Some caveats

1. What I will say today holds for psycholinguistics; I don't speak for all of cognitive psychology (but the ideas presented here have relevance or cogpsych).
2. There are many different approaches to using Bayesian methods; I will present my own perspective, which is influenced by Andrew Gelman's work.
3. This talk is **only** about data that can be replicated in principle. Examples of non-replicable "experiments": political elections, football match outcomes, earthquakes.

What is replication?

Standard frequentist definition:

Repeatedly obtaining "significant" results

Bayesian (Gelman) perspective:

"Consistency" across studies

Before discussing the role of replicability in Bayes, it is important to be clear on the differences between frequentist and Bayesian approaches.

The frequentist procedure

Imagine that you have some independent and identically distributed data: $x_1, x_2, \ldots, x_n$

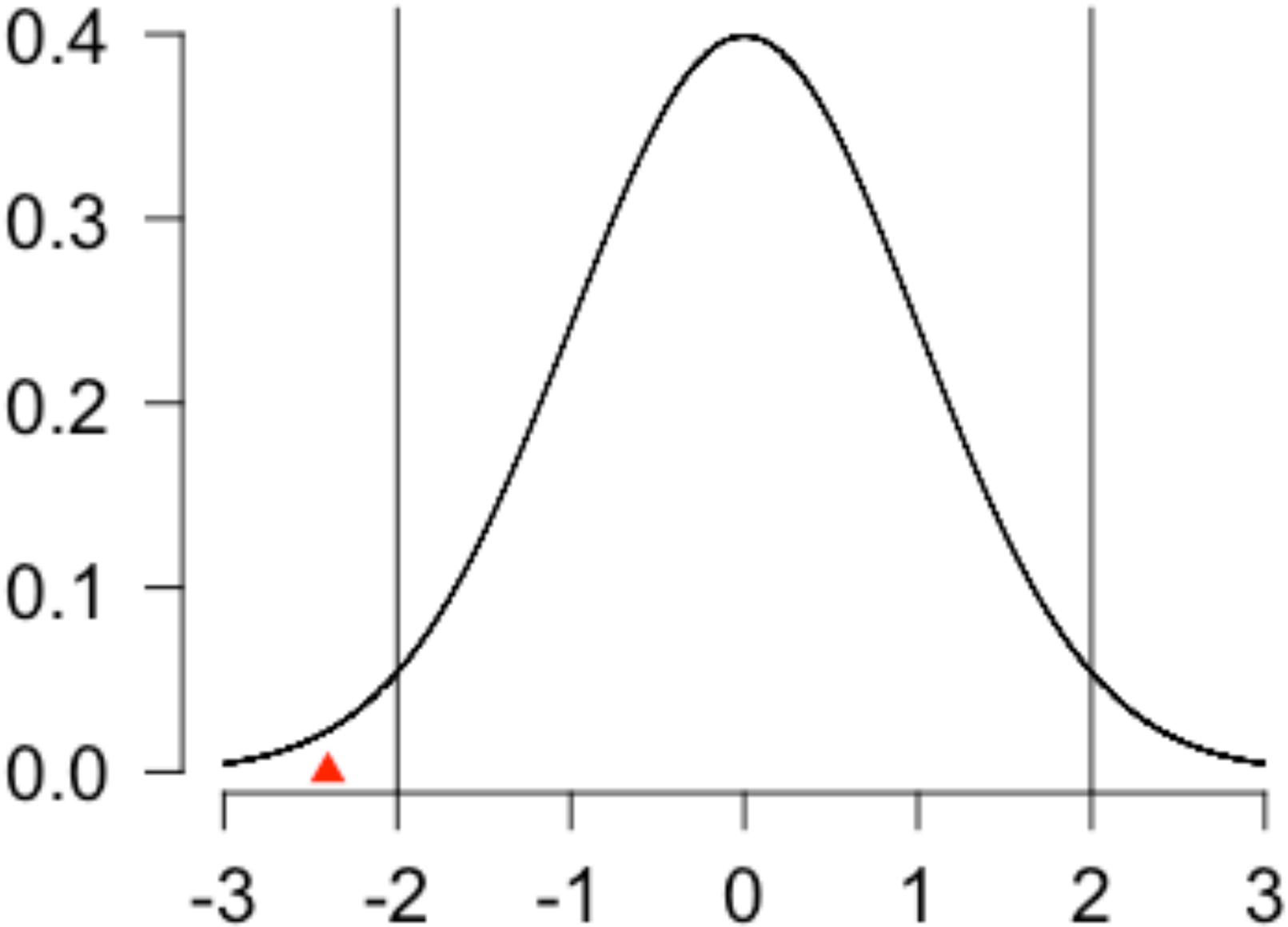
$$X \sim Normal(\mu, \sigma)$$

1. Set up a null hypothesis: $H_0 : \mu = \mu_0$
2. Check how far sample mean $\bar{x}$ is from μ_0 in SE units:
   $t_{observed} \times SE = \bar{x} - \mu_0$
3. If $t_{observed}$ is large enough, reject null hypothesis

Statistical data analysis is reduced to checking for significance (is $p<0.05$?)

The frequentist procedure

Decision: Reject null and publish



The frequentist procedure

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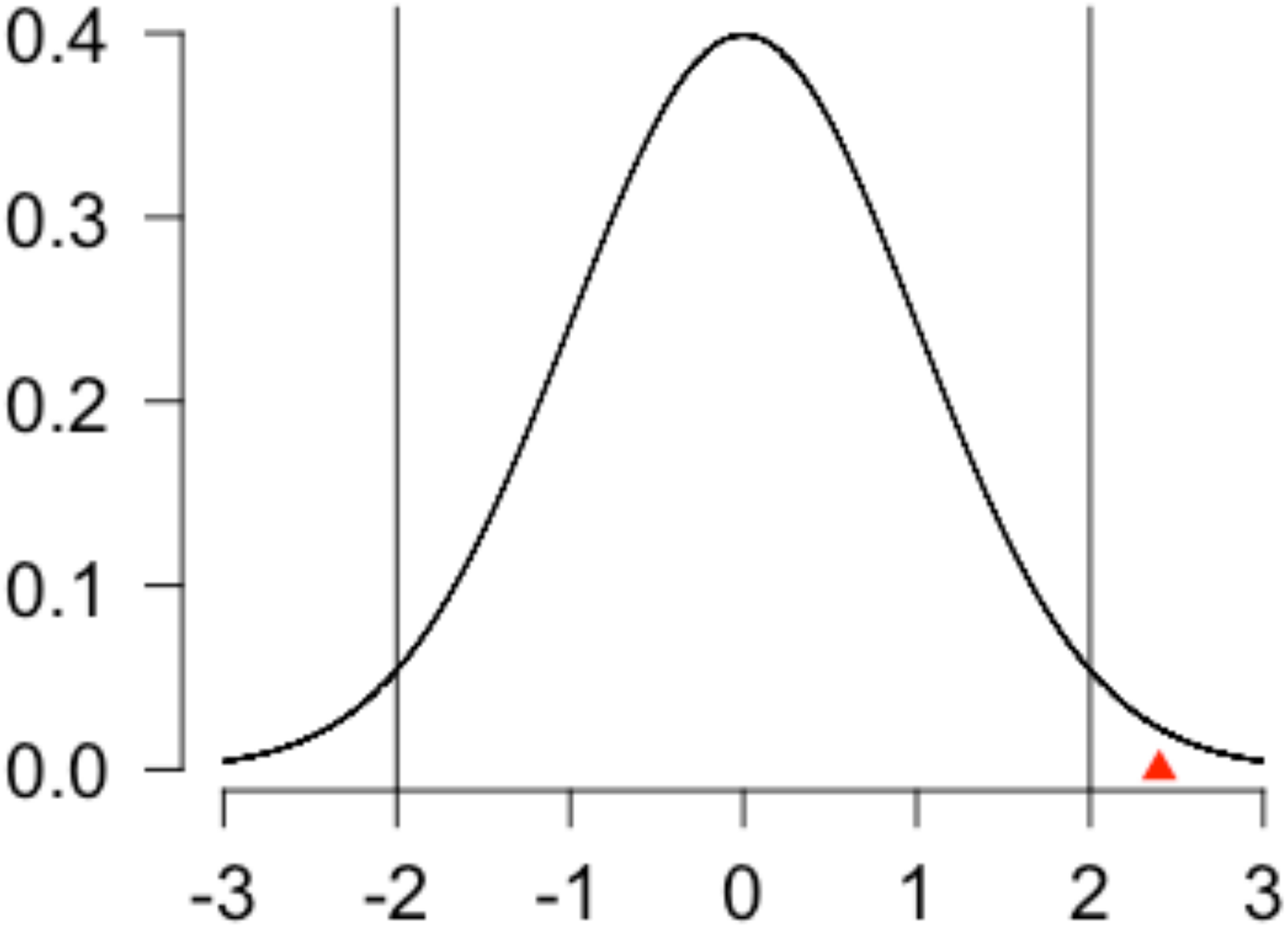
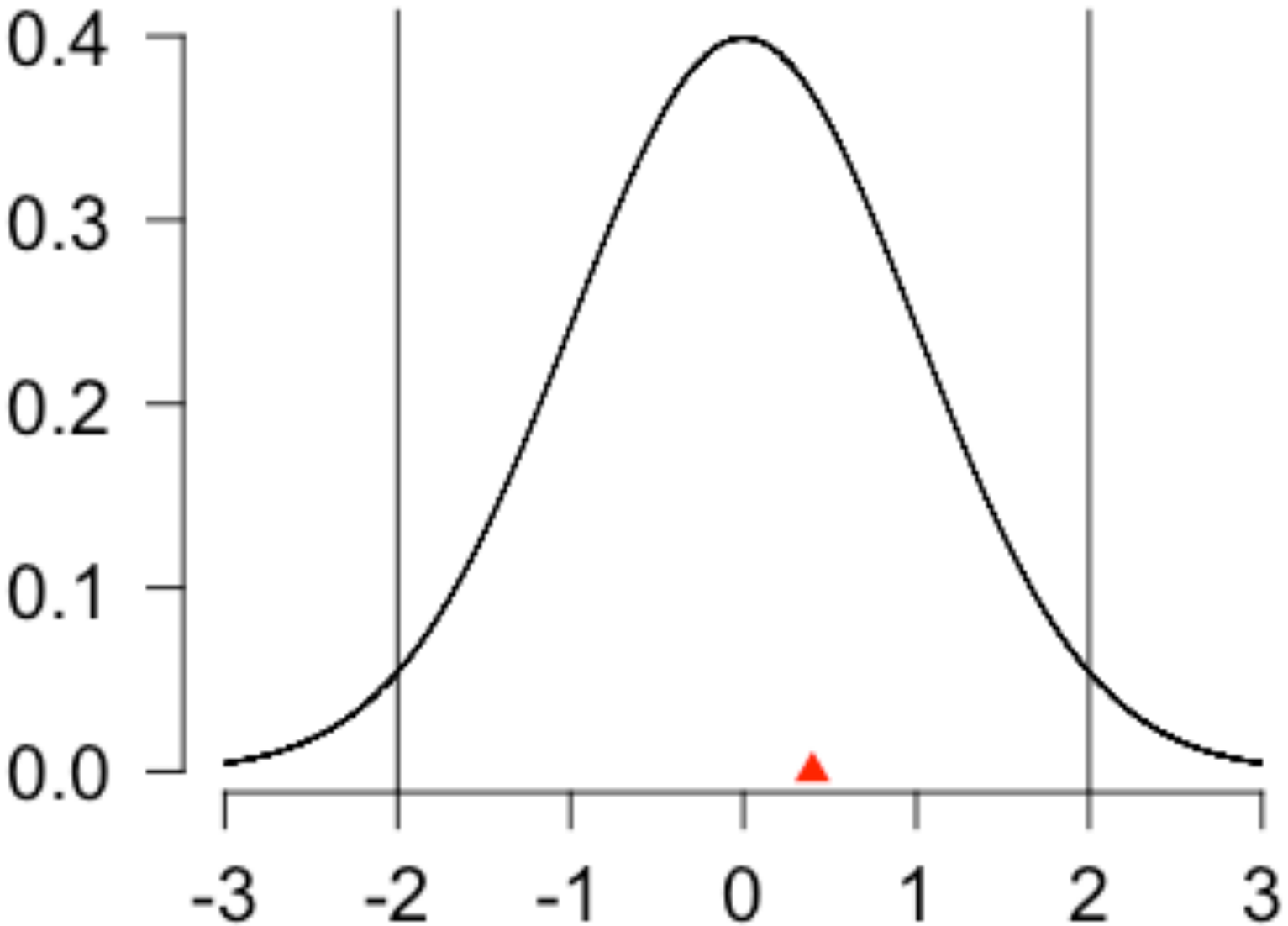


The frequentist procedure

Accept null?



The frequentist procedure

Power: the probability of detecting a particular effect

Power depends on:

- effect size (+ experiment design)
- standard deviation(s)
- sample size

The frequentist paradigm works well when power is high (80% or higher).

The frequentist paradigm is not designed to be used in low power situations.

Example: agreement attraction

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Interference patterns in subject-verb agreement and reflexives revisited: A large-sample study☆

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Jäger, Mertzen, Van Dyke & Vasishth, JML, 2020

Example: agreement attraction

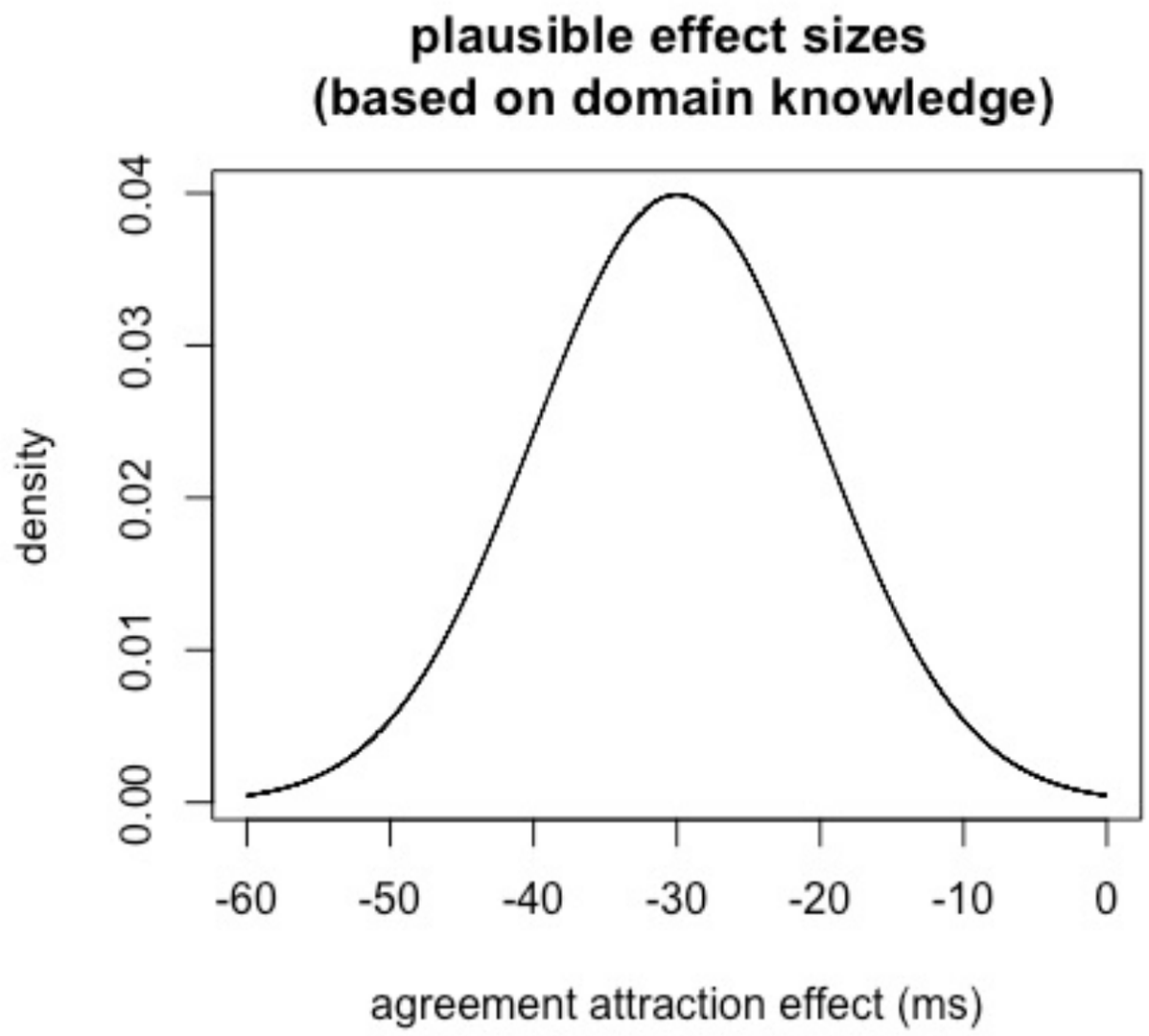
a. *The bodybuilder$^{-plural}_{+subject}$ who met the trainers$^{+plural}_{-subject}$ were$\{^{plural}_{subject}\}$

...

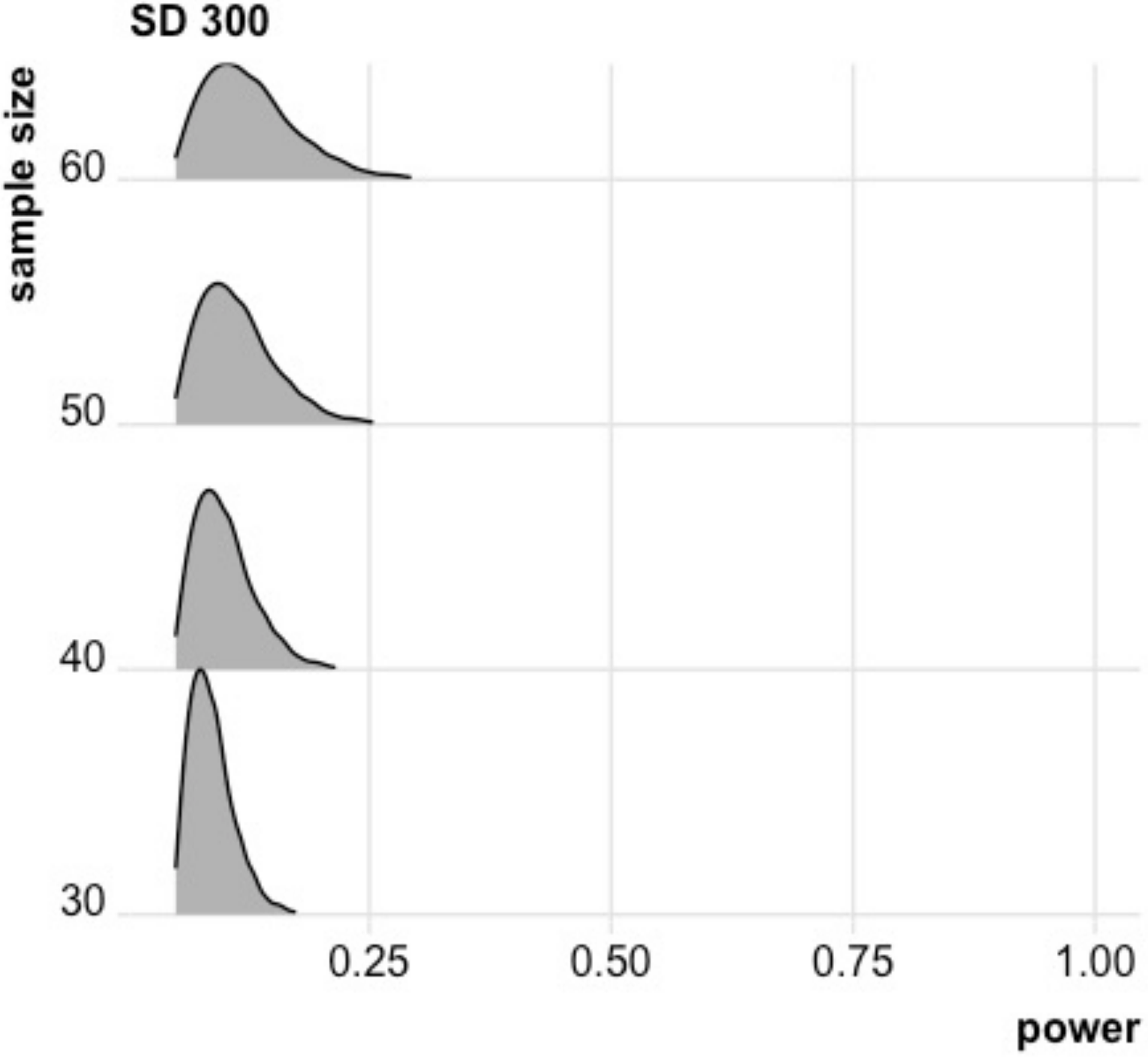
b. *The bodybuilder$^{-plural}_{+subject}$ who met the trainer$^{-plural}_{-subject}$ were$\{^{plural}_{subject}\}$

...

a is read faster than b at *were*



Power in reading studies on agreement attraction



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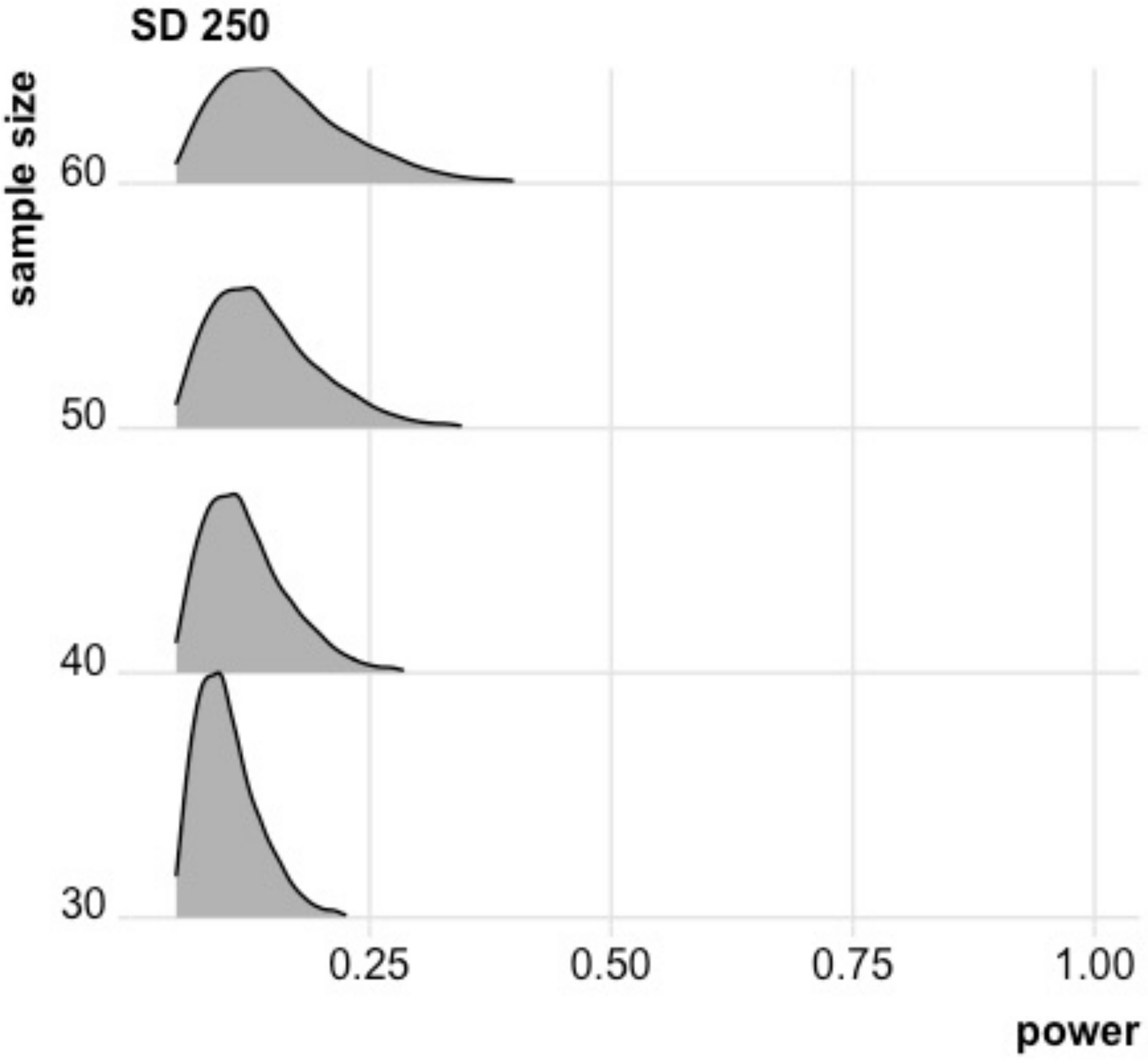

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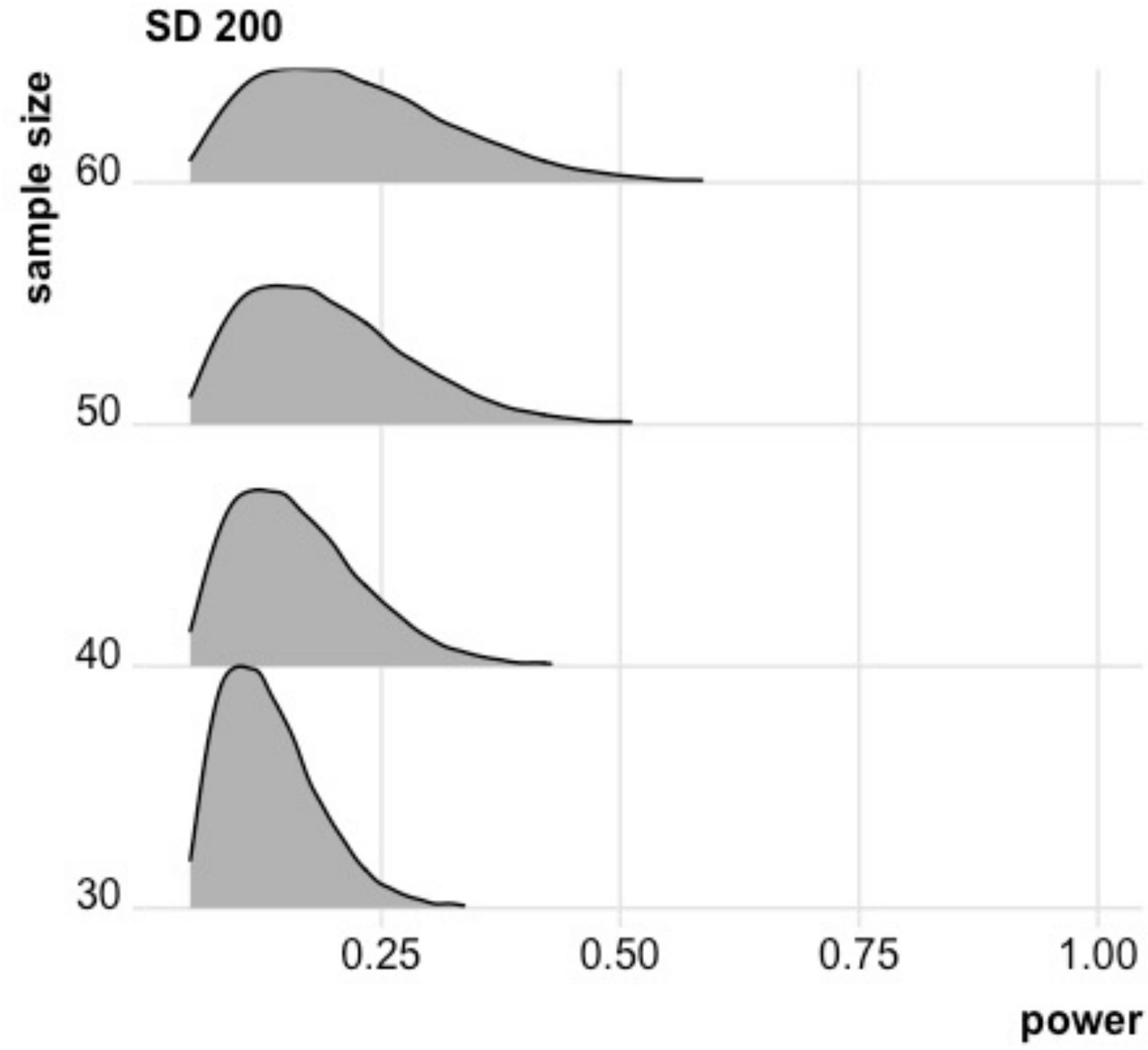

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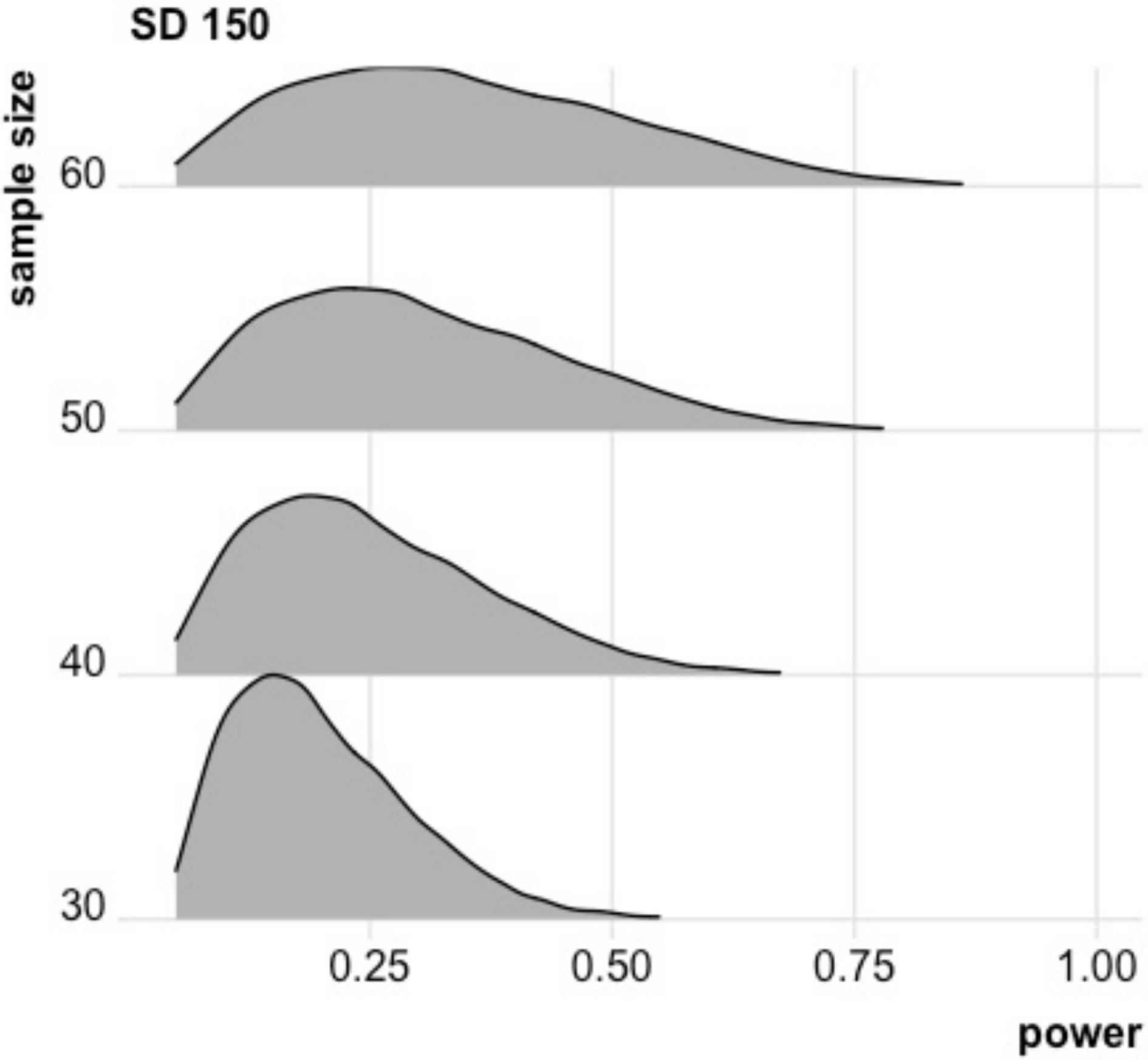
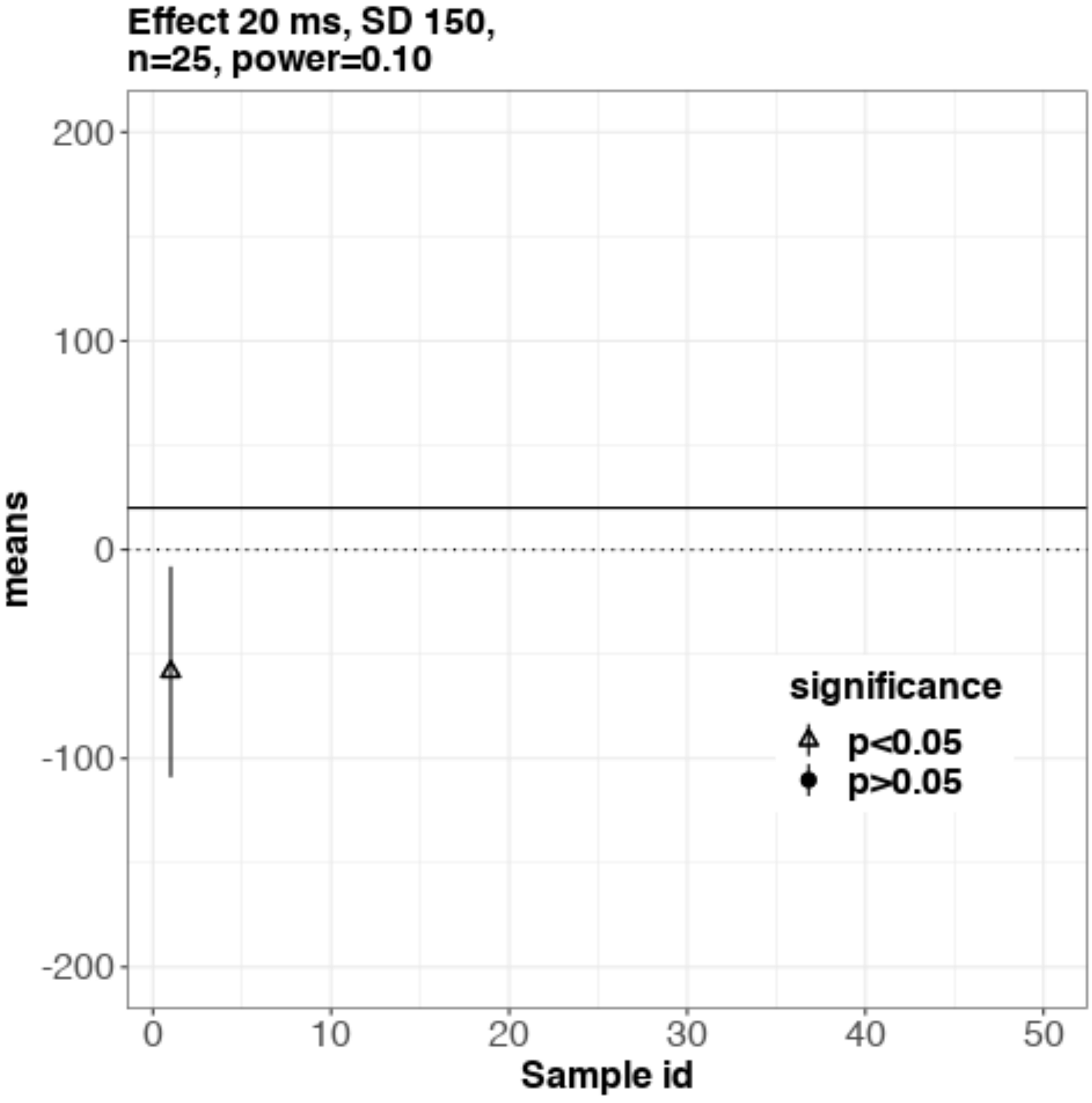


Low power leads to exaggerated estimates: Type M error



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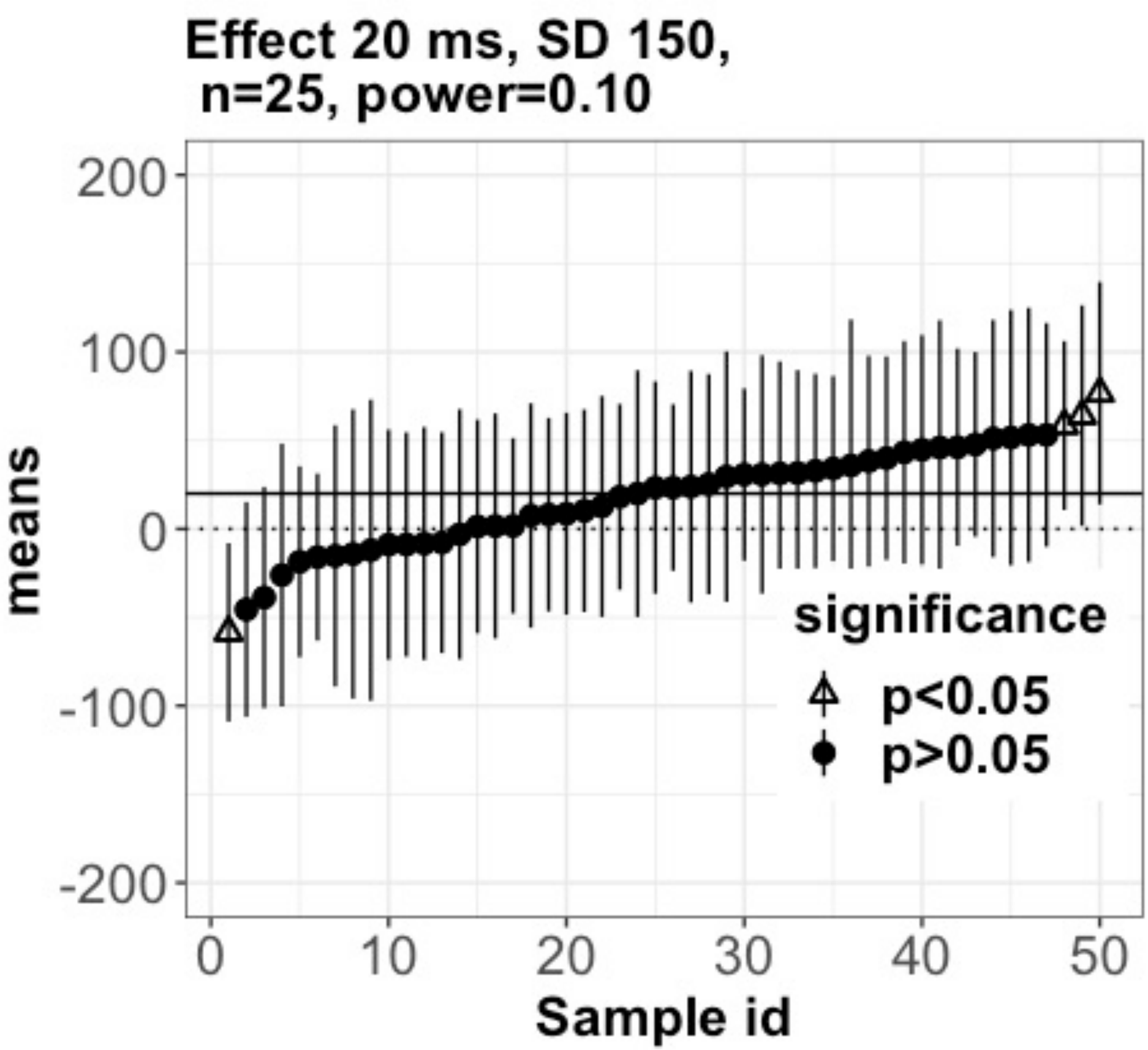
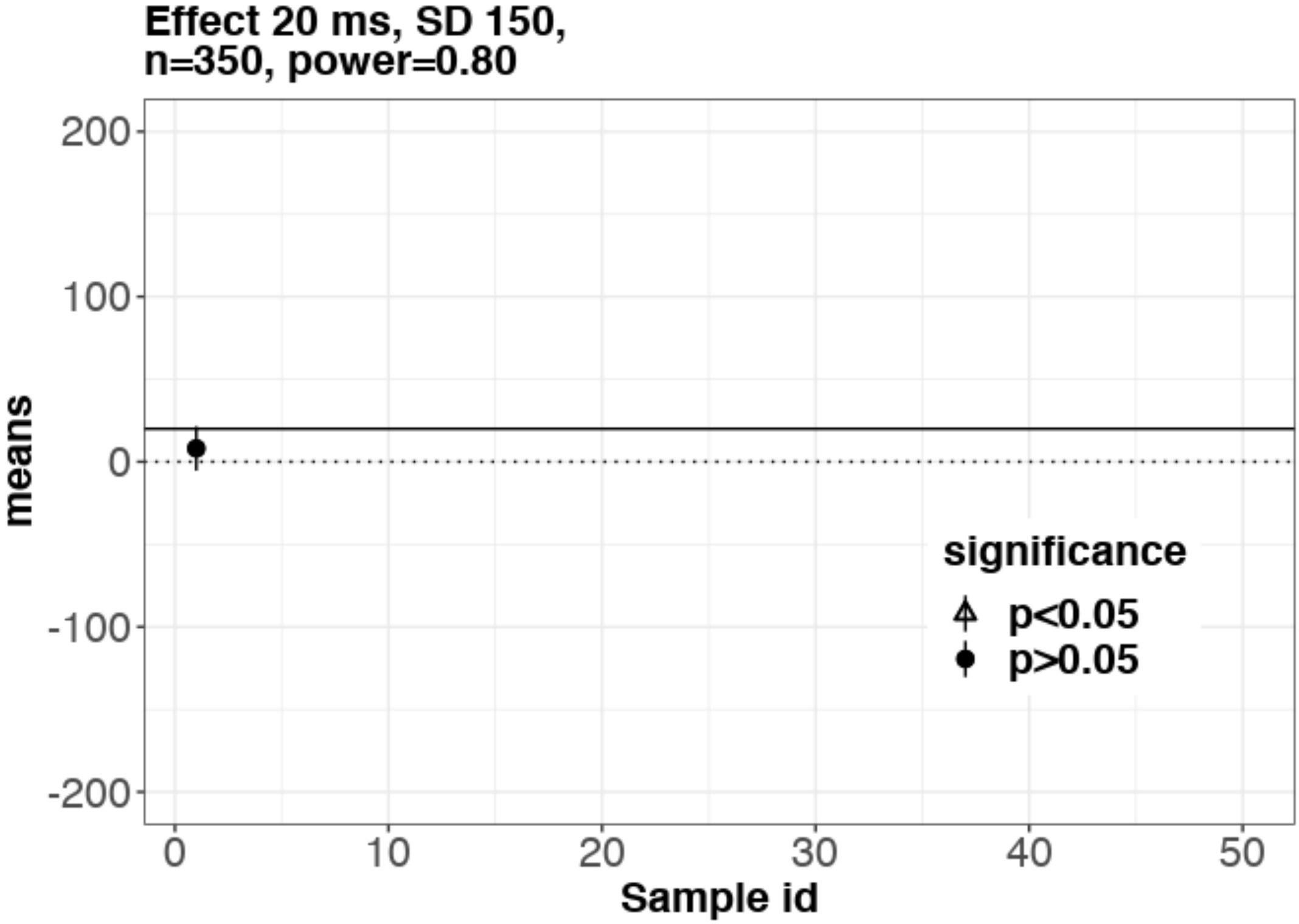


Compare with a high power situation



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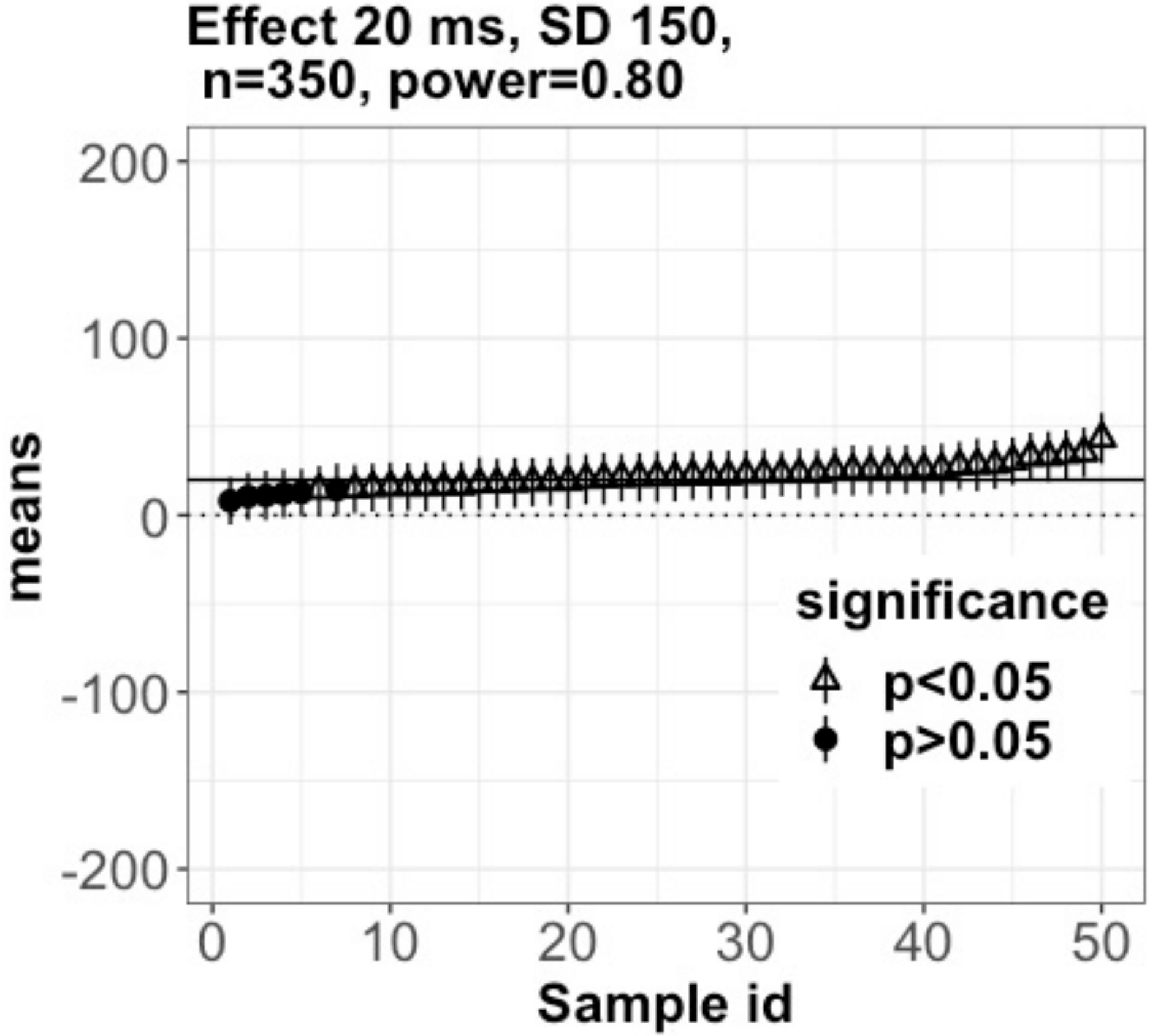


The frequentist paradigm breaks down when power is low

1. Null results are inconclusive
2. Significant results are based on biased estimates (Type M error)

Consequences:

1. Non-replicable results
2. Incorrect inferences

The Bayesian approach

Imagine that you have some independent and identically distributed data: $x_1, x_2, \ldots, x_n$

$$X \sim Normal(\mu, \sigma)$$

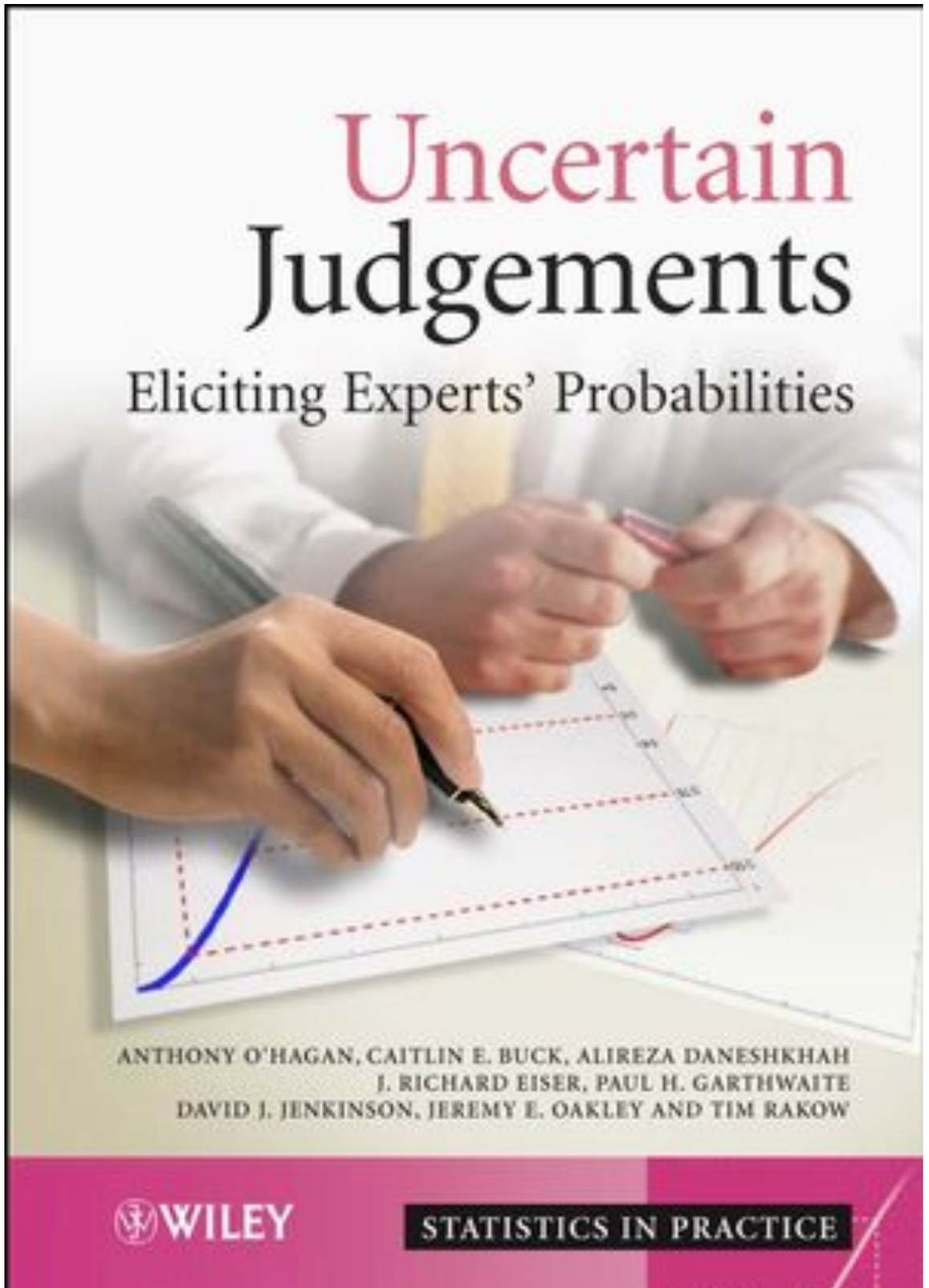
1. Define **prior distributions** for the parameters (here, μ, σ)
2. Derive **posterior distributions** of the parameters of interest using Bayes' rule:

$$f(\mu \mid data) \propto f(data \mid \mu) \times f(\mu)$$

posterior likelihood prior

3. Carry out inference based on the posterior

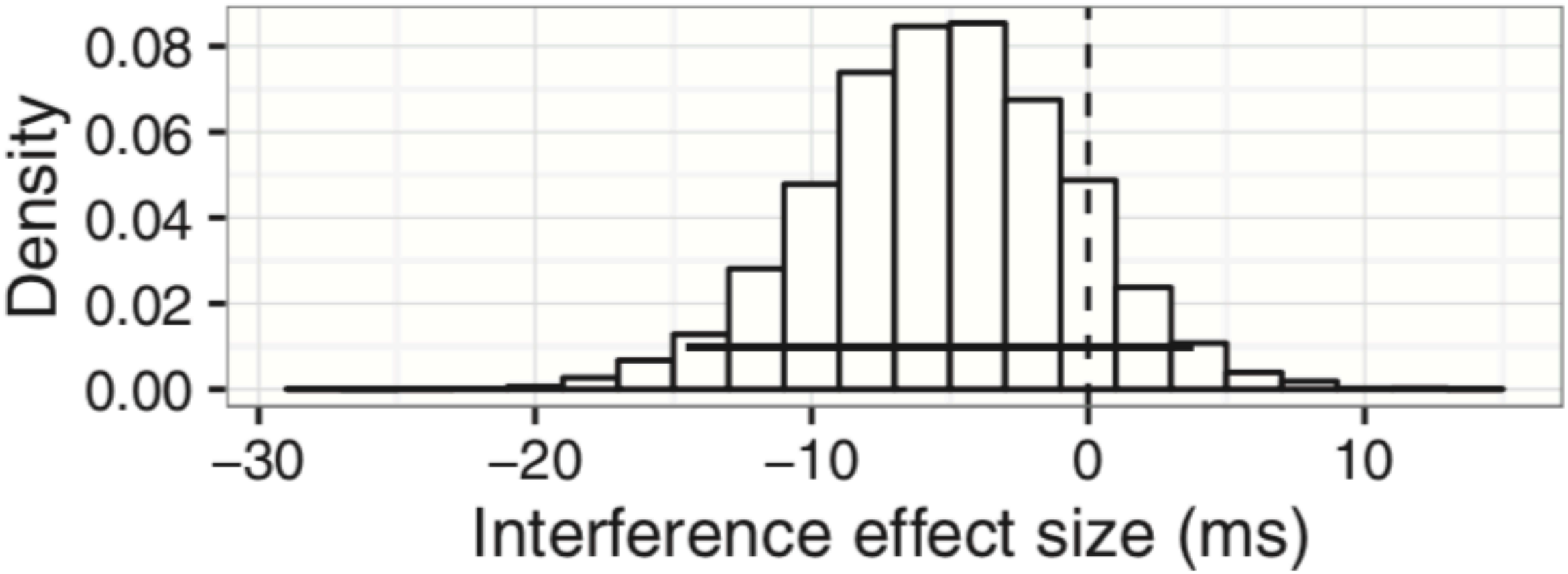
Deriving prior distributions requires domain knowledge



The Bayesian approach

The end result of a Bayesian analysis is a posterior distribution of the parameter of interest

Agreement attraction effect (meta-analysis estimate)



Comparison of Frequentist vs Bayesian approaches

	Frequentist	Bayesian
Parameters	Fixed	Random*
Data	Random	Fixed
Prior knowledge used	No	Yes
Type I, II error	relevant	irrelevant**
Hypothesis testing	reject null	Bayes factor
Uncertainty quantification	No***	Yes

* Random variables

** Type I, II error could be seen as relevant for Bayes

*** Confidence intervals can be a proxy

ANOVA vs Bayes factors

$$BF_{12} = \frac{Likelihood(Data \mid Model_1)}{Likelihood(Data \mid Model_2)}$$

The Bayes factor is similar to the frequentist likelihood ratio test (or ANOVA), with the difference that in the Bayes factor, the likelihood is integrated over the parameter space.

The BF can be highly sensitive to the priors.
A Bayes factor analysis must come with a sensitivity analysis.

Why is the Bayesian approach useful?

1. Handles sparse data without any problems
2. Highly customised models can be defined
3. The focus is on **uncertainty quantification**

But Bayes comes with a cost

1. You have to think about your prior knowledge/belief

2. There is no one answer corresponding to $p<0.05$ or $p>0.05$.

3. You have to learn to think about **uncertainty**:

   Compare:

   "50% probability of rain tomorrow"

   "95% sure that probability for rain is between 40-60%"

   "95% sure that probability for rain is between 5-95%"

Summarizing the Bayesian/frequentist divide

"[Bayesian data analysis] is a method for summarizing **uncertainty** and making estimates and predictions using probability statements conditional on observed data and an assumed model.

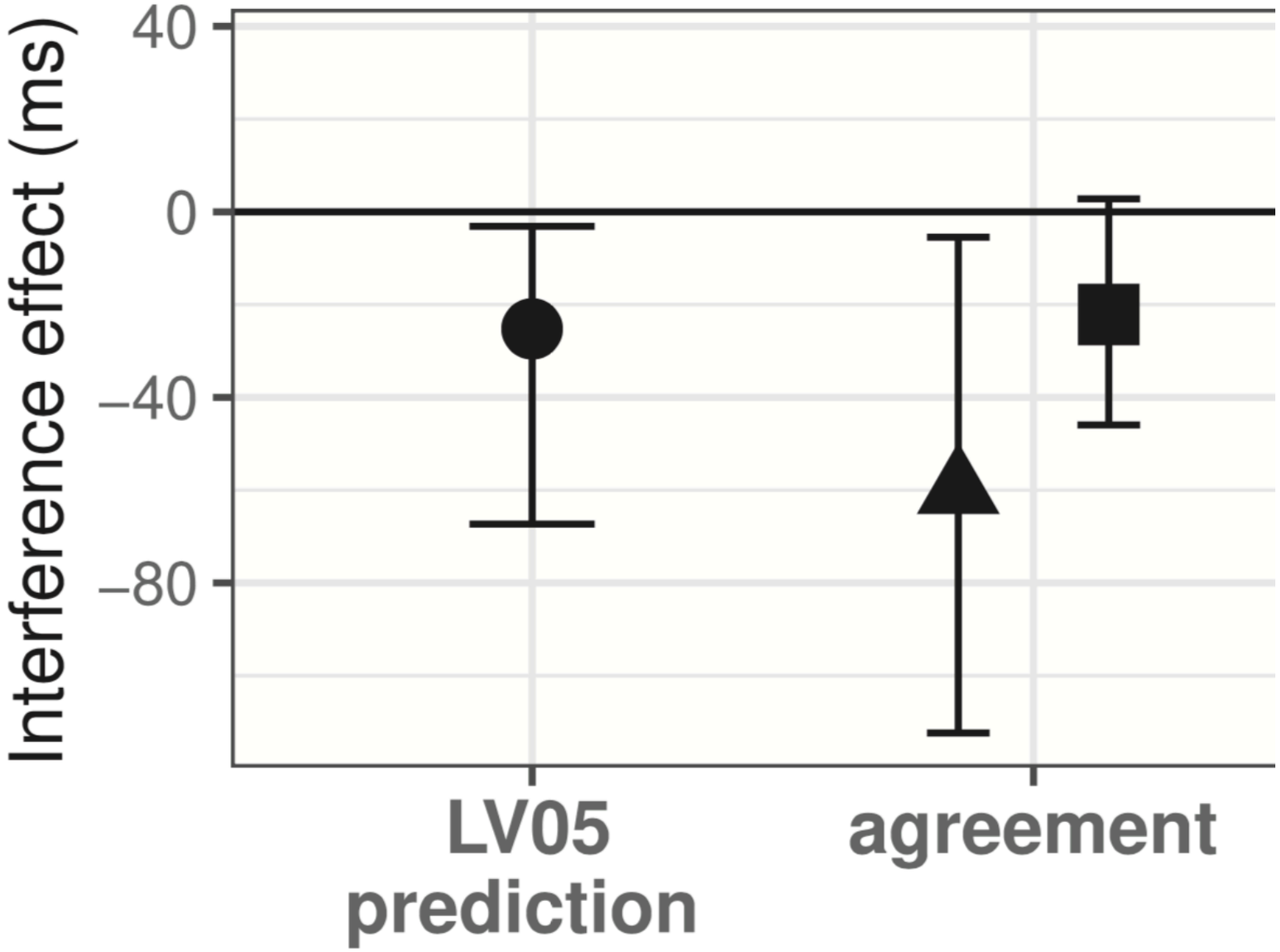
"Frequentist statistics … is an approach for evaluating statistical **procedures** conditional on some family of posited probability models."

Gelman, 2008. Rejoinder. *Bayesian Analysis*.

Is replication important in Bayesian data analysis?

Let's look at a concrete example.

Example: a high-powered replication attempt of the agreement attraction effect



Jäger, Mertzen, Van Dyke & Vasishth, JML, 2020

Example of a replication attempt of a low-powered study

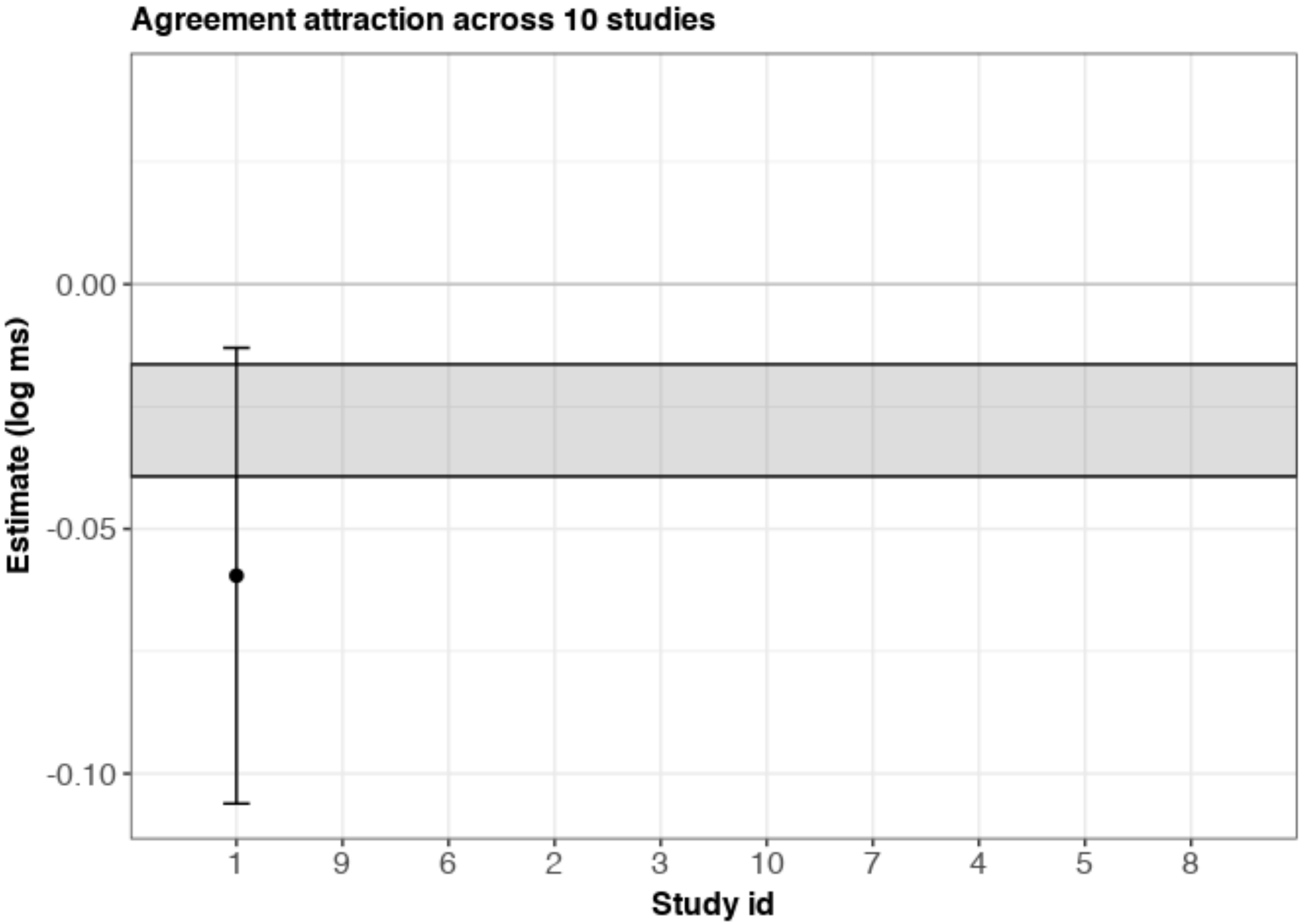
1. The model prediction (LV05) is quite constrained
2. The N=40 agreement estimate has very wide uncertainty but shows a significant effect
3. **Question**: Does the N=181 study replicate the significant agreement effect from N=40?

Frequentist answer: no

Bayesian answer: yes, because the original estimates had a very wide 95% confidence interval! It was compatible with a broad range of values!

Jäger, Mertzen, Van Dyke & Vasishth, JML, 2020

We can use frequentist confidence intervals to quantify uncertainty



Vasishth, Gelman, https://psyarxiv.com/zcf8s/

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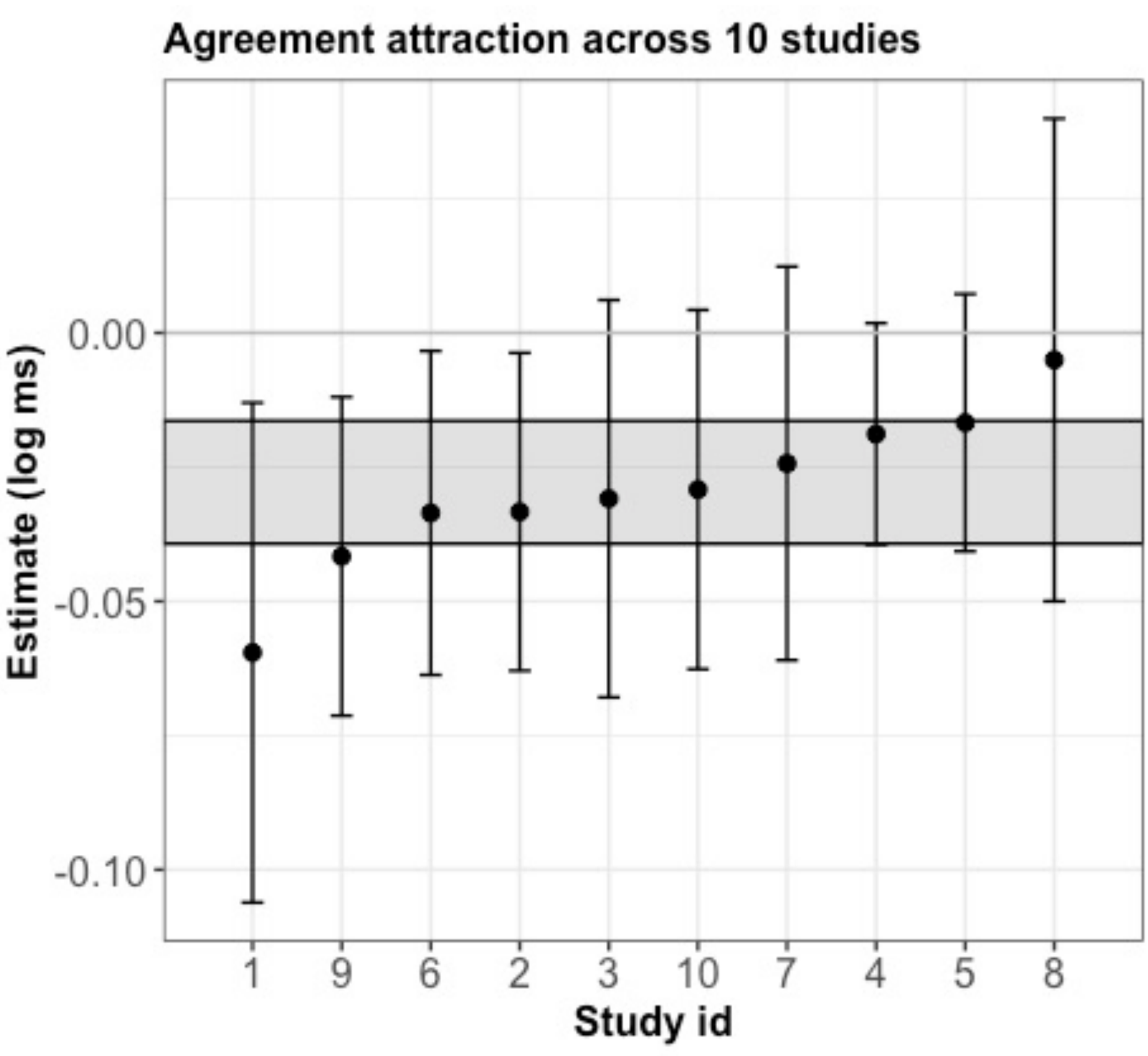
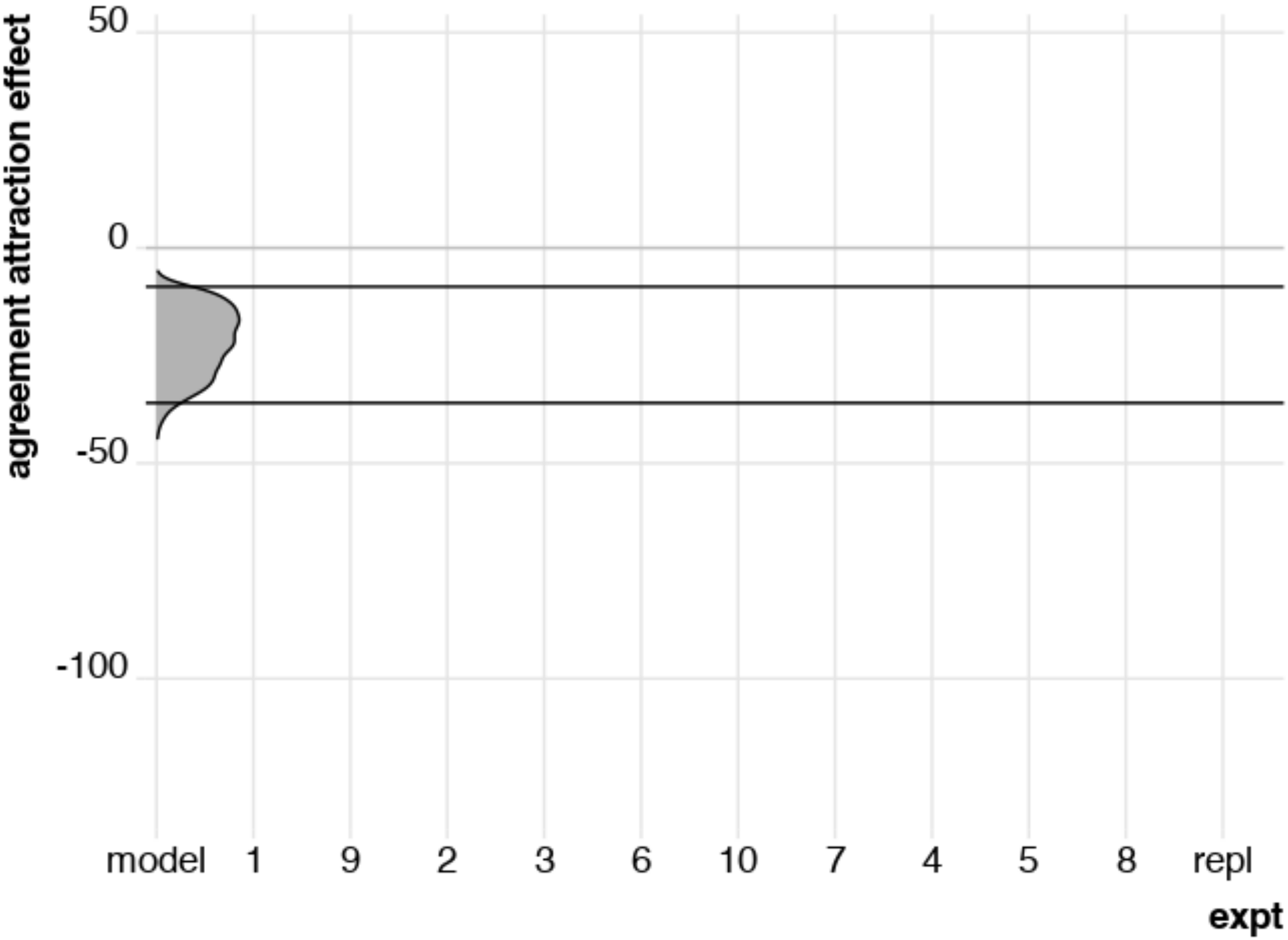


Bayesian approach to studying replicability



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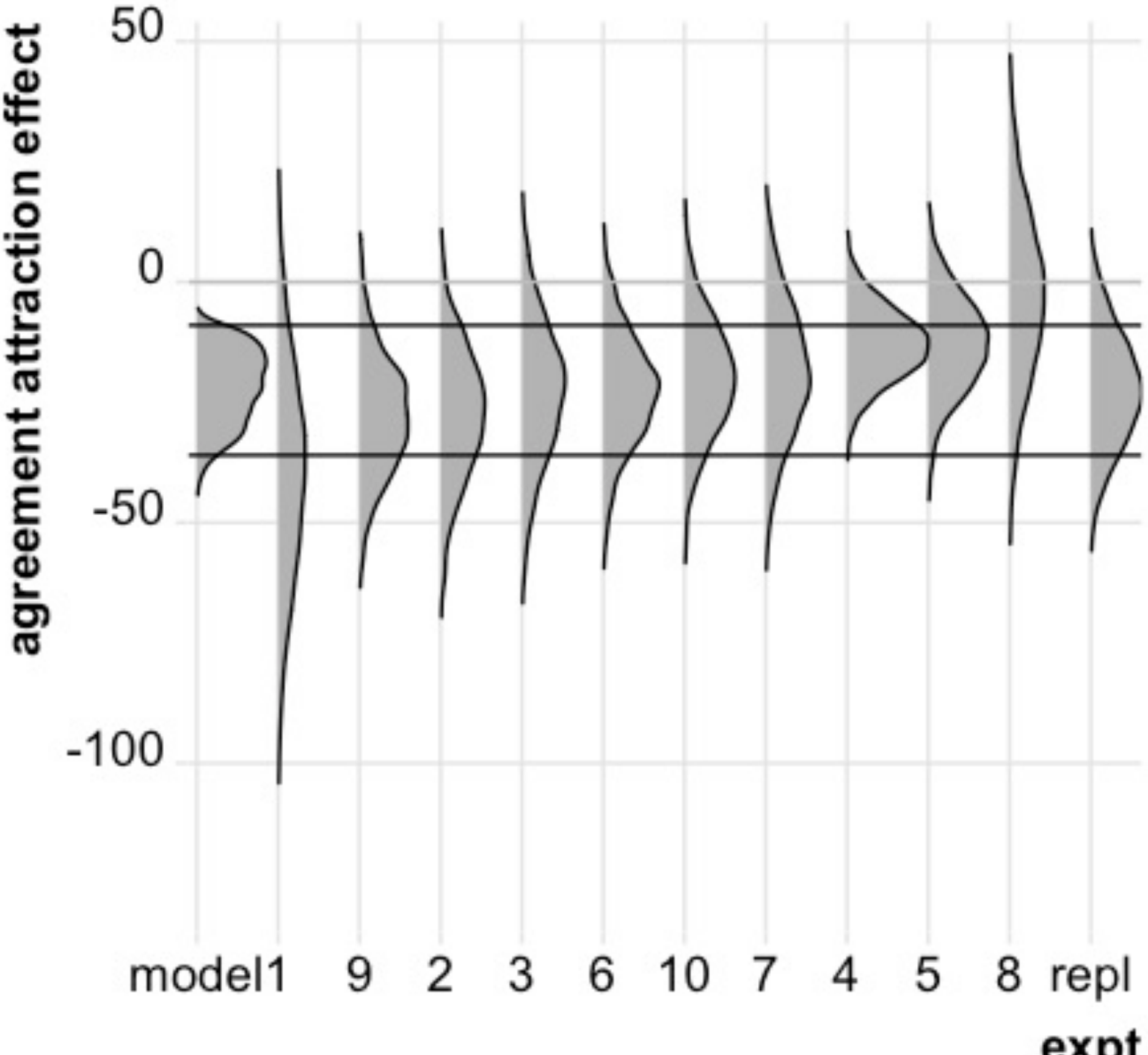

Concluding remarks

1. Replicability is equally important in frequentist and Bayesian methods.
2. Low-powered designs will be misleading or uninformative in isolation.
3. With Bayes, one can accumulate evidence and use it in future studies.
4. We can establish replicability by (a) showing consistency across studies, (b) accumulating evidence through meta-analyses, (c) running higher-precision studies **informed by accumulated knowledge**.
5. Bayes provides the machinery for achieving these goals.

Second example of accumulating knowledge
Nicenboim, Vasishth, Rösler (under review)

It was a windy day.
The boy went out to fly a/an _____
DeLong et al 2005, *Nature Neuroscience*

Are words pre-activated probabilistically during sentence comprehension? Evidence from new data and a Bayesian random-effects meta-analysis using publicly available data

Bruno Nicenboim[1], Shravan Vasishth[1], & Frank Rösler[2]

[1] University of Potsdam
[2] University of Hamburg

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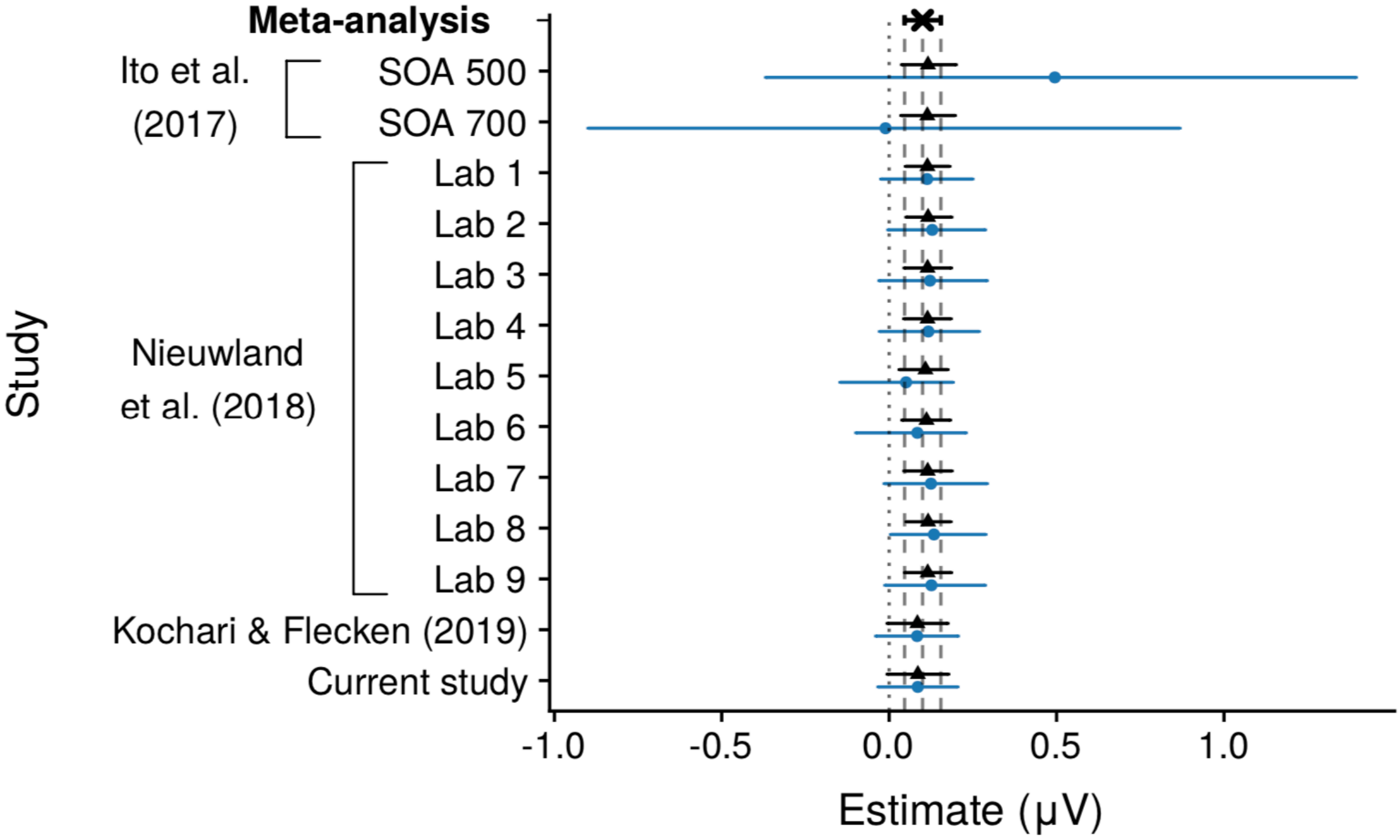

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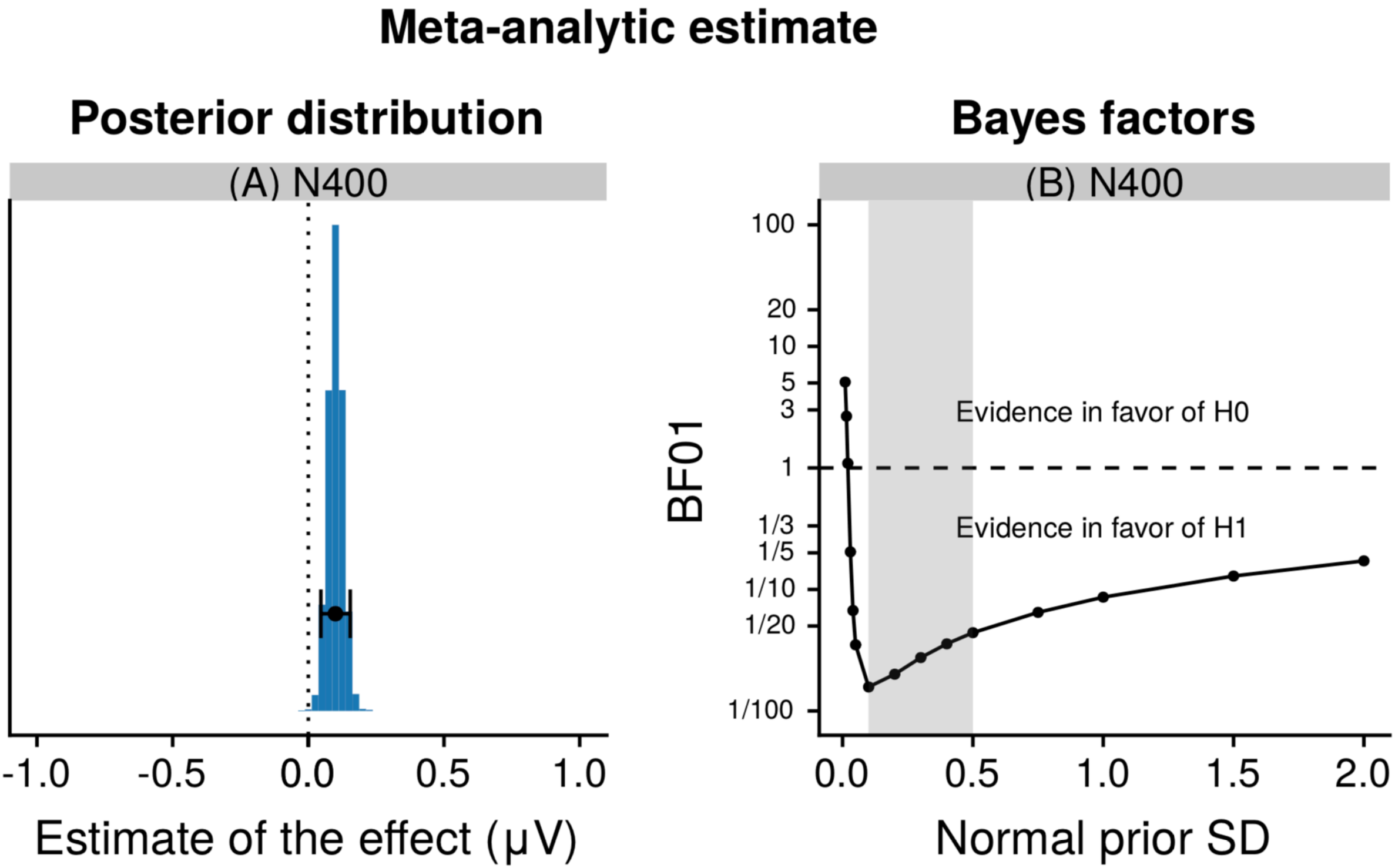

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All code and data from this talk are available from:

https://osf.io/p8amv/

All code and data from the Nicenboim, Vasishth, & Rösler study are available from:

https://osf.io/w85gc/