Fundamentals of Wireless Communications

Multiuser Capacity & Opportunistic Communication

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Introduction

- We studied that fast-fading channels exploit the fluctuations of the fading channel to increase performance gain of the point-to-point system.
- This chapter is about multiuser capacity. We want to analyze the performance limits of multiuser communication and suggest optimal multiple access strategies.
- The multiuser setting tackles when to transmit, which users to transmit and the amount of power to allocate between users, giving it more opportunities to exploit hence further increase in performance gain.

Uplink AWGN Channel

- For a discrete-time model for the uplink AWGN channel with two users:

$$y[m] = x_1[m] + x_2[m] + w[m] \quad \sim CN(0, N_O)\ is\ i.i.d,$$

$$user\ k\ has\ an\ average\ power\ constraint\ of\ P_k\ J/s$$

- In the point-to-point case: $R < C\ \sim reliable\ communication$
- In multiuser case, a capacity region, C is set such the two users can communicate simultaneously and reliably at rates (R_1, R_2).
- NB: Since the two users share the same bandwidth, there is naturally a tradeoff between the reliable communication rates of users.

Uplink AWGN Channel

- From this capacity region, one can derive other scalar performance measures of interest:
- The symmetric capacity: $C_{sym} = \max_{(R,R)\epsilon C} R$ ~ Maximum common rate for simultaneous reliable communication
- The sum capacity: $C_{sum} = \max_{(R,R)\epsilon C} R_1 + R_2$ ~ Maximum total throughput
- The capacity region C, of the uplink AWGN channel is the set of all rates (R_1, R_2) satisfying these three constraints:

$$
\begin{aligned}
R_1 &< \log\left(1 + \frac{P_1}{N_0}\right), \\
R_2 &< \log\left(1 + \frac{P_2}{N_0}\right), \\
R_1 + R_2 &< \log\left(1 + \frac{P_1 + P_2}{N_0}\right).
\end{aligned}
$$

Uplink AWGN Channel

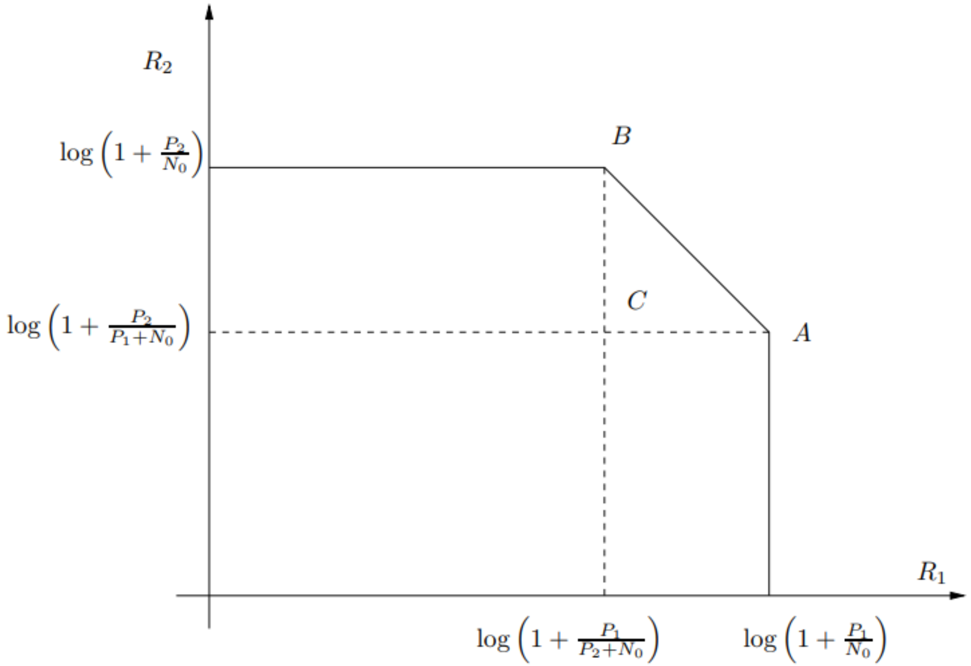
- The first two constraints say that the rate of the individual user cannot exceed the single-user bounds.
- The third constraint tells us that it is not possible for the two users to simultaneously reliably communicate at the point-to-point capacity.
- Even though there must be a tradeoff between performance of the two users, user 1 can achieve its single-user bound while at the same time user 2 can get a high non-zero rate:

$$R_2{}^* = \log\left(1 + \frac{P_1 + P_2}{N_o}\right) - \log\left(1 + \frac{P_1}{N_o}\right) = \log\left(1 + \frac{P_2}{P_1 + N_o}\right)$$

- This can be achieved by successive interference cancellation(SIC).

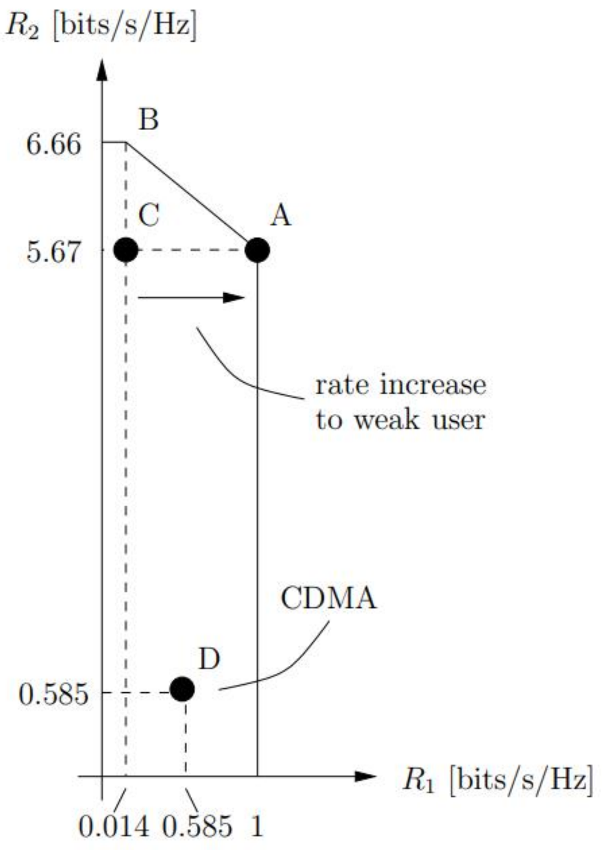
Uplink AWGN Channel

- The segment AB contains all the “optimal” operating points of the channel.
- Getting optimal operating points on the segment AB depends on the objective of the system under analysis.



Uplink AWGN Channel

- **Comparison with Conventional CDMA**
- In CDMA, every user is decoded treating the other users as interference. On the other hand, in the SIC receiver, one of the users, say user 1's signal is decoded treating user 2 as interference, but user 2's signal is decoded with the benefit of the signal of user 1 already removed.
- We can conclude that the performance of the conventional CDMA receiver is sub-optimal.
- CDMA has a near-far problem whiles SIC turns this into a near-far advantage.



Uplink AWGN Channel

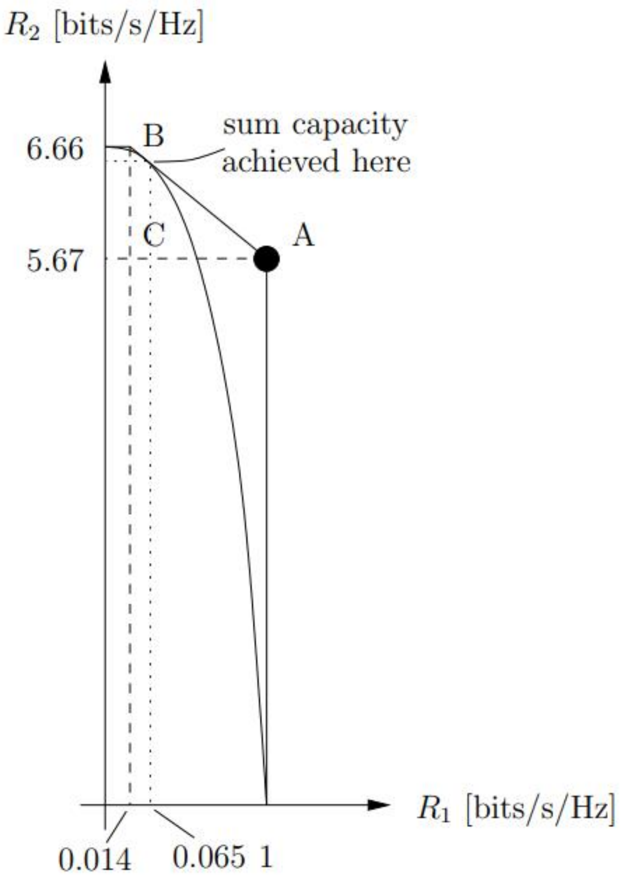
- **Comparison with Orthogonal Multiple Access**

- The orthogonal scheme allocates a fraction of degrees of freedom to each user, α and 1-α.

$$R_1 = \alpha W\log\left(1 + \frac{P_1}{P_2|h_1|^2 + N_O}\right), \quad R_2 = (1-\alpha)W\log\left(1 + \frac{P_2|h_2|^2}{N_O}\right)$$

- Orthogonal schemes are in general sub-optimal, except for one point which highly unfair when there is a large disparity between the powers of the users.

- The scheme can approach the performance of SIC for the weak user only by nearly sacrificing all the rate of the strong user.



Uplink AWGN Channel

- For a K-user system, the capacity region is described by $2^K - 1$ constraints and the single user bounds given by:

$$\sum_{k \epsilon S} R_k < \log\left(1 + \frac{\sum_{k \epsilon S} P_k}{N_o}\right)$$

- The sum capacity is

$$C_{sum} = \log\left(1 + \frac{\sum_{k=1} P_k}{N_o}\right) \frac{bits}{s} / Hz$$

- In a case where equal received power is received $(P_1 = \cdots = P_1 = P)$, the sum capacity is

$$C_{sum} = \log\left(1 + \frac{KP}{N_o}\right)$$

Uplink AWGN Channel

- The symmetric capacity is

$$C_{sum} = \frac{1}{K}\log\left(1 + \frac{KP}{N_o}\right)$$

- Note: This rate can be obtained via orthogonal multiplexing: each user is allocated a fraction, ${}^{1}/_{K}$ of the total degrees of freedom.

- We can conclude that under equal received powers, the OFDM scheme has a better performance than the CDMA scheme (which uses conventional CDMA receiver)

Uplink AWGN Channel

- It is observed that the sum capacity is unbounded as the number of users grow.
- In the CDMA receiver, the sum rate is:

$$K \log\left(1 + \frac{P}{(K-1)P + N_O}\right) \frac{bits}{s} /Hz$$

- As $K \to \infty$, the above expression is approximately $log_2 e \approx 1.442$.
- The growing interference is eventually the limiting factor, and such a rate is said to be interference-limited.

Downlink AWGN Channel

- The baseband downlink AWGN channel with two users:

$$y_k[m] = h_k x[m] + w[m] \quad \sim CN(0, N_O)\ is\ i.i.d,$$

$$user\ x[m]\ has\ an\ average\ power\ constraint\ of\ P\ J/s$$

- Assumption: the channel gain per user is known both to the transmitter and the user k

- The single user bounds as in the uplink channel is expressed as:

$$R_k < \log\left(1 + \frac{P|h_k|^2}{N_o}\right)$$

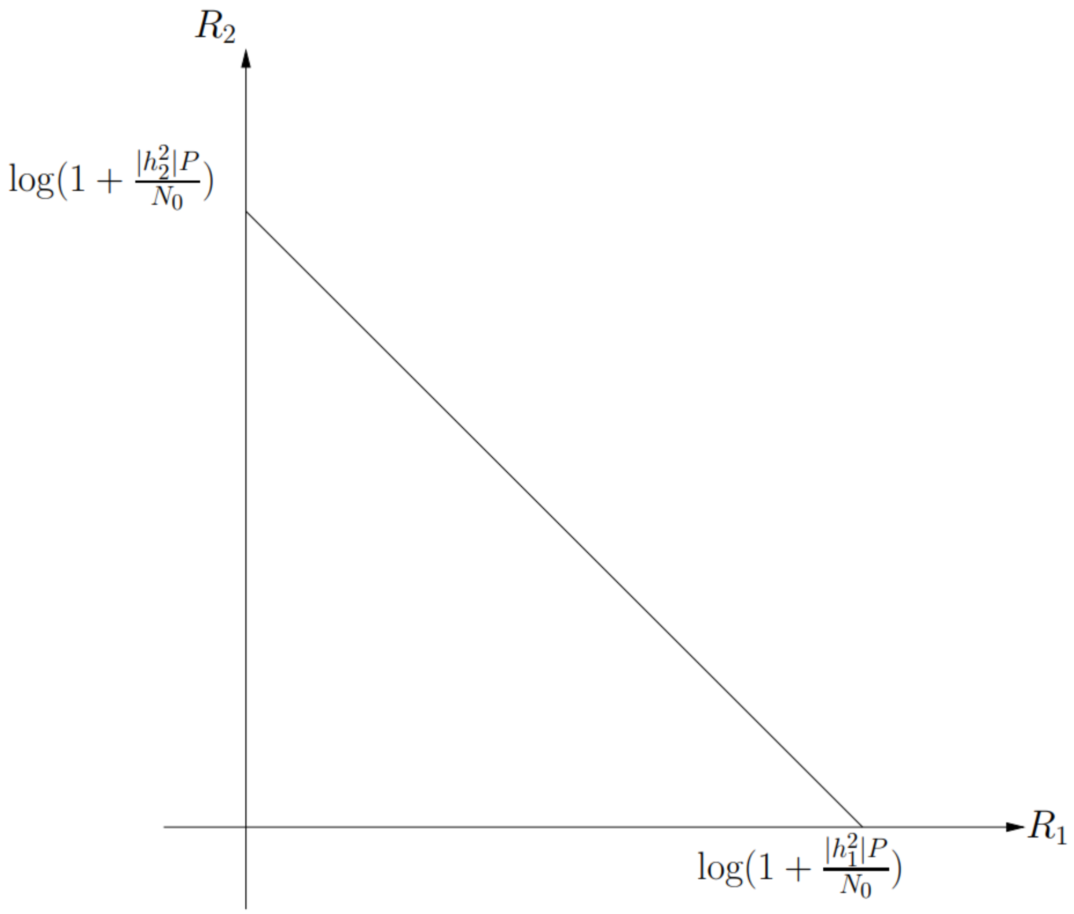
- The upper bound on this rate can be attained by using all the power and degrees of freedom to communicate to user k (with the other user getting zero rate).

Downlink AWGN Channel

- Symmetric case $(|h_1| = |h_2|)$
- The sum information rate must be bounded by the single-user capacity:

$$R_1 + R_2 < \log\left(1 + \frac{P|h_1|^2}{N_o}\right)$$

- The rate pairs can be achieved by sharing the degrees of freedom between the two users.
- This suggests a natural approach. Another approach is to superpose the signals of both users and perform SIC.



Downlink AWGN Channel

- General case $(|h_1| < |h_2|)$
- The superposition coding scheme is used after which the user with the better channel performs SIC.
- With a power split of $P = P_1 + P_2$, the following rate pair can be achieved:

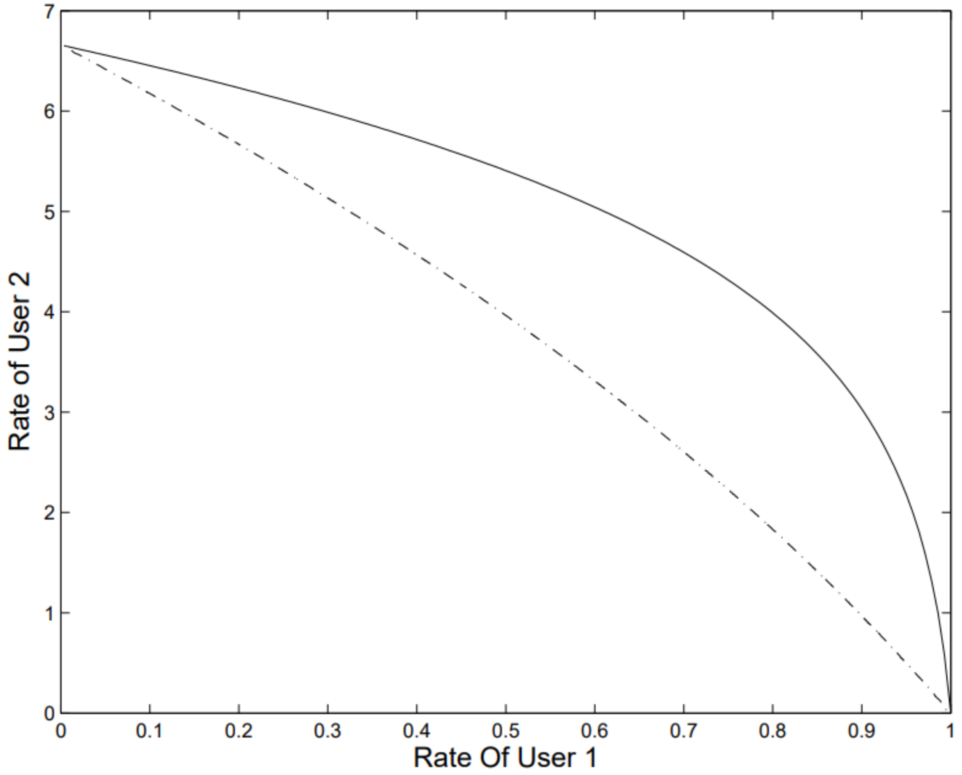
$$R_1 = \log\left(1 + \frac{P_1|h_1|^2}{P_2|h_1|^2 + N_O}\right), \quad R_2 = \log\left(1 + \frac{P_2|h_2|^2}{N_O}\right)$$

- For the orthogonal scheme, the rates below are achieved with the same power split:

$$R_1 = \alpha\log\left(1 + \frac{P_1|h_1|^2}{\alpha N_o}\right), \quad R_2 = (1-\alpha)\log\left(1 + \frac{P_2|h_2|^2}{(1-\alpha)N_o}\right)$$

Downlink AWGN Channel

- Comparing superposition coding (solid line) and orthogonal schemes (dashed line)

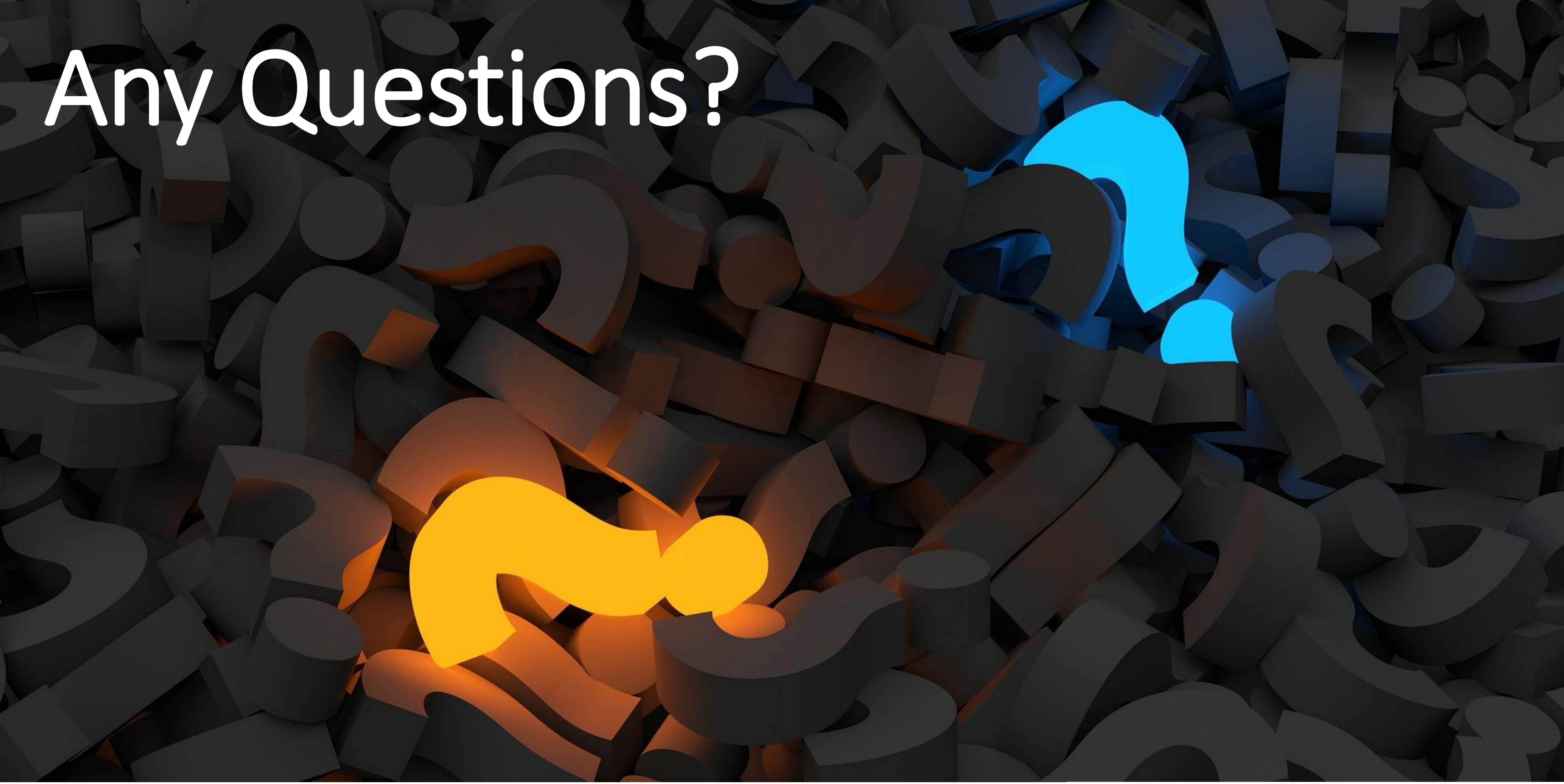


Downlink AWGN Channel

- Superposition-coding allows the strong user to use the full degrees of freedom of the channel while being allocated only a small amount of transmit power, thus causing small amount of interference to the weak user.

- In contrast, an orthogonal scheme must allocate a significant fraction of the degrees of freedom to the weak user to achieve near single-user performance, and this causes a large degradation in the performance of the strong user.

Any Questions?



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