# CS598HS Advanced Social & Information Networks
# Information Diffusion in Grid-like Social Networks

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# Overview

1. Introduction
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- In our model, the information can only be transmitted between agents of the same type, and that there exists a source of information for each different agent type.
- Finally, there is a constant probability $p$ that the information will leak through a single interaction. This implies that an agent $j$ who interacts with frequency $f_{ij}^*(t)$ at time $t$ with another agent $i$ who has the information, has a probability $pf_{ij}^*(t)$ of getting the information.

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- Our problem is thus motivated by these examples, and we assume that there exist $k$ external service providers, who provide services of different quality to agents of different type, but of the same quality to agents of the same type.
- Thus, the information that is propagated through the network is assumed to be the identity of the highest quality service provider for a specific type of agents.

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- We seek to compare the **Diffusion Rate (DR)** of information between these graphs, which is defined as the expected time it will take for the information to spread to all agents in the graph who can potentially learn it.
- We seek to both develop a model and obtain some theoretical results about the DR between these graphs, as well as perform several simulations that possibly confirm our theoretical analysis.

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## Diffusion Rate

For any graph $G$

$$DR \leq n \sum_{t=1}^{\infty} \left( t \left(1 - (1-\delta)^{|C_t|}\right) \prod_{k=1}^{t-1} (1-\delta)^{|C_k|} \right)$$

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- Similarly, the increase in DR between BG and UG is exponential, and is dependent on the bias $b$.
- Specifically, for $b = 2$ we expect the difference in DR between BG and UG to be larger than the difference in DR between UG and PG.

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# Diffusion Rate Simulations

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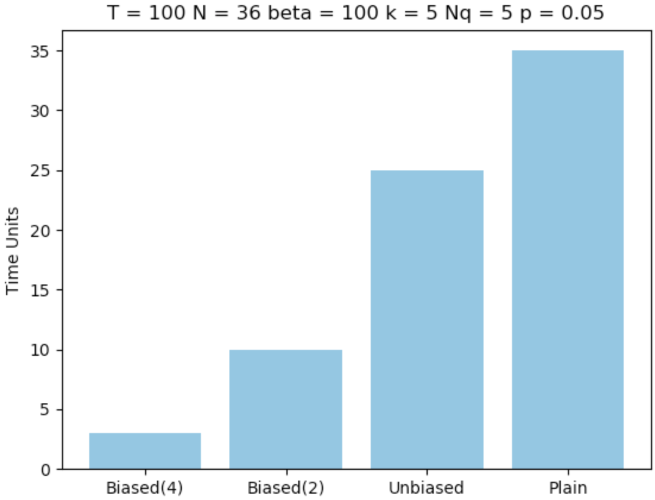

# QUESTIONS ?