Introduction to Bayesian Data Analysis
Lecture 1-1: Discrete random variables

Prof. Dr. Shravan Vasishth
Professor, Linguistics
Cognitive Science / Linguistics, Uni Potsdam, Germany

HPI Hasso Plattner Institut
Digital Engineering • Universität Potsdam

Random variables

A random variable X is a function $X : S \rightarrow \mathbb{R}$ that associates to each outcome $\omega \in S$ exactly one number $X(\omega) = x$.

S_X is all the x's (all the possible values of X, the **support of X**). I.e., $x \in S_X$. We can also sloppily write $X \in S_X$.

An example of a discrete random variable: the number of coin tosses till H

- $X : \omega \rightarrow x$
- ω: H, TH, TTH,. . . (infinite)
- $X(H) = 1, X(TH) = 2, X(TTH) = 3, \dots$
- $x = 0, 1, 2, \dots; x \in S_X$

Random variables

A second example of a discrete random variable: tossing a coin once

- $X : \omega \rightarrow x$
- ω: H, T
- $X(T) = 0, X(H) = 1$
- $x = 0, 1, \ldots; x \in S_X$

Random variables: PMFs and PDFs

- Every discrete (continuous) random variable X has associated with it a **probability mass (density) function (pmf, pdf)**.
- PMF is used for discrete distributions and PDF for continuous.
- (I may sometimes use lower case for pdf and sometimes upper case. Some books use pdf for both discrete and continuous distributions.)

Thinking just about discrete random variables for now:

$$p_X : S_X \rightarrow [0, 1] \tag{1}$$

defined by

$$p_X(x) = P(X(\omega) = x), x \in S_X \tag{2}$$

Random variables: PMFs and PDFs

Example of a PMF: a random variable X representing tossing a coin once.

- In the case of a fair coin, x can be 0 or 1, and the probability of each outcome is 0.5.
- Formally:

$$p_X(x) = P(X(\omega) = x), x \in S_X \tag{3}$$

- The probability mass function defines the probability of each outcome: $p_X(0) = p_X(1) = 0.5$.
- The cumulative distribution function (CDF) $F(X \leq x)$ gives the cumulative probability of observing all the outcomes $X \leq x$.

$$F(x=1) = \text{Prob}(X \leq 1) = \sum_{x=0}^{1} p_X(x) = p_X(x=0) + p_X(x=1) = 1$$

$$F(x=0) = \text{Prob}(X \leq 0) = \sum_{x=0}^{1} p_X(x) = 0.5$$

Random variables: Demo code

Let’s look at some example code to make this concrete: L1_1_Code.Rmd.

The main points of this lecture

- The definition of a random variable
- Examples of discrete random variables (e.g., the Bernoulli)
- Using the PMF to compute the probabilities of different possible outcomes (the dbern function)
- Computing the cumulative distribution function using pbern.

You can start reading chapter 1 of the textbook!

The next lecture

In the next lecture, we will look at continuous random variables.